Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.7-Miscellaneous/135-4.7.1-c-trig-^m-d-trig-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [254]. This is test number [135].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (254)	0.00 (0)
Mathematica	99.21 (252)	0.79 (2)
Maple	84.65 (215)	15.35 (39)
Fricas	84.25 (214)	15.75 (40)
Mupad	66.54 (169)	33.46 (85)
Giac	63.39 (161)	36.61 (93)
Maxima	62.60 (159)	37.40 (95)
Sympy	27.56 (70)	72.44 (184)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

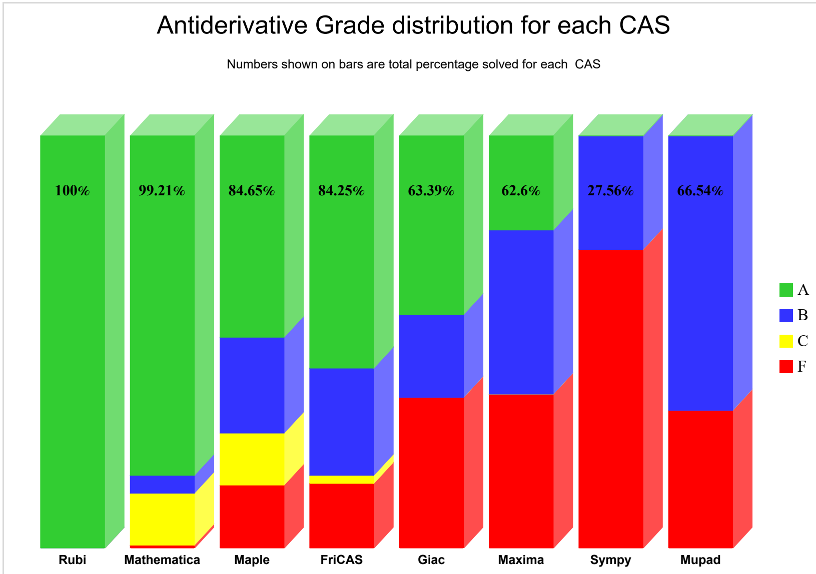
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

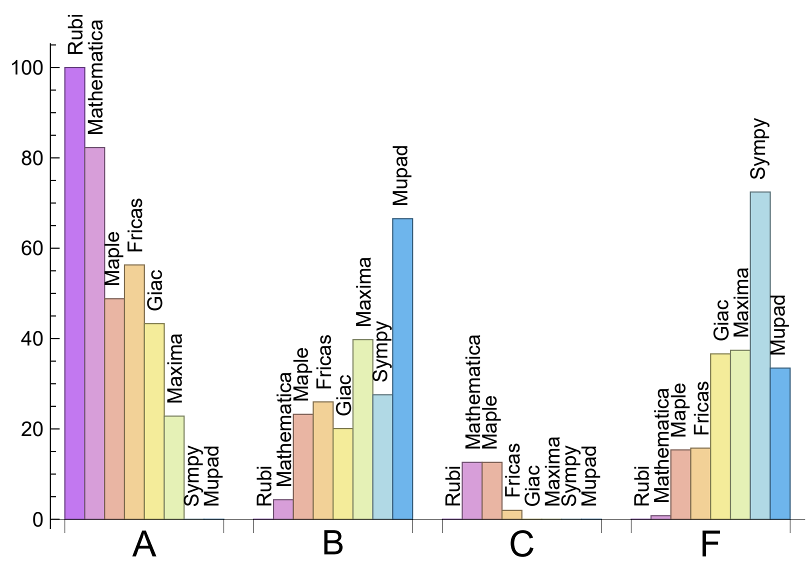
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	82.28	4.33	12.60	0.79
Fricas	56.30	25.98	1.97	15.75
Maple	48.82	23.23	12.60	15.35
Giac	43.31	20.08	0.00	36.61
Maxima	22.83	39.76	0.00	37.40
Mupad	N/A	66.54	0.00	33.46
Sympy	0.00	27.56	0.00	72.44

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	39	82.05 %	17.95 %	0.00 %
Fricas	40	82.50 %	0.00 %	17.50 %
Giac	93	87.10 %	10.75 %	2.15 %
Maxima	95	100.00 %	0.00 %	0.00 %
Sympy	184	20.65 %	66.85 %	12.50 %
Mupad	85	83.53 %	16.47 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

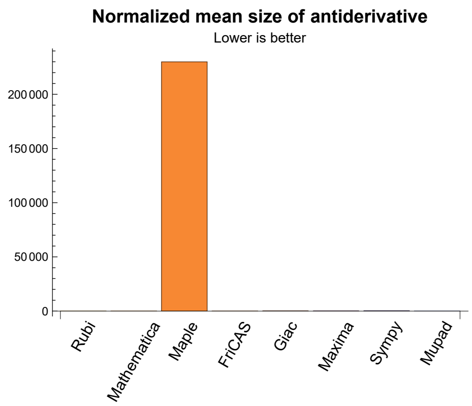
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

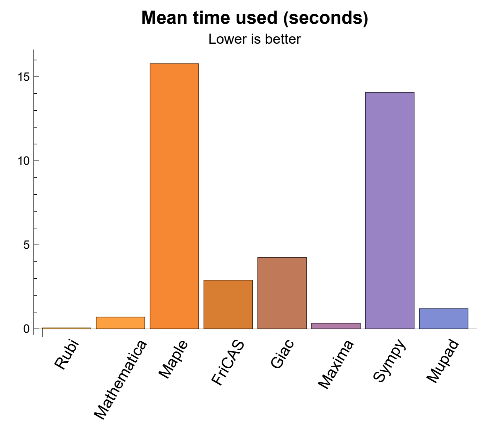
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.06	72.75	1.00	60.00	1.00
Mathematica	0.71	99.83	1.48	62.00	0.95
Maple	15.78	16872315.33	229938.75	89.00	1.50
Maxima	0.34	702.83	10.38	124.00	6.38
Fricas	2.90	110.13	1.80	73.00	1.33
Sympy	14.07	2655.17	141.31	406.50	9.01
Giac	4.25	722.25	15.96	68.00	1.09
Mupad	1.20	123.10	1.96	49.00	1.00

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
200000
150000
100000
50000
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
15
10
5
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{123, 124, 126, 127, 128, 187, 188\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

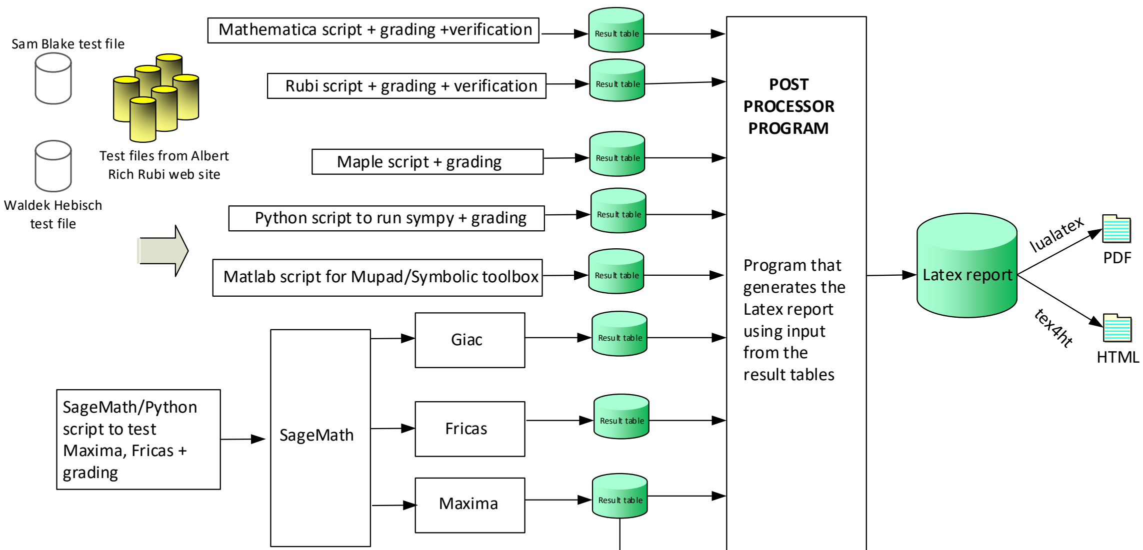
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 190, 191, 192, 193, 194, 195, 197, 198, 199, 200, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 229, 230, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 252, 253, 254 }

B grade: { 11, 40, 42, 44, 61, 72, 136, 138, 140, 158, 160 }

C grade: { 10, 12, 32, 41, 43, 69, 71, 123, 124, 125, 126, 127, 128, 137, 139, 159, 187, 188, 189, 196, 214, 227, 228, 231, 232, 233, 234, 242, 248, 249, 250, 251 }

F grade: { 201, 205 }

2.1.3 Maple

A grade: { 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 19, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 87, 105, 106, 107, 108, 112, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 144, 146, 147, 148, 149, 150, 152, 153, 154, 156, 157, 158, 159, 160, 175, 190, 192, 193, 194, 202, 203, 204, 206, 207, 208, 210, 211, 212, 220, 221, 222, 224, 225, 226, 229, 238, 239, 240, 243, 244, 245, 246 }

B grade: { 1, 3, 13, 15, 17, 23, 27, 73, 74, 75, 76, 77, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 109, 111, 129, 131, 141, 143, 145, 151, 155, 161, 162, 163, 164, 165, 170, 171, 172, 173, 174, 180, 195, 196, 197, 198, 199, 200, 213, 214, 215, 216, 217, 218, 227, 228, 241, 242 }

C grade: { 78, 93, 97, 98, 99, 100, 101, 102, 103, 104, 115, 116, 117, 118, 119, 121, 166, 181, 185, 186, 230, 231, 232, 233, 234, 235, 247, 248, 249, 250, 251, 252 }

F grade: { 79, 80, 94, 95, 96, 110, 113, 114, 120, 122, 123, 124, 125, 126, 127, 128, 167, 168, 169, 176, 177, 178, 179, 182, 183, 184, 187, 188, 189, 191, 201, 205, 209, 219, 223, 236, 237, 253, 254 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 16, 17, 23, 24, 25, 26, 33, 34, 35, 36, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50, 51, 59, 60, 61, 62, 63, 64, 66, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 145, 151, 152, 153, 154, 190, 194, 212, 240 }

B grade: { 8, 9, 10, 11, 12, 18, 19, 20, 21, 22, 27, 28, 29, 30, 31, 32, 41, 42, 43, 44, 52, 53, 54, 55, 56, 57, 58, 65, 67, 68, 69, 70, 71, 72, 136, 137, 138, 139, 140, 146, 147, 148, 149, 150, 155, 156, 157, 158, 159, 160, 192, 193, 195, 196, 197, 198, 199, 200, 202, 203, 204, 206, 207, 208, 210, 211, 213, 214, 215, 216, 217, 218, 220, 221, 222, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252 }

C grade: { }

F grade: { 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 201, 205, 209, 219, 223, 236, 237, 253, 254 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 72, 77, 78, 79, 80, 93, 94, 95, 96, 101, 102, 103, 104, 115, 116, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 145, 146, 147, 149, 151, 152, 153, 154, 155, 156, 157, 165, 166, 167, 168, 182, 183, 184, 190, 192, 193, 194, 195, 196, 197, 199, 202, 203, 204, 206, 207, 208, 210, 211, 212, 213, 215, 216, 217, 218, 220, 221, 222, 224, 225, 226, 227, 229, 238, 239, 240, 241, 242, 243, 244, 245, 246 }

B grade: { 8, 9, 20, 22, 27, 30, 41, 42, 43, 53, 55, 57, 69, 70, 71, 73, 74, 75, 76, 89, 90, 91, 92, 97, 98, 99, 100, 117, 118, 119, 136, 137, 138, 139, 148, 150, 158, 159, 160, 161, 162, 163, 164, 177, 178, 179, 180, 181, 185, 186, 198, 200, 214, 228, 230, 231, 232, 233, 234, 235, 247, 248, 249, 250, 251, 252 }

C grade: { 87, 110, 112, 114, 175 }

F grade: { 81, 82, 83, 84, 85, 86, 88, 105, 106, 107, 108, 109, 111, 113, 123, 124, 125, 126, 127, 128, 169, 170, 171, 172, 173, 174, 176, 187, 188, 189, 191, 201, 205, 209, 219, 223, 236, 237, 253, 254 }

2.1.6 Sympy

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 13, 14, 15, 16, 17, 23, 24, 25, 26, 27, 39, 40, 52, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 145, 151, 152, 153, 154, 155, 190, 192, 193, 194, 195, 196, 202, 203, 204, 206, 207, 208, 210, 211, 212, 213, 214, 220, 221, 222, 224, 225, 226, 227, 228, 238, 239, 240, 241, 242, 244, 245, 246 }

C grade: { }

F grade: { 8, 9, 10, 11, 12, 18, 19, 20, 21, 22, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 136, 137, 138, 139, 140, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 197, 198, 199, 200, 201, 205, 209, 215, 216, 217, 218, 219, 223, 229, 230, 231, 232, 233, 234, 235, 236, 237, 243, 247, 248, 249, 250, 251, 252, 253, 254 }

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A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 32, 34, 36, 38, 40, 41, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 60, 62, 64, 66, 68, 69, 71, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 190, 192, 193, 194, 202, 203, 204, 206, 207, 208, 210, 211, 212, 220, 221, 222, 224, 225, 226, 238, 239, 240, 244, 245, 246 }

B grade: { 8, 9, 11, 29, 31, 33, 35, 37, 39, 42, 44, 59, 61, 63, 65, 67, 70, 72, 77, 78, 79, 93, 136, 165, 166, 167, 181, 195, 196, 197, 198, 199, 200, 213, 214, 215, 216, 217, 218, 227, 228, 229, 233, 234, 235, 241, 242, 243, 250, 251, 252 }

C grade: { }

F grade: { 73, 74, 75, 76, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 191, 201, 205, 209, 219, 223, 230, 231, 232, 236, 237, 247, 248, 249, 253, 254 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 77, 78, 79, 80, 93, 94, 95, 96, 101, 102, 103, 104, 119, 120, 121, 122, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 165, 166, 167, 168, 181, 182, 183, 184, 190, 192, 193, 194, 195, 196, 202, 203, 204, 206, 207, 208, 210, 211, 212, 213, 214, 220, 221, 222, 224, 225, 226, 227, 228, 231, 232, 233, 234, 238, 239, 240, 241, 242, 244, 245, 246, 248, 249, 250, 251 }

C grade: { }

F grade: { 73, 74, 75, 76, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 97, 98, 99, 100, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 123, 124, 125, 126, 127, 128, 161, 162, 163, 164, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 191, 197, 198, 199, 200, 201, 205, 209, 215, 216, 217, 218, 219, 223, 229, 230, 235, 236, 237, 243, 247, 252, 253, 254 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	111	91	83	269	46	45
N.S.	1	1.00	0.77	1.82	1.49	1.36	4.41	0.75	0.74
time (sec)	N/A	0.040	0.490	0.178	0.302	3.906	34.296	0.434	0.079

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	97	80	46	235	80	46
N.S.	1	1.00	0.77	1.59	1.31	0.75	3.85	1.31	0.75
time (sec)	N/A	0.041	0.349	0.076	0.293	4.136	15.180	0.394	0.134

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	83	69	63	197	36	36
N.S.	1	1.00	0.80	1.80	1.50	1.37	4.28	0.78	0.78
time (sec)	N/A	0.039	0.276	0.087	0.313	4.110	6.570	0.430	0.088

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	69	58	36	163	58	36
N.S.	1	1.00	0.80	1.50	1.26	0.78	3.54	1.26	0.78
time (sec)	N/A	0.038	0.164	0.066	0.304	4.190	2.776	0.413	0.140

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	55	47	41	126	26	26
N.S.	1	1.00	0.87	1.77	1.52	1.32	4.06	0.84	0.84
time (sec)	N/A	0.033	0.107	0.066	0.304	4.308	1.105	0.513	0.051

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	41	36	26	92	36	26
N.S.	1	1.00	0.87	1.32	1.16	0.84	2.97	1.16	0.84
time (sec)	N/A	0.035	0.075	0.072	0.299	2.928	0.431	0.442	0.106

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	15	27	26	21	51	13	44
N.S.	1	1.00	0.50	0.90	0.87	0.70	1.70	0.43	1.47
time (sec)	N/A	0.008	0.038	0.048	0.299	3.565	0.196	0.399	0.360

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	20	115	28	0	28	12
N.S.	1	1.00	1.00	1.43	8.21	2.00	0.00	2.00	0.86
time (sec)	N/A	0.012	0.007	0.060	0.525	3.441	0.000	0.403	0.149

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	50	31	236	52	0	52	26
N.S.	1	1.00	1.79	1.11	8.43	1.86	0.00	1.86	0.93
time (sec)	N/A	0.028	0.047	0.108	0.280	3.618	0.000	0.440	0.111

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	29	51	808	85	0	63	48
N.S.	1	1.00	0.59	1.04	16.49	1.73	0.00	1.29	0.98
time (sec)	N/A	0.048	0.020	0.112	0.518	3.561	0.000	0.480	0.121

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	205	71	2174	112	0	160	60
N.S.	1	1.00	3.11	1.08	32.94	1.70	0.00	2.42	0.91
time (sec)	N/A	0.050	0.481	0.132	0.347	3.080	0.000	0.470	0.098

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	31	87	3088	140	0	85	79
N.S.	1	1.00	0.35	0.98	34.70	1.57	0.00	0.96	0.89
time (sec)	N/A	0.052	0.034	0.120	0.640	2.808	0.000	0.441	0.172

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	68	86	72	46	593	36	46
N.S.	1	1.00	1.55	1.95	1.64	1.05	13.48	0.82	1.05
time (sec)	N/A	0.051	0.370	0.092	0.269	4.384	15.575	0.429	0.128

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	75	65	67	434	68	110
N.S.	1	1.00	0.82	0.99	0.86	0.88	5.71	0.89	1.45
time (sec)	N/A	0.050	0.220	0.120	0.277	2.534	6.708	0.451	1.675

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	48	58	50	36	359	26	33
N.S.	1	1.00	1.66	2.00	1.72	1.24	12.38	0.90	1.14
time (sec)	N/A	0.044	0.135	0.069	0.278	2.295	2.807	0.538	0.109

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	40	47	41	46	231	44	43
N.S.	1	1.00	0.82	0.96	0.84	0.94	4.71	0.90	0.88
time (sec)	N/A	0.041	0.083	0.065	0.285	2.249	1.141	0.448	0.302

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	26	24	131	13	13
N.S.	1	1.00	1.00	2.00	1.73	1.60	8.73	0.87	0.87
time (sec)	N/A	0.025	0.007	0.075	0.269	4.476	0.436	0.436	0.096

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	55	14	0	18	12
N.S.	1	1.00	1.00	0.93	3.93	1.00	0.00	1.29	0.86
time (sec)	N/A	0.019	0.015	0.063	0.273	2.924	0.000	0.410	0.084

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	53	19	0	11	11
N.S.	1	1.00	1.00	0.92	4.08	1.46	0.00	0.85	0.85
time (sec)	N/A	0.025	0.010	0.092	0.274	3.425	0.000	0.437	0.109

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	36	24	641	56	0	41	35
N.S.	1	1.00	1.20	0.80	21.37	1.87	0.00	1.37	1.17
time (sec)	N/A	0.035	0.041	0.096	0.294	3.883	0.000	0.503	0.146

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	48	51	308	43	0	32	33
N.S.	1	1.00	1.14	1.21	7.33	1.02	0.00	0.76	0.79
time (sec)	N/A	0.044	0.063	0.114	0.282	3.932	0.000	0.459	0.135

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	62	3164	112	0	74	74
N.S.	1	1.00	0.93	1.03	52.73	1.87	0.00	1.23	1.23
time (sec)	N/A	0.051	0.258	0.107	0.357	2.819	0.000	0.439	0.155

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	97	80	73	447	36	36
N.S.	1	1.00	0.80	2.11	1.74	1.59	9.72	0.78	0.78
time (sec)	N/A	0.046	0.410	0.167	0.279	3.481	34.913	0.425	0.074

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	83	69	46	366	69	46
N.S.	1	1.00	0.77	1.36	1.13	0.75	6.00	1.13	0.75
time (sec)	N/A	0.051	0.254	0.105	0.272	3.955	15.632	0.421	0.118

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	55	47	53	284	26	26
N.S.	1	1.00	0.87	1.77	1.52	1.71	9.16	0.84	0.84
time (sec)	N/A	0.043	0.159	0.102	0.292	3.738	6.692	0.546	0.053

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	55	47	36	202	47	36
N.S.	1	1.00	0.80	1.20	1.02	0.78	4.39	1.02	0.78
time (sec)	N/A	0.046	0.123	0.060	0.276	3.549	2.777	0.446	0.125

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	34	31	117	13	13
N.S.	1	1.00	1.00	2.73	2.27	2.07	7.80	0.87	0.87
time (sec)	N/A	0.025	0.008	0.056	0.267	3.380	1.110	0.402	0.099

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	29	124	36	0	36	23
N.S.	1	1.00	0.96	1.04	4.43	1.29	0.00	1.29	0.82
time (sec)	N/A	0.027	0.016	0.059	0.507	4.708	0.000	0.400	0.124

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	83	13	0	28	13
N.S.	1	1.00	1.00	1.08	6.38	1.00	0.00	2.15	1.00
time (sec)	N/A	0.026	0.013	0.046	0.314	4.173	0.000	0.453	0.091

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	37	480	61	0	48	36
N.S.	1	1.00	1.12	1.09	14.12	1.79	0.00	1.41	1.06
time (sec)	N/A	0.030	0.014	0.116	0.530	3.504	0.000	0.466	0.167

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	61	41	987	67	0	98	37
N.S.	1	1.00	1.42	0.95	22.95	1.56	0.00	2.28	0.86
time (sec)	N/A	0.039	0.035	0.108	0.320	2.856	0.000	0.473	0.075

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	29	69	1805	95	0	73	67
N.S.	1	1.00	0.41	0.99	25.79	1.36	0.00	1.04	0.96
time (sec)	N/A	0.054	0.042	0.100	0.594	2.632	0.000	0.420	0.192

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	119	71	91	46	0	270	46
N.S.	1	1.00	1.95	1.16	1.49	0.75	0.00	4.43	0.75
time (sec)	N/A	0.046	0.105	0.091	0.292	3.269	0.000	0.429	0.153

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	48	97	80	73	0	46	45
N.S.	1	1.00	0.79	1.59	1.31	1.20	0.00	0.75	0.74
time (sec)	N/A	0.046	0.223	0.108	0.283	2.924	0.000	0.418	0.128

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	89	53	69	36	0	204	36
N.S.	1	1.00	1.93	1.15	1.50	0.78	0.00	4.43	0.78
time (sec)	N/A	0.044	0.068	0.088	0.289	3.355	0.000	0.428	0.136

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	69	58	53	0	36	36
N.S.	1	1.00	0.83	1.50	1.26	1.15	0.00	0.78	0.78
time (sec)	N/A	0.042	0.102	0.096	0.272	2.266	0.000	0.405	0.067

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	59	35	47	26	0	138	26
N.S.	1	1.00	1.90	1.13	1.52	0.84	0.00	4.45	0.84
time (sec)	N/A	0.039	0.047	0.070	0.282	3.176	0.000	0.405	0.053

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	28	41	36	33	0	26	26
N.S.	1	1.00	0.90	1.32	1.16	1.06	0.00	0.84	0.84
time (sec)	N/A	0.039	0.056	0.094	0.319	2.897	0.000	0.553	0.108

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	23	13	104225	52	13
N.S.	1	1.00	1.00	0.93	1.53	0.87	6948.33	3.47	0.87
time (sec)	N/A	0.035	0.009	0.059	0.266	2.675	75.379	0.446	0.034

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	23	12	11	11	3636	11	11
N.S.	1	1.00	2.09	1.09	1.00	1.00	330.55	1.00	1.00
time (sec)	N/A	0.012	0.012	0.040	0.266	2.061	10.956	0.399	0.024

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	29	31	233	50	0	38	26
N.S.	1	1.00	1.04	1.11	8.32	1.79	0.00	1.36	0.93
time (sec)	N/A	0.030	0.021	0.070	0.506	2.630	0.000	0.421	0.114

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	143	53	974	96	0	137	49
N.S.	1	1.00	2.92	1.08	19.88	1.96	0.00	2.80	1.00
time (sec)	N/A	0.045	0.284	0.102	0.290	2.989	0.000	0.451	0.129

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	31	69	1780	130	0	72	61
N.S.	1	1.00	0.47	1.05	26.97	1.97	0.00	1.09	0.92
time (sec)	N/A	0.050	0.024	0.092	0.552	3.449	0.000	0.503	0.186

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	268	89	3846	148	0	206	78
N.S.	1	1.00	3.01	1.00	43.21	1.66	0.00	2.31	0.88
time (sec)	N/A	0.053	0.527	0.134	0.415	3.349	0.000	0.438	0.122

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	85	111	87	87	0	95	149
N.S.	1	1.00	0.55	0.72	0.56	0.56	0.00	0.61	0.96
time (sec)	N/A	0.134	0.265	0.125	0.279	3.404	0.000	0.419	2.272

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	48	53	72	36	0	46	35
N.S.	1	1.00	1.09	1.20	1.64	0.82	0.00	1.05	0.80
time (sec)	N/A	0.050	0.190	0.073	0.274	4.333	0.000	0.426	0.149

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	62	83	65	66	0	75	109
N.S.	1	1.00	0.56	0.75	0.59	0.59	0.00	0.68	0.98
time (sec)	N/A	0.096	0.225	0.102	0.280	2.888	0.000	0.416	1.690

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	48	35	50	26	0	36	25
N.S.	1	1.00	1.66	1.21	1.72	0.90	0.00	1.24	0.86
time (sec)	N/A	0.044	0.149	0.062	0.309	3.714	0.000	0.404	0.052

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	40	55	43	47	0	55	65
N.S.	1	1.00	0.67	0.92	0.72	0.78	0.00	0.92	1.08
time (sec)	N/A	0.058	0.112	0.106	0.279	1.923	0.000	0.468	0.490

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	26	13	0	13	13
N.S.	1	1.00	1.00	1.08	2.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.031	0.008	0.049	0.266	3.530	0.000	0.551	0.111

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	20	28	18	22	0	29	17
N.S.	1	1.00	0.95	1.33	0.86	1.05	0.00	1.38	0.81
time (sec)	N/A	0.026	0.029	0.059	0.258	2.526	0.000	0.488	0.148

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	81	14	18894	13	13
N.S.	1	1.00	1.67	1.08	6.75	1.17	1574.50	1.08	1.08
time (sec)	N/A	0.015	0.021	0.059	0.268	3.915	155.399	0.424	0.125

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	34	24	656	65	0	37	36
N.S.	1	1.00	1.13	0.80	21.87	2.17	0.00	1.23	1.20
time (sec)	N/A	0.032	0.059	0.079	0.293	4.319	0.000	0.432	0.137

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	48	51	308	54	0	35	37
N.S.	1	1.00	1.14	1.21	7.33	1.29	0.00	0.83	0.88
time (sec)	N/A	0.047	0.093	0.104	0.279	2.797	0.000	0.452	0.140

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	62	3188	138	0	74	82
N.S.	1	1.00	0.90	1.03	53.13	2.30	0.00	1.23	1.37
time (sec)	N/A	0.051	0.381	0.105	0.345	3.320	0.000	0.504	0.219

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	90	87	1227	86	0	56	55
N.S.	1	1.00	1.25	1.21	17.04	1.19	0.00	0.78	0.76
time (sec)	N/A	0.053	0.064	0.122	0.341	2.683	0.000	0.432	0.381

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	76	98	7650	194	0	94	114
N.S.	1	1.00	0.84	1.09	85.00	2.16	0.00	1.04	1.27
time (sec)	N/A	0.061	0.456	0.122	0.601	3.047	0.000	0.424	0.294

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	132	123	2710	118	0	76	83
N.S.	1	1.00	1.29	1.21	26.57	1.16	0.00	0.75	0.81
time (sec)	N/A	0.059	0.081	0.122	0.462	2.787	0.000	0.448	0.281

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	119	71	91	46	0	314	46
N.S.	1	1.00	1.95	1.16	1.49	0.75	0.00	5.15	0.75
time (sec)	N/A	0.050	0.160	0.098	0.271	2.681	0.000	0.461	0.132

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	58	107	91	83	0	56	55
N.S.	1	1.00	0.76	1.41	1.20	1.09	0.00	0.74	0.72
time (sec)	N/A	0.053	0.354	0.121	0.267	2.813	0.000	0.437	0.115

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	104	53	80	36	0	248	36
N.S.	1	1.00	2.26	1.15	1.74	0.78	0.00	5.39	0.78
time (sec)	N/A	0.046	0.112	0.081	0.269	3.146	0.000	0.437	0.137

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	48	79	69	63	0	46	45
N.S.	1	1.00	0.79	1.30	1.13	1.03	0.00	0.75	0.74
time (sec)	N/A	0.050	0.183	0.101	0.269	3.257	0.000	0.428	0.125

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	35	47	26	0	182	26
N.S.	1	1.00	0.87	1.13	1.52	0.84	0.00	5.87	0.84
time (sec)	N/A	0.042	0.175	0.068	0.263	3.214	0.000	0.430	0.054

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	51	47	43	0	36	36
N.S.	1	1.00	0.80	1.11	1.02	0.93	0.00	0.78	0.78
time (sec)	N/A	0.045	0.127	0.092	0.272	4.789	0.000	0.432	0.124

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	34	13	0	74	13
N.S.	1	1.00	1.00	0.93	2.27	0.87	0.00	4.93	0.87
time (sec)	N/A	0.032	0.011	0.063	0.268	2.792	0.000	0.415	0.047

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	28	22	23	21	0	22	24
N.S.	1	1.00	1.04	0.81	0.85	0.78	0.00	0.81	0.89
time (sec)	N/A	0.030	0.015	0.086	0.273	2.803	0.000	0.624	0.102

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	44	29	92	38	0	54	22
N.S.	1	1.00	1.83	1.21	3.83	1.58	0.00	2.25	0.92
time (sec)	N/A	0.030	0.028	0.086	0.272	5.110	0.000	0.452	0.112

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	84	13	0	13	13
N.S.	1	1.00	1.00	1.27	7.64	1.18	0.00	1.18	1.18
time (sec)	N/A	0.020	0.014	0.046	0.276	2.475	0.000	0.443	0.038

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	41	834	94	0	52	38
N.S.	1	1.00	0.72	0.95	19.40	2.19	0.00	1.21	0.88
time (sec)	N/A	0.036	0.023	0.095	0.516	2.513	0.000	0.427	0.126

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	129	71	2237	132	0	160	66
N.S.	1	1.00	1.84	1.01	31.96	1.89	0.00	2.29	0.94
time (sec)	N/A	0.055	4.665	0.104	0.330	2.250	0.000	0.503	0.150

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	31	87	3095	166	0	82	71
N.S.	1	1.00	0.38	1.07	38.21	2.05	0.00	1.01	0.88
time (sec)	N/A	0.055	0.039	0.105	0.650	2.829	0.000	0.476	0.179

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	278	107	4268	194	0	268	100
N.S.	1	1.00	2.48	0.96	38.11	1.73	0.00	2.39	0.89
time (sec)	N/A	0.065	0.871	0.135	0.471	2.212	0.000	0.458	0.227

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	98	183652438	0	291	0	0	-1
N.S.	1	1.00	0.72	1350385.57	0.00	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.316	52.961	0.000	3.616	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	86	73720488	0	280	0	0	-1
N.S.	1	1.00	0.78	670186.25	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.221	28.838	0.000	6.041	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	72	6806586	0	266	0	0	-1
N.S.	1	1.00	0.86	81030.79	0.00	3.17	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.087	1.437	0.000	3.644	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	50	18282335	0	240	0	0	-1
N.S.	1	1.00	0.86	315212.67	0.00	4.14	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.060	1.558	0.000	3.014	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	67736131	0	39	0	2029	34
N.S.	1	1.00	0.96	2945049.17	0.00	1.70	0.00	88.22	1.48
time (sec)	N/A	0.014	0.021	10.372	0.000	2.335	0.000	12.610	0.284

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	597	0	69	0	7875	108
N.S.	1	1.00	0.81	11.26	0.00	1.30	0.00	148.58	2.04
time (sec)	N/A	0.028	0.129	59.401	0.000	2.690	0.000	48.012	3.080

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	52	0	0	88	0	18022	131
N.S.	1	1.00	0.66	0.00	0.00	1.11	0.00	228.13	1.66
time (sec)	N/A	0.044	0.213	180.000	0.000	4.576	0.000	162.483	3.391

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	67	0	0	113	0	0	351
N.S.	1	1.00	0.64	0.00	0.00	1.08	0.00	0.00	3.34
time (sec)	N/A	0.062	0.166	180.000	0.000	5.989	0.000	0.000	4.335

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	96	519395265	0	0	0	0	-1
N.S.	1	1.00	0.98	5299951.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	3.153	233.506	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	306311267	0	0	0	0	-1
N.S.	1	1.00	0.96	4439293.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.241	130.065	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	76	183042750	0	0	0	0	-1
N.S.	1	1.00	1.10	2652793.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.330	64.299	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	17203919	0	0	0	0	-1
N.S.	1	1.00	0.85	430097.98	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.101	3.293	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	75	58561095	0	0	0	0	-1
N.S.	1	1.00	1.88	1464027.38	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.264	7.409	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	96383272	0	0	0	0	-1
N.S.	1	1.00	0.91	2141850.49	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.106	12.414	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	83	123	0	92	0	0	-1
N.S.	1	1.00	1.73	2.56	0.00	1.92	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.203	53.144	0.000	0.642	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	227	0	0	0	0	-1
N.S.	1	1.00	0.86	2.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.865	247.569	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	98	370168303	0	290	0	0	-1
N.S.	1	1.00	0.72	2721825.76	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.367	149.501	0.000	3.073	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	86	57690707	0	281	0	0	-1
N.S.	1	1.00	0.78	524460.97	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.241	17.743	0.000	4.012	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	74	174944974	0	268	0	0	-1
N.S.	1	1.00	0.88	2082678.26	0.00	3.19	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.158	49.987	0.000	3.533	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	72	172831627	0	296	0	0	-1
N.S.	1	1.00	0.89	2133723.79	0.00	3.65	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.106	38.636	0.000	2.420	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	727	0	48	0	15648	85
N.S.	1	1.00	0.96	25.96	0.00	1.71	0.00	558.86	3.04
time (sec)	N/A	0.021	0.063	170.078	0.000	4.319	0.000	88.458	2.194

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	35	0	0	55	0	0	88
N.S.	1	1.00	0.64	0.00	0.00	1.00	0.00	0.00	1.60
time (sec)	N/A	0.036	0.107	180.000	0.000	5.515	0.000	0.000	3.260

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	0	0	79	0	0	300
N.S.	1	1.00	0.68	0.00	0.00	0.98	0.00	0.00	3.70
time (sec)	N/A	0.053	0.121	180.000	0.000	3.565	0.000	0.000	3.713

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	62	0	0	98	0	0	383
N.S.	1	1.00	0.58	0.00	0.00	0.92	0.00	0.00	3.58
time (sec)	N/A	0.075	0.184	180.000	0.000	3.336	0.000	0.000	5.161

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	98	973	0	290	0	0	-1
N.S.	1	1.00	0.72	7.15	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.338	9.660	0.000	4.229	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	86	243	0	281	0	0	-1
N.S.	1	1.00	0.78	2.21	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.212	6.507	0.000	3.904	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	362	0	266	0	0	-1
N.S.	1	1.00	0.86	4.47	0.00	3.28	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.092	4.280	0.000	3.640	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	157	0	242	0	0	-1
N.S.	1	1.00	0.98	2.96	0.00	4.57	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.050	2.819	0.000	2.401	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	308	0	39	0	0	24
N.S.	1	1.00	0.96	12.83	0.00	1.62	0.00	0.00	1.00
time (sec)	N/A	0.017	0.053	3.863	0.000	2.563	0.000	0.000	0.308

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	194	0	74	0	0	103
N.S.	1	1.00	0.81	3.66	0.00	1.40	0.00	0.00	1.94
time (sec)	N/A	0.046	0.119	7.459	0.000	3.757	0.000	0.000	2.962

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	52	481	0	103	0	0	136
N.S.	1	1.00	0.66	6.09	0.00	1.30	0.00	0.00	1.72
time (sec)	N/A	0.068	0.138	31.787	0.000	3.414	0.000	0.000	3.362

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	67	222	0	118	0	0	350
N.S.	1	1.00	0.64	2.11	0.00	1.12	0.00	0.00	3.33
time (sec)	N/A	0.085	0.152	143.652	0.000	2.727	0.000	0.000	4.130

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	66	204	0	0	0	0	-1
N.S.	1	1.00	0.62	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.334	20.416	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	76	139	0	0	0	0	-1
N.S.	1	1.00	0.72	1.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.194	14.146	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	34	137	0	0	0	0	-1
N.S.	1	1.00	0.45	1.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.091	10.839	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	111	0	0	0	0	-1
N.S.	1	1.00	1.04	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.929	7.872	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	37	176	0	0	0	0	-1
N.S.	1	1.00	0.84	4.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.140	10.641	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	82	0	0	101	0	0	-1
N.S.	1	1.00	1.71	0.00	0.00	2.10	0.00	0.00	-0.02
time (sec)	N/A	0.029	1.031	180.000	0.000	1.222	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	64	227	0	0	0	0	-1
N.S.	1	1.00	0.83	2.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.612	42.560	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	154	0	177	0	0	-1
N.S.	1	1.00	0.86	2.00	0.00	2.30	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.475	224.974	0.000	0.550	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	85	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.849	180.000	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	86	0	0	233	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	2.20	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.362	180.000	0.000	0.942	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	100	441	0	291	0	0	-1
N.S.	1	1.00	0.53	2.32	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.374	286.109	0.000	2.634	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	84	973	0	280	0	0	-1
N.S.	1	1.00	0.51	5.93	0.00	1.71	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.230	129.376	0.000	2.185	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	70	243	0	268	0	0	-1
N.S.	1	1.00	0.55	1.91	0.00	2.11	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.153	33.215	0.000	2.465	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	68	542	0	295	0	0	-1
N.S.	1	1.00	0.65	5.21	0.00	2.84	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.105	83.900	0.000	2.100	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	192	0	53	0	0	95
N.S.	1	1.00	0.96	6.86	0.00	1.89	0.00	0.00	3.39
time (sec)	N/A	0.020	0.056	25.585	0.000	2.252	0.000	0.000	1.500

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	35	0	0	76	0	0	93
N.S.	1	1.00	0.64	0.00	0.00	1.38	0.00	0.00	1.69
time (sec)	N/A	0.041	0.100	180.000	0.000	2.869	0.000	0.000	3.162

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	222	0	104	0	0	302
N.S.	1	1.00	0.68	2.74	0.00	1.28	0.00	0.00	3.73
time (sec)	N/A	0.071	0.129	172.617	0.000	2.121	0.000	0.000	3.700

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	62	0	0	131	0	0	383
N.S.	1	1.00	0.58	0.00	0.00	1.22	0.00	0.00	3.58
time (sec)	N/A	0.091	0.107	180.000	0.000	2.993	0.000	0.000	4.989

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	602	0	0	0	0	0	-1
N.S.	1	1.00	7.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	5.724	0.569	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	602	0	0	0	0	0	-1
N.S.	1	1.00	7.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	3.770	0.362	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	152	0	0	0	0	0	-1
N.S.	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.571	0.104	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	254	0	0	0	0	0	-1
N.S.	1	1.00	3.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.976	0.213	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	938	0	0	0	0	0	-1
N.S.	1	1.00	11.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	7.920	0.231	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	2308	0	0	0	0	0	-1
N.S.	1	1.00	27.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	18.958	0.244	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	111	91	46	270	46	46
N.S.	1	1.00	0.77	1.82	1.49	0.75	4.43	0.75	0.75
time (sec)	N/A	0.044	0.488	0.217	0.286	2.847	33.359	0.409	0.035

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	97	80	73	233	80	45
N.S.	1	1.00	0.77	1.59	1.31	1.20	3.82	1.31	0.74
time (sec)	N/A	0.043	0.356	0.320	0.269	3.983	14.949	0.404	0.132

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	83	69	36	199	36	36
N.S.	1	1.00	0.80	1.80	1.50	0.78	4.33	0.78	0.78
time (sec)	N/A	0.039	0.296	0.151	0.271	2.631	6.472	0.396	0.146

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	69	58	53	162	58	36
N.S.	1	1.00	0.80	1.50	1.26	1.15	3.52	1.26	0.78
time (sec)	N/A	0.037	0.157	0.186	0.270	2.807	2.724	0.398	0.141

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	55	47	26	128	26	26
N.S.	1	1.00	0.87	1.77	1.52	0.84	4.13	0.84	0.84
time (sec)	N/A	0.038	0.107	0.113	0.273	2.872	1.075	0.519	0.032

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	41	36	33	90	36	26
N.S.	1	1.00	0.87	1.32	1.16	1.06	2.90	1.16	0.84
time (sec)	N/A	0.034	0.077	0.193	0.280	1.960	0.404	0.420	0.028

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	15	27	26	13	53	13	43
N.S.	1	1.00	0.50	0.90	0.87	0.43	1.77	0.43	1.43
time (sec)	N/A	0.008	0.007	0.087	0.268	2.212	0.165	0.402	0.192

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	42	22	84	30	0	28	12
N.S.	1	1.00	3.00	1.57	6.00	2.14	0.00	2.00	0.86
time (sec)	N/A	0.013	0.021	0.142	0.289	2.636	0.000	0.433	0.022

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	29	31	233	50	0	38	26
N.S.	1	1.00	1.04	1.11	8.32	1.79	0.00	1.36	0.93
time (sec)	N/A	0.029	0.026	0.161	0.516	3.341	0.000	0.441	0.024

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	143	53	974	96	0	63	49
N.S.	1	1.00	2.92	1.08	19.88	1.96	0.00	1.29	1.00
time (sec)	N/A	0.039	0.276	0.182	0.292	2.525	0.000	0.534	0.083

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	31	69	1780	130	0	72	61
N.S.	1	1.00	0.47	1.05	26.97	1.97	0.00	1.09	0.92
time (sec)	N/A	0.044	0.027	0.194	0.572	2.052	0.000	0.442	0.098

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	268	89	3846	148	0	85	78
N.S.	1	1.00	3.01	1.00	43.21	1.66	0.00	0.96	0.88
time (sec)	N/A	0.051	0.538	0.237	0.419	2.139	0.000	0.433	0.119

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	68	86	72	36	597	36	35
N.S.	1	1.00	1.55	1.95	1.64	0.82	13.57	0.82	0.80
time (sec)	N/A	0.051	0.443	0.241	0.264	2.685	15.365	0.411	0.175

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	75	65	66	434	68	109
N.S.	1	1.00	0.82	0.99	0.86	0.87	5.71	0.89	1.43
time (sec)	N/A	0.054	0.215	0.267	0.279	4.238	6.472	0.406	1.727

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	48	58	50	26	362	26	26
N.S.	1	1.00	1.71	2.07	1.79	0.93	12.93	0.93	0.93
time (sec)	N/A	0.043	0.138	0.142	0.259	3.653	2.720	0.527	0.144

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	40	47	43	47	231	46	43
N.S.	1	1.00	0.82	0.96	0.88	0.96	4.71	0.94	0.88
time (sec)	N/A	0.041	0.111	0.141	0.267	2.016	1.131	0.453	0.313

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	26	13	133	13	13
N.S.	1	1.00	1.00	2.00	1.73	0.87	8.87	0.87	0.87
time (sec)	N/A	0.025	0.005	0.141	0.286	2.710	0.428	0.414	0.153

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	22	13	82	14	0	13	14
N.S.	1	1.00	1.57	0.93	5.86	1.00	0.00	0.93	1.00
time (sec)	N/A	0.019	0.022	0.125	0.274	2.823	0.000	0.406	0.157

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	53	19	0	13	11
N.S.	1	1.00	1.00	0.92	4.08	1.46	0.00	1.00	0.85
time (sec)	N/A	0.027	0.018	0.149	0.267	2.441	0.000	0.456	0.142

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	34	24	656	65	0	41	36
N.S.	1	1.00	1.13	0.80	21.87	2.17	0.00	1.37	1.20
time (sec)	N/A	0.036	0.058	0.163	0.293	2.753	0.000	0.531	0.178

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	48	51	308	54	0	35	37
N.S.	1	1.00	1.14	1.21	7.33	1.29	0.00	0.83	0.88
time (sec)	N/A	0.047	0.064	0.208	0.284	2.842	0.000	0.441	0.184

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	62	3188	138	0	74	82
N.S.	1	1.00	0.90	1.03	53.13	2.30	0.00	1.23	1.37
time (sec)	N/A	0.051	0.386	0.216	0.356	2.470	0.000	0.439	0.144

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	97	80	36	447	36	36
N.S.	1	1.00	0.80	2.11	1.74	0.78	9.72	0.78	0.78
time (sec)	N/A	0.046	0.443	0.329	0.275	1.892	33.479	0.415	0.071

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	47	83	69	63	366	69	45
N.S.	1	1.00	0.77	1.36	1.13	1.03	6.00	1.13	0.74
time (sec)	N/A	0.050	0.254	0.289	0.276	1.581	14.965	0.409	0.061

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	27	55	47	26	284	26	26
N.S.	1	1.00	0.87	1.77	1.52	0.84	9.16	0.84	0.84
time (sec)	N/A	0.044	0.155	0.184	0.279	1.913	6.368	0.546	0.152

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	55	47	43	202	47	36
N.S.	1	1.00	0.80	1.20	1.02	0.93	4.39	1.02	0.78
time (sec)	N/A	0.046	0.105	0.164	0.288	2.365	2.709	0.456	0.160

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	34	13	117	13	13
N.S.	1	1.00	1.00	2.73	2.27	0.87	7.80	0.87	0.87
time (sec)	N/A	0.025	0.008	0.099	0.273	1.740	1.037	0.405	0.142

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	46	29	92	38	0	36	22
N.S.	1	1.00	1.64	1.04	3.29	1.36	0.00	1.29	0.79
time (sec)	N/A	0.026	0.024	0.154	0.272	2.886	0.000	0.418	0.053

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	84	13	0	13	13
N.S.	1	1.00	1.00	1.08	6.46	1.00	0.00	1.00	1.00
time (sec)	N/A	0.027	0.015	0.090	0.273	3.099	0.000	0.462	0.023

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	79	39	558	72	0	48	36
N.S.	1	1.00	2.32	1.15	16.41	2.12	0.00	1.41	1.06
time (sec)	N/A	0.030	0.020	0.217	0.293	2.230	0.000	0.549	0.059

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	41	834	94	0	52	38
N.S.	1	1.00	0.72	0.95	19.40	2.19	0.00	1.21	0.88
time (sec)	N/A	0.040	0.022	0.194	0.531	2.157	0.000	0.473	0.064

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	195	71	2237	132	0	73	66
N.S.	1	1.00	2.79	1.01	31.96	1.89	0.00	1.04	0.94
time (sec)	N/A	0.053	0.370	0.229	0.332	2.457	0.000	0.451	0.186

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	98	221660564	0	290	0	0	-1
N.S.	1	1.00	0.72	1629857.09	0.00	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.375	99.630	0.000	2.429	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	86	85899870	0	281	0	0	-1
N.S.	1	1.00	0.78	780907.91	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.211	24.427	0.000	3.698	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	5537888	0	266	0	0	-1
N.S.	1	1.00	0.83	65927.24	0.00	3.17	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.115	2.474	0.000	2.298	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	52	18450099	0	242	0	0	-1
N.S.	1	1.00	0.90	318105.16	0.00	4.17	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.058	2.801	0.000	1.624	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	23	63401108	0	39	0	2029	24
N.S.	1	1.00	0.96	2641712.83	0.00	1.62	0.00	84.54	1.00
time (sec)	N/A	0.014	0.023	10.362	0.000	1.593	0.000	12.481	0.192

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	194	0	74	0	7875	104
N.S.	1	1.00	0.81	3.66	0.00	1.40	0.00	148.58	1.96
time (sec)	N/A	0.027	0.122	17.645	0.000	1.541	0.000	45.071	3.168

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	52	0	0	103	0	18022	136
N.S.	1	1.00	0.66	0.00	0.00	1.30	0.00	228.13	1.72
time (sec)	N/A	0.044	0.143	180.000	0.000	2.252	0.000	155.497	3.233

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	67	0	0	118	0	0	350
N.S.	1	1.00	0.64	0.00	0.00	1.12	0.00	0.00	3.33
time (sec)	N/A	0.064	0.164	180.000	0.000	1.536	0.000	0.000	3.866

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	96	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.377	180.000	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	358108730	0	0	0	0	-1
N.S.	1	1.00	0.96	5189981.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.230	186.905	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	76	189673602	0	0	0	0	-1
N.S.	1	1.00	1.10	2748892.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.333	71.639	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	34	26159849	0	0	0	0	-1
N.S.	1	1.00	0.85	653996.22	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.069	6.631	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	76	66249372	0	0	0	0	-1
N.S.	1	1.00	1.90	1656234.30	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.999	15.168	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	39	106313389	0	0	0	0	-1
N.S.	1	1.00	0.85	2311160.63	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.131	17.612	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	82	123	0	101	0	0	-1
N.S.	1	1.00	1.71	2.56	0.00	2.10	0.00	0.00	-0.02
time (sec)	N/A	0.028	1.141	82.682	0.000	0.439	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	64	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.625	180.000	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	99	0	0	291	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.368	180.000	0.000	1.949	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	84	0	0	280	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.210	180.000	0.000	2.848	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	73	0	0	268	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	3.19	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.129	180.000	0.000	3.260	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	179323150	0	295	0	0	-1
N.S.	1	1.00	0.85	2186867.68	0.00	3.60	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.107	23.230	0.000	3.663	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	192	0	53	0	15292	94
N.S.	1	1.00	0.96	6.86	0.00	1.89	0.00	546.14	3.36
time (sec)	N/A	0.021	0.064	175.777	0.000	2.700	0.000	92.584	1.168

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	35	0	0	76	0	0	93
N.S.	1	1.00	0.64	0.00	0.00	1.38	0.00	0.00	1.69
time (sec)	N/A	0.035	0.112	180.000	0.000	3.164	0.000	0.000	3.075

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	55	0	0	104	0	0	302
N.S.	1	1.00	0.68	0.00	0.00	1.28	0.00	0.00	3.73
time (sec)	N/A	0.053	0.134	180.000	0.000	2.243	0.000	0.000	3.661

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	62	0	0	131	0	0	383
N.S.	1	1.00	0.58	0.00	0.00	1.22	0.00	0.00	3.58
time (sec)	N/A	0.068	0.112	180.000	0.000	2.188	0.000	0.000	4.881

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	29	98	0	137	0	0	-1
N.S.	1	1.00	0.94	3.16	0.00	4.42	0.00	0.00	-0.03
time (sec)	N/A	0.010	0.030	0.175	0.000	2.690	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	99	0	137	0	0	-1
N.S.	1	1.00	1.00	3.96	0.00	5.48	0.00	0.00	-0.04
time (sec)	N/A	0.024	0.021	0.145	0.000	2.910	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	2472	0	0	0	0	0	-1
N.S.	1	1.00	29.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	13.375	0.177	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	890	0	0	0	0	0	-1
N.S.	1	1.00	10.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	8.231	0.173	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	149	0	0	0	0	0	-1
N.S.	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.301	0.082	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	37	69	58	36	318	36	36
N.S.	1	1.00	0.80	1.50	1.26	0.78	6.91	0.78	0.78
time (sec)	N/A	0.070	0.182	0.217	0.272	2.700	14.954	0.455	0.205

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	199	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.594	0.939	0.020	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	86	84	916	122	918	84	311
N.S.	1	1.00	0.95	0.92	10.07	1.34	10.09	0.92	3.42
time (sec)	N/A	0.059	0.579	0.211	0.316	2.846	2.359	0.395	1.567

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	414	71	405	56	98
N.S.	1	1.00	1.11	0.92	6.68	1.15	6.53	0.90	1.58
time (sec)	N/A	0.038	0.749	0.224	0.296	2.164	0.796	0.405	0.736

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	40	42	153	40	84
N.S.	1	1.00	1.00	0.93	0.93	0.98	3.56	0.93	1.95
time (sec)	N/A	0.030	0.214	0.116	0.268	3.001	0.309	0.403	1.063

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	161	108	31	335	236	111
N.S.	1	1.00	1.00	6.19	4.15	1.19	12.88	9.08	4.27
time (sec)	N/A	0.020	0.173	0.376	0.298	2.846	4.102	0.413	0.869

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	90	347	454	71	3264	349	252
N.S.	1	1.00	2.50	9.64	12.61	1.97	90.67	9.69	7.00
time (sec)	N/A	0.024	0.116	0.617	0.312	2.735	60.428	0.401	5.209

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	120	399	47	0	145	-1
N.S.	1	1.00	0.90	3.08	10.23	1.21	0.00	3.72	-0.03
time (sec)	N/A	0.031	0.228	0.948	0.280	2.029	0.000	0.426	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	1748	1773	141	0	2221	-1
N.S.	1	1.00	1.00	26.09	26.46	2.10	0.00	33.15	-0.01
time (sec)	N/A	0.034	0.614	1.884	0.353	3.452	0.000	0.441	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	58	321	1076	75	0	301	-1
N.S.	1	1.00	0.97	5.35	17.93	1.25	0.00	5.02	-0.02
time (sec)	N/A	0.033	0.407	2.851	0.318	2.683	0.000	0.421	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	79	6746	3879	197	0	8035	-1
N.S.	1	1.00	0.84	71.77	41.27	2.10	0.00	85.48	-0.01
time (sec)	N/A	0.045	1.228	6.250	0.537	2.906	0.000	0.481	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.693	0.432	0.079	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	80	63	371	69	410	61	105
N.S.	1	1.00	1.18	0.93	5.46	1.01	6.03	0.90	1.54
time (sec)	N/A	0.041	0.371	0.214	0.314	3.171	0.851	0.406	0.751

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	106	83	620	118	1027	80	177
N.S.	1	1.00	1.20	0.94	7.05	1.34	11.67	0.91	2.01
time (sec)	N/A	0.047	0.831	0.178	0.307	3.073	1.927	0.410	0.886

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1362	192	2006	129	469
N.S.	1	1.00	1.10	0.92	9.46	1.33	13.93	0.90	3.26
time (sec)	N/A	0.076	1.610	0.231	0.372	2.158	6.588	0.426	1.873

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.186	0.604	0.086	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	91	90	789	115	937	89	494
N.S.	1	1.00	0.94	0.93	8.13	1.19	9.66	0.92	5.09
time (sec)	N/A	0.058	0.584	0.224	0.333	2.827	2.385	0.433	1.717

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	1360	189	2020	124	437
N.S.	1	1.00	1.11	0.92	9.86	1.37	14.64	0.90	3.17
time (sec)	N/A	0.073	1.648	0.220	0.355	2.226	6.558	0.409	2.127

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	177	184	2612	291	3580	181	997
N.S.	1	1.00	0.91	0.94	13.39	1.49	18.36	0.93	5.11
time (sec)	N/A	0.097	1.709	0.382	0.437	1.957	21.217	0.421	4.315

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	203	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.407	0.931	0.027	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	87	84	912	106	921	84	297
N.S.	1	1.00	0.96	0.92	10.02	1.16	10.12	0.92	3.26
time (sec)	N/A	0.048	0.529	0.184	0.324	2.105	2.361	0.393	1.615

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	71	57	414	66	408	56	97
N.S.	1	1.00	1.15	0.92	6.68	1.06	6.58	0.90	1.56
time (sec)	N/A	0.034	0.834	0.208	0.304	2.963	0.800	0.413	0.767

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	40	42	155	40	85
N.S.	1	1.00	1.00	0.93	0.93	0.98	3.60	0.93	1.98
time (sec)	N/A	0.026	0.210	0.116	0.265	3.971	0.317	0.415	0.842

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	161	73	31	435	158	112
N.S.	1	1.00	1.00	5.96	2.70	1.15	16.11	5.85	4.15
time (sec)	N/A	0.012	0.169	0.398	0.289	3.872	5.171	0.438	0.894

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	88	346	387	69	5545	248	254
N.S.	1	1.00	2.59	10.18	11.38	2.03	163.09	7.29	7.47
time (sec)	N/A	0.021	0.110	0.649	0.551	2.916	91.129	0.437	5.345

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	34	121	391	42	0	174	-1
N.S.	1	1.00	0.89	3.18	10.29	1.11	0.00	4.58	-0.03
time (sec)	N/A	0.030	0.200	0.973	0.289	3.599	0.000	0.422	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	1782	1424	94	0	495	-1
N.S.	1	1.00	0.96	26.60	21.25	1.40	0.00	7.39	-0.01
time (sec)	N/A	0.035	0.482	2.023	0.586	2.172	0.000	0.437	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	48	324	1074	53	0	327	-1
N.S.	1	1.00	0.81	5.49	18.20	0.90	0.00	5.54	-0.02
time (sec)	N/A	0.031	0.371	2.681	0.291	3.019	0.000	0.428	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	78	6742	3096	107	0	756	-1
N.S.	1	1.00	0.83	71.72	32.94	1.14	0.00	8.04	-0.01
time (sec)	N/A	0.044	1.046	6.898	0.690	2.948	0.000	0.441	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	249	0	0	0	0	0	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.485	2.195	0.074	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	76	63	371	70	410	61	105
N.S.	1	1.00	1.12	0.93	5.46	1.03	6.03	0.90	1.54
time (sec)	N/A	0.038	0.857	0.233	0.286	3.622	0.790	0.440	0.809

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	108	83	620	108	1027	80	177
N.S.	1	1.00	1.23	0.94	7.05	1.23	11.67	0.91	2.01
time (sec)	N/A	0.050	0.806	0.181	0.316	2.932	1.874	0.442	0.929

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1362	174	2006	129	495
N.S.	1	1.00	1.10	0.92	9.46	1.21	13.93	0.90	3.44
time (sec)	N/A	0.069	1.847	0.253	0.352	1.706	6.635	0.417	1.969

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	568	568	355	0	0	0	0	0	-1
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.843	32.932	0.082	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	90	785	116	937	89	471
N.S.	1	1.00	0.93	0.93	8.09	1.20	9.66	0.92	4.86
time (sec)	N/A	0.050	0.562	0.185	0.319	3.763	2.268	0.415	1.613

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	153	127	1360	179	2030	124	438
N.S.	1	1.00	1.11	0.92	9.86	1.30	14.71	0.90	3.17
time (sec)	N/A	0.067	1.654	0.219	0.329	3.260	6.343	0.425	2.032

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	176	184	2612	264	3582	181	951
N.S.	1	1.00	0.90	0.94	13.39	1.35	18.37	0.93	4.88
time (sec)	N/A	0.096	1.602	0.332	0.413	2.775	20.858	0.412	4.136

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	58	162	106	30	335	482	115
N.S.	1	1.00	2.15	6.00	3.93	1.11	12.41	17.85	4.26
time (sec)	N/A	0.010	0.202	0.384	0.289	2.648	3.999	0.436	0.703

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	90	408	450	71	3264	893	252
N.S.	1	1.00	2.57	11.66	12.86	2.03	93.26	25.51	7.20
time (sec)	N/A	0.022	0.108	0.667	0.303	1.972	60.012	0.538	5.210

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	55	395	45	0	327	-1
N.S.	1	1.00	0.92	1.45	10.39	1.18	0.00	8.61	-0.03
time (sec)	N/A	0.030	0.231	0.865	0.273	2.120	0.000	0.456	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	70	186	1027	376	0	0	-1
N.S.	1	1.00	0.97	2.58	14.26	5.22	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.418	0.164	0.573	2.586	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	109	143	520	315	0	0	294
N.S.	1	1.00	2.48	3.25	11.82	7.16	0.00	0.00	6.68
time (sec)	N/A	0.026	0.112	0.123	0.551	1.753	0.000	0.000	5.306

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	94	99	131	188	0	0	227
N.S.	1	1.00	3.24	3.41	4.52	6.48	0.00	0.00	7.83
time (sec)	N/A	0.012	0.057	0.095	0.532	2.019	0.000	0.000	5.487

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	93	95	105	197	0	226	233
N.S.	1	1.00	3.21	3.28	3.62	6.79	0.00	7.79	8.03
time (sec)	N/A	0.015	0.066	0.107	0.293	2.075	0.000	0.410	4.846

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	111	143	612	316	0	577	290
N.S.	1	1.00	2.41	3.11	13.30	6.87	0.00	12.54	6.30
time (sec)	N/A	0.028	0.116	0.138	0.301	2.212	0.000	0.447	5.262

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	71	184	1254	372	0	870	-1
N.S.	1	1.00	0.96	2.49	16.95	5.03	0.00	11.76	-0.01
time (sec)	N/A	0.055	0.412	0.161	0.319	2.302	0.000	0.452	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	116	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	1.446	0.035	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	260	0	0	0	0	0	-1
N.S.	1	1.00	1.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	3.141	0.039	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	85	84	914	109	921	84	313
N.S.	1	1.00	0.93	0.92	10.04	1.20	10.12	0.92	3.44
time (sec)	N/A	0.049	0.548	0.217	0.337	2.010	2.295	0.388	1.949

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	69	57	416	63	408	56	98
N.S.	1	1.00	1.11	0.92	6.71	1.02	6.58	0.90	1.58
time (sec)	N/A	0.036	0.813	0.229	0.339	2.385	0.800	0.390	1.017

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	40	40	42	153	40	84
N.S.	1	1.00	1.00	0.93	0.93	0.98	3.56	0.93	1.95
time (sec)	N/A	0.026	0.199	0.129	0.276	2.745	0.317	0.393	1.295

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	161	74	31	435	440	109
N.S.	1	1.00	1.00	6.19	2.85	1.19	16.73	16.92	4.19
time (sec)	N/A	0.011	0.140	0.408	0.295	2.144	5.128	0.441	1.039

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	89	407	391	69	5545	1341	246
N.S.	1	1.00	2.54	11.63	11.17	1.97	158.43	38.31	7.03
time (sec)	N/A	0.020	0.099	0.688	0.561	1.314	88.158	0.499	6.525

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	56	382	40	0	315	-1
N.S.	1	1.00	0.92	1.47	10.05	1.05	0.00	8.29	-0.03
time (sec)	N/A	0.032	0.223	0.899	0.287	2.048	0.000	0.458	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	158	133	1362	163	2003	129	495
N.S.	1	1.00	1.10	0.92	9.46	1.13	13.91	0.90	3.44
time (sec)	N/A	0.069	1.659	0.276	0.348	1.672	6.531	0.407	2.445

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	105	83	620	105	1027	80	177
N.S.	1	1.00	1.19	0.94	7.05	1.19	11.67	0.91	2.01
time (sec)	N/A	0.049	0.775	0.176	0.304	2.592	1.904	0.406	1.058

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	176	184	2614	240	3582	181	999
N.S.	1	1.00	0.90	0.94	13.41	1.23	18.37	0.93	5.12
time (sec)	N/A	0.093	1.711	0.379	0.424	3.146	21.025	0.418	4.542

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	70	181	1027	366	0	0	-1
N.S.	1	1.00	0.97	2.51	14.26	5.08	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.406	0.160	0.592	3.094	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	111	149	526	316	0	0	285
N.S.	1	1.00	2.41	3.24	11.43	6.87	0.00	0.00	6.20
time (sec)	N/A	0.030	0.100	0.126	0.550	2.656	0.000	0.000	5.243

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	93	97	131	196	0	0	237
N.S.	1	1.00	3.10	3.23	4.37	6.53	0.00	0.00	7.90
time (sec)	N/A	0.011	0.066	0.098	0.546	3.163	0.000	0.000	4.727

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	94	93	105	190	0	234	231
N.S.	1	1.00	3.24	3.21	3.62	6.55	0.00	8.07	7.97
time (sec)	N/A	0.011	0.059	0.127	0.282	2.742	0.000	0.431	5.386

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	112	145	613	316	0	627	289
N.S.	1	1.00	2.43	3.15	13.33	6.87	0.00	13.63	6.28
time (sec)	N/A	0.026	0.108	0.151	0.307	3.229	0.000	0.447	5.439

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	71	179	1254	385	0	963	-1
N.S.	1	1.00	0.97	2.45	17.18	5.27	0.00	13.19	-0.01
time (sec)	N/A	0.053	0.368	0.165	0.319	3.544	0.000	0.462	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	197	0	0	0	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	1.709	0.038	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	108	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	1.876	0.046	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [190] had the largest ratio of [28]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	18	0.167
2	A	4	3	1.00	18	0.167
3	A	4	3	1.00	18	0.167
4	A	4	3	1.00	18	0.167
5	A	4	3	1.00	18	0.167
6	A	4	3	1.00	18	0.167
7	A	1	1	1.00	16	0.062
8	A	2	2	1.00	16	0.125
9	A	4	4	1.00	18	0.222
10	A	5	5	1.00	18	0.278
11	A	6	5	1.00	18	0.278
12	A	7	5	1.00	18	0.278
13	A	5	4	1.00	20	0.200
14	A	6	5	1.00	20	0.250
15	A	4	3	1.00	20	0.150
16	A	5	5	1.00	20	0.250
17	A	3	3	1.00	18	0.167
18	A	2	2	1.00	18	0.111
19	A	3	3	1.00	20	0.150
20	A	4	3	1.00	20	0.150
21	A	4	3	1.00	20	0.150
22	A	5	4	1.00	20	0.200
23	A	4	3	1.00	20	0.150
24	A	4	3	1.00	20	0.150
25	A	4	3	1.00	20	0.150

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	4	3	1.00	20	0.150
27	A	3	3	1.00	18	0.167
28	A	4	4	1.00	18	0.222
29	A	3	3	1.00	20	0.150
30	A	3	3	1.00	20	0.150
31	A	5	4	1.00	20	0.200
32	A	6	5	1.00	20	0.250
33	A	4	3	1.00	18	0.167
34	A	4	3	1.00	18	0.167
35	A	4	3	1.00	18	0.167
36	A	4	3	1.00	18	0.167
37	A	4	3	1.00	18	0.167
38	A	4	3	1.00	18	0.167
39	A	3	3	1.00	18	0.167
40	A	2	2	1.00	16	0.125
41	A	4	4	1.00	16	0.250
42	A	5	5	1.00	18	0.278
43	A	6	5	1.00	18	0.278
44	A	7	5	1.00	18	0.278
45	A	9	4	1.00	20	0.200
46	A	5	4	1.00	20	0.200
47	A	7	4	1.00	20	0.200
48	A	4	3	1.00	20	0.150
49	A	5	4	1.00	20	0.200
50	A	3	3	1.00	20	0.150
51	A	3	3	1.00	20	0.150
52	A	2	2	1.00	18	0.111
53	A	4	3	1.00	18	0.167
54	A	4	3	1.00	20	0.150
55	A	5	4	1.00	20	0.200
56	A	4	3	1.00	20	0.150
57	A	5	4	1.00	20	0.200
58	A	4	3	1.00	20	0.150
59	A	4	3	1.00	20	0.150
60	A	4	3	1.00	20	0.150

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	3	1.00	20	0.150
62	A	4	3	1.00	20	0.150
63	A	4	3	1.00	20	0.150
64	A	4	3	1.00	20	0.150
65	A	3	3	1.00	20	0.150
66	A	3	2	1.00	20	0.100
67	A	4	4	1.00	20	0.200
68	A	3	3	1.00	18	0.167
69	A	5	4	1.00	18	0.222
70	A	6	5	1.00	20	0.250
71	A	6	5	1.00	20	0.250
72	A	8	5	1.00	20	0.250
73	A	4	3	1.00	20	0.150
74	A	3	3	1.00	20	0.150
75	A	2	2	1.00	20	0.100
76	A	1	1	1.00	20	0.050
77	A	1	1	1.00	20	0.050
78	A	2	2	1.00	20	0.100
79	A	3	3	1.00	20	0.150
80	A	4	3	1.00	20	0.150
81	A	4	3	1.00	22	0.136
82	A	3	3	1.00	22	0.136
83	A	3	3	1.00	22	0.136
84	A	2	2	1.00	22	0.091
85	A	2	2	1.00	22	0.091
86	A	2	2	1.00	22	0.091
87	A	2	2	1.00	22	0.091
88	A	3	3	1.00	22	0.136
89	A	4	4	1.00	22	0.182
90	A	3	3	1.00	22	0.136
91	A	2	2	1.00	22	0.091
92	A	3	3	1.00	22	0.136
93	A	1	1	1.00	22	0.045
94	A	2	2	1.00	22	0.091
95	A	3	3	1.00	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	4	1.00	22	0.182
97	A	5	4	1.00	20	0.200
98	A	4	4	1.00	20	0.200
99	A	3	3	1.00	20	0.150
100	A	2	2	1.00	20	0.100
101	A	1	1	1.00	20	0.050
102	A	3	3	1.00	20	0.150
103	A	4	4	1.00	20	0.200
104	A	5	4	1.00	20	0.200
105	A	4	3	1.00	22	0.136
106	A	4	3	1.00	22	0.136
107	A	3	3	1.00	22	0.136
108	A	3	3	1.00	22	0.136
109	A	2	2	1.00	22	0.091
110	A	2	2	1.00	22	0.091
111	A	3	3	1.00	22	0.136
112	A	3	3	1.00	22	0.136
113	A	4	3	1.00	22	0.136
114	A	4	3	1.00	22	0.136
115	A	7	5	1.00	22	0.227
116	A	6	5	1.00	22	0.227
117	A	5	5	1.00	22	0.227
118	A	4	4	1.00	22	0.182
119	A	1	1	1.00	22	0.045
120	A	2	2	1.00	22	0.091
121	A	4	4	1.00	22	0.182
122	A	5	5	1.00	22	0.227
123	A	2	2	1.00	20	0.100
124	A	2	2	1.00	20	0.100
125	A	2	2	1.00	18	0.111
126	A	2	2	1.00	18	0.111
127	A	2	2	1.00	20	0.100
128	A	2	2	1.00	20	0.100
129	A	4	3	1.00	18	0.167
130	A	4	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	3	1.00	18	0.167
132	A	4	3	1.00	18	0.167
133	A	4	3	1.00	18	0.167
134	A	4	3	1.00	18	0.167
135	A	1	1	1.00	16	0.062
136	A	2	2	1.00	16	0.125
137	A	4	4	1.00	18	0.222
138	A	5	5	1.00	18	0.278
139	A	6	5	1.00	18	0.278
140	A	7	5	1.00	18	0.278
141	A	5	4	1.00	20	0.200
142	A	6	5	1.00	20	0.250
143	A	4	3	1.00	20	0.150
144	A	5	5	1.00	20	0.250
145	A	3	3	1.00	18	0.167
146	A	2	2	1.00	18	0.111
147	A	3	3	1.00	20	0.150
148	A	4	3	1.00	20	0.150
149	A	4	3	1.00	20	0.150
150	A	5	4	1.00	20	0.200
151	A	4	3	1.00	20	0.150
152	A	4	3	1.00	20	0.150
153	A	4	3	1.00	20	0.150
154	A	4	3	1.00	20	0.150
155	A	3	3	1.00	18	0.167
156	A	4	4	1.00	18	0.222
157	A	3	3	1.00	20	0.150
158	A	3	3	1.00	20	0.150
159	A	5	4	1.00	20	0.200
160	A	6	5	1.00	20	0.250
161	A	4	3	1.00	20	0.150
162	A	3	3	1.00	20	0.150
163	A	2	2	1.00	20	0.100
164	A	1	1	1.00	20	0.050
165	A	1	1	1.00	20	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	2	2	1.00	20	0.100
167	A	3	3	1.00	20	0.150
168	A	4	3	1.00	20	0.150
169	A	4	3	1.00	22	0.136
170	A	3	3	1.00	22	0.136
171	A	3	3	1.00	22	0.136
172	A	2	2	1.00	22	0.091
173	A	2	2	1.00	22	0.091
174	A	2	2	1.00	22	0.091
175	A	2	2	1.00	22	0.091
176	A	3	3	1.00	22	0.136
177	A	4	4	1.00	22	0.182
178	A	3	3	1.00	22	0.136
179	A	2	2	1.00	22	0.091
180	A	3	3	1.00	22	0.136
181	A	1	1	1.00	22	0.045
182	A	2	2	1.00	22	0.091
183	A	3	3	1.00	22	0.136
184	A	4	4	1.00	22	0.182
185	A	1	1	1.00	11	0.091
186	A	2	2	1.00	11	0.182
187	A	2	2	1.00	20	0.100
188	A	2	2	1.00	20	0.100
189	A	2	2	1.00	18	0.111
190	A	4	3	1.00	28	0.107
191	A	10	5	1.00	15	0.333
192	A	6	2	1.00	15	0.133
193	A	5	2	1.00	15	0.133
194	A	4	2	1.00	13	0.154
195	A	3	3	1.00	13	0.231
196	A	4	4	1.00	15	0.267
197	A	5	5	1.00	15	0.333
198	A	5	5	1.00	15	0.333
199	A	5	4	1.00	15	0.267
200	A	6	5	1.00	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	15	10	1.00	17	0.588
202	A	5	2	1.00	15	0.133
203	A	6	2	1.00	17	0.118
204	A	8	2	1.00	17	0.118
205	A	18	5	1.00	17	0.294
206	A	6	2	1.00	15	0.133
207	A	8	2	1.00	17	0.118
208	A	10	2	1.00	17	0.118
209	A	8	4	1.00	15	0.267
210	A	6	2	1.00	15	0.133
211	A	5	2	1.00	15	0.133
212	A	4	2	1.00	13	0.154
213	A	3	3	1.00	13	0.231
214	A	4	4	1.00	15	0.267
215	A	5	5	1.00	15	0.333
216	A	5	5	1.00	15	0.333
217	A	5	4	1.00	15	0.267
218	A	6	5	1.00	15	0.333
219	A	11	7	1.00	17	0.412
220	A	5	2	1.00	15	0.133
221	A	6	2	1.00	17	0.118
222	A	8	2	1.00	17	0.118
223	A	14	4	1.00	17	0.235
224	A	6	2	1.00	15	0.133
225	A	8	2	1.00	17	0.118
226	A	10	2	1.00	17	0.118
227	A	3	3	1.00	13	0.231
228	A	4	4	1.00	15	0.267
229	A	5	5	1.00	15	0.333
230	A	9	7	1.00	15	0.467
231	A	6	6	1.00	15	0.400
232	A	3	3	1.00	13	0.231
233	A	3	3	1.00	13	0.231
234	A	6	6	1.00	15	0.400
235	A	9	7	1.00	15	0.467

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	6	3	1.00	13	0.231
237	A	6	3	1.00	13	0.231
238	A	6	2	1.00	15	0.133
239	A	5	2	1.00	15	0.133
240	A	4	2	1.00	13	0.154
241	A	3	3	1.00	13	0.231
242	A	4	4	1.00	15	0.267
243	A	5	5	1.00	15	0.333
244	A	8	2	1.00	17	0.118
245	A	6	2	1.00	17	0.118
246	A	10	2	1.00	17	0.118
247	A	9	7	1.00	15	0.467
248	A	6	6	1.00	15	0.400
249	A	3	3	1.00	13	0.231
250	A	3	3	1.00	13	0.231
251	A	6	6	1.00	15	0.400
252	A	9	7	1.00	15	0.467
253	A	6	3	1.00	13	0.231
254	A	6	3	1.00	13	0.231

Chapter 3

Listing of integrals

Local contents

3.1 $\int \sin(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{128\sin^9(a+bx)}{9b}-\frac{384\sin^{11}(a+bx)}{11b}+\frac{384\sin^{13}(a+bx)}{13b}-\frac{128\sin^{15}(a+bx)}{15b}$$

```
[Out] 128/9*sin(b*x+a)^9/b-384/11*sin(b*x+a)^11/b+384/13*sin(b*x+a)^13/b-128/15*s
in(b*x+a)^15/b
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4373, 2644, 276}

$$-\frac{128\sin^{15}(a+bx)}{15b}+\frac{384\sin^{13}(a+bx)}{13b}-\frac{384\sin^{11}(a+bx)}{11b}+\frac{128\sin^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*Sin[a + b*x]^9)/(9*b) - (384*Sin[a + b*x]^11)/(11*b) + (384*Sin[a + b*
x]^13)/(13*b) - (128*Sin[a + b*x]^15)/(15*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \sin(a+bx)\sin^7(2a+2bx)\,dx = 128\int \cos^7(a+bx)\sin^8(a+bx)\,dx$$
$$= \frac{128\text{Subst}\left(\int x^8(1-x^2)^3\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{128\text{Subst}\left(\int (x^8-3x^{10}+3x^{12}-x^{14})\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{128\sin^9(a+bx)}{9b} - \frac{384\sin^{11}(a+bx)}{11b} + \frac{384\sin^{13}(a+bx)}{13b} - \frac{128\sin^{15}(a+}{15b}$$

Mathematica [A]
time = 0.49, size = 47, normalized size = 0.77

$$\frac{4(8330+10755\cos(2(a+bx))+3366\cos(4(a+bx))+429\cos(6(a+bx)))\sin^9(a+bx)}{6435b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (4*(8330 + 10755*Cos[2*(a + b*x)] + 3366*Cos[4*(a + b*x)] + 429*Cos[6*(a + b*x)])*Sin[a + b*x]^9)/(6435*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(53) = 106$.
time = 0.18, size = 111, normalized size = 1.82

method	result
default	$\frac{35\sin(xb+a)}{128b} - \frac{35\sin(3xb+3a)}{384b} - \frac{21\sin(5xb+5a)}{640b} + \frac{3\sin(7xb+7a)}{128b} + \frac{7\sin(9xb+9a)}{1152b} - \frac{7\sin(11xb+11a)}{1408b} - \frac{\sin(13xb+13a)}{1664b}$
risch	$\frac{35\sin(xb+a)}{128b} - \frac{35\sin(3xb+3a)}{384b} - \frac{21\sin(5xb+5a)}{640b} + \frac{3\sin(7xb+7a)}{128b} + \frac{7\sin(9xb+9a)}{1152b} - \frac{7\sin(11xb+11a)}{1408b} - \frac{\sin(13xb+13a)}{1664b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 35/128*sin(b*x+a)/b-35/384*sin(3*b*x+3*a)/b-21/640/b*sin(5*b*x+5*a)+3/128/b
*sin(7*b*x+7*a)+7/1152/b*sin(9*b*x+9*a)-7/1408/b*sin(11*b*x+11*a)-1/1664/b*
sin(13*b*x+13*a)+1/1920/b*sin(15*b*x+15*a)
```

Maxima [A]
time = 0.30, size = 91, normalized size = 1.49

$$\frac{429\sin(15bx+15a)-495\sin(13bx+13a)-4095\sin(11bx+11a)+5005\sin(9bx+9a)+19305\sin(7bx+7a)-27027\sin(5bx+5a)-75075\sin(3bx+3a)+225225\sin(bx+a)}{823680b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

```
[Out] 1/823680*(429*sin(15*b*x + 15*a) - 495*sin(13*b*x + 13*a) - 4095*sin(11*b*x
 + 11*a) + 5005*sin(9*b*x + 9*a) + 19305*sin(7*b*x + 7*a) - 27027*sin(5*b*x
 + 5*a) - 75075*sin(3*b*x + 3*a) + 225225*sin(b*x + a))/b
```

Fricas [A]
time = 3.91, size = 83, normalized size = 1.36

$$\frac{128\,\left(429\,\cos\left(bx+a\right)^{14}-1518\,\cos\left(bx+a\right)^{12}+1854\,\cos\left(bx+a\right)^{10}-800\,\cos\left(bx+a\right)^{8}+5\,\cos\left(bx+a\right)^{6}+6\,\cos\left(bx+a\right)^{4}+8\,\cos\left(bx+a\right)^{2}+16\right)\sin\left(bx+a\right)}{6435\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

```
[Out] 128/6435*(429*cos(b*x + a)^14 - 1518*cos(b*x + a)^12 + 1854*cos(b*x + a)^10
 - 800*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*cos(b*x + a)^4 + 8*cos(b*x + a
)^2 + 16)*sin(b*x + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(53) = 106$.
time = 34.30, size = 269, normalized size = 4.41

$$\begin{cases} -\frac{3838\sin(a+bx)\sin^{6}(2a+2bx)\cos(2a+2bx)}{6435b}-\frac{1648\sin(a+bx)\sin^{4}(2a+2bx)\cos^{3}(2a+2bx)}{1287b}-\frac{768\sin(a+bx)\sin^{2}(2a+2bx)\cos^{5}(2a+2bx)}{715b}-\frac{2048\sin(a+bx)\cos^{7}(2a+2bx)}{6435b}+\frac{1241\sin^{7}(2a+2bx)\cos(a+bx)}{6435b}+\frac{376\sin^{5}(2a+2bx)\cos(a+bx)\cos^{2}(2a+2bx)}{715b}+\frac{640\sin^{3}(2a+2bx)\cos(a+bx)\cos^{4}(2a+2bx)}{1287b}+\frac{1024\sin(2a+2bx)\cos(a+bx)\cos^{6}(2a+2bx)}{6435b} & \text{for}\ b\neq 0\\ x\sin(a)\sin^{7}(2a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**7,x)
```

```
[Out] Piecewise((-3838*sin(a + b*x)*sin(2*a + 2*b*x)**6*cos(2*a + 2*b*x)/(6435*b)
 - 1648*sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(2*a + 2*b*x)**3/(1287*b) - 768
*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**5/(715*b) - 2048*sin(a
+ b*x)*cos(2*a + 2*b*x)**7/(6435*b) + 1241*sin(2*a + 2*b*x)**7*cos(a + b*x)
/(6435*b) + 376*sin(2*a + 2*b*x)**5*cos(a + b*x)*cos(2*a + 2*b*x)**2/(715*b
) + 640*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**4/(1287*b) + 102
4*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**6/(6435*b), Ne(b, 0)), (x
*sin(a)*sin(2*a)**7, True))
```

Giac [A]
time = 0.43, size = 46, normalized size = 0.75

$$-\frac{128\,\left(429\,\sin\left(bx+a\right)^{15}-1485\,\sin\left(bx+a\right)^{13}+1755\,\sin\left(bx+a\right)^{11}-715\,\sin\left(bx+a\right)^{9}\right)}{6435\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

```
[Out] -128/6435*(429*sin(b*x + a)^15 - 1485*sin(b*x + a)^13 + 1755*sin(b*x + a)^1
1 - 715*sin(b*x + a)^9)/b
```

Mupad [B]
time = 0.08, size = 45, normalized size = 0.74

$$\frac{-\frac{128\sin(a+b\,x)^{15}}{15}+\frac{384\sin(a+b\,x)^{13}}{13}-\frac{384\sin(a+b\,x)^{11}}{11}+\frac{128\sin(a+b\,x)^9}{9}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^7,x)
```

```
[Out] ((128*sin(a + b*x)^9)/9 - (384*sin(a + b*x)^11)/11 + (384*sin(a + b*x)^13)/
13 - (128*sin(a + b*x)^15)/15)/b
```

3.2 $\int \sin(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{64\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{3b}-\frac{192\cos^{11}(a+bx)}{11b}+\frac{64\cos^{13}(a+bx)}{13b}$$

```
[Out] -64/7*cos(b*x+a)^7/b+64/3*cos(b*x+a)^9/b-192/11*cos(b*x+a)^11/b+64/13*cos(b
*x+a)^13/b
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4373, 2645, 276}

$$\frac{64\cos^{13}(a+bx)}{13b}-\frac{192\cos^{11}(a+bx)}{11b}+\frac{64\cos^9(a+bx)}{3b}-\frac{64\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (-64*Cos[a + b*x]^7)/(7*b) + (64*Cos[a + b*x]^9)/(3*b) - (192*Cos[a + b*x]^
11)/(11*b) + (64*Cos[a + b*x]^13)/(13*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^6(2a+2bx)\,dx &= 64\int \cos^6(a+bx)\sin^7(a+bx)\,dx \\ &= -\frac{64\text{Subst}\left(\int x^6(1-x^2)^3\,dx, x, \cos(a+bx)\right)}{b} \\ &= -\frac{64\text{Subst}\left(\int (x^6-3x^8+3x^{10}-x^{12})\,dx, x, \cos(a+bx)\right)}{b} \\ &= -\frac{64\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{3b}-\frac{192\cos^{11}(a+bx)}{11b}+\frac{64\cos^{13}(a+\end{aligned}$$

Mathematica [A]
time = 0.35, size = 47, normalized size = 0.77

$$\frac{2\cos^7(a+bx)(-5230+6377\cos(2(a+bx))-1890\cos(4(a+bx))+231\cos(6(a+bx)))}{3003b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (2*Cos[a + b*x]^7*(-5230 + 6377*Cos[2*(a + b*x)] - 1890*Cos[4*(a + b*x)] +
231*Cos[6*(a + b*x)]))/(3003*b)
```

Maple [A]
time = 0.08, size = 97, normalized size = 1.59

method	result
default	$-\frac{5\cos(xb+a)}{16b}-\frac{5\cos(3xb+3a)}{64b}+\frac{3\cos(5xb+5a)}{64b}+\frac{3\cos(7xb+7a)}{224b}-\frac{\cos(9xb+9a)}{96b}-\frac{\cos(11xb+11a)}{704b}+\frac{\cos(13xb+13a)}{832b}$
risch	$-\frac{5\cos(xb+a)}{16b}-\frac{5\cos(3xb+3a)}{64b}+\frac{3\cos(5xb+5a)}{64b}+\frac{3\cos(7xb+7a)}{224b}-\frac{\cos(9xb+9a)}{96b}-\frac{\cos(11xb+11a)}{704b}+\frac{\cos(13xb+13a)}{832b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -5/16*cos(b*x+a)/b-5/64*cos(3*b*x+3*a)/b+3/64*cos(5*b*x+5*a)/b+3/224*cos(7*
b*x+7*a)/b-1/96*cos(9*b*x+9*a)/b-1/704*cos(11*b*x+11*a)/b+1/832*cos(13*b*x+
13*a)/b
```

Maxima [A]
time = 0.29, size = 80, normalized size = 1.31

$$\frac{231\cos(13bx+13a)-273\cos(11bx+11a)-2002\cos(9bx+9a)+2574\cos(7bx+7a)+9009\cos(5bx+5a)-15015\cos(3bx+3a)-60060\cos(bx+a)}{192192b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] 1/192192*(231*cos(13*b*x + 13*a) - 273*cos(11*b*x + 11*a) - 2002*cos(9*b*x
+ 9*a) + 2574*cos(7*b*x + 7*a) + 9009*cos(5*b*x + 5*a) - 15015*cos(3*b*x +
3*a) - 60060*cos(b*x + a))/b
```

Fricas [A]
time = 4.14, size = 46, normalized size = 0.75

$$\frac{64\left(231\cos\left(bx+a\right)^{13}-819\cos\left(bx+a\right)^{11}+1001\cos\left(bx+a\right)^{9}-429\cos\left(bx+a\right)^{7}\right)}{3003\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] 64/3003*(231*cos(b*x + a)^13 - 819*cos(b*x + a)^11 + 1001*cos(b*x + a)^9 -
429*cos(b*x + a)^7)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(53) = 106.
time = 15.18, size = 235, normalized size = 3.85

$$\begin{cases} -\frac{1084\sin\left(a+bx\right)\sin^{5}\left(2a+2bx\right)\cos\left(2a+2bx\right)}{3003b}-\frac{64\sin\left(a+bx\right)\sin^{3}\left(2a+2bx\right)\cos^{3}\left(2a+2bx\right)}{143b}-\frac{512\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos^{5}\left(2a+2bx\right)}{3003b}-\frac{835\sin^{6}\left(2a+2bx\right)\cos\left(a+bx\right)}{3003b}-\frac{2776\sin^{4}\left(2a+2bx\right)\cos\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{3003b}-\frac{2944\sin^{2}\left(2a+2bx\right)\cos\left(a+bx\right)\cos^{4}\left(2a+2bx\right)}{3003b}-\frac{1024\cos\left(a+bx\right)\cos^{6}\left(2a+2bx\right)}{3003b} & \text{for}\ b\neq 0 \\ x\sin\left(a\right)\sin^{6}\left(2a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**6,x)
```

```
[Out] Piecewise((-1084*sin(a + b*x)*sin(2*a + 2*b*x)**5*cos(2*a + 2*b*x)/(3003*b)
 - 64*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**3/(143*b) - 512*si
n(a + b*x)*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**5/(3003*b) - 835*sin(2*a + 2*
b*x)**6*cos(a + b*x)/(3003*b) - 2776*sin(2*a + 2*b*x)**4*cos(a + b*x)*cos(2
*a + 2*b*x)**2/(3003*b) - 2944*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2
*b*x)**4/(3003*b) - 1024*cos(a + b*x)*cos(2*a + 2*b*x)**6/(3003*b), Ne(b, 0
)), (x*sin(a)*sin(2*a)**6, True))
```

Giac [A]
time = 0.39, size = 80, normalized size = 1.31

$$\frac{231\cos\left(13\,bx+13\,a\right)-273\cos\left(11\,bx+11\,a\right)-2002\cos\left(9\,bx+9\,a\right)+2574\cos\left(7\,bx+7\,a\right)+9009\cos\left(5\,bx+5\,a\right)-15015\cos\left(3\,bx+3\,a\right)-60060\cos\left(bx+a\right)}{192192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] 1/192192*(231*cos(13*b*x + 13*a) - 273*cos(11*b*x + 11*a) - 2002*cos(9*b*x
+ 9*a) + 2574*cos(7*b*x + 7*a) + 9009*cos(5*b*x + 5*a) - 15015*cos(3*b*x +
3*a) - 60060*cos(b*x + a))/b
```

Mupad [B]
time = 0.13, size = 46, normalized size = 0.75

$$-\frac{-\frac{64\cos(a+b\,x)^{13}}{13}+\frac{192\cos(a+b\,x)^{11}}{11}-\frac{64\cos(a+b\,x)^{9}}{3}+\frac{64\cos(a+b\,x)^{7}}{7}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^6,x)
```

```
[Out] -((64*cos(a + b*x)^7)/7 - (64*cos(a + b*x)^9)/3 + (192*cos(a + b*x)^11)/11
- (64*cos(a + b*x)^13)/13)/b
```

3.3 $\int \sin(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{32\sin^7(a+bx)}{7b}-\frac{64\sin^9(a+bx)}{9b}+\frac{32\sin^{11}(a+bx)}{11b}$$

```
[Out] 32/7*sin(b*x+a)^7/b-64/9*sin(b*x+a)^9/b+32/11*sin(b*x+a)^11/b
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4373, 2644, 276}

$$\frac{32\sin^{11}(a+bx)}{11b}-\frac{64\sin^9(a+bx)}{9b}+\frac{32\sin^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (32*Sin[a + b*x]^7)/(7*b) - (64*Sin[a + b*x]^9)/(9*b) + (32*Sin[a + b*x]^11
)/(11*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \sin(a+bx)\sin^5(2a+2bx)\,dx = 32\int \cos^5(a+bx)\sin^6(a+bx)\,dx$$
$$= \frac{32\text{Subst}\left(\int x^6(1-x^2)^2\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{32\text{Subst}\left(\int (x^6-2x^8+x^{10})\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{32\sin^7(a+bx)}{7b} - \frac{64\sin^9(a+bx)}{9b} + \frac{32\sin^{11}(a+bx)}{11b}$$

Mathematica [A]
time = 0.28, size = 37, normalized size = 0.80

$$\frac{4(365+364\cos(2(a+bx))+63\cos(4(a+bx)))\sin^7(a+bx)}{693b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*(365 + 364*Cos[2*(a + b*x)] + 63*Cos[4*(a + b*x)])*Sin[a + b*x]^7)/(693*
b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(40) = 80$.
time = 0.09, size = 83, normalized size = 1.80

method	result	size
default	$\frac{5\sin(xb+a)}{16b} - \frac{5\sin(3xb+3a)}{48b} - \frac{\sin(5xb+5a)}{32b} + \frac{5\sin(7xb+7a)}{224b} + \frac{\sin(9xb+9a)}{288b} - \frac{\sin(11xb+11a)}{352b}$	83
risch	$\frac{5\sin(xb+a)}{16b} - \frac{5\sin(3xb+3a)}{48b} - \frac{\sin(5xb+5a)}{32b} + \frac{5\sin(7xb+7a)}{224b} + \frac{\sin(9xb+9a)}{288b} - \frac{\sin(11xb+11a)}{352b}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 5/16*sin(b*x+a)/b-5/48*sin(3*b*x+3*a)/b-1/32/b*sin(5*b*x+5*a)+5/224/b*sin(7
*b*x+7*a)+1/288/b*sin(9*b*x+9*a)-1/352/b*sin(11*b*x+11*a)
```

Maxima [A]
time = 0.31, size = 69, normalized size = 1.50

$$-\frac{63\,\sin(11\,bx+11\,a) - 77\,\sin(9\,bx+9\,a) - 495\,\sin(7\,bx+7\,a) + 693\,\sin(5\,bx+5\,a) + 2310\,\sin(3\,bx+3\,a) - 6930\,\sin(bx+a)}{22176\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/22176*(63*sin(11*b*x + 11*a) - 77*sin(9*b*x + 9*a) - 495*sin(7*b*x + 7*a
) + 693*sin(5*b*x + 5*a) + 2310*sin(3*b*x + 3*a) - 6930*sin(b*x + a))/b
```

Fricas [A]
time = 4.11, size = 63, normalized size = 1.37

$$-\frac{32\left(63\,\cos\left(bx+a\right)^{10}-161\,\cos\left(bx+a\right)^{8}+113\,\cos\left(bx+a\right)^{6}-3\,\cos\left(bx+a\right)^{4}-4\,\cos\left(bx+a\right)^{2}-8\right)\sin\left(bx+a\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -32/693*(63*cos(b*x + a)^10 - 161*cos(b*x + a)^8 + 113*cos(b*x + a)^6 - 3*c
os(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b*x + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(39) = 78$.
time = 6.57, size = 197, normalized size = 4.28

$$\begin{cases} -\frac{422\sin\left(a+bx\right)\sin^{4}\left(2a+2bx\right)\cos\left(2a+2bx\right)}{693b}-\frac{608\sin\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos^{3}\left(2a+2bx\right)}{693b}-\frac{256\sin\left(a+bx\right)\cos^{5}\left(2a+2bx\right)}{693b}+\frac{151\sin^{5}\left(2a+2bx\right)\cos\left(a+bx\right)}{693b}+\frac{272\sin^{3}\left(2a+2bx\right)\cos\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{693b}+\frac{128\sin\left(2a+2bx\right)\cos\left(a+bx\right)\cos^{4}\left(2a+2bx\right)}{693b} & \text{for}\: b\neq 0 \\ x\sin\left(a\right)\sin^{5}\left(2a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**5,x)
```

```
[Out] Piecewise((-422*sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(2*a + 2*b*x)/(693*b) -
 608*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**3/(693*b) - 256*sin
(a + b*x)*cos(2*a + 2*b*x)**5/(693*b) + 151*sin(2*a + 2*b*x)**5*cos(a + b*x
)/(693*b) + 272*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**2/(693*b
) + 128*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(693*b), Ne(b, 0)
), (x*sin(a)*sin(2*a)**5, True))
```

Giac [A]
time = 0.43, size = 36, normalized size = 0.78

$$\frac{32\left(63\,\sin\left(bx+a\right)^{11}-154\,\sin\left(bx+a\right)^{9}+99\,\sin\left(bx+a\right)^{7}\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 32/693*(63*sin(b*x + a)^11 - 154*sin(b*x + a)^9 + 99*sin(b*x + a)^7)/b
```

Mupad [B]
time = 0.09, size = 36, normalized size = 0.78

$$\frac{32\left(63\sin\left(a+b\,x\right)^{11}-154\sin\left(a+b\,x\right)^{9}+99\sin\left(a+b\,x\right)^{7}\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^5,x)
[Out] (32*(99*sin(a + b*x)^7 - 154*sin(a + b*x)^9 + 63*sin(a + b*x)^11))/(693*b)
```

3.4 $\int \sin(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{16\cos^5(a+bx)}{5b}+\frac{32\cos^7(a+bx)}{7b}-\frac{16\cos^9(a+bx)}{9b}$$

```
[Out] -16/5*cos(b*x+a)^5/b+32/7*cos(b*x+a)^7/b-16/9*cos(b*x+a)^9/b
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {4373, 2645, 276}

$$-\frac{16\cos^9(a+bx)}{9b}+\frac{32\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-16*Cos[a + b*x]^5)/(5*b) + (32*Cos[a + b*x]^7)/(7*b) - (16*Cos[a + b*x]^9
)/(9*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \sin(a+bx)\sin^4(2a+2bx)\,dx &= 16\int \cos^4(a+bx)\sin^5(a+bx)\,dx \\
&= -\frac{16\text{Subst}\left(\int x^4(1-x^2)^2\,dx, x, \cos(a+bx)\right)}{b} \\
&= -\frac{16\text{Subst}\left(\int (x^4-2x^6+x^8)\,dx, x, \cos(a+bx)\right)}{b} \\
&= -\frac{16\cos^5(a+bx)}{5b}+\frac{32\cos^7(a+bx)}{7b}-\frac{16\cos^9(a+bx)}{9b}
\end{aligned}
$$

Mathematica [A]
time = 0.16, size = 37, normalized size = 0.80

$$\frac{2\cos^5(a+bx)(-249+220\cos(2(a+bx))-35\cos(4(a+bx)))}{315b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (2*Cos[a + b*x]^5*(-249 + 220*Cos[2*(a + b*x)] - 35*Cos[4*(a + b*x)]))/(315
*b)
```

Maple [A]
time = 0.07, size = 69, normalized size = 1.50

method	result	size
default	$-\frac{3\cos(xb+a)}{8b}-\frac{\cos(3xb+3a)}{12b}+\frac{\cos(5xb+5a)}{20b}+\frac{\cos(7xb+7a)}{112b}-\frac{\cos(9xb+9a)}{144b}$	69
risch	$-\frac{3\cos(xb+a)}{8b}-\frac{\cos(3xb+3a)}{12b}+\frac{\cos(5xb+5a)}{20b}+\frac{\cos(7xb+7a)}{112b}-\frac{\cos(9xb+9a)}{144b}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*cos(b*x+a)/b-1/12*cos(3*b*x+3*a)/b+1/20*cos(5*b*x+5*a)/b+1/112*cos(7*b
*x+7*a)/b-1/144*cos(9*b*x+9*a)/b
```

Maxima [A]
time = 0.30, size = 58, normalized size = 1.26

$$-\frac{35\cos(9\,bx+9\,a)-45\cos(7\,bx+7\,a)-252\cos(5\,bx+5\,a)+420\cos(3\,bx+3\,a)+1890\cos(bx+a)}{5040\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] -1/5040*(35*cos(9*b*x + 9*a) - 45*cos(7*b*x + 7*a) - 252*cos(5*b*x + 5*a) +
 420*cos(3*b*x + 3*a) + 1890*cos(b*x + a))/b
```

Fricas [A]
time = 4.19, size = 36, normalized size = 0.78

$$-\frac{16\,\left(35\,\cos\left(bx+a\right)^{9}-90\,\cos\left(bx+a\right)^{7}+63\,\cos\left(bx+a\right)^{5}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -16/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(39) = 78.
time = 2.78, size = 163, normalized size = 3.54

$$\begin{cases} -\frac{104\sin\left(a+bx\right)\sin^{3}\left(2a+2bx\right)\cos\left(2a+2bx\right)}{315b}-\frac{64\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos^{3}\left(2a+2bx\right)}{315b}-\frac{107\sin^{4}\left(2a+2bx\right)\cos\left(a+bx\right)}{315b}-\frac{16\sin^{2}\left(2a+2bx\right)\cos\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{21b}-\frac{128\cos\left(a+bx\right)\cos^{4}\left(2a+2bx\right)}{315b} & \text{for}\ b\neq 0 \\ x\sin\left(a\right)\sin^{4}\left(2a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**4,x)
```

```
[Out] Piecewise((-104*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)/(315*b) -
 64*sin(a + b*x)*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(315*b) - 107*sin(2*a
 + 2*b*x)**4*cos(a + b*x)/(315*b) - 16*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos
(2*a + 2*b*x)**2/(21*b) - 128*cos(a + b*x)*cos(2*a + 2*b*x)**4/(315*b), Ne(
b, 0)), (x*sin(a)*sin(2*a)**4, True))
```

Giac [A]
time = 0.41, size = 58, normalized size = 1.26

$$-\frac{35\,\cos\left(9\,bx+9\,a\right)-45\,\cos\left(7\,bx+7\,a\right)-252\,\cos\left(5\,bx+5\,a\right)+420\,\cos\left(3\,bx+3\,a\right)+1890\,\cos\left(bx+a\right)}{5040\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/5040*(35*cos(9*b*x + 9*a) - 45*cos(7*b*x + 7*a) - 252*cos(5*b*x + 5*a) +
 420*cos(3*b*x + 3*a) + 1890*cos(b*x + a))/b
```

Mupad [B]
time = 0.14, size = 36, normalized size = 0.78

$$-\frac{16\,\left(35\cos\left(a+b\,x\right)^{9}-90\cos\left(a+b\,x\right)^{7}+63\cos\left(a+b\,x\right)^{5}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^4,x)
```

```
[Out] -(16*(63*cos(a + b*x)^5 - 90*cos(a + b*x)^7 + 35*cos(a + b*x)^9))/(315*b)
```

3.5 $\int \sin(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{8\sin^5(a+bx)}{5b}-\frac{8\sin^7(a+bx)}{7b}$$

```
[Out] 8/5*sin(b*x+a)^5/b-8/7*sin(b*x+a)^7/b
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4373, 2644, 14}

$$\frac{8\sin^5(a+bx)}{5b}-\frac{8\sin^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (8*Sin[a + b*x]^5)/(5*b) - (8*Sin[a + b*x]^7)/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\sin^4(a+bx)\,dx\\ &= \frac{8\text{Subst}\left(\int x^4(1-x^2)\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{8\text{Subst}\left(\int (x^4-x^6)\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{8\sin^5(a+bx)}{5b}-\frac{8\sin^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 27, normalized size = 0.87

$$\frac{4(9+5\cos(2(a+bx)))\sin^5(a+bx)}{35b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (4*(9 + 5*Cos[2*(a + b*x)])*Sin[a + b*x]^5)/(35*b)
```

Maple [A]
time = 0.07, size = 55, normalized size = 1.77

method	result	size
default	$\frac{3\sin(xb+a)}{8b}-\frac{\sin(3xb+3a)}{8b}-\frac{\sin(5xb+5a)}{40b}+\frac{\sin(7xb+7a)}{56b}$	55
risch	$\frac{3\sin(xb+a)}{8b}-\frac{\sin(3xb+3a)}{8b}-\frac{\sin(5xb+5a)}{40b}+\frac{\sin(7xb+7a)}{56b}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*sin(b*x+a)/b-1/8*sin(3*b*x+3*a)/b-1/40/b*sin(5*b*x+5*a)+1/56/b*sin(7*b*
x+7*a)
```

Maxima [A]
time = 0.30, size = 47, normalized size = 1.52

$$\frac{5\sin(7\,bx+7\,a)-7\sin(5\,bx+5\,a)-35\sin(3\,bx+3\,a)+105\sin(bx+a)}{280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/280*(5*sin(7*b*x + 7*a) - 7*sin(5*b*x + 5*a) - 35*sin(3*b*x + 3*a) + 105*
sin(b*x + a))/b
```

Fricas [A]
time = 4.31, size = 41, normalized size = 1.32

$$\frac{8\left(5\cos\left(bx+a\right)^{6}-8\cos\left(bx+a\right)^{4}+\cos\left(bx+a\right)^{2}+2\right)\sin\left(bx+a\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 8/35*(5*cos(b*x + a)^6 - 8*cos(b*x + a)^4 + cos(b*x + a)^2 + 2)*sin(b*x + a
)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(26) = 52$.
time = 1.10, size = 126, normalized size = 4.06

$$\begin{cases} -\frac{22\sin\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos\left(2a+2bx\right)}{35b}-\frac{16\sin\left(a+bx\right)\cos^{3}\left(2a+2bx\right)}{35b}+\frac{9\sin^{3}\left(2a+2bx\right)\cos\left(a+bx\right)}{35b}+\frac{8\sin\left(2a+2bx\right)\cos\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{35b} & \text{for}\ b\neq 0 \\ x\sin\left(a\right)\sin^{3}\left(2a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((-22*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)/(35*b) - 1
6*sin(a + b*x)*cos(2*a + 2*b*x)**3/(35*b) + 9*sin(2*a + 2*b*x)**3*cos(a + b
*x)/(35*b) + 8*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(35*b), Ne
(b, 0)), (x*sin(a)*sin(2*a)**3, True))
```

Giac [A]
time = 0.51, size = 26, normalized size = 0.84

$$-\frac{8\left(5\sin\left(bx+a\right)^{7}-7\sin\left(bx+a\right)^{5}\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -8/35*(5*sin(b*x + a)^7 - 7*sin(b*x + a)^5)/b
```

Mupad [B]
time = 0.05, size = 26, normalized size = 0.84

$$\frac{8\left(7\sin\left(a+b\,x\right)^{5}-5\sin\left(a+b\,x\right)^{7}\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^3,x)
```

```
[Out] (8*(7*sin(a + b*x)^5 - 5*sin(a + b*x)^7))/(35*b)
```

3.6 $\int \sin(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=31

$$-\frac{4\cos^3(a+bx)}{3b}+\frac{4\cos^5(a+bx)}{5b}$$

```
[Out] -4/3*cos(b*x+a)^3/b+4/5*cos(b*x+a)^5/b
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4373, 2645, 14}

$$\frac{4\cos^5(a+bx)}{5b}-\frac{4\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*Cos[a + b*x]^3)/(3*b) + (4*Cos[a + b*x]^5)/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \sin(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^2(a+bx)\sin^3(a+bx)\,dx \\
&= -\frac{4\text{Subst}\left(\int x^2(1-x^2)\,dx, x, \cos(a+bx)\right)}{b} \\
&= -\frac{4\text{Subst}\left(\int (x^2-x^4)\,dx, x, \cos(a+bx)\right)}{b} \\
&= -\frac{4\cos^3(a+bx)}{3b}+\frac{4\cos^5(a+bx)}{5b}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 27, normalized size = 0.87

$$\frac{2\cos^3(a+bx)(-7+3\cos(2(a+bx)))}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (2*Cos[a + b*x]^3*(-7 + 3*Cos[2*(a + b*x)]))/(15*b)
```

Maple [A]
time = 0.07, size = 41, normalized size = 1.32

method	result	si
default	$-\frac{\cos(xb+a)}{2b}-\frac{\cos(3xb+3a)}{12b}+\frac{\cos(5xb+5a)}{20b}$	41
risch	$-\frac{\cos(xb+a)}{2b}-\frac{\cos(3xb+3a)}{12b}+\frac{\cos(5xb+5a)}{20b}$	41
norman	$\frac{-\frac{16}{15b}-\frac{16\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)}{15b}+\frac{16\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan^3(xb+a)\right)}{15b}-\frac{16\left(\tan^4(xb+a)\right)}{15b}-\frac{28\left(\tan^2(xb+a)\right)}{15b}-\frac{4\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)\right)\left(\tan^2(xb+a)\right)}{15b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)\right)\left(1+\tan^2(xb+a)\right)^2}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cos(b*x+a)/b-1/12*cos(3*b*x+3*a)/b+1/20*cos(5*b*x+5*a)/b
```

Maxima [A]
time = 0.30, size = 36, normalized size = 1.16

$$\frac{3\,\cos\left(5\,bx+5\,a\right)-5\,\cos\left(3\,bx+3\,a\right)-30\,\cos\left(bx+a\right)}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 1/60*(3*cos(5*b*x + 5*a) - 5*cos(3*b*x + 3*a) - 30*cos(b*x + a))/b
```

Fricas [A]
time = 2.93, size = 26, normalized size = 0.84

$$\frac{4\left(3\,\cos\left(bx+a\right)^{5}-5\,\cos\left(bx+a\right)^{3}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 4/15*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(26) = 52$.
time = 0.43, size = 92, normalized size = 2.97

$$\begin{cases} -\frac{4\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos\left(2a+2bx\right)}{15b}-\frac{7\sin^{2}\left(2a+2bx\right)\cos\left(a+bx\right)}{15b}-\frac{8\cos\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{15b} & \text{for}\ b\neq 0\\ x\sin\left(a\right)\sin^{2}\left(2a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((-4*sin(a + b*x)*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)/(15*b) - 7*sin
(2*a + 2*b*x)**2*cos(a + b*x)/(15*b) - 8*cos(a + b*x)*cos(2*a + 2*b*x)**2/(
15*b), Ne(b, 0)), (x*sin(a)*sin(2*a)**2, True))
```

Giac [A]
time = 0.44, size = 36, normalized size = 1.16

$$\frac{3\,\cos\left(5\,bx+5\,a\right)-5\,\cos\left(3\,bx+3\,a\right)-30\,\cos\left(bx+a\right)}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 1/60*(3*cos(5*b*x + 5*a) - 5*cos(3*b*x + 3*a) - 30*cos(b*x + a))/b
```

Mupad [B]
time = 0.11, size = 26, normalized size = 0.84

$$-\frac{4\,\left(5\cos\left(a+b\,x\right)^{3}-3\cos\left(a+b\,x\right)^{5}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^2,x)
```

```
[Out] -(4*(5*cos(a + b*x)^3 - 3*cos(a + b*x)^5))/(15*b)
```

3.7 $\int \sin(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=30

$$\frac{\sin(a+bx)}{2b}-\frac{\sin(3a+3bx)}{6b}$$

```
[Out] 1/2*sin(b*x+a)/b-1/6*sin(3*b*x+3*a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4367}

$$\frac{\sin(a+bx)}{2b}-\frac{\sin(3a+3bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x],x]
[Out] Sin[a + b*x]/(2*b) - Sin[3*a + 3*b*x]/(6*b)
```

Rule 4367

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(a+bx)\sin(2a+2bx)\,dx=\frac{\sin(a+bx)}{2b}-\frac{\sin(3a+3bx)}{6b}$$

Mathematica [A]
time = 0.04, size = 15, normalized size = 0.50

$$\frac{2\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x],x]
[Out] (2*Sin[a + b*x]^3)/(3*b)
```

Maple [A]
time = 0.05, size = 27, normalized size = 0.90

method	result	size
default	$\frac{\sin(xb+a)}{2b}-\frac{\sin(3xb+3a)}{6b}$	27
risch	$\frac{\sin(xb+a)}{2b}-\frac{\sin(3xb+3a)}{6b}$	27
norman	$\frac{-\frac{4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{3b}+\frac{2\tan(xb+a)}{3b}+\frac{4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan^2(xb+a)\right)}{3b}-\frac{2\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)\right)\tan(xb+a)}{3b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)\right)(1+\tan^2(xb+a))}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(b*x+a)/b-1/6*sin(3*b*x+3*a)/b
```

Maxima [A]
time = 0.30, size = 26, normalized size = 0.87

$$-\frac{\sin\left(3\,bx+3\,a\right)}{6\,b}+\frac{\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/6*sin(3*b*x + 3*a)/b + 1/2*sin(b*x + a)/b
```

Fricas [A]
time = 3.56, size = 21, normalized size = 0.70

$$-\frac{2\left(\cos\left(bx+a\right)^2-1\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(b*x + a)^2 - 1)*sin(b*x + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.
time = 0.20, size = 51, normalized size = 1.70

$$\begin{cases} -\frac{2\sin\left(a+bx\right)\cos\left(2a+2bx\right)}{3b}+\frac{\sin\left(2a+2bx\right)\cos\left(a+bx\right)}{3b} & \text{for}\ b\neq 0 \\ x\sin\left(a\right)\sin\left(2a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-2*sin(a + b*x)*cos(2*a + 2*b*x)/(3*b) + sin(2*a + 2*b*x)*cos(a
+ b*x)/(3*b), Ne(b, 0)), (x*sin(a)*sin(2*a), True))
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.43

$$\frac{2\sin\left(bx+a\right)^{3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 2/3*sin(b*x + a)^3/b
```

Mupad [B]
time = 0.36, size = 44, normalized size = 1.47

$$\begin{cases} 2\,x\,\left(\cos\left(a\right)-\cos\left(a\right)^3\right) & \text{if } b=0\\ \frac{3\,\sin(a+b\,x)-\sin(3\,a+3\,b\,x)}{6\,b} & \text{if } b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x),x)
```

```
[Out] piecewise(b == 0, 2*x*(cos(a) - cos(a)^3), b ~= 0, (3*sin(a + b*x) - sin(3*
a + 3*b*x))/(6*b))
```

3.8 $\int \csc(2a+2bx)\sin(a+bx)\,dx$

Optimal. Leaf size=14

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

```
[Out] 1/2*arctanh(sin(b*x+a))/b
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4373, 3855}

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]*Sin[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(2a+2bx)\sin(a+bx)\,dx &= \frac{1}{2}\int \sec(a+bx)\,dx\\ &= \frac{\tanh^{-1}(\sin(a+bx))}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b)
```

Maple [A]
time = 0.06, size = 20, normalized size = 1.43

method	result	size
default	$\frac{\ln(\sec(xb+a)+\tan(xb+a))}{2b}$	20
risch	$-\frac{\ln(e^{i(xb+a)}-i)}{2b}+\frac{\ln(i+e^{i(xb+a)})}{2b}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(12) = 24.
time = 0.52, size = 115, normalized size = 8.21

$$-\frac{\log\left(\frac{\cos(bx+2a)^2+\cos(a)^2-2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2+2\cos(bx+2a)\sin(a)+\sin(a)^2}{\cos(bx+2a)^2+\cos(a)^2+2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2-2\cos(bx+2a)\sin(a)+\sin(a)^2}\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/4*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2
+ 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + si
n(a)^2))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.
time = 3.44, size = 28, normalized size = 2.00

$$\frac{\log(\sin(bx+a)+1)-\log(-\sin(bx+a)+1)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.
time = 0.40, size = 28, normalized size = 2.00

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

Mupad [**B**]
time = 0.15, size = 12, normalized size = 0.86

$$\frac{\mathrm{atanh}(\sin\left(a+b\,x\right))}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x),x)
```

```
[Out] atanh(sin(a + b*x))/(2*b)
```

3.9 $\int \csc^2(2a+2bx)\sin(a+bx)\,dx$

Optimal. Leaf size=28

$$-\frac{\tanh^{-1}(\cos(a+bx))}{4b}+\frac{\sec(a+bx)}{4b}$$

```
[Out] -1/4*arctanh(cos(b*x+a))/b+1/4*sec(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {4373, 2702, 327, 213}

$$\frac{\sec(a+bx)}{4b}-\frac{\tanh^{-1}(\cos(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/4*ArcTanh[Cos[a + b*x]]/b + Sec[a + b*x]/(4*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
```

```
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(2a+2bx)\sin(a+bx)\,dx &= \frac{1}{4}\int \csc(a+bx)\sec^2(a+bx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{4b}\\ &= \frac{\sec(a+bx)}{4b} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{4b}\\ &= -\frac{\tanh^{-1}(\cos(a+bx))}{4b} + \frac{\sec(a+bx)}{4b}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 1.79

$$-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{4b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{4b} + \frac{\sec(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x],x]
[Out] -1/4*Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/(4*b) + Sec[a + b*x]/(
4*b)
```

Maple [A]
time = 0.11, size = 31, normalized size = 1.11

method	result	size
default	$\frac{\frac{1}{\cos(xb+a)}+\ln(\csc(xb+a)-\cot(xb+a))}{4b}$	31
risch	$\frac{e^{i(xb+a)}}{2b\left(e^{2i(xb+a)}+1\right)}-\frac{\ln\left(e^{i(xb+a)}+1\right)}{4b}+\frac{\ln\left(e^{i(xb+a)}-1\right)}{4b}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/4/b*(1/cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(24) = 48.
time = 0.28, size = 236, normalized size = 8.43

$$\frac{4\cos(2bx+2a)\cos(bx+a)-\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)+1\right)\log\left(\cos(bx)^2+2\cos(bx)\cos(a)+\cos(a)^2+\sin(bx)^2-2\sin(bx)\sin(a)+\sin(a)^2\right)+\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)+1\right)\log\left(\cos(bx)^2-2\cos(bx)\cos(a)+\cos(a)^2+\sin(bx)^2+2\sin(bx)\sin(a)+\sin(a)^2\right)+4\sin(2bx+2a)\sin(bx+a)+4\cos(bx+a)}{8\left(b\cos(2bx+2a)^2+b\sin(2bx+2a)^2+2b\cos(2bx+2a)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(4*cos(2*b*x + 2*a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*
a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^
2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + (cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a)
 + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(2*b*x + 2*
a)*sin(b*x + a) + 4*cos(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a
)^2 + 2*b*cos(2*b*x + 2*a) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(24) = 48$.
time = 3.62, size = 52, normalized size = 1.86

$$-\frac{\cos\left(bx+a\right)\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-\cos\left(bx+a\right)\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-2}{8\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(cos(b*x + a)*log(1/2*cos(b*x + a) + 1/2) - cos(b*x + a)*log(-1/2*cos(
b*x + a) + 1/2) - 2)/(b*cos(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(24) = 48$.
time = 0.44, size = 52, normalized size = 1.86

$$\frac{\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1}+\log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + log(-(cos(b*x + a) - 1
)/(cos(b*x + a) + 1)))/b
```

Mupad [B]
time = 0.11, size = 26, normalized size = 0.93

$$\frac{1}{4\,b\,\cos\left(a+b\,x\right)}-\frac{\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^2,x)
[Out] 1/(4*b*cos(a + b*x)) - atanh(cos(a + b*x))/(4*b)
```

3.10 $\int \csc^3(2a+2bx)\sin(a+bx)\,dx$

Optimal. Leaf size=49

$$\frac{3\tanh^{-1}(\sin(a+bx))}{16b}-\frac{3\csc(a+bx)}{16b}+\frac{\csc(a+bx)\sec^2(a+bx)}{16b}$$

```
[Out] 3/16*arctanh(sin(b*x+a))/b-3/16*csc(b*x+a)/b+1/16*csc(b*x+a)*sec(b*x+a)^2/b
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.278$, Rules used = {4373, 2701, 294, 327, 213}

$$-\frac{3\csc(a+bx)}{16b}+\frac{3\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc(a+bx)\sec^2(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x],x]
```

```
[Out] (3*ArcTanh[Sin[a + b*x]])/(16*b) - (3*Csc[a + b*x])/(16*b) + (Csc[a + b*x]*
Sec[a + b*x]^2)/(16*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc^3(2a+2bx)\sin(a+bx)\,dx &= \frac{1}{8}\int \csc^2(a+bx)\sec^3(a+bx)\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{8b} \\
&= \frac{\csc(a+bx)\sec^2(a+bx)}{16b} - \frac{3\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b} \\
&= -\frac{3\csc(a+bx)}{16b} + \frac{\csc(a+bx)\sec^2(a+bx)}{16b} - \frac{3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a\right.}{16b} \\
&= \frac{3\tanh^{-1}(\sin(a+bx))}{16b} - \frac{3\csc(a+bx)}{16b} + \frac{\csc(a+bx)\sec^2(a+bx)}{16b}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 29, normalized size = 0.59

$$-\frac{\csc(a+bx)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\sin^2(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x],x]
[Out] -1/8*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b
```

Maple [A]
time = 0.11, size = 51, normalized size = 1.04

method	result	size

default	$\frac{\frac{1}{2\sin(xb+a)\cos(xb+a)^2}-\frac{3}{2\sin(xb+a)}+\frac{3\ln(\sec(xb+a)+\tan(xb+a))}{2}}{8b}$	51
risch	$-\frac{i\left(3\,\mathrm{e}^{5i(xb+a)}+2\,\mathrm{e}^{3i(xb+a)}+3\,\mathrm{e}^{i(xb+a)}\right)}{8b\left(\mathrm{e}^{2i(xb+a)}+1\right)^2\left(\mathrm{e}^{2i(xb+a)}-1\right)}+\frac{3\ln\left(i+\mathrm{e}^{i(xb+a)}\right)}{16b}-\frac{3\ln\left(\mathrm{e}^{i(xb+a)}-i\right)}{16b}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(1/2/sin(b*x+a)/cos(b*x+a)^2-3/2/sin(b*x+a)+3/2*ln(sec(b*x+a)+tan(b*x
+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(43) = 86$.
time = 0.52, size = 808, normalized size = 16.49

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/32*(4*(3*sin(5*b*x + 5*a) + 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*b*
x + 6*a) - 12*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 4*(2
*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(4*b*x + 4*a) - 3*(2*(cos(4*b*x + 4*
a) - cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + cos(6*b*x + 6*a)^2 - 2*(cos(2
*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + cos(2*b*x + 2*a)^2
 + 2*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + sin(6*b*x + 6
*a)^2 + sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + sin(2*b*
x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*c
os(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^
2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^
2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(3*cos(5*b*x + 5*a) + 2*cos(3*
b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a) + 12*(cos(4*b*x + 4*a) - cos(
2*b*x + 2*a) - 1)*sin(5*b*x + 5*a) - 4*(2*cos(3*b*x + 3*a) + 3*cos(b*x + a)
)*sin(4*b*x + 4*a) - 8*(cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a) + 8*cos(3*b*
x + 3*a)*sin(2*b*x + 2*a) + 12*cos(b*x + a)*sin(2*b*x + 2*a) - 12*cos(2*b*x
 + 2*a)*sin(b*x + a) - 12*sin(b*x + a))/(b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x
 + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(6*b*x + 6*a)^2 + b*sin(4*b*x + 4*a
)^2 - 2*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a)^2 + 2*(b*c
os(4*b*x + 4*a) - b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) - 2*(b*cos(2*b*x
 + 2*a) + b)*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) + 2*(b*sin(4*b*x + 4*a
) - b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [A]
time = 3.56, size = 85, normalized size = 1.73

$$\frac{3\cos(bx+a)^2\log(\sin(bx+a)+1)\sin(bx+a)-3\cos(bx+a)^2\log(-\sin(bx+a)+1)\sin(bx+a)-6\cos(bx+a)^2+2}{32\,b\cos(bx+a)^2\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(3*cos(b*x + a)^2*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*cos(b*x + a)^
2*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 2)/(b*cos(b*x +
a)^2*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.48, size = 63, normalized size = 1.29

$$-\frac{\frac{2\left(3\sin(bx+a)^2-2\right)}{\sin(bx+a)^3-\sin(bx+a)} - 3\log\left(\sin\left(bx+a\right)+1\right) + 3\log\left(-\sin\left(bx+a\right)+1\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/32*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin
(b*x + a) + 1) + 3*log(-sin(b*x + a) + 1))/b
```

Mupad [**B**]
time = 0.12, size = 48, normalized size = 0.98

$$\frac{3\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{16\,b}+\frac{\frac{3\sin(a+b\,x)^2}{16}-\frac{1}{8}}{b\,\left(\sin\left(a+b\,x\right)-\sin\left(a+b\,x\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^3,x)
```

```
[Out] (3*atanh(sin(a + b*x)))/(16*b) + ((3*sin(a + b*x)^2)/16 - 1/8)/(b*(sin(a +
b*x) - sin(a + b*x)^3))
```

3.11 $\int \csc^4(2a+2bx)\sin(a+bx)\,dx$

Optimal. Leaf size=66

$$-\frac{5\tanh^{-1}(\cos(a+bx))}{32b}+\frac{5\sec(a+bx)}{32b}+\frac{5\sec^3(a+bx)}{96b}-\frac{\csc^2(a+bx)\sec^3(a+bx)}{32b}$$

```
[Out] -5/32*arctanh(cos(b*x+a))/b+5/32*sec(b*x+a)/b+5/96*sec(b*x+a)^3/b-1/32*csc(
b*x+a)^2*sec(b*x+a)^3/b
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {4373, 2702, 294, 308, 213}

$$\frac{5\sec^3(a+bx)}{96b}+\frac{5\sec(a+bx)}{32b}-\frac{5\tanh^{-1}(\cos(a+bx))}{32b}-\frac{\csc^2(a+bx)\sec^3(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x],x]
```

```
[Out] (-5*ArcTanh[Cos[a + b*x]])/(32*b) + (5*Sec[a + b*x])/(32*b) + (5*Sec[a + b*
x]^3)/(96*b) - (Csc[a + b*x]^2*Sec[a + b*x]^3)/(32*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
```

```
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc^4(2a+2bx)\sin(a+bx)\,dx &= \frac{1}{16}\int \csc^3(a+bx)\sec^4(a+bx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{16b} \\
&= -\frac{\csc^2(a+bx)\sec^3(a+bx)}{32b} + \frac{5\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{32b} \\
&= -\frac{\csc^2(a+bx)\sec^3(a+bx)}{32b} + \frac{5\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \sec(a+b
\end{aligned}$$

$$\begin{aligned}
&= \frac{5\sec(a+bx)}{32b} + \frac{5\sec^3(a+bx)}{96b} - \frac{\csc^2(a+bx)\sec^3(a+bx)}{32b} + \frac{5\text{Subst}\left(\int \frac{}{-}\right.}{} \\
&= -\frac{5\tanh^{-1}(\cos(a+bx))}{32b} + \frac{5\sec(a+bx)}{32b} + \frac{5\sec^3(a+bx)}{96b} - \frac{\csc^2(a+bx)}{32}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 205 vs. 2(66) = 132.
time = 0.48, size = 205, normalized size = 3.11

$$\frac{\csc^8(a+bx)\left(22-40\cos(2(a+bx))+13\cos(3(a+bx))-30\cos(4(a+bx))+13\cos(5(a+bx))+15\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)+15\cos(5(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-15\cos(3(a+bx))\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)-15\cos(5(a+bx))\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)+\cos(a+bx)\left(-26-30\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)+30\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)\right)\right)}{24b\left(\csc^2\left(\frac{1}{2}(a+bx)\right)-\sec^2\left(\frac{1}{2}(a+bx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x],x]
```

```
[Out] (Csc[a + b*x]^8*(22 - 40*Cos[2*(a + b*x)] + 13*Cos[3*(a + b*x)] - 30*Cos[4*
(a + b*x)] + 13*Cos[5*(a + b*x)] + 15*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]
] + 15*Cos[5*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 15*Cos[3*(a + b*x)]*Log[Sin
[(a + b*x)/2]] - 15*Cos[5*(a + b*x)]*Log[Sin[(a + b*x)/2]] + Cos[a + b*x]*(
-26 - 30*Log[Cos[(a + b*x)/2]] + 30*Log[Sin[(a + b*x)/2]])))/(24*b*(Csc[(a
+ b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Maple [A]
time = 0.13, size = 71, normalized size = 1.08

method	result	size
default	$\frac{\frac{1}{3\sin(xb+a)^2\cos(xb+a)^3}-\frac{5}{6\sin(xb+a)^2\cos(xb+a)}+\frac{5}{2\cos(xb+a)}+\frac{5\ln(\csc(xb+a)-\cot(xb+a))}{2}}{16b}$	71
risch	$\frac{15\,e^{9i(xb+a)}+20\,e^{7i(xb+a)}-22\,e^{5i(xb+a)}+20\,e^{3i(xb+a)}+15\,e^{i(xb+a)}}{48b\left(e^{2i(xb+a)}+1\right)^3\left(e^{2i(xb+a)}-1\right)^2}+\frac{5\ln\left(e^{i(xb+a)}-1\right)}{32b}-\frac{5\ln\left(e^{i(xb+a)}+1\right)}{32b}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/b*(1/3/sin(b*x+a)^2/cos(b*x+a)^3-5/6/sin(b*x+a)^2/cos(b*x+a)+5/2/cos(b
*x+a)+5/2*ln(csc(b*x+a)-cot(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2174 vs. $2(58) = 116$.
time = 0.35, size = 2174, normalized size = 32.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/192*(4*(15*cos(9*b*x + 9*a) + 20*cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a) +
 20*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) + 60*(cos(8*b*x
+ 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)*co
s(9*b*x + 9*a) + 4*(20*cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a) + 20*cos(3*b*
x + 3*a) + 15*cos(b*x + a))*cos(8*b*x + 8*a) - 80*(2*cos(6*b*x + 6*a) + 2*c
os(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(7*b*x + 7*a) + 8*(22*cos(5*b*x
+ 5*a) - 20*cos(3*b*x + 3*a) - 15*cos(b*x + a))*cos(6*b*x + 6*a) + 88*(2*co
s(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) - 40*(4*cos(3*b*x +
 3*a) + 3*cos(b*x + a))*cos(4*b*x + 4*a) + 80*(cos(2*b*x + 2*a) + 1)*cos(3*
b*x + 3*a) + 60*cos(2*b*x + 2*a)*cos(b*x + a) - 15*(2*(cos(8*b*x + 8*a) - 2
*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)*cos(10*b*x +
 10*a) + cos(10*b*x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) + 2*cos(4*b*x + 4*a)
- cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*x + 8*a)^2 + 4*(2*cos(4*
b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + 4*cos(6*b*x + 6*a)^2
- 4*(cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*cos(4*b*x + 4*a)^2 + cos(2*
b*x + 2*a)^2 + 2*(sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a
) + sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + sin(10*b*x + 10*a)^2 - 2*(2*sin(
6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + si
n(8*b*x + 8*a)^2 + 4*(2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6*
a) + 4*sin(6*b*x + 6*a)^2 + 4*sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 +
```

```
2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2)
+ 15*(2*(cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + cos(2
*b*x + 2*a) + 1)*cos(10*b*x + 10*a) + cos(10*b*x + 10*a)^2 - 2*(2*cos(6*b*x
 + 6*a) + 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos
(8*b*x + 8*a)^2 + 4*(2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(6*b*x +
 6*a) + 4*cos(6*b*x + 6*a)^2 - 4*(cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) +
4*cos(4*b*x + 4*a)^2 + cos(2*b*x + 2*a)^2 + 2*(sin(8*b*x + 8*a) - 2*sin(6*b
*x + 6*a) - 2*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + sin
(10*b*x + 10*a)^2 - 2*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - sin(2*b*x
+ 2*a))*sin(8*b*x + 8*a) + sin(8*b*x + 8*a)^2 + 4*(2*sin(4*b*x + 4*a) - sin
(2*b*x + 2*a))*sin(6*b*x + 6*a) + 4*sin(6*b*x + 6*a)^2 + 4*sin(4*b*x + 4*a)
^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
 + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(a) + sin(a)^2) + 4*(15*sin(9*b*x + 9*a) + 20*sin(7*b*x + 7*a)
 - 22*sin(5*b*x + 5*a) + 20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(10*b*x
+ 10*a) + 60*(sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) +
sin(2*b*x + 2*a))*sin(9*b*x + 9*a) + 4*(20*sin(7*b*x + 7*a) - 22*sin(5*b*x
+ 5*a) + 20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(8*b*x + 8*a) - 80*(2*si
n(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(7*b*x + 7*a) +
8*(22*sin(5*b*x + 5*a) - 20*sin(3*b*x + 3*a) - 15*sin(b*x + a))*sin(6*b*x +
 6*a) + 88*(2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(5*b*x + 5*a) - 40*(4
*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(4*b*x + 4*a) + 80*sin(3*b*x + 3*a)*
sin(2*b*x + 2*a) + 60*sin(2*b*x + 2*a)*sin(b*x + a) + 60*cos(b*x + a))/(b*c
os(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*c
os(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(10*b*x + 10*a)^2 + b*sin(8
*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x + 4*a)^2 - 4*b*sin(4
*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a)^2 + 2*(b*cos(8*b*x + 8*a)
 - 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) + b)*co
s(10*b*x + 10*a) - 2*(2*b*cos(6*b*x + 6*a) + 2*b*cos(4*b*x + 4*a) - b*cos(2
*b*x + 2*a) - b)*cos(8*b*x + 8*a) + 4*(2*b*cos(4*b*x + 4*a) - b*cos(2*b*x +
 2*a) - b)*cos(6*b*x + 6*a) - 4*(b*cos(2*b*x + 2*a) + b)*cos(4*b*x + 4*a) +
 2*b*cos(2*b*x + 2*a) + 2*(b*sin(8*b*x + 8*a) - 2*b*sin(6*b*x + 6*a) - 2*b*
sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 2*(2*b*sin(6*b*
x + 6*a) + 2*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + 4*
(2*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [**A**]
time = 3.08, size = 112, normalized size = 1.70

$$\frac{30\cos\left(bx+a\right)^{4}-20\cos\left(bx+a\right)^{2}-15\left(\cos\left(bx+a\right)^{5}-\cos\left(bx+a\right)^{3}\right)\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)+15\left(\cos\left(bx+a\right)^{5}-\cos\left(bx+a\right)^{3}\right)\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-4}{192\left(b\cos\left(bx+a\right)^{5}-b\cos\left(bx+a\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/192*(30*cos(b*x + a)^4 - 20*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - cos(b*x
```

```
 + a)^3)*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 - cos(b*x + a)^3)
*log(-1/2*cos(b*x + a) + 1/2) - 4)/(b*cos(b*x + a)^5 - b*cos(b*x + a)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**4*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(58) = 116.
time = 0.47, size = 160, normalized size = 2.42

$$-\frac{\frac{3\left(\frac{10(\cos(bx+a)-1)}{\cos(bx+a)+1}-1\right)(\cos(bx+a)+1)}{\cos(bx+a)-1}+\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{16\left(\frac{12(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{9(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+7\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}-30\log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/384*(3*(10*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)*(cos(b*x + a) + 1)
/(cos(b*x + a) - 1) + 3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 16*(12*(cos
(b*x + a) - 1)/(cos(b*x + a) + 1) + 9*(cos(b*x + a) - 1)^2/(cos(b*x + a) +
1)^2 + 7)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3 - 30*log(-(cos(b*x
+ a) - 1)/(cos(b*x + a) + 1)))/b
```

Mupad [**B**]
time = 0.10, size = 60, normalized size = 0.91

$$\frac{-\frac{5\cos(a+b\,x)^4}{32}+\frac{5\cos(a+b\,x)^2}{48}+\frac{1}{48}}{b\,\left(\cos\left(a+b\,x\right)^3-\cos\left(a+b\,x\right)^5\right)}-\frac{5\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^4,x)
```

```
[Out] ((5*cos(a + b*x)^2)/48 - (5*cos(a + b*x)^4)/32 + 1/48)/(b*(cos(a + b*x)^3 -
 cos(a + b*x)^5)) - (5*atanh(cos(a + b*x)))/(32*b)
```

3.12 $\int \csc^5(2a+2bx)\sin(a+bx)\,dx$

Optimal. Leaf size=89

$$\frac{35\tanh^{-1}(\sin(a+bx))}{256b}-\frac{35\csc(a+bx)}{256b}-\frac{35\csc^3(a+bx)}{768b}+\frac{7\csc^3(a+bx)\sec^2(a+bx)}{256b}+\frac{\csc^3(a+bx)\sec^4(a+bx)}{128b}$$

```
[Out] 35/256*arctanh(sin(b*x+a))/b-35/256*csc(b*x+a)/b-35/768*csc(b*x+a)^3/b+7/25
6*csc(b*x+a)^3*sec(b*x+a)^2/b+1/128*csc(b*x+a)^3*sec(b*x+a)^4/b
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {4373, 2701, 294, 308, 213}

$$-\frac{35\csc^3(a+bx)}{768b}-\frac{35\csc(a+bx)}{256b}+\frac{35\tanh^{-1}(\sin(a+bx))}{256b}+\frac{\csc^3(a+bx)\sec^4(a+bx)}{128b}+\frac{7\csc^3(a+bx)\sec^2(a+bx)}{256b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x],x]
```

```
[Out] (35*ArcTanh[Sin[a + b*x]])/(256*b) - (35*Csc[a + b*x])/(256*b) - (35*Csc[a
+ b*x]^3)/(768*b) + (7*Csc[a + b*x]^3*Sec[a + b*x]^2)/(256*b) + (Csc[a + b*
x]^3*Sec[a + b*x]^4)/(128*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^5(2a+2bx)\sin(a+bx)\,dx = \frac{1}{32}\int \csc^4(a+bx)\sec^5(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, \csc(a+bx)\right)}{32b}$$
$$= \frac{\csc^3(a+bx)\sec^4(a+bx)}{128b} - \frac{7\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{128b}$$
$$= \frac{7\csc^3(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{128b} - \frac{35\text{Subst}\left(\int \frac{}{-}\right.}{}$$
$$= \frac{7\csc^3(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc^3(a+bx)\sec^4(a+bx)}{128b} - \frac{35\text{Subst}\left(\int (1\right.}{}$$
$$= -\frac{35\csc(a+bx)}{256b} - \frac{35\csc^3(a+bx)}{768b} + \frac{7\csc^3(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc^3}{}$$
$$= \frac{35\tanh^{-1}(\sin(a+bx))}{256b} - \frac{35\csc(a+bx)}{256b} - \frac{35\csc^3(a+bx)}{768b} + \frac{7\csc^3(a+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 31, normalized size = 0.35

$$-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\sin^2(a+bx)\right)}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x],x]
[Out] -1/96*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 3, -1/2, Sin[a + b*x]^2])/b
```

Maple [A]
time = 0.12, size = 87, normalized size = 0.98

method	result
default	$\frac{\frac{1}{4\sin(xb+a)^3\cos(xb+a)^4}-\frac{7}{12\sin(xb+a)^3\cos(xb+a)^2}+\frac{35}{24\sin(xb+a)\cos(xb+a)^2}-\frac{35}{8\sin(xb+a)}+\frac{35\ln(\sec(xb+a)+\tan(xb+a))}{8}}{32b}$
risch	$-\frac{i\left(105\,\mathrm{e}^{13i(xb+a)}+70\,\mathrm{e}^{11i(xb+a)}-329\,\mathrm{e}^{9i(xb+a)}-204\,\mathrm{e}^{7i(xb+a)}-329\,\mathrm{e}^{5i(xb+a)}+70\,\mathrm{e}^{3i(xb+a)}+105\,\mathrm{e}^{i(xb+a)}\right)}{384b\left(\mathrm{e}^{2i(xb+a)}+1\right)^4\left(\mathrm{e}^{2i(xb+a)}-1\right)^3}-\frac{35\ln\left(\mathrm{e}^{i(xb+a)}-i\right)}{256b}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^5*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/b*(1/4/sin(b*x+a)^3/cos(b*x+a)^4-7/12/sin(b*x+a)^3/cos(b*x+a)^2+35/24/
sin(b*x+a)/cos(b*x+a)^2-35/8/sin(b*x+a)+35/8*ln(sec(b*x+a)+tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3088 vs. 2(79) = 158.
time = 0.64, size = 3088, normalized size = 34.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/1536*(4*(105*sin(13*b*x + 13*a) + 70*sin(11*b*x + 11*a) - 329*sin(9*b*x +
 9*a) - 204*sin(7*b*x + 7*a) - 329*sin(5*b*x + 5*a) + 70*sin(3*b*x + 3*a) +
 105*sin(b*x + a))*cos(14*b*x + 14*a) - 420*(sin(12*b*x + 12*a) - 3*sin(10*
b*x + 10*a) - 3*sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a)
- sin(2*b*x + 2*a))*cos(13*b*x + 13*a) + 4*(70*sin(11*b*x + 11*a) - 329*sin
(9*b*x + 9*a) - 204*sin(7*b*x + 7*a) - 329*sin(5*b*x + 5*a) + 70*sin(3*b*x
+ 3*a) + 105*sin(b*x + a))*cos(12*b*x + 12*a) + 280*(3*sin(10*b*x + 10*a) +
 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + sin(2*b*x +
 2*a))*cos(11*b*x + 11*a) + 12*(329*sin(9*b*x + 9*a) + 204*sin(7*b*x + 7*a)
 + 329*sin(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) - 105*sin(b*x + a))*cos(10*b*
x + 10*a) - 1316*(3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4
*a) + sin(2*b*x + 2*a))*cos(9*b*x + 9*a) + 12*(204*sin(7*b*x + 7*a) + 329*s
in(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) - 105*sin(b*x + a))*cos(8*b*x + 8*a)
+ 816*(3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(7*b*
x + 7*a) - 84*(47*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) - 15*sin(b*x + a))
*cos(6*b*x + 6*a) + 1316*(3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x
+ 5*a) + 420*(2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(4*b*x + 4*a) - 105*(
2*(cos(12*b*x + 12*a) - 3*cos(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) + 3*cos(6
*b*x + 6*a) + 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) - 1)*cos(14*b*x + 14*a)
 + cos(14*b*x + 14*a)^2 - 2*(3*cos(10*b*x + 10*a) + 3*cos(8*b*x + 8*a) - 3*
cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)*cos(12*b*x +
```

12*a) + cos(12*b*x + 12*a)^2 + 6*(3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) -
 3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) + 9*cos(10*b
*x + 10*a)^2 - 6*(3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a
) - 1)*cos(8*b*x + 8*a) + 9*cos(8*b*x + 8*a)^2 + 6*(3*cos(4*b*x + 4*a) - co
s(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + 9*cos(6*b*x + 6*a)^2 - 6*(cos(2*b*x
 + 2*a) + 1)*cos(4*b*x + 4*a) + 9*cos(4*b*x + 4*a)^2 + cos(2*b*x + 2*a)^2 +
2*(sin(12*b*x + 12*a) - 3*sin(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) + 3*sin(6
*b*x + 6*a) + 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(14*b*x + 14*a) + s
in(14*b*x + 14*a)^2 - 2*(3*sin(10*b*x + 10*a) + 3*sin(8*b*x + 8*a) - 3*sin(
6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(12*b*x + 12*a) +
sin(12*b*x + 12*a)^2 + 6*(3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) - 3*sin(4
*b*x + 4*a) + sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 9*sin(10*b*x + 10*a)^2
 - 6*(3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x
 + 8*a) + 9*sin(8*b*x + 8*a)^2 + 6*(3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*
sin(6*b*x + 6*a) + 9*sin(6*b*x + 6*a)^2 + 9*sin(4*b*x + 4*a)^2 - 6*sin(4*b*
x + 4*a)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*lo
g((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2
 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos
(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)
) - 4*(105*cos(13*b*x + 13*a) + 70*cos(11*b*x + 11*a) - 329*cos(9*b*x + 9*a
) - 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) + 70*cos(3*b*x + 3*a) + 105
*cos(b*x + a))*sin(14*b*x + 14*a) + 420*(cos(12*b*x + 12*a) - 3*cos(10*b*x
 + 10*a) - 3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) - co
s(2*b*x + 2*a) - 1)*sin(13*b*x + 13*a) - 4*(70*cos(11*b*x + 11*a) - 329*cos
(9*b*x + 9*a) - 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) + 70*cos(3*b*x
 + 3*a) + 105*cos(b*x + a))*sin(12*b*x + 12*a) - 280*(3*cos(10*b*x + 10*a) +
 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) + cos(2*b*x +
 2*a) + 1)*sin(11*b*x + 11*a) - 12*(329*cos(9*b*x + 9*a) + 204*cos(7*b*x +
7*a) + 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) - 105*cos(b*x + a))*sin(1
0*b*x + 10*a) + 1316*(3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) - 3*cos(4*b*x
 + 4*a) + cos(2*b*x + 2*a) + 1)*sin(9*b*x + 9*a) - 12*(204*cos(7*b*x + 7*a)
 + 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) - 105*cos(b*x + a))*sin(8*b*x
 + 8*a) - 816*(3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) -
 1)*sin(7*b*x + 7*a) + 84*(47*cos(5*b*x + 5*a) - 10*cos(3*b*x + 3*a) - 15*c
os(b*x + a))*sin(6*b*x + 6*a) - 1316*(3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a)
 - 1)*sin(5*b*x + 5*a) - 420*(2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(4*b*
x + 4*a) - 280*(cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a) + 280*cos(3*b*x + 3*
a)*sin(2*b*x + 2*a) + 420*cos(b*x + a)*sin(2*b*x + 2*a) - 420*cos(2*b*x + 2
*a)*sin(b*x + a) - 420*sin(b*x + a))/(b*cos(14*b*x + 14*a)^2 + b*cos(12*b*x
 + 12*a)^2 + 9*b*cos(10*b*x + 10*a)^2 + 9*b*cos(8*b*x + 8*a)^2 + 9*b*cos(6*
b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(14*b*x
 + 14*a)^2 + b*sin(12*b*x + 12*a)^2 + 9*b*sin(10*b*x + 10*a)^2 + 9*b*sin(8*
b*x + 8*a)^2 + 9*b*sin(6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 - 6*b*sin(4*
b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a)^2 + 2*(b*cos(12*b*x + 12*a
) - 3*b*cos(10*b*x + 10*a) - 3*b*cos(8*b*x + 8*a) + 3*b*cos(6*b*x + 6*a) +

```
3*b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) - b)*...
```

Fricas [A]
time = 2.81, size = 140, normalized size = 1.57

$$-\frac{210 \cos(bx+a)^6 - 280 \cos(bx+a)^4 - 105 \left(\cos(bx+a)^6 - \cos(bx+a)^4\right) \log(\sin(bx+a)+1) \sin(bx+a) + 105 \left(\cos(bx+a)^6 - \cos(bx+a)^4\right) \log(-\sin(bx+a)+1) \sin(bx+a) + 42 \cos(bx+a)^2 + 12}{1536 \left(b \cos(bx+a)^6 - b \cos(bx+a)^4\right) \sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/1536*(210*cos(b*x + a)^6 - 280*cos(b*x + a)^4 - 105*(cos(b*x + a)^6 - co
s(b*x + a)^4)*log(sin(b*x + a) + 1)*sin(b*x + a) + 105*(cos(b*x + a)^6 - co
s(b*x + a)^4)*log(-sin(b*x + a) + 1)*sin(b*x + a) + 42*cos(b*x + a)^2 + 12)
/((b*cos(b*x + a)^6 - b*cos(b*x + a)^4)*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**5*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 85, normalized size = 0.96

$$-\frac{\frac{6 \left(11 \sin(bx+a)^3 - 13 \sin(bx+a)\right)}{\left(\sin(bx+a)^2 - 1\right)^2} + \frac{16 \left(9 \sin(bx+a)^2 + 1\right)}{\sin(bx+a)^3} - 105 \log(\sin(bx+a)+1) + 105 \log(-\sin(bx+a)+1)}{1536 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/1536*(6*(11*sin(b*x + a)^3 - 13*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 + 1
6*(9*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 105*log(sin(b*x + a) + 1) + 105*l
og(-sin(b*x + a) + 1))/b
```

Mupad [B]
time = 0.17, size = 79, normalized size = 0.89

$$\frac{35 \operatorname{atanh}(\sin(a+b\,x))}{256\,b} - \frac{\frac{35 \sin(a+b\,x)^6}{256} - \frac{175 \sin(a+b\,x)^4}{768} + \frac{7 \sin(a+b\,x)^2}{96} + \frac{1}{96}}{b \left(\sin(a+b\,x)^7 - 2 \sin(a+b\,x)^5 + \sin(a+b\,x)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^5,x)
```

```
[Out] (35*atanh(sin(a + b*x)))/(256*b) - ((7*sin(a + b*x)^2)/96 - (175*sin(a + b*
x)^4)/768 + (35*sin(a + b*x)^6)/256 + 1/96)/(b*(sin(a + b*x)^3 - 2*sin(a +
b*x)^5 + sin(a + b*x)^7))
```

3.13 $\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=44

$$\frac{4\sin^8(a+bx)}{b}-\frac{32\sin^{10}(a+bx)}{5b}+\frac{8\sin^{12}(a+bx)}{3b}$$

```
[Out] 4*sin(b*x+a)^8/b-32/5*sin(b*x+a)^10/b+8/3*sin(b*x+a)^12/b
```

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {4373, 2644, 272, 45}

$$\frac{8\sin^{12}(a+bx)}{3b}-\frac{32\sin^{10}(a+bx)}{5b}+\frac{4\sin^8(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*Sin[a + b*x]^8)/b - (32*Sin[a + b*x]^10)/(5*b) + (8*Sin[a + b*x]^12)/(3*
b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
```

```
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^5(a+bx)\sin^7(a+bx)\,dx\\ &= \frac{32\text{Subst}\left(\int x^7(1-x^2)^2\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{16\text{Subst}\left(\int (1-x)^2x^3\,dx, x, \sin^2(a+bx)\right)}{b}\\ &= \frac{16\text{Subst}\left(\int \left(x^3-2x^4+x^5\right)\,dx, x, \sin^2(a+bx)\right)}{b}\\ &= \frac{4\sin^8(a+bx)}{b}-\frac{32\sin^{10}(a+bx)}{5b}+\frac{8\sin^{12}(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.37, size = 68, normalized size = 1.55

$$\frac{-600\cos(2(a+bx))+75\cos(4(a+bx))+100\cos(6(a+bx))-30\cos(8(a+bx))-12\cos(10(a+bx))+5\cos(12(a+bx))}{3840b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-600*Cos[2*(a + b*x)] + 75*Cos[4*(a + b*x)] + 100*Cos[6*(a + b*x)] - 30*Co
s[8*(a + b*x)] - 12*Cos[10*(a + b*x)] + 5*Cos[12*(a + b*x)])/(3840*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(40) = 80.
time = 0.09, size = 86, normalized size = 1.95

method	result	size
default	$-\frac{5\cos(2xb+2a)}{32b}+\frac{5\cos(4xb+4a)}{256b}+\frac{5\cos(6xb+6a)}{192b}-\frac{\cos(8xb+8a)}{128b}-\frac{\cos(10xb+10a)}{320b}+\frac{\cos(12xb+12a)}{768b}$	86
risch	$-\frac{5\cos(2xb+2a)}{32b}+\frac{5\cos(4xb+4a)}{256b}+\frac{5\cos(6xb+6a)}{192b}-\frac{\cos(8xb+8a)}{128b}-\frac{\cos(10xb+10a)}{320b}+\frac{\cos(12xb+12a)}{768b}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -5/32*cos(2*b*x+2*a)/b+5/256*cos(4*b*x+4*a)/b+5/192*cos(6*b*x+6*a)/b-1/128*
cos(8*b*x+8*a)/b-1/320*cos(10*b*x+10*a)/b+1/768*cos(12*b*x+12*a)/b
```

Maxima [A]
time = 0.27, size = 72, normalized size = 1.64

$$\frac{5\cos(12bx+12a)-12\cos(10bx+10a)-30\cos(8bx+8a)+100\cos(6bx+6a)+75\cos(4bx+4a)-600\cos(2bx+2a)}{3840b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/3840*(5*cos(12*b*x + 12*a) - 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a)
+ 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) - 600*cos(2*b*x + 2*a))/b
```

Fricas [A]
time = 4.38, size = 46, normalized size = 1.05

$$\frac{4\left(10\cos(bx+a)^{12}-36\cos(bx+a)^{10}+45\cos(bx+a)^{8}-20\cos(bx+a)^{6}\right)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 4/15*(10*cos(b*x + a)^12 - 36*cos(b*x + a)^10 + 45*cos(b*x + a)^8 - 20*cos(
b*x + a)^6)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. 2(37) = 74.
time = 15.57, size = 593, normalized size = 13.48

$$\begin{cases} \frac{5x\sin^2(a+bx)\sin^5(2a+2bx)}{32}+\frac{5x\sin^2(a+bx)\sin^3(2a+2bx)\cos^2(2a+2bx)}{16}+\frac{5x\sin^2(a+bx)\sin(2a+2bx)\cos^4(2a+2bx)}{32}+\frac{5x\sin(a+bx)\sin^4(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{16}+\frac{5x\sin(a+bx)\sin^2(2a+2bx)\cos(a+bx)\cos^3(2a+2bx)}{8}+\frac{5x\sin(a+bx)\cos(a+bx)\cos^5(2a+2bx)}{16}-\frac{5x\sin^5(2a+2bx)\cos^2(a+bx)}{32}-\frac{5x\sin^3(2a+2bx)\cos^2(a+bx)\cos^2(2a+2bx)}{16}-\frac{5x\sin(2a+2bx)\cos^2(a+bx)\cos^4(2a+2bx)}{32}-\frac{65\sin^2(a+bx)\sin^4(2a+2bx)\cos(2a+2bx)}{128b}-\frac{2\sin^2(a+bx)\sin^2(2a+2bx)\cos^3(2a+2bx)}{3b}-\frac{167\sin^2(a+bx)\cos^5(2a+2bx)}{640b}+\frac{11\sin(a+bx)\sin^5(2a+2bx)\cos(a+bx)}{64b}+\frac{\sin(a+bx)\sin^3(2a+2bx)\cos(a+bx)\cos^2(2a+2bx)}{4b}+\frac{19\sin(a+bx)\sin(2a+2bx)\cos(a+bx)\cos^4(2a+2bx)}{192b}+\frac{\sin^4(2a+2bx)\cos^2(a+bx)\cos(2a+2bx)}{128b}-\frac{11\cos^2(a+bx)\cos^5(2a+2bx)}{1920b} & \text{for } b \neq 0 \\ x\sin^2(a)\sin^5(2a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**5,x)
```

```
[Out] Piecewise((5*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**5/32 + 5*x*sin(a + b*x)**2
*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**2/16 + 5*x*sin(a + b*x)**2*sin(2*a +
 2*b*x)*cos(2*a + 2*b*x)**4/32 + 5*x*sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(a
 + b*x)*cos(2*a + 2*b*x)/16 + 5*x*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a +
b*x)*cos(2*a + 2*b*x)**3/8 + 5*x*sin(a + b*x)*cos(a + b*x)*cos(2*a + 2*b*x)
**5/16 - 5*x*sin(2*a + 2*b*x)**5*cos(a + b*x)**2/32 - 5*x*sin(2*a + 2*b*x)*
*3*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 - 5*x*sin(2*a + 2*b*x)*cos(a + b*
x)**2*cos(2*a + 2*b*x)**4/32 - 65*sin(a + b*x)**2*sin(2*a + 2*b*x)**4*cos(2
*a + 2*b*x)/(128*b) - 2*sin(a + b*x)**2*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x
)**3/(3*b) - 167*sin(a + b*x)**2*cos(2*a + 2*b*x)**5/(640*b) + 11*sin(a + b
*x)*sin(2*a + 2*b*x)**5*cos(a + b*x)/(64*b) + sin(a + b*x)*sin(2*a + 2*b*x)
**3*cos(a + b*x)*cos(2*a + 2*b*x)**2/(4*b) + 19*sin(a + b*x)*sin(2*a + 2*b*
x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(192*b) + sin(2*a + 2*b*x)**4*cos(a + b
```

```
*x)**2*cos(2*a + 2*b*x)/(128*b) - 11*cos(a + b*x)**2*cos(2*a + 2*b*x)**5/(1
920*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**5, True))
```

Giac [A]
time = 0.43, size = 36, normalized size = 0.82

$$\frac{4 \left(10 \sin \left(bx+a\right)^{12}-24 \sin \left(bx+a\right)^{10}+15 \sin \left(bx+a\right)^{8}\right)}{15 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 4/15*(10*sin(b*x + a)^12 - 24*sin(b*x + a)^10 + 15*sin(b*x + a)^8)/b
```

Mupad [B]
time = 0.13, size = 46, normalized size = 1.05

$$-\frac{-\frac{8\cos(a+b\,x)^{12}}{3}+\frac{48\cos(a+b\,x)^{10}}{5}-12\cos\left(a+b\,x\right)^8+\frac{16\cos(a+b\,x)^6}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^5,x)
```

```
[Out] -((16*cos(a + b*x)^6)/3 - 12*cos(a + b*x)^8 + (48*cos(a + b*x)^10)/5 - (8*c
os(a + b*x)^12)/3)/b
```

3.14 $\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=76

$$\frac{3x}{16}-\frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b}-\frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b}-\frac{\sin^5(2a+2bx)}{20b}$$

```
[Out] 3/16*x-3/32*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b-1/16*cos(2*b*x+2*a)*sin(2*b*x+2
*a)^3/b-1/20*sin(2*b*x+2*a)^5/b
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {4371, 2715, 8, 2644, 30}

$$-\frac{\sin^5(2a+2bx)}{20b}-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{16b}-\frac{3\sin(2a+2bx)\cos(2a+2bx)}{32b}+\frac{3x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (3*x)/16 - (3*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(32*b) - (Cos[2*a + 2*b*x]
*Sin[2*a + 2*b*x]^3)/(16*b) - Sin[2*a + 2*b*x]^5/(20*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4371

```
Int[sin[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Dist[1/2, Int[(g*Sin[c + d*x])^p, x], x] - Dist[1/2, Int[Cos[c + d*x]
*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d,
0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Rubi steps

$$\int \sin^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{1}{2}\int \sin^4(2a+2bx)\,dx - \frac{1}{2}\int \cos(2a+2bx)\sin^4(2a+2bx)\,dx$$
$$= -\frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} + \frac{3}{8}\int \sin^2(2a+2bx)\,dx - \frac{\text{Subst}\left(\int x^4\,dx\right.}{}$$
$$= -\frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b} - \frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} - \frac{\sin^5(2}{}$$
$$= \frac{3x}{16} - \frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b} - \frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} - \frac{\text{si}}{}$$

Mathematica [A]
time = 0.22, size = 62, normalized size = 0.82

$$\frac{120bx - 20\sin(2(a+bx)) - 40\sin(4(a+bx)) + 10\sin(6(a+bx)) + 5\sin(8(a+bx)) - 2\sin(10(a+bx))}{640b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
[Out] (120*b*x - 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] + 10*Sin[6*(a + b*x)]
+ 5*Sin[8*(a + b*x)] - 2*Sin[10*(a + b*x)])/(640*b)
```

Maple [A]
time = 0.12, size = 75, normalized size = 0.99

method	result	size
default	$\frac{3x}{16}-\frac{\sin(2xb+2a)}{32b}-\frac{\sin(4xb+4a)}{16b}+\frac{\sin(6xb+6a)}{64b}+\frac{\sin(8xb+8a)}{128b}-\frac{\sin(10xb+10a)}{320b}$	75
risch	$\frac{3x}{16}-\frac{\sin(2xb+2a)}{32b}-\frac{\sin(4xb+4a)}{16b}+\frac{\sin(6xb+6a)}{64b}+\frac{\sin(8xb+8a)}{128b}-\frac{\sin(10xb+10a)}{320b}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
[Out] 3/16*x-1/32*sin(2*b*x+2*a)/b-1/16/b*sin(4*b*x+4*a)+1/64/b*sin(6*b*x+6*a)+1/
128/b*sin(8*b*x+8*a)-1/320/b*sin(10*b*x+10*a)
```

Maxima [A]
time = 0.28, size = 65, normalized size = 0.86

$$\frac{120\,bx - 2\,\sin\left(10\,bx + 10\,a\right) + 5\,\sin\left(8\,bx + 8\,a\right) + 10\,\sin\left(6\,bx + 6\,a\right) - 40\,\sin\left(4\,bx + 4\,a\right) - 20\,\sin\left(2\,bx + 2\,a\right)}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/640*(120*b*x - 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) + 10*sin(6*b*x +
 6*a) - 40*sin(4*b*x + 4*a) - 20*sin(2*b*x + 2*a))/b
```

Fricas [A]
time = 2.53, size = 67, normalized size = 0.88

$$\frac{15\,bx - \left(128\,\cos\left(bx + a\right)^{9} - 336\,\cos\left(bx + a\right)^{7} + 248\,\cos\left(bx + a\right)^{5} - 10\,\cos\left(bx + a\right)^{3} - 15\,\cos\left(bx + a\right)\right)\sin\left(bx + a\right)}{80\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/80*(15*b*x - (128*cos(b*x + a)^9 - 336*cos(b*x + a)^7 + 248*cos(b*x + a)^
5 - 10*cos(b*x + a)^3 - 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(70) = 140.
time = 6.71, size = 434, normalized size = 5.71

$$\begin{cases} \frac{3x\sin^{2}(a+bx)\sin^{4}(2a+2bx)}{16} + \frac{3x\sin^{2}(a+bx)\sin^{2}(2a+2bx)\cos^{2}(2a+2bx)}{8} + \frac{3x\sin^{2}(a+bx)\cos^{4}(2a+2bx)}{16} + \frac{3x\sin^{4}(2a+2bx)\cos^{2}(a+bx)}{16} + \frac{3x\sin^{2}(2a+2bx)\cos^{2}(a+bx)\cos^{2}(2a+2bx)}{8} + \frac{3x\cos^{2}(a+bx)\cos^{4}(2a+2bx)}{16} - \frac{57\sin^{2}(a+bx)\sin^{3}(2a+2bx)\cos(2a+2bx)}{160b} - \frac{109\sin^{2}(a+bx)\sin(2a+2bx)\cos^{3}(2a+2bx)}{480b} - \frac{\sin(a+bx)\sin^{4}(2a+2bx)\cos(a+bx)}{10b} - \frac{2\sin(a+bx)\sin^{2}(2a+2bx)\cos(a+bx)\cos^{2}(2a+2bx)}{5b} - \frac{4\sin(a+bx)\cos(a+bx)\cos^{4}(2a+2bx)}{15b} + \frac{7\sin^{3}(2a+2bx)\cos^{2}(a+bx)\cos(2a+2bx)}{160b} + \frac{19\sin(2a+2bx)\cos^{2}(a+bx)\cos^{3}(2a+2bx)}{480b} & \text{for}\: b \neq 0 \\ x\sin^{2}(a)\sin^{4}(2a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**4/16 + 3*x*sin(a + b*x)**2
*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*sin(a + b*x)**2*cos(2*a +
2*b*x)**4/16 + 3*x*sin(2*a + 2*b*x)**4*cos(a + b*x)**2/16 + 3*x*sin(2*a + 2
*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*cos(a + b*x)**2*cos(2*
a + 2*b*x)**4/16 - 57*sin(a + b*x)**2*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)/
(160*b) - 109*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(480*b)
- sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(a + b*x)/(10*b) - 2*sin(a + b*x)*sin
(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(5*b) - 4*sin(a + b*x)*co
s(a + b*x)*cos(2*a + 2*b*x)**4/(15*b) + 7*sin(2*a + 2*b*x)**3*cos(a + b*x)*
*2*cos(2*a + 2*b*x)/(160*b) + 19*sin(2*a + 2*b*x)*cos(a + b*x)**2*cos(2*a +
 2*b*x)**3/(480*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**4, True))
```

Giac [A]
time = 0.45, size = 68, normalized size = 0.89

$$\frac{120\,bx + 120\,a - 2\,\sin\left(10\,bx + 10\,a\right) + 5\,\sin\left(8\,bx + 8\,a\right) + 10\,\sin\left(6\,bx + 6\,a\right) - 40\,\sin\left(4\,bx + 4\,a\right) - 20\,\sin\left(2\,bx + 2\,a\right)}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 1/640*(120*b*x + 120*a - 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) + 10*sin
(6*b*x + 6*a) - 40*sin(4*b*x + 4*a) - 20*sin(2*b*x + 2*a))/b
```

Mupad [B]
time = 1.68, size = 110, normalized size = 1.45

$$\frac{3\,x}{16}-\frac{-\frac{3\,\tan\left(a+b\,x\right)^9}{16}-\frac{7\,\tan\left(a+b\,x\right)^7}{8}+\frac{8\,\tan\left(a+b\,x\right)^5}{5}+\frac{7\,\tan\left(a+b\,x\right)^3}{8}+\frac{3\,\tan\left(a+b\,x\right)}{16}}{b\,\left({\tan\left(a+b\,x\right)}^{10}+5\,{\tan\left(a+b\,x\right)}^8+10\,{\tan\left(a+b\,x\right)}^6+10\,{\tan\left(a+b\,x\right)}^4+5\,{\tan\left(a+b\,x\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^4,x)
```

```
[Out] (3*x)/16 - ((3*tan(a + b*x))/16 + (7*tan(a + b*x)^3)/8 + (8*tan(a + b*x)^5)
/5 - (7*tan(a + b*x)^7)/8 - (3*tan(a + b*x)^9)/16)/(b*(5*tan(a + b*x)^2 + 1
0*tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*x)^10 +
 1))
```

3.15 $\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=29

$$\frac{4\sin^6(a+bx)}{3b}-\frac{\sin^8(a+bx)}{b}$$

```
[Out] 4/3*sin(b*x+a)^6/b-sin(b*x+a)^8/b
```

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {4373, 2644, 14}

$$\frac{4\sin^6(a+bx)}{3b}-\frac{\sin^8(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (4*Sin[a + b*x]^6)/(3*b) - Sin[a + b*x]^8/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \sin^2(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\sin^5(a+bx)\,dx \\
&= \frac{8\text{Subst}\left(\int x^5(1-x^2)\,dx, x, \sin(a+bx)\right)}{b} \\
&= \frac{8\text{Subst}\left(\int (x^5-x^7)\,dx, x, \sin(a+bx)\right)}{b} \\
&= \frac{4\sin^6(a+bx)}{3b}-\frac{\sin^8(a+bx)}{b}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 48, normalized size = 1.66

$$
\frac{-72\cos(2(a+bx))+12\cos(4(a+bx))+8\cos(6(a+bx))-3\cos(8(a+bx))}{384b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-72*Cos[2*(a + b*x)] + 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] - 3*Cos[8*
(a + b*x)])/(384*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(27) = 54$.
time = 0.07, size = 58, normalized size = 2.00

method	result	size
default	$-\frac{3\cos(2xb+2a)}{16b}+\frac{\cos(4xb+4a)}{32b}+\frac{\cos(6xb+6a)}{48b}-\frac{\cos(8xb+8a)}{128b}$	58
risch	$-\frac{3\cos(2xb+2a)}{16b}+\frac{\cos(4xb+4a)}{32b}+\frac{\cos(6xb+6a)}{48b}-\frac{\cos(8xb+8a)}{128b}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*cos(2*b*x+2*a)/b+1/32*cos(4*b*x+4*a)/b+1/48*cos(6*b*x+6*a)/b-1/128*co
s(8*b*x+8*a)/b
```

Maxima [A]
time = 0.28, size = 50, normalized size = 1.72

$$
-\frac{3\cos(8\,bx+8\,a)-8\cos(6\,bx+6\,a)-12\cos(4\,bx+4\,a)+72\cos(2\,bx+2\,a)}{384\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] -1/384*(3*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) + 72*
cos(2*b*x + 2*a))/b
```

Fricas [A]
time = 2.30, size = 36, normalized size = 1.24

$$-\frac{3\cos(bx+a)^8 - 8\cos(bx+a)^6 + 6\cos(bx+a)^4}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(b*x + a)^8 - 8*cos(b*x + a)^6 + 6*cos(b*x + a)^4)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(22) = 44$.
time = 2.81, size = 359, normalized size = 12.38

$$\begin{cases} \frac{3x\sin^2(a+bx)\sin^3(2a+2bx)}{16} + \frac{3x\sin^2(a+bx)\sin(2a+2bx)\cos^2(2a+2bx)}{16} + \frac{3x\sin(a+bx)\sin^2(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{8} + \frac{3x\sin(a+bx)\cos(a+bx)\cos^3(2a+2bx)}{8} - \frac{3x\sin^3(2a+2bx)\cos^2(a+bx)}{16} - \frac{3x\sin(2a+2bx)\cos^2(a+bx)\cos^2(2a+2bx)}{16} - \frac{\sin^2(a+bx)\sin^2(2a+2bx)\cos(2a+2bx)}{2b} - \frac{31\sin^2(a+bx)\cos^3(2a+2bx)}{96b} + \frac{3\sin(a+bx)\sin^3(2a+2bx)\cos(a+bx)}{16b} + \frac{\sin(a+bx)\sin(2a+2bx)\cos(a+bx)\cos^2(2a+2bx)}{8b} - \frac{\cos^2(a+bx)\cos^3(2a+2bx)}{96b} & \text{for}\ b \neq 0 \\ x\sin^2(a)\sin^3(2a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**3/16 + 3*x*sin(a + b*x)**2
*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/16 + 3*x*sin(a + b*x)*sin(2*a + 2*b*x
)**2*cos(a + b*x)*cos(2*a + 2*b*x)/8 + 3*x*sin(a + b*x)*cos(a + b*x)*cos(2*
a + 2*b*x)**3/8 - 3*x*sin(2*a + 2*b*x)**3*cos(a + b*x)**2/16 - 3*x*sin(2*a
+ 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 - sin(a + b*x)**2*sin(2*a +
 2*b*x)**2*cos(2*a + 2*b*x)/(2*b) - 31*sin(a + b*x)**2*cos(2*a + 2*b*x)**3/
(96*b) + 3*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(a + b*x)/(16*b) + sin(a + b
*x)*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(8*b) - cos(a + b*x)*
*2*cos(2*a + 2*b*x)**3/(96*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**3, True))
```

Giac [A]
time = 0.54, size = 26, normalized size = 0.90

$$-\frac{3\sin(bx+a)^8 - 4\sin(bx+a)^6}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -1/3*(3*sin(b*x + a)^8 - 4*sin(b*x + a)^6)/b
```

Mupad [B]
time = 0.11, size = 33, normalized size = 1.14

$$-\frac{\cos\left(a+b\,x\right)^4\,\left(\cos\left(a+b\,x\right)^4-\frac{8\,\cos\left(a+b\,x\right)^2}{3}+2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^3,x)
[Out] -(cos(a + b*x)^4*(cos(a + b*x)^4 - (8*cos(a + b*x)^2)/3 + 2))/b
```

3.16 $\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=49

$$\frac{x}{4}-\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b}-\frac{\sin^3(2a+2bx)}{12b}$$

```
[Out] 1/4*x-1/8*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b-1/12*sin(2*b*x+2*a)^3/b
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4371, 2715, 8, 2644, 30}

$$-\frac{\sin^3(2a+2bx)}{12b}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{8b}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] x/4 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(8*b) - Sin[2*a + 2*b*x]^3/(12*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4371

```
Int[sin[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Dist[1/2, Int[(g*Sin[c + d*x])^p, x], x] - Dist[1/2, Int[Cos[c + d*x]
*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d,
0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin^2(2a+2bx)\,dx &= \frac{1}{2}\int \sin^2(2a+2bx)\,dx-\frac{1}{2}\int \cos(2a+2bx)\sin^2(2a+2bx)\,dx\\ &= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b}+\frac{\int 1\,dx}{4}-\frac{\text{Subst}\left(\int x^2\,dx,x,\sin(2a+2bx\right.}{4b}\\ &= \frac{x}{4}-\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b}-\frac{\sin^3(2a+2bx)}{12b}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 40, normalized size = 0.82

$$\frac{12bx-3\sin(2(a+bx))-3\sin(4(a+bx))+\sin(6(a+bx))}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
[Out] (12*b*x - 3*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)])/(48*b
)
```

Maple [A]
time = 0.06, size = 47, normalized size = 0.96

method	result	size
default	$\frac{x}{4}-\frac{\sin(2xb+2a)}{16b}-\frac{\sin(4xb+4a)}{16b}+\frac{\sin(6xb+6a)}{48b}$	47
risch	$\frac{x}{4}-\frac{\sin(2xb+2a)}{16b}-\frac{\sin(4xb+4a)}{16b}+\frac{\sin(6xb+6a)}{48b}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
[Out] 1/4*x-1/16*sin(2*b*x+2*a)/b-1/16/b*sin(4*b*x+4*a)+1/48/b*sin(6*b*x+6*a)
```

Maxima [A]
time = 0.29, size = 41, normalized size = 0.84

$$\frac{12\,bx+\sin\left(6\,bx+6\,a\right)-3\,\sin\left(4\,bx+4\,a\right)-3\,\sin\left(2\,bx+2\,a\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(12*b*x + sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))/
b
```

Fricas [A]
time = 2.25, size = 46, normalized size = 0.94

$$\frac{3\,bx+\left(8\,\cos\left(bx+a\right)^{5}-14\,\cos\left(bx+a\right)^{3}+3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*x + (8*cos(b*x + a)^5 - 14*cos(b*x + a)^3 + 3*cos(b*x + a))*sin(b
*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(41) = 82$.
time = 1.14, size = 231, normalized size = 4.71

$$\begin{cases} \frac{x\sin^{2}(a+bx)\sin^{2}(2a+2bx)}{4}+\frac{x\sin^{2}(a+bx)\cos^{2}(2a+2bx)}{4}+\frac{x\sin^{2}(2a+2bx)\cos^{2}(a+bx)}{4}+\frac{x\cos^{2}(a+bx)\cos^{2}(2a+2bx)}{4}-\frac{7\sin^{2}(a+bx)\sin(2a+2bx)\cos(2a+2bx)}{24b}-\frac{\sin(a+bx)\sin^{2}(2a+2bx)\cos(a+bx)}{6b}-\frac{\sin(a+bx)\cos(a+bx)\cos^{2}(2a+2bx)}{3b}+\frac{\sin(2a+2bx)\cos^{2}(a+bx)\cos(2a+2bx)}{24b} & \text{for}\ b\neq 0\\ x\sin^{2}(a)\sin^{2}(2a) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)**2/4 + x*sin(a + b*x)**2*cos(
2*a + 2*b*x)**2/4 + x*sin(2*a + 2*b*x)**2*cos(a + b*x)**2/4 + x*cos(a + b*x
)**2*cos(2*a + 2*b*x)**2/4 - 7*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a + 2
*b*x)/(24*b) - sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)/(6*b) - sin(a
+ b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(3*b) + sin(2*a + 2*b*x)*cos(a + b*
x)**2*cos(2*a + 2*b*x)/(24*b), Ne(b, 0)), (x*sin(a)**2*sin(2*a)**2, True))
```

Giac [A]
time = 0.45, size = 44, normalized size = 0.90

$$\frac{12\,bx+12\,a+\sin\left(6\,bx+6\,a\right)-3\,\sin\left(4\,bx+4\,a\right)-3\,\sin\left(2\,bx+2\,a\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 1/48*(12*b*x + 12*a + sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 3*sin(2*b*x +
 2*a))/b
```

Mupad [B]
time = 0.30, size = 43, normalized size = 0.88

$$\frac{x}{4}-\frac{\frac{\sin(2\,a+2\,b\,x)}{16}+\frac{\sin(4\,a+4\,b\,x)}{16}-\frac{\sin(6\,a+6\,b\,x)}{48}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^2,x)
[Out] x/4 - (sin(2*a + 2*b*x)/16 + sin(4*a + 4*b*x)/16 - sin(6*a + 6*b*x)/48)/b
```

3.17 $\int \sin^2(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=15

$$\frac{\sin^4(a+bx)}{2b}$$

[Out] 1/2*sin(b*x+a)^4/b

Rubi [A]
time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4373, 2644, 30}

$$\frac{\sin^4(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x],x]

[Out] Sin[a + b*x]^4/(2*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \sin^2(a+bx)\sin(2a+2bx)\,dx = 2\int \cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{2\text{Subst}\left(\int x^3\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{\sin^4(a+bx)}{2b}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{\sin^4(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

```
[Out] Sin[a + b*x]^4/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.
time = 0.08, size = 30, normalized size = 2.00

method	result
default	$-\frac{\cos(2xb+2a)}{4b}+\frac{\cos(4xb+4a)}{16b}$
risch	$-\frac{\cos(2xb+2a)}{4b}+\frac{\cos(4xb+4a)}{16b}$
norman	$\frac{x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+x\left(\tan^3\left(\frac{a}{2}+\frac{xb}{2}\right)\right)\left(\tan^2(xb+a)\right)+\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)}{b}-x\left(\tan^3\left(\frac{a}{2}+\frac{xb}{2}\right)\right)-\frac{x\tan(xb+a)}{2}-x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan^2(xb+a)\right)}{\left(1+\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*cos(2*b*x+2*a)/b+1/16*cos(4*b*x+4*a)/b
```

Maxima [A]
time = 0.27, size = 26, normalized size = 1.73

$$\frac{\cos(4\,bx+4\,a)-4\,\cos(2\,bx+2\,a)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/16*(cos(4*b*x + 4*a) - 4*cos(2*b*x + 2*a))/b
```

Fricas [A]
time = 4.48, size = 24, normalized size = 1.60

$$\frac{\cos\left(bx+a\right)^4-2\,\cos\left(bx+a\right)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(b*x + a)^4 - 2*cos(b*x + a)^2)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(10) = 20$.
time = 0.44, size = 131, normalized size = 8.73

$$\begin{cases} \frac{x\sin^2\left(a+bx\right)\sin\left(2a+2bx\right)}{4}+\frac{x\sin\left(a+bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{2}-\frac{x\sin\left(2a+2bx\right)\cos^2\left(a+bx\right)}{4}-\frac{\sin^2\left(a+bx\right)\cos\left(2a+2bx\right)}{2b}+\frac{\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos\left(a+bx\right)}{4b} & \text{for}\ b\neq 0 \\ x\sin^2\left(a\right)\sin\left(2a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)/4 + x*sin(a + b*x)*cos(a + b*
x)*cos(2*a + 2*b*x)/2 - x*sin(2*a + 2*b*x)*cos(a + b*x)**2/4 - sin(a + b*x)
**2*cos(2*a + 2*b*x)/(2*b) + sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)/(4*
b), Ne(b, 0)), (x*sin(a)**2*sin(2*a), True))
```

Giac [A]
time = 0.44, size = 13, normalized size = 0.87

$$\frac{\sin\left(bx+a\right)^4}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 1/2*sin(b*x + a)^4/b
```

Mupad [B]
time = 0.10, size = 13, normalized size = 0.87

$$\frac{\sin\left(a+b\,x\right)^4}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x),x)
```

```
[Out] sin(a + b*x)^4/(2*b)
```

3.18 $\int \csc(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=14

$$-\frac{\log(\cos(a+bx))}{2b}$$

```
[Out] -1/2*ln(cos(b*x+a))/b
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4373, 3556}

$$-\frac{\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -1/2*Log[Cos[a + b*x]]/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(2a+2bx)\sin^2(a+bx)\,dx &= \frac{1}{2}\int \tan(a+bx)\,dx\\ &= -\frac{\log(\cos(a+bx))}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-\frac{\log(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -1/2*Log[Cos[a + b*x]]/b
```

Maple [A]
time = 0.06, size = 13, normalized size = 0.93

method	result	size
default	$-\frac{\ln(\cos(xb+a))}{2b}$	13
risch	$\frac{ix}{2}+\frac{ia}{b}-\frac{\ln\left(e^{2i(xb+a)}+1\right)}{2b}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln(cos(b*x+a))/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(12) = 24.
time = 0.27, size = 55, normalized size = 3.93

$$-\frac{\log\left(\cos\left(2\,bx\right)^{2}+2\,\cos\left(2\,bx\right)\cos\left(2\,a\right)+\cos\left(2\,a\right)^{2}+\sin\left(2\,bx\right)^{2}-2\,\sin\left(2\,bx\right)\sin\left(2\,a\right)+\sin\left(2\,a\right)^{2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 -
 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2)/b
```

Fricas [A]
time = 2.92, size = 14, normalized size = 1.00

$$-\frac{\log\left(-\cos\left(bx+a\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*log(-cos(b*x + a))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.41, size = 18, normalized size = 1.29

$$-\frac{\log\left(-\sin\left(bx+a\right)^{2}+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*log(-sin(b*x + a)^2 + 1)/b
```

Mupad [B]
time = 0.08, size = 12, normalized size = 0.86

$$-\frac{\ln\left(\cos\left(a+b\,x\right)\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x),x)
```

```
[Out] -log(cos(a + b*x))/(2*b)
```

3.19 $\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=13

$$\frac{\tan(a+bx)}{4b}$$

```
[Out] 1/4*tan(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 3852, 8}

$$\frac{\tan(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(2a+2bx)\sin^2(a+bx)\,dx &= \frac{1}{4}\int \sec^2(a+bx)\,dx\\ &= -\frac{\operatorname{Subst}\left(\int 1\,dx, x, -\tan(a+bx)\right)}{4b}\\ &= \frac{\tan(a+bx)}{4b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\tan(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]/(4*b)
```

Maple [A]
time = 0.09, size = 12, normalized size = 0.92

method	result	size
default	$\frac{\tan(xb+a)}{4b}$	12
risch	$\frac{i}{2b\left(\mathrm{e}^{2i(xb+a)}+1\right)}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*tan(b*x+a)/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(11) = 22.
time = 0.27, size = 53, normalized size = 4.08

$$\frac{\sin(2bx+2a)}{2\left(b\cos(2bx+2a)^2+b\sin(2bx+2a)^2+2b\cos(2bx+2a)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sin(2*b*x + 2*a)/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 + 2*b*cos
(2*b*x + 2*a) + b)
```

Fricas [A]
time = 3.43, size = 19, normalized size = 1.46

$$\frac{\sin(bx+a)}{4b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*sin(b*x + a)/(b*cos(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 11, normalized size = 0.85

$$\frac{\tan\left(bx+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*tan(b*x + a)/b
```

Mupad [**B**]
time = 0.11, size = 11, normalized size = 0.85

$$\frac{\tan(a+b\,x)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^2,x)
```

```
[Out] tan(a + b*x)/(4*b)
```

3.20 $\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=30

$$\frac{\log(\tan(a+bx))}{8b}+\frac{\tan^2(a+bx)}{16b}$$

```
[Out] 1/8*ln(tan(b*x+a))/b+1/16*tan(b*x+a)^2/b
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2700, 14}

$$\frac{\tan^2(a+bx)}{16b}+\frac{\log(\tan(a+bx))}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] Log[Tan[a + b*x]]/(8*b) + Tan[a + b*x]^2/(16*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc^3(2a+2bx)\sin^2(a+bx)\,dx &= \frac{1}{8}\int \csc(a+bx)\sec^3(a+bx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, \tan(a+bx)\right)}{8b} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx, x, \tan(a+bx)\right)}{8b} \\
&= \frac{\log(\tan(a+bx))}{8b} + \frac{\tan^2(a+bx)}{16b}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 36, normalized size = 1.20

$$
-\frac{2\log(\cos(a+bx)) - 2\log(\sin(a+bx)) - \sec^2(a+bx)}{16b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/16*(2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]] - Sec[a + b*x]^2)/b
```

Maple [A]
time = 0.10, size = 24, normalized size = 0.80

method	result	size
default	$\frac{\frac{1}{2\cos(xb+a)^2}+\ln(\tan(xb+a))}{8b}$	24
risch	$\frac{e^{2i(xb+a)}}{4b\left(e^{2i(xb+a)}+1\right)^2} + \frac{\ln\left(e^{2i(xb+a)}-1\right)}{8b} - \frac{\ln\left(e^{2i(xb+a)}+1\right)}{8b}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(1/2/cos(b*x+a)^2+ln(tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(26) = 52.
time = 0.29, size = 641, normalized size = 21.37

$$
\frac{4\cos(4bx+4a)\cos(2bx+2a) + 8\cos(2bx+2a)^2 - \left(2\left(2\cos(2bx+2a)+1\right)\cos(4bx+4a) + \cos(4bx+4a)^2 + 4\cos(2bx+2a)^2 + \sin(4bx+4a)^2 + 4\sin(4bx+4a)\sin(2bx+2a) + 4\sin(2bx+2a)^2 + 4\cos(2bx+2a) + 1\right)\log\left(\cos(2bx)^2 + 2\cos(2bx)\cos(2a) + \cos(2a)^2 + \sin(2bx)^2 - 2\sin(2bx)\sin(2a) + \sin(2a)^2\right) + \left(2\left(2\cos(2bx+2a)+1\right)\cos(4bx+4a) + \cos(4bx+4a)^2 + 4\cos(2bx+2a)^2 + \sin(4bx+4a)^2 + 4\sin(4bx+4a)\sin(2bx+2a) + 4\sin(2bx+2a)^2 + 4\cos(2bx+2a) + 1\right)\log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2\right) + \left(2\left(2\cos(2bx+2a)+1\right)\cos(4bx+4a) + \cos(4bx+4a)^2 + 4\cos(2bx+2a)^2 + \sin(4bx+4a)^2 + 4\sin(4bx+4a)\sin(2bx+2a) + 4\sin(2bx+2a)^2 + 4\cos(2bx+2a) + 1\right)\log\left(\cos(bx)^2 - 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 + 2\sin(bx)\sin(a) + \sin(a)^2\right) + 4\sin(4bx+4a)\sin(2bx+2a) + 8\sin(2bx+2a)^2 + 4\cos(2bx+2a)}{16\left(b\cos(4bx+4a)^2 + 4b\cos(2bx+2a)^2 + b\sin(4bx+4a)^2 + 4b\sin(4bx+4a)\sin(2bx+2a) + 4b\sin(2bx+2a)^2 + 2\left(2b\cos(2bx+2a)+b\right)\cos(4bx+4a) + 4b\cos(2bx+2a) + b\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) + 8*cos(2*b*x + 2*a)^2 - (2*(2*co
s(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2
*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*
b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(
2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) + (2
*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b
*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*
sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 + 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + (2*(2*cos(2*
b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^
2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x
+ 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos
(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b*cos(4*b*x + 4
*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a
)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(2*b*x + 2*a) + b)*
cos(4*b*x + 4*a) + 4*b*cos(2*b*x + 2*a) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.
time = 3.88, size = 56, normalized size = 1.87

$$-\frac{\cos\left(bx+a\right)^{2}\log\left(\cos\left(bx+a\right)^{2}\right)-\cos\left(bx+a\right)^{2}\log\left(-\frac{1}{4}\,\cos\left(bx+a\right)^{2}+\frac{1}{4}\right)-1}{16\,b\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(cos(b*x + a)^2*log(cos(b*x + a)^2) - cos(b*x + a)^2*log(-1/4*cos(b*x
 + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 41, normalized size = 1.37

$$-\frac{\frac{1}{\sin\left(bx+a\right)^{2}-1}+\log\left(-\sin\left(bx+a\right)^{2}+1\right)-2\,\log\left(\left|\sin\left(bx+a\right)\right|\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*(1/(sin(b*x + a)^2 - 1) + log(-sin(b*x + a)^2 + 1) - 2*log(abs(sin(b*
x + a))))/b
```

Mupad [B]
time = 0.15, size = 35, normalized size = 1.17

$$\frac{\frac{\ln\left(\sin(a+b\,x)^2\right)}{16}-\frac{\ln(\cos(a+b\,x))}{8}+\frac{1}{16\cos(a+b\,x)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^3,x)
```

```
[Out] (log(sin(a + b*x)^2)/16 - log(cos(a + b*x))/8 + 1/(16*cos(a + b*x)^2))/b
```

3.21 $\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=42

$$-\frac{\cot(a+bx)}{16b}+\frac{\tan(a+bx)}{8b}+\frac{\tan^3(a+bx)}{48b}$$

```
[Out] -1/16*cot(b*x+a)/b+1/8*tan(b*x+a)/b+1/48*tan(b*x+a)^3/b
```

Rubi [A]
time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2700, 276}

$$\frac{\tan^3(a+bx)}{48b}+\frac{\tan(a+bx)}{8b}-\frac{\cot(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^2,x]
```

```
[Out] -1/16*Cot[a + b*x]/b + Tan[a + b*x]/(8*b) + Tan[a + b*x]^3/(48*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^4(2a+2bx)\sin^2(a+bx)\,dx = \frac{1}{16}\int \csc^2(a+bx)\sec^4(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2}\,dx, x, \tan(a+bx)\right)}{16b}$$
$$= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)\,dx, x, \tan(a+bx)\right)}{16b}$$
$$= -\frac{\cot(a+bx)}{16b}+\frac{\tan(a+bx)}{8b}+\frac{\tan^3(a+bx)}{48b}$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 1.14

$$-\frac{\cot(a+bx)}{16b}+\frac{5\tan(a+bx)}{48b}+\frac{\sec^2(a+bx)\tan(a+bx)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^2,x]
```

```
[Out] -1/16*Cot[a + b*x]/b + (5*Tan[a + b*x])/(48*b) + (Sec[a + b*x]^2*Tan[a + b*
x])/(48*b)
```

Maple [A]
time = 0.11, size = 51, normalized size = 1.21

method	result	size
risch	$-\frac{i\left(2\,{\mathrm e}^{2 i (x b+a)}+1\right)}{3 b \left({\mathrm e}^{2 i (x b+a)}+1\right)^{3} \left({\mathrm e}^{2 i (x b+a)}-1\right)}$	46
default	$\frac{\frac{1}{3 \sin(x b+a)\cos(x b+a)^{3}}+\frac{4}{3 \sin(x b+a)\cos(x b+a)}-\frac{8 \cot(x b+a)}{3}}{16 b}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/b*(1/3/sin(b*x+a)/cos(b*x+a)^3+4/3/sin(b*x+a)/cos(b*x+a)-8/3*cot(b*x+a
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(36) = 72.
time = 0.28, size = 308, normalized size = 7.33

$$-\frac{(2\cos(2bx+2a)+1)\sin(8bx+8a)+2(2\cos(2bx+2a)+1)\sin(6bx+6a)-2\cos(8bx+8a)\sin(2bx+2a)-4\cos(6bx+6a)\sin(2bx+2a)}{3\left(b\cos(8bx+8a)^2+4b\cos(6bx+6a)^2+4b\cos(2bx+2a)^2+b\sin(8bx+8a)^2+4b\sin(6bx+6a)^2-8b\sin(6bx+6a)\sin(2bx+2a)+4b\sin(2bx+2a)^2+2(2b\cos(6bx+6a)-2b\cos(2bx+2a)-b)\cos(8bx+8a)-4(2b\cos(2bx+2a)+b)\cos(6bx+6a)+4b\cos(2bx+2a)+4(b\sin(6bx+6a)-b\sin(2bx+2a))\sin(8bx+8a)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*((2*cos(2*b*x + 2*a) + 1)*sin(8*b*x + 8*a) + 2*(2*cos(2*b*x + 2*a) + 1
)*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) - 4*cos(6*b*x + 6*
a)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*c
os(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b*sin
(6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(6*b*x
+ 6*a) - 2*b*cos(2*b*x + 2*a) - b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*x + 2*
a) + b)*cos(6*b*x + 6*a) + 4*b*cos(2*b*x + 2*a) + 4*(b*sin(6*b*x + 6*a) - b
*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

Fricas [A]
time = 3.93, size = 43, normalized size = 1.02

$$-\frac{8\,\cos\left(bx+a\right)^{4}-4\,\cos\left(bx+a\right)^{2}-1}{48\,b\cos\left(bx+a\right)^{3}\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^3*sin(b*x +
 a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**4*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 32, normalized size = 0.76

$$\frac{\tan\left(bx+a\right)^{3}-\frac{3}{\tan\left(bx+a\right)}+6\,\tan\left(bx+a\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/48*(tan(b*x + a)^3 - 3/tan(b*x + a) + 6*tan(b*x + a))/b
```

Mupad [B]
time = 0.13, size = 33, normalized size = 0.79

$$\frac{\tan(a+b\,x)^4+6\tan(a+b\,x)^2-3}{48\,b\tan\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^4,x)
[Out] (6*tan(a + b*x)^2 + tan(a + b*x)^4 - 3)/(48*b*tan(a + b*x))
```

3.22 $\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=60

$$-\frac{\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}+\frac{3\tan^2(a+bx)}{64b}+\frac{\tan^4(a+bx)}{128b}$$

```
[Out] -1/64*cot(b*x+a)^2/b+3/32*ln(tan(b*x+a))/b+3/64*tan(b*x+a)^2/b+1/128*tan(b*
x+a)^4/b
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4373, 2700, 272, 45}

$$\frac{\tan^4(a+bx)}{128b}+\frac{3\tan^2(a+bx)}{64b}-\frac{\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^2,x]
```

```
[Out] -1/64*Cot[a + b*x]^2/b + (3*Log[Tan[a + b*x]])/(32*b) + (3*Tan[a + b*x]^2)/
(64*b) + Tan[a + b*x]^4/(128*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
```

```
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^5(2a+2bx)\sin^2(a+bx)\,dx &= \frac{1}{32}\int \csc^3(a+bx)\sec^5(a+bx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^3}\,dx, x, \tan(a+bx)\right)}{32b}\\ &= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^2}\,dx, x, \tan^2(a+bx)\right)}{64b}\\ &= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^2}+\frac{3}{x}+x\right)\,dx, x, \tan^2(a+bx)\right)}{64b}\\ &= -\frac{\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}+\frac{3\tan^2(a+bx)}{64b}+\frac{\tan^4(a+bx)}{128b}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 56, normalized size = 0.93

$$-\frac{2\csc^2(a+bx)+12\log(\cos(a+bx))-12\log(\sin(a+bx))-4\sec^2(a+bx)-\sec^4(a+bx)}{128b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^2,x]
```

```
[Out] -1/128*(2*Csc[a + b*x]^2 + 12*Log[Cos[a + b*x]] - 12*Log[Sin[a + b*x]] - 4*
Sec[a + b*x]^2 - Sec[a + b*x]^4)/b
```

Maple [A]
time = 0.11, size = 62, normalized size = 1.03

method	result	size
default	$\frac{\frac{1}{4\sin(xb+a)^2\cos(xb+a)^4}+\frac{3}{4\sin(xb+a)^2\cos(xb+a)^2}-\frac{3}{2\sin(xb+a)^2}+3\ln(\tan(xb+a))}{32b}$	62
risch	$\frac{3\,\mathrm{e}^{10i(xb+a)}+6\,\mathrm{e}^{8i(xb+a)}-2\,\mathrm{e}^{6i(xb+a)}+6\,\mathrm{e}^{4i(xb+a)}+3\,\mathrm{e}^{2i(xb+a)}}{16b\left(\mathrm{e}^{2i(xb+a)}+1\right)^4\left(\mathrm{e}^{2i(xb+a)}-1\right)^2}-\frac{3\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)}{32b}+\frac{3\ln\left(\mathrm{e}^{2i(xb+a)}-1\right)}{32b}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/b*(1/4/sin(b*x+a)^2/cos(b*x+a)^4+3/4/sin(b*x+a)^2/cos(b*x+a)^2-3/2/sin
(b*x+a)^2+3*ln(tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3164 vs. $2(52) = 104$.
time = 0.36, size = 3164, normalized size = 52.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(4*(3*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) + 6
*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b*x
 + 8*a) - 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) + 12*cos(2*b*x + 2*a) +
3)*cos(10*b*x + 10*a) + 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a) +
12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) - 6)*cos(8*b*x + 8*a) - 24*cos(8*b
*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) + 8*cos(2*b*x + 2*a) + 1)*cos(6*b*x +
6*a) + 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) + 2)*cos(4*b*x + 4*a)
 - 24*cos(4*b*x + 4*a)^2 + 24*cos(2*b*x + 2*a)^2 - 3*(2*(2*cos(10*b*x + 10*
a) - cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) + 2*cos(2*b*x
 + 2*a) + 1)*cos(12*b*x + 12*a) + cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a
) + 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(10*
b*x + 10*a) + 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) + cos(4*b*x +
4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*x + 8*a)^2 + 8*(c
os(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + 16*cos(6*b*x +
 6*a)^2 - 2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2
+ 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) - sin(8*b*x + 8*a) - 4*sin
(6*b*x + 6*a) - sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) +
 sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) + 4*sin(6*b*x + 6*a) + sin(4*b*
x + 4*a) - 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 4*sin(10*b*x + 10*a)^2
+ 2*(4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x
+ 8*a) + sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin
(6*b*x + 6*a) + 16*sin(6*b*x + 6*a)^2 + sin(4*b*x + 4*a)^2 - 4*sin(4*b*x +
4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(
cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*
b*x)*sin(2*a) + sin(2*a)^2) + 3*(2*(2*cos(10*b*x + 10*a) - cos(8*b*x + 8*a)
 - 4*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) + 1)*cos(12*b
*x + 12*a) + cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a) + 4*cos(6*b*x + 6*a
) + cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) + 4*cos(1
0*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 2*cos(2*b*x +
2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*x + 8*a)^2 + 8*(cos(4*b*x + 4*a) - 2*c
os(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) + 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*
b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^
2 + 2*(2*sin(10*b*x + 10*a) - sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) - sin(4
*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) + sin(12*b*x + 12*a)^2
 - 4*(sin(8*b*x + 8*a) + 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 2*sin(2*b*
x + 2*a))*sin(10*b*x + 10*a) + 4*sin(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*
```

```
a) + sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + sin(8*b*x +
8*a)^2 + 8*(sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + 16*si
n(6*b*x + 6*a)^2 + sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
 + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(b*x)^2 + 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*(2*(2
*cos(10*b*x + 10*a) - cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) - cos(4*b*x + 4
*a) + 2*cos(2*b*x + 2*a) + 1)*cos(12*b*x + 12*a) + cos(12*b*x + 12*a)^2 - 4
*(cos(8*b*x + 8*a) + 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 2*cos(2*b*x +
2*a) - 1)*cos(10*b*x + 10*a) + 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*
a) + cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + cos(8*b*
x + 8*a)^2 + 8*(cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a)
 + 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + co
s(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) - sin(8*b
*x + 8*a) - 4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) + sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) + 4*sin(6*b*x
+ 6*a) + sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 4*sin(
10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 2*sin(2*b*x +
 2*a))*sin(8*b*x + 8*a) + sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) - 2*sin(
2*b*x + 2*a))*sin(6*b*x + 6*a) + 16*sin(6*b*x + 6*a)^2 + sin(4*b*x + 4*a)^2
 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x
 + 2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a)
 - 2*sin(6*b*x + 6*a) + 6*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x
 + 12*a) + 4*(9*sin(8*b*x + 8*a) - 16*sin(6*b*x + 6*a) + 9*sin(4*b*x + 4*a)
 + 12*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + 24*sin(10*b*x + 10*a)^2 - 4*(2
2*sin(6*b*x + 6*a) + 12*sin(4*b*x + 4*a) - 9*sin(2*b*x + 2*a))*sin(8*b*x +
8*a) - 24*sin(8*b*x + 8*a)^2 - 8*(11*sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))
*sin(6*b*x + 6*a) + 32*sin(6*b*x + 6*a)^2 - 24*sin(4*b*x + 4*a)^2 + 36*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) + 24*sin(2*b*x + 2*a)^2 + 12*cos(2*b*x + 2*a)
)/(b*cos(12*b*x + 12*a)^2 + 4*b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2
 + 16*b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x + 4*a)...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(52) = 104$.
time = 2.82, size = 112, normalized size = 1.87

$$\frac{6\cos(bx+a)^4 - 3\cos(bx+a)^2 - 6\left(\cos(bx+a)^6 - \cos(bx+a)^4\right)\log\left(\cos(bx+a)^2\right) + 6\left(\cos(bx+a)^6 - \cos(bx+a)^4\right)\log\left(-\frac{1}{4}\cos(bx+a)^2 + \frac{1}{4}\right) - 1}{128\left(b\cos(bx+a)^6 - b\cos(bx+a)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/128*(6*cos(b*x + a)^4 - 3*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - cos(b*x +
a)^4)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - cos(b*x + a)^4)*log(-1/4*co
s(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^6 - b*cos(b*x + a)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**5*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 74, normalized size = 1.23

$$-\frac{\frac{6\,\sin(bx+a)^4-9\,\sin(bx+a)^2+2}{\left(\sin(bx+a)^2-1\right)^2\sin(bx+a)^2}+6\,\log\left(-\sin\left(bx+a\right)^2+1\right)-12\,\log\left(\left|\sin\left(bx+a\right)\right|\right)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/128*((6*sin(b*x + a)^4 - 9*sin(b*x + a)^2 + 2)/((sin(b*x + a)^2 - 1)^2*s
in(b*x + a)^2) + 6*log(-sin(b*x + a)^2 + 1) - 12*log(abs(sin(b*x + a))))/b
```

Mupad [**B**]
time = 0.16, size = 74, normalized size = 1.23

$$\frac{3\,\ln\left(\sin\left(a+b\,x\right)^2\right)}{64\,b}-\frac{3\,\ln\left(\cos\left(a+b\,x\right)\right)}{32\,b}+\frac{-\frac{3\cos(a+b\,x)^4}{64}+\frac{3\cos(a+b\,x)^2}{128}+\frac{1}{128}}{b\,\left(\cos\left(a+b\,x\right)^4-\cos\left(a+b\,x\right)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^5,x)
```

```
[Out] (3*log(sin(a + b*x)^2))/(64*b) - (3*log(cos(a + b*x)))/(32*b) + ((3*cos(a +
 b*x)^2)/128 - (3*cos(a + b*x)^4)/64 + 1/128)/(b*(cos(a + b*x)^4 - cos(a +
b*x)^6))
```

3.23 $\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{32\sin^9(a+bx)}{9b}-\frac{64\sin^{11}(a+bx)}{11b}+\frac{32\sin^{13}(a+bx)}{13b}$$

```
[Out] 32/9*sin(b*x+a)^9/b-64/11*sin(b*x+a)^11/b+32/13*sin(b*x+a)^13/b
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {4373, 2644, 276}

$$\frac{32\sin^{13}(a+bx)}{13b}-\frac{64\sin^{11}(a+bx)}{11b}+\frac{32\sin^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (32*Sin[a + b*x]^9)/(9*b) - (64*Sin[a + b*x]^11)/(11*b) + (32*Sin[a + b*x]^
13)/(13*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \sin^3(a+bx)\sin^5(2a+2bx)\,dx = 32\int \cos^5(a+bx)\sin^8(a+bx)\,dx$$
$$= \frac{32\text{Subst}\left(\int x^8(1-x^2)^2\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{32\text{Subst}\left(\int (x^8-2x^{10}+x^{12})\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{32\sin^9(a+bx)}{9b} - \frac{64\sin^{11}(a+bx)}{11b} + \frac{32\sin^{13}(a+bx)}{13b}$$

Mathematica [A]
time = 0.41, size = 37, normalized size = 0.80

$$\frac{4(505+540\cos(2(a+bx))+99\cos(4(a+bx)))\sin^9(a+bx)}{1287b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*(505 + 540*Cos[2*(a + b*x)] + 99*Cos[4*(a + b*x)])*Sin[a + b*x]^9)/(1287
*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(40) = 80.
time = 0.17, size = 97, normalized size = 2.11

method	result	size
default	$\frac{5\sin(xb+a)}{32b} - \frac{25\sin(3xb+3a)}{384b} - \frac{\sin(5xb+5a)}{128b} + \frac{\sin(7xb+7a)}{64b} - \frac{\sin(9xb+9a)}{576b} - \frac{3\sin(11xb+11a)}{1408b} + \frac{\sin(13xb+13a)}{1664b}$	97
risch	$\frac{5\sin(xb+a)}{32b} - \frac{25\sin(3xb+3a)}{384b} - \frac{\sin(5xb+5a)}{128b} + \frac{\sin(7xb+7a)}{64b} - \frac{\sin(9xb+9a)}{576b} - \frac{3\sin(11xb+11a)}{1408b} + \frac{\sin(13xb+13a)}{1664b}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 5/32*sin(b*x+a)/b-25/384*sin(3*b*x+3*a)/b-1/128/b*sin(5*b*x+5*a)+1/64/b*sin
(7*b*x+7*a)-1/576/b*sin(9*b*x+9*a)-3/1408/b*sin(11*b*x+11*a)+1/1664/b*sin(1
3*b*x+13*a)
```

Maxima [A]
time = 0.28, size = 80, normalized size = 1.74

$$\frac{99\sin(13bx+13a) - 351\sin(11bx+11a) - 286\sin(9bx+9a) + 2574\sin(7bx+7a) - 1287\sin(5bx+5a) - 10725\sin(3bx+3a) + 25740\sin(bx+a)}{164736b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/164736*(99*sin(13*b*x + 13*a) - 351*sin(11*b*x + 11*a) - 286*sin(9*b*x +
9*a) + 2574*sin(7*b*x + 7*a) - 1287*sin(5*b*x + 5*a) - 10725*sin(3*b*x + 3*
a) + 25740*sin(b*x + a))/b
```

Fricas [A]
time = 3.48, size = 73, normalized size = 1.59

$$\frac{32\left(99\cos\left(bx+a\right)^{12}-360\cos\left(bx+a\right)^{10}+458\cos\left(bx+a\right)^{8}-212\cos\left(bx+a\right)^{6}+3\cos\left(bx+a\right)^{4}+4\cos\left(bx+a\right)^{2}+8\right)\sin\left(bx+a\right)}{1287\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 32/1287*(99*cos(b*x + a)^12 - 360*cos(b*x + a)^10 + 458*cos(b*x + a)^8 - 21
2*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*x + a)^2 + 8)*sin(b*x + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(39) = 78$.
time = 34.91, size = 447, normalized size = 9.72

$$\begin{cases} -\frac{1366\sin^{3}(a+bx)\sin^{4}(2a+2bx)\cos(2a+2bx)}{3003b}-\frac{4960\sin^{3}(a+bx)\sin^{2}(2a+2bx)\cos^{3}(2a+2bx)}{9009b}-\frac{256\sin^{3}(a+bx)\cos^{5}(2a+2bx)}{1287b}-\frac{271\sin^{2}(a+bx)\sin^{5}(2a+2bx)\cos(a+bx)}{3003b}-\frac{48\sin^{2}(a+bx)\sin^{3}(2a+2bx)\cos(a+bx)\cos^{2}(2a+2bx)}{143b}-\frac{640\sin^{2}(a+bx)\sin(2a+2bx)\cos(a+bx)\cos^{4}(2a+2bx)}{3003b}-\frac{1388\sin(a+bx)\sin^{4}(2a+2bx)\cos^{2}(a+bx)\cos(2a+2bx)}{3003b}-\frac{2944\sin(a+bx)\sin^{2}(2a+2bx)\cos^{2}(a+bx)\cos^{3}(2a+2bx)}{3003b}-\frac{512\sin(a+bx)\cos^{2}(a+bx)\cos^{5}(2a+2bx)}{1001b}+\frac{2234\sin^{5}(2a+2bx)\cos^{3}(a+bx)}{9009b}+\frac{4544\sin^{3}(2a+2bx)\cos^{3}(a+bx)\cos^{2}(2a+2bx)}{9009b}+\frac{256\sin(2a+2bx)\cos^{3}(a+bx)\cos^{4}(2a+2bx)}{1001b} & \text{for}\: b \neq 0 \\ x\sin^{3}(a)\sin^{5}(2a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**5,x)
```

```
[Out] Piecewise((-1366*sin(a + b*x)**3*sin(2*a + 2*b*x)**4*cos(2*a + 2*b*x)/(3003
*b) - 4960*sin(a + b*x)**3*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**3/(9009*b)
 - 256*sin(a + b*x)**3*cos(2*a + 2*b*x)**5/(1287*b) - 271*sin(a + b*x)**2*s
in(2*a + 2*b*x)**5*cos(a + b*x)/(3003*b) - 48*sin(a + b*x)**2*sin(2*a + 2*b
*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**2/(143*b) - 640*sin(a + b*x)**2*sin(2
*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(3003*b) - 1388*sin(a + b*x)*s
in(2*a + 2*b*x)**4*cos(a + b*x)**2*cos(2*a + 2*b*x)/(3003*b) - 2944*sin(a +
 b*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)**3/(3003*b) - 51
2*sin(a + b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**5/(1001*b) + 2234*sin(2*a
+ 2*b*x)**5*cos(a + b*x)**3/(9009*b) + 4544*sin(2*a + 2*b*x)**3*cos(a + b*x
)**3*cos(2*a + 2*b*x)**2/(9009*b) + 256*sin(2*a + 2*b*x)*cos(a + b*x)**3*co
s(2*a + 2*b*x)**4/(1001*b), Ne(b, 0)), (x*sin(a)**3*sin(2*a)**5, True))
```

Giac [A]
time = 0.43, size = 36, normalized size = 0.78

$$\frac{32\left(99\sin\left(bx+a\right)^{13}-234\sin\left(bx+a\right)^{11}+143\sin\left(bx+a\right)^{9}\right)}{1287\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 32/1287*(99*sin(b*x + a)^13 - 234*sin(b*x + a)^11 + 143*sin(b*x + a)^9)/b
```

Mupad [B]
time = 0.07, size = 36, normalized size = 0.78

$$\frac{32\,\left(99\sin\left(a+b\,x\right)^{13}-234\sin\left(a+b\,x\right)^{11}+143\sin\left(a+b\,x\right)^{9}\right)}{1287\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^5,x)
```

```
[Out] (32*(143*sin(a + b*x)^9 - 234*sin(a + b*x)^11 + 99*sin(a + b*x)^13))/(1287*
b)
```

3.24 $\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{16\cos^5(a+bx)}{5b}+\frac{48\cos^7(a+bx)}{7b}-\frac{16\cos^9(a+bx)}{3b}+\frac{16\cos^{11}(a+bx)}{11b}$$

```
[Out] -16/5*cos(b*x+a)^5/b+48/7*cos(b*x+a)^7/b-16/3*cos(b*x+a)^9/b+16/11*cos(b*x+
a)^11/b
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2645, 276}

$$\frac{16\cos^{11}(a+bx)}{11b}-\frac{16\cos^9(a+bx)}{3b}+\frac{48\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-16*Cos[a + b*x]^5)/(5*b) + (48*Cos[a + b*x]^7)/(7*b) - (16*Cos[a + b*x]^9
)/(3*b) + (16*Cos[a + b*x]^11)/(11*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \sin^3(a+bx)\sin^4(2a+2bx)\,dx = 16\int \cos^4(a+bx)\sin^7(a+bx)\,dx$$
$$= -\frac{16\text{Subst}\left(\int x^4(1-x^2)^3\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{16\text{Subst}\left(\int (x^4-3x^6+3x^8-x^{10})\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{16\cos^5(a+bx)}{5b}+\frac{48\cos^7(a+bx)}{7b}-\frac{16\cos^9(a+bx)}{3b}+\frac{16\cos^{11}(a+bx)}{11b}$$

Mathematica [A]
time = 0.25, size = 47, normalized size = 0.77

$$\frac{\cos^5(a+bx)(-3042+3335\cos(2(a+bx))-910\cos(4(a+bx))+105\cos(6(a+bx)))}{2310b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (Cos[a + b*x]^5*(-3042 + 3335*Cos[2*(a + b*x)] - 910*Cos[4*(a + b*x)] + 105
*Cos[6*(a + b*x)]))/(2310*b)
```

Maple [A]
time = 0.10, size = 83, normalized size = 1.36

method	result	size
default	$-\frac{7\cos(xb+a)}{32b}-\frac{\cos(3xb+3a)}{32b}+\frac{11\cos(5xb+5a)}{320b}-\frac{\cos(7xb+7a)}{448b}-\frac{\cos(9xb+9a)}{192b}+\frac{\cos(11xb+11a)}{704b}$	83
risch	$-\frac{7\cos(xb+a)}{32b}-\frac{\cos(3xb+3a)}{32b}+\frac{11\cos(5xb+5a)}{320b}-\frac{\cos(7xb+7a)}{448b}-\frac{\cos(9xb+9a)}{192b}+\frac{\cos(11xb+11a)}{704b}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -7/32*cos(b*x+a)/b-1/32*cos(3*b*x+3*a)/b+11/320*cos(5*b*x+5*a)/b-1/448*cos(
7*b*x+7*a)/b-1/192*cos(9*b*x+9*a)/b+1/704*cos(11*b*x+11*a)/b
```

Maxima [A]
time = 0.27, size = 69, normalized size = 1.13

$$\frac{105\cos(11bx+11a)-385\cos(9bx+9a)-165\cos(7bx+7a)+2541\cos(5bx+5a)-2310\cos(3bx+3a)-16170\cos(bx+a)}{73920b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/73920*(105*cos(11*b*x + 11*a) - 385*cos(9*b*x + 9*a) - 165*cos(7*b*x + 7*
a) + 2541*cos(5*b*x + 5*a) - 2310*cos(3*b*x + 3*a) - 16170*cos(b*x + a))/b
```

Fricas [A]
time = 3.96, size = 46, normalized size = 0.75

$$\frac{16\left(105\cos\left(bx+a\right)^{11}-385\cos\left(bx+a\right)^{9}+495\cos\left(bx+a\right)^{7}-231\cos\left(bx+a\right)^{5}\right)}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 16/1155*(105*cos(b*x + a)^11 - 385*cos(b*x + a)^9 + 495*cos(b*x + a)^7 - 23
1*cos(b*x + a)^5)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(53) = 106$.
time = 15.63, size = 366, normalized size = 6.00

$$\begin{cases} - \frac{472 \sin^{3}{\left(a + bx \right)} \sin^{3}{\left(2a + 2bx \right)} \cos{\left(2a + 2bx \right)}}{1155b} - \frac{64 \sin^{3}{\left(a + bx \right)} \sin{\left(2a + 2bx \right)} \cos^{3}{\left(2a + 2bx \right)}}{231b} - \frac{211 \sin^{2}{\left(a + bx \right)} \sin^{4}{\left(2a + 2bx \right)} \cos{\left(a + bx \right)}}{1155b} - \frac{304 \sin^{2}{\left(a + bx \right)} \sin^{2}{\left(2a + 2bx \right)} \cos{\left(a + bx \right)} \cos^{2}{\left(2a + 2bx \right)}}{385b} - \frac{128 \sin^{2}{\left(a + bx \right)} \cos{\left(a + bx \right)} \cos^{4}{\left(2a + 2bx \right)}}{231b} + \frac{272 \sin{\left(a + bx \right)} \sin^{3}{\left(2a + 2bx \right)} \cos^{2}{\left(a + bx \right)} \cos{\left(2a + 2bx \right)}}{1155b} + \frac{256 \sin{\left(a + bx \right)} \sin{\left(2a + 2bx \right)} \cos^{2}{\left(a + bx \right)} \cos^{3}{\left(2a + 2bx \right)}}{1155b} - \frac{46 \sin^{4}{\left(2a + 2bx \right)} \cos^{3}{\left(a + bx \right)}}{165b} - \frac{192 \sin^{2}{\left(2a + 2bx \right)} \cos^{3}{\left(a + bx \right)} \cos^{2}{\left(2a + 2bx \right)}}{385b} - \frac{256 \cos^{3}{\left(a + bx \right)} \cos^{4}{\left(2a + 2bx \right)}}{1155b} & \text{for}\: b \neq 0 \\x \sin^{3}{\left(a \right)} \sin^{4}{\left(2a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**4,x)
```

```
[Out] Piecewise((-472*sin(a + b*x)**3*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)/(1155*
b) - 64*sin(a + b*x)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(231*b) - 211*
sin(a + b*x)**2*sin(2*a + 2*b*x)**4*cos(a + b*x)/(1155*b) - 304*sin(a + b*x
)**2*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(385*b) - 128*sin
(a + b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**4/(231*b) + 272*sin(a + b*x)*si
n(2*a + 2*b*x)**3*cos(a + b*x)**2*cos(2*a + 2*b*x)/(1155*b) + 256*sin(a + b
*x)*sin(2*a + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**3/(1155*b) - 46*sin(
2*a + 2*b*x)**4*cos(a + b*x)**3/(165*b) - 192*sin(2*a + 2*b*x)**2*cos(a + b
*x)**3*cos(2*a + 2*b*x)**2/(385*b) - 256*cos(a + b*x)**3*cos(2*a + 2*b*x)**
4/(1155*b), Ne(b, 0)), (x*sin(a)**3*sin(2*a)**4, True))
```

Giac [A]
time = 0.42, size = 69, normalized size = 1.13

$$\frac{105\cos\left(11\,bx+11\,a\right)-385\cos\left(9\,bx+9\,a\right)-165\cos\left(7\,bx+7\,a\right)+2541\cos\left(5\,bx+5\,a\right)-2310\cos\left(3\,bx+3\,a\right)-16170\cos\left(bx+a\right)}{73920\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 1/73920*(105*cos(11*b*x + 11*a) - 385*cos(9*b*x + 9*a) - 165*cos(7*b*x + 7*
a) + 2541*cos(5*b*x + 5*a) - 2310*cos(3*b*x + 3*a) - 16170*cos(b*x + a))/b
```

Mupad [B]
time = 0.12, size = 46, normalized size = 0.75

$$-\frac{-\frac{16\cos(a+b\,x)^{11}}{11}+\frac{16\cos(a+b\,x)^9}{3}-\frac{48\cos(a+b\,x)^7}{7}+\frac{16\cos(a+b\,x)^5}{5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^4,x)
[Out] -((16*cos(a + b*x)^5)/5 - (48*cos(a + b*x)^7)/7 + (16*cos(a + b*x)^9)/3 - (
16*cos(a + b*x)^11)/11)/b
```

3.25 $\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{8\sin^7(a+bx)}{7b}-\frac{8\sin^9(a+bx)}{9b}$$

```
[Out] 8/7*sin(b*x+a)^7/b-8/9*sin(b*x+a)^9/b
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2644, 14}

$$\frac{8\sin^7(a+bx)}{7b}-\frac{8\sin^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (8*Sin[a + b*x]^7)/(7*b) - (8*Sin[a + b*x]^9)/(9*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin^3(2a+2bx)\,dx &= 8\int \cos^3(a+bx)\sin^6(a+bx)\,dx\\ &= \frac{8\text{Subst}\left(\int x^6(1-x^2)\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{8\text{Subst}\left(\int (x^6-x^8)\,dx, x, \sin(a+bx)\right)}{b}\\ &= \frac{8\sin^7(a+bx)}{7b}-\frac{8\sin^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 27, normalized size = 0.87

$$\frac{4(11+7\cos(2(a+bx)))\sin^7(a+bx)}{63b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
[Out] (4*(11 + 7*Cos[2*(a + b*x)])*Sin[a + b*x]^7)/(63*b)
```

Maple [A]
time = 0.10, size = 55, normalized size = 1.77

method	result	size
default	$\frac{3\sin(xb+a)}{16b}-\frac{\sin(3xb+3a)}{12b}+\frac{3\sin(7xb+7a)}{224b}-\frac{\sin(9xb+9a)}{288b}$	55
risch	$\frac{3\sin(xb+a)}{16b}-\frac{\sin(3xb+3a)}{12b}+\frac{3\sin(7xb+7a)}{224b}-\frac{\sin(9xb+9a)}{288b}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
[Out] 3/16*sin(b*x+a)/b-1/12*sin(3*b*x+3*a)/b+3/224/b*sin(7*b*x+7*a)-1/288/b*sin(
9*b*x+9*a)
```

Maxima [A]
time = 0.29, size = 47, normalized size = 1.52

$$-\frac{7\sin(9\,bx+9\,a)-27\sin(7\,bx+7\,a)+168\sin(3\,bx+3\,a)-378\sin(bx+a)}{2016\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] -1/2016*(7*sin(9*b*x + 9*a) - 27*sin(7*b*x + 7*a) + 168*sin(3*b*x + 3*a) -
378*sin(b*x + a))/b
```

Fricas [A]
time = 3.74, size = 53, normalized size = 1.71

$$-\frac{8\left(7\cos\left(bx+a\right)^{8}-19\cos\left(bx+a\right)^{6}+15\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}-2\right)\sin\left(bx+a\right)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -8/63*(7*cos(b*x + a)^8 - 19*cos(b*x + a)^6 + 15*cos(b*x + a)^4 - cos(b*x +
 a)^2 - 2)*sin(b*x + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(26) = 52$.
time = 6.69, size = 284, normalized size = 9.16

$$\begin{cases} -\frac{46\sin^{3}\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos\left(2a+2bx\right)}{105b}-\frac{16\sin^{3}\left(a+bx\right)\cos^{3}\left(2a+2bx\right)}{63b}-\frac{13\sin^{2}\left(a+bx\right)\sin^{3}\left(2a+2bx\right)\cos\left(a+bx\right)}{105b}-\frac{8\sin^{2}\left(a+bx\right)\sin\left(2a+2bx\right)\cos\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{35b}-\frac{4\sin\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos^{2}\left(a+bx\right)\cos\left(2a+2bx\right)}{7b}-\frac{64\sin\left(a+bx\right)\cos^{2}\left(a+bx\right)\cos^{3}\left(2a+2bx\right)}{105b}+\frac{94\sin^{3}\left(2a+2bx\right)\cos^{3}\left(a+bx\right)}{315b}+\frac{32\sin\left(2a+2bx\right)\cos^{3}\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{105b} & \text{for}\ b\neq 0 \\ x\sin^{3}\left(a\right)\sin^{3}\left(2a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((-46*sin(a + b*x)**3*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)/(105*b)
 - 16*sin(a + b*x)**3*cos(2*a + 2*b*x)**3/(63*b) - 13*sin(a + b*x)**2*sin(2
*a + 2*b*x)**3*cos(a + b*x)/(105*b) - 8*sin(a + b*x)**2*sin(2*a + 2*b*x)*co
s(a + b*x)*cos(2*a + 2*b*x)**2/(35*b) - 4*sin(a + b*x)*sin(2*a + 2*b*x)**2*
cos(a + b*x)**2*cos(2*a + 2*b*x)/(7*b) - 64*sin(a + b*x)*cos(a + b*x)**2*co
s(2*a + 2*b*x)**3/(105*b) + 94*sin(2*a + 2*b*x)**3*cos(a + b*x)**3/(315*b)
+ 32*sin(2*a + 2*b*x)*cos(a + b*x)**3*cos(2*a + 2*b*x)**2/(105*b), Ne(b, 0)
), (x*sin(a)**3*sin(2*a)**3, True))
```

Giac [A]
time = 0.55, size = 26, normalized size = 0.84

$$-\frac{8\left(7\sin\left(bx+a\right)^{9}-9\sin\left(bx+a\right)^{7}\right)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -8/63*(7*sin(b*x + a)^9 - 9*sin(b*x + a)^7)/b
```

Mupad [B]
time = 0.05, size = 26, normalized size = 0.84

$$\frac{8\left(9\sin\left(a+b\,x\right)^{7}-7\sin\left(a+b\,x\right)^{9}\right)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^3,x)
[Out] (8*(9*sin(a + b*x)^7 - 7*sin(a + b*x)^9))/(63*b)
```

3.26 $\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{4\cos^3(a+bx)}{3b}+\frac{8\cos^5(a+bx)}{5b}-\frac{4\cos^7(a+bx)}{7b}$$

```
[Out] -4/3*cos(b*x+a)^3/b+8/5*cos(b*x+a)^5/b-4/7*cos(b*x+a)^7/b
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2645, 276}

$$-\frac{4\cos^7(a+bx)}{7b}+\frac{8\cos^5(a+bx)}{5b}-\frac{4\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*Cos[a + b*x]^3)/(3*b) + (8*Cos[a + b*x]^5)/(5*b) - (4*Cos[a + b*x]^7)/(
7*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \sin^3(a+bx)\sin^2(2a+2bx)\,dx = 4\int \cos^2(a+bx)\sin^5(a+bx)\,dx$$
$$= -\frac{4\text{Subst}\left(\int x^2(1-x^2)^2\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{4\text{Subst}\left(\int (x^2-2x^4+x^6)\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{4\cos^3(a+bx)}{3b}+\frac{8\cos^5(a+bx)}{5b}-\frac{4\cos^7(a+bx)}{7b}$$

Mathematica [A]
time = 0.12, size = 37, normalized size = 0.80

$$\frac{\cos^3(a+bx)(-157+108\cos(2(a+bx))-15\cos(4(a+bx)))}{210b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (Cos[a + b*x]^3*(-157 + 108*Cos[2*(a + b*x)] - 15*Cos[4*(a + b*x)]))/(210*b
)
```

Maple [A]
time = 0.06, size = 55, normalized size = 1.20

method	result	size
default	$-\frac{5\cos(xb+a)}{16b}-\frac{\cos(3xb+3a)}{48b}+\frac{3\cos(5xb+5a)}{80b}-\frac{\cos(7xb+7a)}{112b}$	55
risch	$-\frac{5\cos(xb+a)}{16b}-\frac{\cos(3xb+3a)}{48b}+\frac{3\cos(5xb+5a)}{80b}-\frac{\cos(7xb+7a)}{112b}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -5/16*cos(b*x+a)/b-1/48*cos(3*b*x+3*a)/b+3/80*cos(5*b*x+5*a)/b-1/112*cos(7*
b*x+7*a)/b
```

Maxima [A]
time = 0.28, size = 47, normalized size = 1.02

$$-\frac{15\cos(7\,bx+7\,a)-63\cos(5\,bx+5\,a)+35\cos(3\,bx+3\,a)+525\cos(bx+a)}{1680\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/1680*(15*cos(7*b*x + 7*a) - 63*cos(5*b*x + 5*a) + 35*cos(3*b*x + 3*a) +
525*cos(b*x + a))/b
```

Fricas [A]
time = 3.55, size = 36, normalized size = 0.78

$$-\frac{4\left(15\cos\left(bx+a\right)^{7}-42\cos\left(bx+a\right)^{5}+35\cos\left(bx+a\right)^{3}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -4/105*(15*cos(b*x + a)^7 - 42*cos(b*x + a)^5 + 35*cos(b*x + a)^3)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(39) = 78$.
time = 2.78, size = 202, normalized size = 4.39

$$\begin{cases} -\frac{12\sin^{3}\left(a+bx\right)\sin\left(2a+2bx\right)\cos\left(2a+2bx\right)}{35b}-\frac{11\sin^{2}\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos\left(a+bx\right)}{35b}-\frac{24\sin^{2}\left(a+bx\right)\cos\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{35b}+\frac{8\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos^{2}\left(a+bx\right)\cos\left(2a+2bx\right)}{35b}-\frac{38\sin^{2}\left(2a+2bx\right)\cos^{3}\left(a+bx\right)}{105b}-\frac{32\cos^{3}\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{105b} & \text{for}\: b\neq 0 \\ x\sin^{3}\left(a\right)\sin^{2}\left(2a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((-12*sin(a + b*x)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)/(35*b) - 1
1*sin(a + b*x)**2*sin(2*a + 2*b*x)**2*cos(a + b*x)/(35*b) - 24*sin(a + b*x)
**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(35*b) + 8*sin(a + b*x)*sin(2*a + 2*b*
x)*cos(a + b*x)**2*cos(2*a + 2*b*x)/(35*b) - 38*sin(2*a + 2*b*x)**2*cos(a +
 b*x)**3/(105*b) - 32*cos(a + b*x)**3*cos(2*a + 2*b*x)**2/(105*b), Ne(b, 0)
), (x*sin(a)**3*sin(2*a)**2, True))
```

Giac [A]
time = 0.45, size = 47, normalized size = 1.02

$$-\frac{15\cos\left(7\,bx+7\,a\right)-63\cos\left(5\,bx+5\,a\right)+35\cos\left(3\,bx+3\,a\right)+525\cos\left(bx+a\right)}{1680\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/1680*(15*cos(7*b*x + 7*a) - 63*cos(5*b*x + 5*a) + 35*cos(3*b*x + 3*a) +
525*cos(b*x + a))/b
```

Mupad [B]
time = 0.12, size = 36, normalized size = 0.78

$$-\frac{4\left(15\cos\left(a+b\,x\right)^{7}-42\cos\left(a+b\,x\right)^{5}+35\cos\left(a+b\,x\right)^{3}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^2,x)
[Out] -(4*(35*cos(a + b*x)^3 - 42*cos(a + b*x)^5 + 15*cos(a + b*x)^7))/(105*b)
```

3.27 $\int \sin^3(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=15

$$\frac{2\sin^5(a+bx)}{5b}$$

```
[Out] 2/5*sin(b*x+a)^5/b
```

Rubi [A]
time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4373, 2644, 30}

$$\frac{2\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*Sin[a + b*x]^5)/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \sin^3(a+bx)\sin(2a+2bx)\,dx = 2\int \cos(a+bx)\sin^4(a+bx)\,dx$$
$$= \frac{2\text{Subst}\left(\int x^4\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{2\sin^5(a+bx)}{5b}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{2\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*Sin[a + b*x]^5)/(5*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(13) = 26$.
time = 0.06, size = 41, normalized size = 2.73

method	result	size
default	$\frac{\sin(xb+a)}{4b} - \frac{\sin(3xb+3a)}{8b} + \frac{\sin(5xb+5a)}{40b}$	41
risch	$\frac{\sin(xb+a)}{4b} - \frac{\sin(3xb+3a)}{8b} + \frac{\sin(5xb+5a)}{40b}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sin(b*x+a)/b-1/8*sin(3*b*x+3*a)/b+1/40/b*sin(5*b*x+5*a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(13) = 26$.
time = 0.27, size = 34, normalized size = 2.27

$$\frac{\sin(5\,bx+5\,a) - 5\,\sin(3\,bx+3\,a) + 10\,\sin(bx+a)}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/40*(sin(5*b*x + 5*a) - 5*sin(3*b*x + 3*a) + 10*sin(b*x + a))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(13) = 26$.
time = 3.38, size = 31, normalized size = 2.07

$$\frac{2\left(\cos\left(bx+a\right)^4 - 2\cos\left(bx+a\right)^2 + 1\right)\sin\left(bx+a\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2/5*(cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1)*sin(b*x + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(12) = 24$.
time = 1.11, size = 117, normalized size = 7.80

$$\begin{cases} -\frac{2\sin^3(a+bx)\cos(2a+2bx)}{5b} - \frac{\sin^2(a+bx)\sin(2a+2bx)\cos(a+bx)}{5b} - \frac{4\sin(a+bx)\cos^2(a+bx)\cos(2a+2bx)}{5b} + \frac{2\sin(2a+2bx)\cos^3(a+bx)}{5b} & \text{for } b \neq 0 \\ x\sin^3(a)\sin(2a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-2*sin(a + b*x)**3*cos(2*a + 2*b*x)/(5*b) - sin(a + b*x)**2*sin(
2*a + 2*b*x)*cos(a + b*x)/(5*b) - 4*sin(a + b*x)*cos(a + b*x)**2*cos(2*a +
2*b*x)/(5*b) + 2*sin(2*a + 2*b*x)*cos(a + b*x)**3/(5*b), Ne(b, 0)), (x*sin(
a)**3*sin(2*a), True))
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.87

$$\frac{2\sin\left(bx+a\right)^5}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 2/5*sin(b*x + a)^5/b
```

Mupad [B]
time = 0.10, size = 13, normalized size = 0.87

$$\frac{2\sin\left(a+b\,x\right)^5}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x),x)
```

```
[Out] (2*sin(a + b*x)^5)/(5*b)
```

3.28 $\int \csc(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=28

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\sin(a+bx)}{2b}$$

```
[Out] 1/2*arctanh(sin(b*x+a))/b-1/2*sin(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {4373, 2672, 327, 212}

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\sin(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]*Sin[a + b*x]^3,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) - Sin[a + b*x]/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
```

```
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(2a+2bx)\sin^3(a+bx)\,dx &= \frac{1}{2}\int \sin(a+bx)\tan(a+bx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b}\\ &= -\frac{\sin(a+bx)}{2b}+\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(a+bx)\right)}{2b}\\ &= \frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 27, normalized size = 0.96

$$\frac{1}{2}\left(\frac{\tanh^{-1}(\sin(a+bx))}{b}-\frac{\sin(a+bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]*Sin[a + b*x]^3,x]
[Out] (ArcTanh[Sin[a + b*x]]/b - Sin[a + b*x]/b)/2
```

Maple [A]
time = 0.06, size = 29, normalized size = 1.04

method	result	size
default	$\frac{-\sin(xb+a)+\ln(\sec(xb+a)+\tan(xb+a))}{2b}$	29
risch	$\frac{i e^{i(xb+a)}}{4b}-\frac{i e^{-i(xb+a)}}{4b}-\frac{\ln\left(e^{i(xb+a)}-i\right)}{2b}+\frac{\ln\left(i+e^{i(xb+a)}\right)}{2b}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/2/b*(-sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(24) = 48.
time = 0.51, size = 124, normalized size = 4.43

$$-\frac{\log\left(\frac{\cos(bx+2a)^2+\cos(a)^2-2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2+2\cos(bx+2a)\sin(a)+\sin(a)^2}{\cos(bx+2a)^2+\cos(a)^2+2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2-2\cos(bx+2a)\sin(a)+\sin(a)^2}\right)+2\sin(bx+a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x
+ 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2
 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + s
in(a)^2)) + 2*sin(b*x + a))/b
```

Fricas [A]
time = 4.71, size = 36, normalized size = 1.29

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)-2\,\sin\left(bx+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1) - 2*sin(b*x + a))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.40, size = 36, normalized size = 1.29

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)-2\,\sin\left(bx+a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1) - 2*sin(b*x + a))/b
```

Mupad [B]
time = 0.12, size = 23, normalized size = 0.82

$$-\frac{\frac{\sin(a+b\,x)}{2}-\frac{\mathrm{atanh}(\sin(a+b\,x))}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x),x)
```

```
[Out] -(sin(a + b*x)/2 - atanh(sin(a + b*x))/2)/b
```

3.29 $\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=13

$$\frac{\sec(a+bx)}{4b}$$

```
[Out] 1/4*sec(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2686, 8}

$$\frac{\sec(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] Sec[a + b*x]/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(2a+2bx)\sin^3(a+bx)\,dx &= \frac{1}{4}\int \sec(a+bx)\tan(a+bx)\,dx \\
&= \frac{\text{Subst}(\int 1\,dx, x, \sec(a+bx))}{4b} \\
&= \frac{\sec(a+bx)}{4b}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\frac{\sec(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] Sec[a + b*x]/(4*b)
```

Maple [A]
time = 0.05, size = 14, normalized size = 1.08

method	result	size
default	$\frac{1}{4\cos(xb+a)b}$	14
risch	$\frac{e^{i(xb+a)}}{2b(e^{2i(xb+a)}+1)}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/cos(b*x+a)/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(11) = 22$.
time = 0.31, size = 83, normalized size = 6.38

$$\frac{\cos(2bx+2a)\cos(bx+a)+\sin(2bx+2a)\sin(bx+a)+\cos(bx+a)}{2\left(b\cos(2bx+2a)^2+b\sin(2bx+2a)^2+2b\cos(2bx+2a)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*
x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 + 2*b*cos(2*b*x + 2*a)
 + b)
```

Fricas [A]
time = 4.17, size = 13, normalized size = 1.00

$$\frac{1}{4b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4/(b*cos(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(11) = 22.
time = 0.45, size = 28, normalized size = 2.15

$$\frac{1}{2\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1))
```

Mupad [**B**]
time = 0.09, size = 13, normalized size = 1.00

$$\frac{1}{4\,b\,\cos\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^2,x)
```

```
[Out] 1/(4*b*cos(a + b*x))
```

3.30 $\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\sec(a+bx)\tan(a+bx)}{16b}$$

```
[Out] 1/16*arctanh(sin(b*x+a))/b+1/16*sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 3853, 3855}

$$\frac{\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\tan(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) + (Sec[a + b*x]*Tan[a + b*x])/(16*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(2a+2bx)\sin^3(a+bx)\,dx &= \frac{1}{8}\int \sec^3(a+bx)\,dx \\ &= \frac{\sec(a+bx)\tan(a+bx)}{16b} + \frac{1}{16}\int \sec(a+bx)\,dx \\ &= \frac{\tanh^{-1}(\sin(a+bx))}{16b} + \frac{\sec(a+bx)\tan(a+bx)}{16b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.12

$$\frac{1}{8}\left(\frac{\tanh^{-1}(\sin(a+bx))}{2b} + \frac{\sec(a+bx)\tan(a+bx)}{2b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^3*Sin[a + b*x]^3,x]
[Out] (ArcTanh[Sin[a + b*x]]/(2*b) + (Sec[a + b*x]*Tan[a + b*x])/(2*b))/8
```

Maple [A]
time = 0.12, size = 37, normalized size = 1.09

method	result	size
default	$\frac{\frac{\sec(xb+a)\tan(xb+a)}{2}+\frac{\ln(\sec(xb+a)+\tan(xb+a))}{2}}{8b}$	37
risch	$-\frac{i\left(e^{3i(xb+a)}-e^{i(xb+a)}\right)}{8b\left(e^{2i(xb+a)}+1\right)^2}-\frac{\ln\left(e^{i(xb+a)}-i\right)}{16b}+\frac{\ln\left(i+e^{i(xb+a)}\right)}{16b}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/8/b*(1/2*sec(b*x+a)*tan(b*x+a)+1/2*ln(sec(b*x+a)+tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(30) = 60.
time = 0.53, size = 480, normalized size = 14.12

$$\frac{4\,(\sin(3\,bx+3\,a)-\sin(bx+a))\cos(4\,bx+4\,a)-(2\,(2\,\cos(2\,bx+2\,a)+1)\cos(4\,bx+4\,a)+\cos(4\,bx+4\,a)^2+4\,\cos(2\,bx+2\,a)^2+\sin(4\,bx+4\,a)^2+4\,\sin(4\,bx+4\,a)\sin(2\,bx+2\,a)+4\,\sin(2\,bx+2\,a)^2+4\,\cos(2\,bx+2\,a)+1)\log\left(\frac{\cos(bx+2\,a)^2+\cos(a)^2-2\,\cos(a)\sin(bx+2\,a)+\sin(bx+2\,a)^2+2\,\cos(bx+2\,a)\sin(a)+\sin(a)^2}{\cos(bx+2\,a)^2+\cos(a)^2+2\,\cos(a)\sin(bx+2\,a)+\sin(bx+2\,a)^2-2\,\cos(bx+2\,a)\sin(a)+\sin(a)^2}\right)-4\,(\cos(3\,bx+3\,a)-\cos(bx+a))\sin(4\,bx+4\,a)+4\,(2\,\cos(2\,bx+2\,a)+1)\sin(3\,bx+3\,a)-8\,\cos(3\,bx+3\,a)\sin(2\,bx+2\,a)+8\,\cos(bx+a)\sin(2\,bx+2\,a)-8\,\cos(2\,bx+2\,a)\sin(bx+a)-4\,\sin(bx+a)}{32\,\left(b\cos(4\,bx+4\,a)^2+4\,b\cos(2\,bx+2\,a)^2+b\sin(4\,bx+4\,a)^2+4\,b\sin(4\,bx+4\,a)\sin(2\,bx+2\,a)+4\,b\sin(2\,bx+2\,a)^2+2\,(2\,b\cos(2\,bx+2\,a)+b)\cos(4\,bx+4\,a)+4\,b\cos(2\,bx+2\,a)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="maxima")
[Out] 1/32*(4*(sin(3*b*x + 3*a) - sin(b*x + a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b*
x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2
```

```
+ sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x +
2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a
)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(
cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 -
2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(cos(3*b*x + 3*a) - cos(b*x + a))*
sin(4*b*x + 4*a) + 4*(2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a) - 8*cos(3*b*
x + 3*a)*sin(2*b*x + 2*a) + 8*cos(b*x + a)*sin(2*b*x + 2*a) - 8*cos(2*b*x +
 2*a)*sin(b*x + a) - 4*sin(b*x + a))/(b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x
+ 2*a)^2 + b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4
*b*sin(2*b*x + 2*a)^2 + 2*(2*b*cos(2*b*x + 2*a) + b)*cos(4*b*x + 4*a) + 4*b
*cos(2*b*x + 2*a) + b)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(30) = 60$.
time = 3.50, size = 61, normalized size = 1.79

$$\frac{\cos(bx+a)^2\log(\sin(bx+a)+1)-\cos(bx+a)^2\log(-\sin(bx+a)+1)+2\sin(bx+a)}{32\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(cos(b*x + a)^2*log(sin(b*x + a) + 1) - cos(b*x + a)^2*log(-sin(b*x +
a) + 1) + 2*sin(b*x + a))/(b*cos(b*x + a)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 48, normalized size = 1.41

$$-\frac{\frac{2\sin(bx+a)}{\sin(bx+a)^2-1}-\log(\sin(bx+a)+1)+\log(-\sin(bx+a)+1)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/32*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) - log(sin(b*x + a) + 1) + log(-s
in(b*x + a) + 1))/b
```

Mupad [B]
time = 0.17, size = 36, normalized size = 1.06

$$\frac{\operatorname{atanh}(\sin(a+b\,x))}{16\,b}-\frac{\sin(a+b\,x)}{16\,b\,\left(\sin(a+b\,x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^3,x)
[Out] atanh(sin(a + b*x))/(16*b) - sin(a + b*x)/(16*b*(sin(a + b*x)^2 - 1))
```

3.31 $\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}(\cos(a+bx))}{16b}+\frac{\sec(a+bx)}{16b}+\frac{\sec^3(a+bx)}{48b}$$

```
[Out] -1/16*arctanh(cos(b*x+a))/b+1/16*sec(b*x+a)/b+1/48*sec(b*x+a)^3/b
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4373, 2702, 308, 213}

$$\frac{\sec^3(a+bx)}{48b}+\frac{\sec(a+bx)}{16b}-\frac{\tanh^{-1}(\cos(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^3,x]
```

```
[Out] -1/16*ArcTanh[Cos[a + b*x]]/b + Sec[a + b*x]/(16*b) + Sec[a + b*x]^3/(48*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc^4(2a+2bx)\sin^3(a+bx)\,dx &= \frac{1}{16}\int \csc(a+bx)\sec^4(a+bx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{16b}\\
&= \frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \sec(a+bx)\right)}{16b}\\
&= \frac{\sec(a+bx)}{16b}+\frac{\sec^3(a+bx)}{48b}+\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{16b}\\
&= -\frac{\tanh^{-1}(\cos(a+bx))}{16b}+\frac{\sec(a+bx)}{16b}+\frac{\sec^3(a+bx)}{48b}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 1.42

$$
\frac{1}{16}\left(-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\sec(a+bx)}{b}+\frac{\sec^3(a+bx)}{3b}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^4*Sin[a + b*x]^3,x]
[Out] (-(Log[Cos[(a + b*x)/2]]/b) + Log[Sin[(a + b*x)/2]]/b + Sec[a + b*x]/b + Se
c[a + b*x]^3/(3*b))/16
```

Maple [A]
time = 0.11, size = 41, normalized size = 0.95

method	result	size
default	$\frac{\frac{1}{3\cos(xb+a)^3}+\frac{1}{\cos(xb+a)}+\ln(\csc(xb+a)-\cot(xb+a))}{16b}$	41
risch	$\frac{3\,e^{5i(xb+a)}+10\,e^{3i(xb+a)}+3\,e^{i(xb+a)}}{24b\left(e^{2i(xb+a)}+1\right)^3}+\frac{\ln\left(e^{i(xb+a)}-1\right)}{16b}-\frac{\ln\left(e^{i(xb+a)}+1\right)}{16b}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
[Out] 1/16/b*(1/3/cos(b*x+a)^3+1/cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 987 vs. 2(37) = 74.
time = 0.32, size = 987, normalized size = 22.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(4*(3*cos(5*b*x + 5*a) + 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*b
*x + 6*a) + 12*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x + 5*
a) + 12*(10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(4*b*x + 4*a) + 40*(3*cos
(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) + 36*cos(2*b*x + 2*a)*cos(b*x + a) - 3*
(2*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) + cos(6*b
*x + 6*a)^2 + 6*(3*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 9*cos(4*b*x + 4
*a)^2 + 9*cos(2*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(
6*b*x + 6*a) + sin(6*b*x + 6*a)^2 + 9*sin(4*b*x + 4*a)^2 + 18*sin(4*b*x + 4
*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) + 1)*log(c
os(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) +
 sin(a)^2) + 3*(2*(3*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x +
 6*a) + cos(6*b*x + 6*a)^2 + 6*(3*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) +
9*cos(4*b*x + 4*a)^2 + 9*cos(2*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) + sin(2*b
*x + 2*a))*sin(6*b*x + 6*a) + sin(6*b*x + 6*a)^2 + 9*sin(4*b*x + 4*a)^2 + 1
8*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x +
2*a) + 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*si
n(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(5*b*x + 5*a) + 10*sin(3*b*x + 3*a) + 3
*sin(b*x + a))*sin(6*b*x + 6*a) + 36*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*
sin(5*b*x + 5*a) + 12*(10*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(4*b*x + 4*
a) + 120*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + 36*sin(2*b*x + 2*a)*sin(b*x +
a) + 12*cos(b*x + a))/(b*cos(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + 9*b*
cos(2*b*x + 2*a)^2 + b*sin(6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 + 18*b*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 + 2*(3*b*cos(4*b*
x + 4*a) + 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) + 6*(3*b*cos(2*b*x +
2*a) + b)*cos(4*b*x + 4*a) + 6*b*cos(2*b*x + 2*a) + 6*(b*sin(4*b*x + 4*a) +
 b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [A]
time = 2.86, size = 67, normalized size = 1.56

$$-\frac{3 \cos \left(bx+a\right)^{3} \log \left(\frac{1}{2} \cos \left(bx+a\right)+\frac{1}{2}\right)-3 \cos \left(bx+a\right)^{3} \log \left(-\frac{1}{2} \cos \left(bx+a\right)+\frac{1}{2}\right)-6 \cos \left(bx+a\right)^{2}-2}{96 \, b \cos \left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(3*cos(b*x + a)^3*log(1/2*cos(b*x + a) + 1/2) - 3*cos(b*x + a)^3*log(
-1/2*cos(b*x + a) + 1/2) - 6*cos(b*x + a)^2 - 2)/(b*cos(b*x + a)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**4*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(37) = 74$.
time = 0.47, size = 98, normalized size = 2.28

$$\frac{\frac{8\left(\frac{3(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+2\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right)^3}+3\log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^4*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/96*(8*(3*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 + 2)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1)^3 + 3*
log(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

Mupad [B]
time = 0.07, size = 37, normalized size = 0.86

$$\frac{\frac{\cos(a+b\,x)^2}{16}+\frac{1}{48}}{b\cos\left(a+b\,x\right)^3}-\frac{\mathrm{atanh}(\cos\left(a+b\,x\right))}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^4,x)
```

```
[Out] (cos(a + b*x)^2/16 + 1/48)/(b*cos(a + b*x)^3) - atanh(cos(a + b*x))/(16*b)
```

3.32 $\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=70

$$\frac{15\tanh^{-1}(\sin(a+bx))}{256b}-\frac{15\csc(a+bx)}{256b}+\frac{5\csc(a+bx)\sec^2(a+bx)}{256b}+\frac{\csc(a+bx)\sec^4(a+bx)}{128b}$$

```
[Out] 15/256*arctanh(sin(b*x+a))/b-15/256*csc(b*x+a)/b+5/256*csc(b*x+a)*sec(b*x+a
)^2/b+1/128*csc(b*x+a)*sec(b*x+a)^4/b
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4373, 2701, 294, 327, 213}

$$-\frac{15\csc(a+bx)}{256b}+\frac{15\tanh^{-1}(\sin(a+bx))}{256b}+\frac{\csc(a+bx)\sec^4(a+bx)}{128b}+\frac{5\csc(a+bx)\sec^2(a+bx)}{256b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^3,x]
```

```
[Out] (15*ArcTanh[Sin[a + b*x]])/(256*b) - (15*Csc[a + b*x])/(256*b) + (5*Csc[a +
 b*x]*Sec[a + b*x]^2)/(256*b) + (Csc[a + b*x]*Sec[a + b*x]^4)/(128*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc^5(2a+2bx)\sin^3(a+bx)\,dx &= \frac{1}{32}\int \csc^2(a+bx)\sec^5(a+bx)\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \csc(a+bx)\right)}{32b} \\
&= \frac{\csc(a+bx)\sec^4(a+bx)}{128b} - \frac{5\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{128b} \\
&= \frac{5\csc(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc(a+bx)\sec^4(a+bx)}{128b} - \frac{15\text{Subst}\left(\int \frac{x}{-1\cdots}\right.}{} \\
&= -\frac{15\csc(a+bx)}{256b} + \frac{5\csc(a+bx)\sec^2(a+bx)}{256b} + \frac{\csc(a+bx)\sec^4(a+b\cdots}{128b} \\
&= \frac{15\tanh^{-1}(\sin(a+bx))}{256b} - \frac{15\csc(a+bx)}{256b} + \frac{5\csc(a+bx)\sec^2(a+bx)}{256b} \cdots
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 29, normalized size = 0.41

$$-\frac{\csc(a+bx)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\sin^2(a+bx)\right)}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*a + 2*b*x]^5*Sin[a + b*x]^3,x]
[Out] -1/32*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 3, 1/2, Sin[a + b*x]^2])/b
```

Maple [A]
time = 0.10, size = 69, normalized size = 0.99

method	result	size
default	$\frac{\frac{1}{4\sin(xb+a)\cos(xb+a)^4}+\frac{5}{8\sin(xb+a)\cos(xb+a)^2}-\frac{15}{8\sin(xb+a)}+\frac{15\ln(\sec(xb+a)+\tan(xb+a))}{8}}{32b}$	69
risch	$-\frac{i(15\,\mathrm{e}^{9i(xb+a)}+40\,\mathrm{e}^{7i(xb+a)}+18\,\mathrm{e}^{5i(xb+a)}+40\,\mathrm{e}^{3i(xb+a)}+15\,\mathrm{e}^{i(xb+a)})}{128b(\mathrm{e}^{2i(xb+a)}+1)^4(\mathrm{e}^{2i(xb+a)}-1)}+\frac{15\ln(i+\mathrm{e}^{i(xb+a)})}{256b}-\frac{15\ln(\mathrm{e}^{i(xb+a)}-i)}{256b}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/b*(1/4/sin(b*x+a)/cos(b*x+a)^4+5/8/sin(b*x+a)/cos(b*x+a)^2-15/8/sin(b*
x+a)+15/8*ln(sec(b*x+a)+tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1805 vs. $2(62) = 124$.
time = 0.59, size = 1805, normalized size = 25.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/512*(4*(15*sin(9*b*x + 9*a) + 40*sin(7*b*x + 7*a) + 18*sin(5*b*x + 5*a) +
 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) - 60*(3*sin(8*b*
x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*co
s(9*b*x + 9*a) + 12*(40*sin(7*b*x + 7*a) + 18*sin(5*b*x + 5*a) + 40*sin(3*b
*x + 3*a) + 15*sin(b*x + a))*cos(8*b*x + 8*a) - 160*(2*sin(6*b*x + 6*a) - 2
*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(18*sin(5*b*x
+ 5*a) + 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(6*b*x + 6*a) + 72*(2*si
n(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(8*sin(3*b*x + 3
*a) + 3*sin(b*x + a))*cos(4*b*x + 4*a) - 15*(2*(3*cos(8*b*x + 8*a) + 2*cos(
6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10
*a) + cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 3
*cos(2*b*x + 2*a) - 1)*cos(8*b*x + 8*a) + 9*cos(8*b*x + 8*a)^2 - 4*(2*cos(4
*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) + 4*cos(6*b*x + 6*a)
^2 + 4*(3*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + 4*cos(4*b*x + 4*a)^2 + 9
*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*
b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) + sin(10*b*x + 10*a)^2
+ 6*(2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(8*b*
x + 8*a) + 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a
))*sin(6*b*x + 6*a) + 4*sin(6*b*x + 6*a)^2 + 4*sin(4*b*x + 4*a)^2 + 12*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) +
 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 +
 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin
```

```
(a)^2)) - 4*(15*cos(9*b*x + 9*a) + 40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a
) + 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*sin(10*b*x + 10*a) + 60*(3*cos(8
*b*x + 8*a) + 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a)
- 1)*sin(9*b*x + 9*a) - 12*(40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a) + 40*
cos(3*b*x + 3*a) + 15*cos(b*x + a))*sin(8*b*x + 8*a) + 160*(2*cos(6*b*x + 6
*a) - 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) - 1)*sin(7*b*x + 7*a) - 8*(18
*cos(5*b*x + 5*a) + 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*sin(6*b*x + 6*a)
 - 72*(2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) + 40*(
8*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(4*b*x + 4*a) - 160*(3*cos(2*b*x +
2*a) + 1)*sin(3*b*x + 3*a) + 480*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + 180*co
s(b*x + a)*sin(2*b*x + 2*a) - 180*cos(2*b*x + 2*a)*sin(b*x + a) - 60*sin(b*
x + a))/(b*cos(10*b*x + 10*a)^2 + 9*b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x +
6*a)^2 + 4*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(10*b*x + 1
0*a)^2 + 9*b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x +
4*a)^2 + 12*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 +
2*(3*b*cos(8*b*x + 8*a) + 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) - 3*b
*cos(2*b*x + 2*a) - b)*cos(10*b*x + 10*a) + 6*(2*b*cos(6*b*x + 6*a) - 2*b*c
os(4*b*x + 4*a) - 3*b*cos(2*b*x + 2*a) - b)*cos(8*b*x + 8*a) - 4*(2*b*cos(4
*b*x + 4*a) + 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) + 4*(3*b*cos(2*b*x
 + 2*a) + b)*cos(4*b*x + 4*a) + 6*b*cos(2*b*x + 2*a) + 2*(3*b*sin(8*b*x + 8
*a) + 2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x + 2*a))*s
in(10*b*x + 10*a) + 6*(2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - 3*b*si
n(2*b*x + 2*a))*sin(8*b*x + 8*a) - 4*(2*b*sin(4*b*x + 4*a) + 3*b*sin(2*b*x
+ 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [**A**]
time = 2.63, size = 95, normalized size = 1.36

$$\frac{15\cos(bx+a)^4\log(\sin(bx+a)+1)\sin(bx+a)-15\cos(bx+a)^4\log(-\sin(bx+a)+1)\sin(bx+a)-30\cos(bx+a)^4+10\cos(bx+a)^2+4}{512\,b\cos(bx+a)^4\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(15*cos(b*x + a)^4*log(sin(b*x + a) + 1)*sin(b*x + a) - 15*cos(b*x +
a)^4*log(-sin(b*x + a) + 1)*sin(b*x + a) - 30*cos(b*x + a)^4 + 10*cos(b*x +
 a)^2 + 4)/(b*cos(b*x + a)^4*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)**5*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 73, normalized size = 1.04

$$-\frac{\frac{2\left(7\sin(bx+a)^3-9\sin(bx+a)\right)}{\left(\sin(bx+a)^2-1\right)^2}+\frac{16}{\sin(bx+a)}-15\log\left(\sin\left(bx+a\right)+1\right)+15\log\left(-\sin\left(bx+a\right)+1\right)}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(2*b*x+2*a)^5*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/512*(2*(7*sin(b*x + a)^3 - 9*sin(b*x + a))/(sin(b*x + a)^2 - 1)^2 + 16/s
in(b*x + a) - 15*log(sin(b*x + a) + 1) + 15*log(-sin(b*x + a) + 1))/b
```

Mupad [B]
time = 0.19, size = 67, normalized size = 0.96

$$\frac{15\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{256\,b}-\frac{\frac{15\sin(a+b\,x)^4}{256}-\frac{25\sin(a+b\,x)^2}{256}+\frac{1}{32}}{b\,\left(\sin\left(a+b\,x\right)^5-2\sin\left(a+b\,x\right)^3+\sin\left(a+b\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^5,x)
```

```
[Out] (15*atanh(sin(a + b*x)))/(256*b) - ((15*sin(a + b*x)^4)/256 - (25*sin(a + b
*x)^2)/256 + 1/32)/(b*(sin(a + b*x) - 2*sin(a + b*x)^3 + sin(a + b*x)^5))
```

3.33 $\int \csc(a+bx)\sin^8(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{256\cos^9(a+bx)}{9b}+\frac{768\cos^{11}(a+bx)}{11b}-\frac{768\cos^{13}(a+bx)}{13b}+\frac{256\cos^{15}(a+bx)}{15b}$$

```
[Out] -256/9*cos(b*x+a)^9/b+768/11*cos(b*x+a)^11/b-768/13*cos(b*x+a)^13/b+256/15*
cos(b*x+a)^15/b
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4373, 2645, 276}

$$\frac{256\cos^{15}(a+bx)}{15b}-\frac{768\cos^{13}(a+bx)}{13b}+\frac{768\cos^{11}(a+bx)}{11b}-\frac{256\cos^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (-256*Cos[a + b*x]^9)/(9*b) + (768*Cos[a + b*x]^11)/(11*b) - (768*Cos[a + b
*x]^13)/(13*b) + (256*Cos[a + b*x]^15)/(15*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc(a+bx)\sin^8(2a+2bx)\,dx = 256\int \cos^8(a+bx)\sin^7(a+bx)\,dx$$
$$= -\frac{256\mathrm{Subst}\left(\int x^8(1-x^2)^3\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{256\mathrm{Subst}\left(\int (x^8-3x^{10}+3x^{12}-x^{14})\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{256\cos^9(a+bx)}{9b} + \frac{768\cos^{11}(a+bx)}{11b} - \frac{768\cos^{13}(a+bx)}{13b} + \frac{256\cos^{15}(a}{15b}$$

Mathematica [A]
time = 0.11, size = 119, normalized size = 1.95

$$-\frac{35\cos(a+bx)}{64b}-\frac{35\cos(3(a+bx))}{192b}+\frac{21\cos(5(a+bx))}{320b}+\frac{3\cos(7(a+bx))}{64b}-\frac{7\cos(9(a+bx))}{576b}-\frac{7\cos(11(a+bx))}{704b}+\frac{\cos(13(a+bx))}{832b}+\frac{\cos(15(a+bx))}{960b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (-35*Cos[a + b*x])/(64*b) - (35*Cos[3*(a + b*x)])/(192*b) + (21*Cos[5*(a +
b*x)])/(320*b) + (3*Cos[7*(a + b*x)])/(64*b) - (7*Cos[9*(a + b*x)])/(576*b)
 - (7*Cos[11*(a + b*x)])/(704*b) + Cos[13*(a + b*x)]/(832*b) + Cos[15*(a +
b*x)]/(960*b)
```

Maple [A]
time = 0.09, size = 71, normalized size = 1.16

method	result
default	$\frac{-\frac{256\left(\sin^6(xb+a)\right)\left(\cos^9(xb+a)\right)}{15}-\frac{512\left(\sin^4(xb+a)\right)\left(\cos^9(xb+a)\right)}{65}-\frac{2048\left(\sin^2(xb+a)\right)\left(\cos^9(xb+a)\right)}{715}-\frac{4096\left(\cos^9(xb+a)\right)}{6435}}{b}$
risch	$-\frac{35\cos(xb+a)}{64b}+\frac{\cos(15xb+15a)}{960b}+\frac{\cos(13xb+13a)}{832b}-\frac{7\cos(11xb+11a)}{704b}-\frac{7\cos(9xb+9a)}{576b}+\frac{3\cos(7xb+7a)}{64b}+\frac{21\cos(5xb+5a}{320b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 256/b*(-1/15*sin(b*x+a)^6*cos(b*x+a)^9-2/65*sin(b*x+a)^4*cos(b*x+a)^9-8/715
*sin(b*x+a)^2*cos(b*x+a)^9-16/6435*cos(b*x+a)^9)
```

Maxima [A]
time = 0.29, size = 91, normalized size = 1.49

$$\frac{429\cos(15bx+15a)+495\cos(13bx+13a)-4095\cos(11bx+11a)-5005\cos(9bx+9a)+19305\cos(7bx+7a)+27027\cos(5bx+5a)-75075\cos(3bx+3a)-225225\cos(bx+a)}{411840b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

```
[Out] 1/411840*(429*cos(15*b*x + 15*a) + 495*cos(13*b*x + 13*a) - 4095*cos(11*b*x
 + 11*a) - 5005*cos(9*b*x + 9*a) + 19305*cos(7*b*x + 7*a) + 27027*cos(5*b*x
 + 5*a) - 75075*cos(3*b*x + 3*a) - 225225*cos(b*x + a))/b
```

Fricas [A]
time = 3.27, size = 46, normalized size = 0.75

$$\frac{256\left(429\cos\left(bx+a\right)^{15}-1485\cos\left(bx+a\right)^{13}+1755\cos\left(bx+a\right)^{11}-715\cos\left(bx+a\right)^{9}\right)}{6435\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

```
[Out] 256/6435*(429*cos(b*x + a)^15 - 1485*cos(b*x + a)^13 + 1755*cos(b*x + a)^11
 - 715*cos(b*x + a)^9)/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. 2(53) = 106.
time = 0.43, size = 270, normalized size = 4.43

$$-\frac{8192\left(\frac{15\left(\cos\left(bx+a\right)-1\right)}{\cos\left(bx+a\right)+1}-\frac{105\left(\cos\left(bx+a\right)-1\right)^{2}}{\left(\cos\left(bx+a\right)+1\right)^{2}}+\frac{455\left(\cos\left(bx+a\right)-1\right)^{3}}{\left(\cos\left(bx+a\right)+1\right)^{3}}+\frac{5070\left(\cos\left(bx+a\right)-1\right)^{4}}{\left(\cos\left(bx+a\right)+1\right)^{4}}+\frac{30030\left(\cos\left(bx+a\right)-1\right)^{5}}{\left(\cos\left(bx+a\right)+1\right)^{5}}+\frac{70070\left(\cos\left(bx+a\right)-1\right)^{6}}{\left(\cos\left(bx+a\right)+1\right)^{6}}+\frac{115830\left(\cos\left(bx+a\right)-1\right)^{7}}{\left(\cos\left(bx+a\right)+1\right)^{7}}+\frac{109395\left(\cos\left(bx+a\right)-1\right)^{8}}{\left(\cos\left(bx+a\right)+1\right)^{8}}+\frac{75075\left(\cos\left(bx+a\right)-1\right)^{9}}{\left(\cos\left(bx+a\right)+1\right)^{9}}+\frac{27027\left(\cos\left(bx+a\right)-1\right)^{10}}{\left(\cos\left(bx+a\right)+1\right)^{10}}+\frac{6435\left(\cos\left(bx+a\right)-1\right)^{11}}{\left(\cos\left(bx+a\right)+1\right)^{11}}-1\right)}{6435\,b\left(\frac{\cos\left(bx+a\right)-1}{\cos\left(bx+a\right)+1}-1\right)^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

```
[Out] -8192/6435*(15*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 105*(cos(b*x + a) -
1)^2/(cos(b*x + a) + 1)^2 + 455*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 +
 5070*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 30030*(cos(b*x + a) - 1)^
5/(cos(b*x + a) + 1)^5 + 70070*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 +
115830*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 + 109395*(cos(b*x + a) - 1
)^8/(cos(b*x + a) + 1)^8 + 75075*(cos(b*x + a) - 1)^9/(cos(b*x + a) + 1)^9
+ 27027*(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 + 6435*(cos(b*x + a) -
1)^11/(cos(b*x + a) + 1)^11 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1)
- 1)^15)
```

Mupad [B]
time = 0.15, size = 46, normalized size = 0.75

$$-\frac{-\frac{256\cos(a+b\,x)^{15}}{15}+\frac{768\cos(a+b\,x)^{13}}{13}-\frac{768\cos(a+b\,x)^{11}}{11}+\frac{256\cos(a+b\,x)^{9}}{9}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^8/sin(a + b*x),x)
```

```
[Out] -((256*cos(a + b*x)^9)/9 - (768*cos(a + b*x)^11)/11 + (768*cos(a + b*x)^13)
/13 - (256*cos(a + b*x)^15)/15)/b
```

3.34 $\int \csc(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{128\sin^7(a+bx)}{7b}-\frac{128\sin^9(a+bx)}{3b}+\frac{384\sin^{11}(a+bx)}{11b}-\frac{128\sin^{13}(a+bx)}{13b}$$

```
[Out] 128/7*sin(b*x+a)^7/b-128/3*sin(b*x+a)^9/b+384/11*sin(b*x+a)^11/b-128/13*sin
(b*x+a)^13/b
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4373, 2644, 276}

$$-\frac{128\sin^{13}(a+bx)}{13b}+\frac{384\sin^{11}(a+bx)}{11b}-\frac{128\sin^9(a+bx)}{3b}+\frac{128\sin^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*Sin[a + b*x]^7)/(7*b) - (128*Sin[a + b*x]^9)/(3*b) + (384*Sin[a + b*x]
^11)/(11*b) - (128*Sin[a + b*x]^13)/(13*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc(a+bx)\sin^7(2a+2bx)\,dx = 128\int \cos^7(a+bx)\sin^6(a+bx)\,dx$$
$$= \frac{128\text{Subst}\left(\int x^6(1-x^2)^3\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{128\text{Subst}\left(\int (x^6-3x^8+3x^{10}-x^{12})\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{128\sin^7(a+bx)}{7b} - \frac{128\sin^9(a+bx)}{3b} + \frac{384\sin^{11}(a+bx)}{11b} - \frac{128\sin^{13}(a+}{13b}$$

Mathematica [A]
time = 0.22, size = 48, normalized size = 0.79

$$\frac{128(429\sin^7(a+bx)-1001\sin^9(a+bx)+819\sin^{11}(a+bx)-231\sin^{13}(a+bx))}{3003b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*(429*Sin[a + b*x]^7 - 1001*Sin[a + b*x]^9 + 819*Sin[a + b*x]^11 - 231*
Sin[a + b*x]^13))/(3003*b)
```

Maple [A]
time = 0.11, size = 97, normalized size = 1.59

method	result
default	$\frac{-\frac{128\left(\sin^5(xb+a)\right)\left(\cos^8(xb+a)\right)}{13}-\frac{640\left(\sin^3(xb+a)\right)\left(\cos^8(xb+a)\right)}{143}-\frac{640\sin(xb+a)\left(\cos^8(xb+a)\right)}{429}+\frac{640\left(\frac{16}{5}+\cos^6(xb+a)+\frac{6\left(\cos^4(xb+a)\right)}{5}+\frac{8\left(\cos^2(\right.}{5}\right.}{3003}}{b}$
risch	$\frac{5\sin(xb+a)}{8b}-\frac{\sin(13xb+13a)}{416b}-\frac{\sin(11xb+11a)}{352b}+\frac{\sin(9xb+9a)}{48b}+\frac{3\sin(7xb+7a)}{112b}-\frac{3\sin(5xb+5a)}{32b}-\frac{5\sin(3xb+3a)}{32b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 128/b*(-1/13*sin(b*x+a)^5*cos(b*x+a)^8-5/143*sin(b*x+a)^3*cos(b*x+a)^8-5/42
9*sin(b*x+a)*cos(b*x+a)^8+5/3003*(16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*co
s(b*x+a)^2)*sin(b*x+a))
```

Maxima [A]
time = 0.28, size = 80, normalized size = 1.31

$$-\frac{231\sin(13bx+13a)+273\sin(11bx+11a)-2002\sin(9bx+9a)-2574\sin(7bx+7a)+9009\sin(5bx+5a)+15015\sin(3bx+3a)-60060\sin(bx+a)}{96096b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")

[Out] -1/96096*(231*sin(13*b*x + 13*a) + 273*sin(11*b*x + 11*a) - 2002*sin(9*b*x + 9*a) - 2574*sin(7*b*x + 7*a) + 9009*sin(5*b*x + 5*a) + 15015*sin(3*b*x + 3*a) - 60060*sin(b*x + a))/b

Fricas [A]
time = 2.92, size = 73, normalized size = 1.20

$$-\frac{128\left(231\cos\left(bx+a\right)^{12}-567\cos\left(bx+a\right)^{10}+371\cos\left(bx+a\right)^{8}-5\cos\left(bx+a\right)^{6}-6\cos\left(bx+a\right)^{4}-8\cos\left(bx+a\right)^{2}-16\right)\sin\left(bx+a\right)}{3003\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")

[Out] -128/3003*(231*cos(b*x + a)^12 - 567*cos(b*x + a)^10 + 371*cos(b*x + a)^8 - 5*cos(b*x + a)^6 - 6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 16)*sin(b*x + a)/b

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**7,x)

[Out] Timed out

Giac [A]
time = 0.42, size = 46, normalized size = 0.75

$$-\frac{128\left(231\sin\left(bx+a\right)^{13}-819\sin\left(bx+a\right)^{11}+1001\sin\left(bx+a\right)^{9}-429\sin\left(bx+a\right)^{7}\right)}{3003\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")

[Out] -128/3003*(231*sin(b*x + a)^13 - 819*sin(b*x + a)^11 + 1001*sin(b*x + a)^9 - 429*sin(b*x + a)^7)/b

Mupad [B]
time = 0.13, size = 45, normalized size = 0.74

$$\frac{-\frac{128\sin(a+b\,x)^{13}}{13}+\frac{384\sin(a+b\,x)^{11}}{11}-\frac{128\sin(a+b\,x)^{9}}{3}+\frac{128\sin(a+b\,x)^{7}}{7}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^7/sin(a + b*x),x)
```

```
[Out] ((128*sin(a + b*x)^7)/7 - (128*sin(a + b*x)^9)/3 + (384*sin(a + b*x)^11)/11
 - (128*sin(a + b*x)^13)/13)/b
```

3.35 $\int \csc(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{64\cos^7(a+bx)}{7b}+\frac{128\cos^9(a+bx)}{9b}-\frac{64\cos^{11}(a+bx)}{11b}$$

```
[Out] -64/7*cos(b*x+a)^7/b+128/9*cos(b*x+a)^9/b-64/11*cos(b*x+a)^11/b
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4373, 2645, 276}

$$-\frac{64\cos^{11}(a+bx)}{11b}+\frac{128\cos^9(a+bx)}{9b}-\frac{64\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (-64*Cos[a + b*x]^7)/(7*b) + (128*Cos[a + b*x]^9)/(9*b) - (64*Cos[a + b*x]^
11)/(11*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc(a+bx)\sin^6(2a+2bx)\,dx = 64\int \cos^6(a+bx)\sin^5(a+bx)\,dx$$
$$= -\frac{64\text{Subst}\left(\int x^6(1-x^2)^2\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{64\text{Subst}\left(\int (x^6-2x^8+x^{10})\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{64\cos^7(a+bx)}{7b}+\frac{128\cos^9(a+bx)}{9b}-\frac{64\cos^{11}(a+bx)}{11b}$$

Mathematica [A]
time = 0.07, size = 89, normalized size = 1.93

$$-\frac{5\cos(a+bx)}{8b}-\frac{5\cos(3(a+bx))}{24b}+\frac{\cos(5(a+bx))}{16b}+\frac{5\cos(7(a+bx))}{112b}-\frac{\cos(9(a+bx))}{144b}-\frac{\cos(11(a+bx))}{176b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (-5*Cos[a + b*x])/(8*b) - (5*Cos[3*(a + b*x)])/(24*b) + Cos[5*(a + b*x)]/(1
6*b) + (5*Cos[7*(a + b*x)])/(112*b) - Cos[9*(a + b*x)]/(144*b) - Cos[11*(a
+ b*x)]/(176*b)
```

Maple [A]
time = 0.09, size = 53, normalized size = 1.15

method	result	size
default	$\frac{-\frac{64\left(\sin^4(xb+a)\right)\left(\cos^7(xb+a)\right)}{11}-\frac{256\left(\sin^2(xb+a)\right)\left(\cos^7(xb+a)\right)}{99}-\frac{512\left(\cos^7(xb+a)\right)}{693}}{b}$	53
risch	$-\frac{5\cos(xb+a)}{8b}-\frac{\cos(11xb+11a)}{176b}-\frac{\cos(9xb+9a)}{144b}+\frac{5\cos(7xb+7a)}{112b}+\frac{\cos(5xb+5a)}{16b}-\frac{5\cos(3xb+3a)}{24b}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 64/b*(-1/11*sin(b*x+a)^4*cos(b*x+a)^7-4/99*sin(b*x+a)^2*cos(b*x+a)^7-8/693*
cos(b*x+a)^7)
```

Maxima [A]
time = 0.29, size = 69, normalized size = 1.50

$$-\frac{63\cos(11\,bx+11\,a)+77\cos(9\,bx+9\,a)-495\cos(7\,bx+7\,a)-693\cos(5\,bx+5\,a)+2310\cos(3\,bx+3\,a)+6930\cos(bx+a)}{11088\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] -1/11088*(63*cos(11*b*x + 11*a) + 77*cos(9*b*x + 9*a) - 495*cos(7*b*x + 7*a
) - 693*cos(5*b*x + 5*a) + 2310*cos(3*b*x + 3*a) + 6930*cos(b*x + a))/b
```

Fricas [A]
time = 3.36, size = 36, normalized size = 0.78

$$-\frac{64\left(63\cos\left(bx+a\right)^{11}-154\cos\left(bx+a\right)^{9}+99\cos\left(bx+a\right)^{7}\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] -64/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**6,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(40) = 80.
time = 0.43, size = 204, normalized size = 4.43

$$-\frac{1024\left(\frac{11\left(\cos(bx+a)-1\right)}{\cos(bx+a)+1}-\frac{55\left(\cos(bx+a)-1\right)^{2}}{\left(\cos(bx+a)+1\right)^{2}}-\frac{297\left(\cos(bx+a)-1\right)^{3}}{\left(\cos(bx+a)+1\right)^{3}}-\frac{1485\left(\cos(bx+a)-1\right)^{4}}{\left(\cos(bx+a)+1\right)^{4}}-\frac{2079\left(\cos(bx+a)-1\right)^{5}}{\left(\cos(bx+a)+1\right)^{5}}-\frac{2541\left(\cos(bx+a)-1\right)^{6}}{\left(\cos(bx+a)+1\right)^{6}}-\frac{1155\left(\cos(bx+a)-1\right)^{7}}{\left(\cos(bx+a)+1\right)^{7}}-\frac{462\left(\cos(bx+a)-1\right)^{8}}{\left(\cos(bx+a)+1\right)^{8}}-1\right)}{693\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] -1024/693*(11*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 55*(cos(b*x + a) - 1)
^2/(cos(b*x + a) + 1)^2 - 297*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 1
485*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 2079*(cos(b*x + a) - 1)^5/(
cos(b*x + a) + 1)^5 - 2541*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 - 1155
*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 462*(cos(b*x + a) - 1)^8/(cos(
b*x + a) + 1)^8 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^11)
```

Mupad [B]
time = 0.14, size = 36, normalized size = 0.78

$$-\frac{64\left(63\cos\left(a+b\,x\right)^{11}-154\cos\left(a+b\,x\right)^{9}+99\cos\left(a+b\,x\right)^{7}\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^6/sin(a + b*x),x)
```

```
[Out] -(64*(99*cos(a + b*x)^7 - 154*cos(a + b*x)^9 + 63*cos(a + b*x)^11))/(693*b)
```

3.36 $\int \csc(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{32\sin^5(a+bx)}{5b}-\frac{64\sin^7(a+bx)}{7b}+\frac{32\sin^9(a+bx)}{9b}$$

```
[Out] 32/5*sin(b*x+a)^5/b-64/7*sin(b*x+a)^7/b+32/9*sin(b*x+a)^9/b
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4373, 2644, 276}

$$\frac{32\sin^9(a+bx)}{9b}-\frac{64\sin^7(a+bx)}{7b}+\frac{32\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (32*Sin[a + b*x]^5)/(5*b) - (64*Sin[a + b*x]^7)/(7*b) + (32*Sin[a + b*x]^9)
/(9*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc(a+bx)\sin^5(2a+2bx)\,dx = 32\int \cos^5(a+bx)\sin^4(a+bx)\,dx$$
$$= \frac{32\text{Subst}\left(\int x^4(1-x^2)^2\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{32\text{Subst}\left(\int (x^4-2x^6+x^8)\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{32\sin^5(a+bx)}{5b} - \frac{64\sin^7(a+bx)}{7b} + \frac{32\sin^9(a+bx)}{9b}$$

Mathematica [A]
time = 0.10, size = 38, normalized size = 0.83

$$\frac{32(63\sin^5(a+bx) - 90\sin^7(a+bx) + 35\sin^9(a+bx))}{315b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (32*(63*Sin[a + b*x]^5 - 90*Sin[a + b*x]^7 + 35*Sin[a + b*x]^9))/(315*b)
```

Maple [A]
time = 0.10, size = 69, normalized size = 1.50

method	result	size
default	$\frac{-\frac{32\left(\sin^3(xb+a)\right)\left(\cos^6(xb+a)\right)}{9}-\frac{32\sin(xb+a)\left(\cos^6(xb+a)\right)}{21}+\frac{32\left(\frac{8}{3}+\cos^4(xb+a)+\frac{4\left(\cos^2(xb+a)\right)}{3}\right)\sin(xb+a)}{105}}{b}$	69
risch	$\frac{3\sin(xb+a)}{4b}+\frac{\sin(9xb+9a)}{72b}+\frac{\sin(7xb+7a)}{56b}-\frac{\sin(5xb+5a)}{10b}-\frac{\sin(3xb+3a)}{6b}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 32/b*(-1/9*sin(b*x+a)^3*cos(b*x+a)^6-1/21*sin(b*x+a)*cos(b*x+a)^6+1/105*(8/
3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A]
time = 0.27, size = 58, normalized size = 1.26

$$\frac{35\,\sin\left(9\,bx+9\,a\right)+45\,\sin\left(7\,bx+7\,a\right)-252\,\sin\left(5\,bx+5\,a\right)-420\,\sin\left(3\,bx+3\,a\right)+1890\,\sin\left(bx+a\right)}{2520\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/2520*(35*sin(9*b*x + 9*a) + 45*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) -
420*sin(3*b*x + 3*a) + 1890*sin(b*x + a))/b
```

Fricas [A]
time = 2.27, size = 53, normalized size = 1.15

$$\frac{32 \left(35 \cos\left(bx+a\right)^{8}-50 \cos\left(bx+a\right)^{6}+3 \cos\left(bx+a\right)^{4}+4 \cos\left(bx+a\right)^{2}+8\right) \sin\left(bx+a\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 32/315*(35*cos(b*x + a)^8 - 50*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*
x + a)^2 + 8)*sin(b*x + a)/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 36, normalized size = 0.78

$$\frac{32 \left(35 \sin\left(bx+a\right)^{9}-90 \sin\left(bx+a\right)^{7}+63 \sin\left(bx+a\right)^{5}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 32/315*(35*sin(b*x + a)^9 - 90*sin(b*x + a)^7 + 63*sin(b*x + a)^5)/b
```

Mupad [B]
time = 0.07, size = 36, normalized size = 0.78

$$\frac{32\,\left(35\,\sin\left(a+b\,x\right)^9-90\,\sin\left(a+b\,x\right)^7+63\,\sin\left(a+b\,x\right)^5\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^5/sin(a + b*x),x)
```

```
[Out] (32*(63*sin(a + b*x)^5 - 90*sin(a + b*x)^7 + 35*sin(a + b*x)^9))/(315*b)
```

3.37 $\int \csc(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=31

$$-\frac{16\cos^5(a+bx)}{5b}+\frac{16\cos^7(a+bx)}{7b}$$

```
[Out] -16/5*cos(b*x+a)^5/b+16/7*cos(b*x+a)^7/b
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4373, 2645, 14}

$$\frac{16\cos^7(a+bx)}{7b}-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-16*Cos[a + b*x]^5)/(5*b) + (16*Cos[a + b*x]^7)/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc(a+bx)\sin^4(2a+2bx)\,dx = 16\int \cos^4(a+bx)\sin^3(a+bx)\,dx$$
$$= -\frac{16\text{Subst}\left(\int x^4(1-x^2)\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{16\text{Subst}\left(\int (x^4-x^6)\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{16\cos^5(a+bx)}{5b} + \frac{16\cos^7(a+bx)}{7b}$$

Mathematica [A]
time = 0.05, size = 59, normalized size = 1.90

$$-\frac{3\cos(a+bx)}{4b} - \frac{\cos(3(a+bx))}{4b} + \frac{\cos(5(a+bx))}{20b} + \frac{\cos(7(a+bx))}{28b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^4,x]
[Out] (-3*Cos[a + b*x])/(4*b) - Cos[3*(a + b*x)]/(4*b) + Cos[5*(a + b*x)]/(20*b)
+ Cos[7*(a + b*x)]/(28*b)
```

Maple [A]
time = 0.07, size = 35, normalized size = 1.13

method	result	size
default	$\frac{-\frac{16\left(\sin^2(xb+a)\right)\left(\cos^5(xb+a)\right)}{7}-\frac{32\left(\cos^5(xb+a)\right)}{35}}{b}$	35
risch	$-\frac{3\cos(xb+a)}{4b}+\frac{\cos(7xb+7a)}{28b}+\frac{\cos(5xb+5a)}{20b}-\frac{\cos(3xb+3a)}{4b}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
[Out] 16/b*(-1/7*sin(b*x+a)^2*cos(b*x+a)^5-2/35*cos(b*x+a)^5)
```

Maxima [A]
time = 0.28, size = 47, normalized size = 1.52

$$\frac{5\cos(7\,bx+7\,a)+7\cos(5\,bx+5\,a)-35\cos(3\,bx+3\,a)-105\cos(bx+a)}{140\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/140*(5*cos(7*b*x + 7*a) + 7*cos(5*b*x + 5*a) - 35*cos(3*b*x + 3*a) - 105*
cos(b*x + a))/b
```

Fricas [A]
time = 3.18, size = 26, normalized size = 0.84

$$\frac{16\left(5\cos\left(bx+a\right)^{7}-7\cos\left(bx+a\right)^{5}\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
[Out] 16/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**4,x)
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(27) = 54.
time = 0.41, size = 138, normalized size = 4.45

$$-\frac{64\left(\frac{7\,(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{14\,(\cos(bx+a)-1)^{2}}{(\cos(bx+a)+1)^{2}}+\frac{70\,(\cos(bx+a)-1)^{3}}{(\cos(bx+a)+1)^{3}}+\frac{35\,(\cos(bx+a)-1)^{4}}{(\cos(bx+a)+1)^{4}}+\frac{35\,(\cos(bx+a)-1)^{5}}{(\cos(bx+a)+1)^{5}}-1\right)}{35\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
[Out] -64/35*(7*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 14*(cos(b*x + a) - 1)^2/(
cos(b*x + a) + 1)^2 + 70*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 35*(co
s(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 35*(cos(b*x + a) - 1)^5/(cos(b*x +
 a) + 1)^5 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^7)
```

Mupad [B]
time = 0.05, size = 26, normalized size = 0.84

$$-\frac{16\left(7\cos\left(a+b\,x\right)^{5}-5\cos\left(a+b\,x\right)^{7}\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^4/sin(a + b*x),x)
[Out] -(16*(7*cos(a + b*x)^5 - 5*cos(a + b*x)^7))/(35*b)
```

3.38 $\int \csc(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{8\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}$$

```
[Out] 8/3*sin(b*x+a)^3/b-8/5*sin(b*x+a)^5/b
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4373, 2644, 14}

$$\frac{8\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (8*Sin[a + b*x]^3)/(3*b) - (8*Sin[a + b*x]^5)/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc(a+bx)\sin^3(2a+2bx)\,dx = 8\int \cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{8\text{Subst}\left(\int x^2(1-x^2)\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{8\text{Subst}\left(\int (x^2-x^4)\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{8\sin^3(a+bx)}{3b} - \frac{8\sin^5(a+bx)}{5b}$$

Mathematica [A]
time = 0.06, size = 28, normalized size = 0.90

$$\frac{8(5\sin^3(a+bx)-3\sin^5(a+bx))}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (8*(5*Sin[a + b*x]^3 - 3*Sin[a + b*x]^5))/(15*b)
```

Maple [A]
time = 0.09, size = 41, normalized size = 1.32

method	result	size
risch	$\frac{\sin(xb+a)}{b} - \frac{\sin(5xb+5a)}{10b} - \frac{\sin(3xb+3a)}{6b}$	40
default	$\frac{-\frac{8\sin(xb+a)\left(\cos^4(xb+a)\right)}{5}+\frac{8\left(2+\cos^2(xb+a)\right)\sin(xb+a)}{15}}{b}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/b*(-1/5*sin(b*x+a)*cos(b*x+a)^4+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A]
time = 0.32, size = 36, normalized size = 1.16

$$-\frac{3\,\sin\left(5\,bx+5\,a\right)+5\,\sin\left(3\,bx+3\,a\right)-30\,\sin\left(bx+a\right)}{30\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] -1/30*(3*sin(5*b*x + 5*a) + 5*sin(3*b*x + 3*a) - 30*sin(b*x + a))/b
```

Fricas [**A**]
time = 2.90, size = 33, normalized size = 1.06

$$-\frac{8\left(3\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}-2\right)\sin\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")
[Out] -8/15*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)*sin(b*x + a)/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**3,x)
[Out] Timed out
```

Giac [**A**]
time = 0.55, size = 26, normalized size = 0.84

$$-\frac{8\left(3\sin\left(bx+a\right)^{5}-5\sin\left(bx+a\right)^{3}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")
[Out] -8/15*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)/b
```

Mupad [**B**]
time = 0.11, size = 26, normalized size = 0.84

$$\frac{8\left(5\sin\left(a+b\,x\right)^{3}-3\sin\left(a+b\,x\right)^{5}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^3/sin(a + b*x),x)
[Out] (8*(5*sin(a + b*x)^3 - 3*sin(a + b*x)^5))/(15*b)
```

3.39 $\int \csc(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=15

$$-\frac{4\cos^3(a+bx)}{3b}$$

[Out] -4/3*cos(b*x+a)^3/b

Rubi [A]
time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4373, 2645, 30}

$$-\frac{4\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^2,x]

[Out] (-4*Cos[a + b*x]^3)/(3*b)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc(a+bx)\sin^2(2a+2bx)\,dx = 4\int \cos^2(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{4\text{Subst}\left(\int x^2\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{4\cos^3(a+bx)}{3b}$$

<u>**Mathematica**</u> [**A**]
time = 0.01, size = 15, normalized size = 1.00

$$-\frac{4\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^2,x]
[Out] (-4*Cos[a + b*x]^3)/(3*b)
```

<u>**Maple**</u> [**A**]
time = 0.06, size = 14, normalized size = 0.93

method	result	size
default	$-\frac{4(\cos^3(xb+a))}{3b}$	14
risch	$-\frac{\cos(xb+a)}{b}-\frac{\cos(3xb+3a)}{3b}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
[Out] -4/3*cos(b*x+a)^3/b
```

<u>**Maxima**</u> [**A**]
time = 0.27, size = 23, normalized size = 1.53

$$-\frac{\cos\left(3\,bx+3\,a\right)+3\,\cos\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
[Out] -1/3*(cos(3*b*x + 3*a) + 3*cos(b*x + a))/b
```

Fricas [A]
time = 2.68, size = 13, normalized size = 0.87

$$-\frac{4 \cos(bx+a)^3}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -4/3*cos(b*x + a)^3/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 15030 vs. $2(14) = 28$.
time = 75.38, size = 104225, normalized size = 6948.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**2,x)
```

```
[Out] 16*Piecewise((0, Eq(a, 0) & Eq(b, 0)), (-cos(b*x)**3/(3*b), Eq(a, 0)), (0,
Eq(b, 0)), (12*log(tan(a/2) + tan(b*x/2))*tan(a/2)**6*tan(b*x/2)**6/(3*b*ta
n(a/2)**8*tan(b*x/2)**6 + 9*b*tan(a/2)**8*tan(b*x/2)**4 + 9*b*tan(a/2)**8*t
an(b*x/2)**2 + 3*b*tan(a/2)**8 + 12*b*tan(a/2)**6*tan(b*x/2)**6 + 36*b*tan(
a/2)**6*tan(b*x/2)**4 + 36*b*tan(a/2)**6*tan(b*x/2)**2 + 12*b*tan(a/2)**6 +
 18*b*tan(a/2)**4*tan(b*x/2)**6 + 54*b*tan(a/2)**4*tan(b*x/2)**4 + 54*b*tan
(a/2)**4*tan(b*x/2)**2 + 18*b*tan(a/2)**4 + 12*b*tan(a/2)**2*tan(b*x/2)**6
+ 36*b*tan(a/2)**2*tan(b*x/2)**4 + 36*b*tan(a/2)**2*tan(b*x/2)**2 + 12*b*ta
n(a/2)**2 + 3*b*tan(b*x/2)**6 + 9*b*tan(b*x/2)**4 + 9*b*tan(b*x/2)**2 + 3*b
) + 36*log(tan(a/2) + tan(b*x/2))*tan(a/2)**6*tan(b*x/2)**4/(3*b*tan(a/2)**
8*tan(b*x/2)**6 + 9*b*tan(a/2)**8*tan(b*x/2)**4 + 9*b*tan(a/2)**8*tan(b*x/2
)**2 + 3*b*tan(a/2)**8 + 12*b*tan(a/2)**6*tan(b*x/2)**6 + 36*b*tan(a/2)**6*
tan(b*x/2)**4 + 36*b*tan(a/2)**6*tan(b*x/2)**2 + 12*b*tan(a/2)**6 + 18*b*ta
n(a/2)**4*tan(b*x/2)**6 + 54*b*tan(a/2)**4*tan(b*x/2)**4 + 54*b*tan(a/2)**4
*tan(b*x/2)**2 + 18*b*tan(a/2)**4 + 12*b*tan(a/2)**2*tan(b*x/2)**6 + 36*b*t
an(a/2)**2*tan(b*x/2)**4 + 36*b*tan(a/2)**2*tan(b*x/2)**2 + 12*b*tan(a/2)**
2 + 3*b*tan(b*x/2)**6 + 9*b*tan(b*x/2)**4 + 9*b*tan(b*x/2)**2 + 3*b) + 36*l
og(tan(a/2) + tan(b*x/2))*tan(a/2)**6*tan(b*x/2)**2/(3*b*tan(a/2)**8*tan(b*
x/2)**6 + 9*b*tan(a/2)**8*tan(b*x/2)**4 + 9*b*tan(a/2)**8*tan(b*x/2)**2 + 3
*b*tan(a/2)**8 + 12*b*tan(a/2)**6*tan(b*x/2)**6 + 36*b*tan(a/2)**6*tan(b*x/
2)**4 + 36*b*tan(a/2)**6*tan(b*x/2)**2 + 12*b*tan(a/2)**6 + 18*b*tan(a/2)**
4*tan(b*x/2)**6 + 54*b*tan(a/2)**4*tan(b*x/2)**4 + 54*b*tan(a/2)**4*tan(b*x
/2)**2 + 18*b*tan(a/2)**4 + 12*b*tan(a/2)**2*tan(b*x/2)**6 + 36*b*tan(a/2)*
*2*tan(b*x/2)**4 + 36*b*tan(a/2)**2*tan(b*x/2)**2 + 12*b*tan(a/2)**2 + 3*b*
tan(b*x/2)**6 + 9*b*tan(b*x/2)**4 + 9*b*tan(b*x/2)**2 + 3*b) + 12*log(tan(a
```

```
/2) + tan(b*x/2))*tan(a/2)**6/(3*b*tan(a/2)**8*tan(b*x/2)**6 + 9*b*tan(a/2)
**8*tan(b*x/2)**4 + 9*b*tan(a/2)**8*tan(b*x/2)**2 + 3*b*tan(a/2)**8 + 12*b*
tan(a/2)**6*tan(b*x/2)**6 + 36*b*tan(a/2)**6*tan(b*x/2)**4 + 36*b*tan(a/2)*
*6*tan(b*x/2)**2 + 12*b*tan(a/2)**6 + 18*b*tan(a/2)**4*tan(b*x/2)**6 + 54*b
*tan(a/2)**4*tan(b*x/2)**4 + 54*b*tan(a/2)**4*tan(b*x/2)**2 + 18*b*tan(a/2)
**4 + 12*b*tan(a/2)**2*tan(b*x/2)**6 + 36*b*tan(a/2)**2*tan(b*x/2)**4 + 36*
b*tan(a/2)**2*tan(b*x/2)**2 + 12*b*tan(a/2)**2 + 3*b*tan(b*x/2)**6 + 9*b*ta
n(b*x/2)**4 + 9*b*tan(b*x/2)**2 + 3*b) - 24*log(tan(a/2) + tan(b*x/2))*tan(
a/2)**4*tan(b*x/2)**6/(3*b*tan(a/2)**8*tan(b*x/2)**6 + 9*b*tan(a/2)**8*tan(
b*x/2)**4 + 9*b*tan(a/2)**8*tan(b*x/2)**2 + 3*b*tan(a/2)**8 + 12*b*tan(a/2)
**6*tan(b*x/2)**6 + 36*b*tan(a/2)**6*tan(b*x/2)**4 + 36*b*tan(a/2)**6*tan(b
*x/2)**2 + 12*b*tan(a/2)**6 + 18*b*tan(a/2)**4*tan(b*x/2)**6 + 54*b*tan(a/2
)**4*tan(b*x/2)**4 + 54*b*tan(a/2)**4*tan(b*x/2)**2 + 18*b*tan(a/2)**4 + 12
*b*tan(a/2)**2*tan(b*x/2)**6 + 36*b*tan(a/2)**2*tan(b*x/2)**4 + 36*b*tan(a/
2)**2*tan(b*x/2)**2 + 12*b*tan(a/2)**2 + 3*b*tan(b*x/2)**6 + 9*b*tan(b*x/2)
**4 + 9*b*tan(b*x/2)**2 + 3*b) - 72*log(tan(a/2) + tan(b*x/2))*tan(a/2)**4*
tan(b*x/2)**4/(3*b*tan(a/2)**8*tan(b*x/2)**6 + 9*b*tan(a/2)**8*tan(b*x/2)**
4 + 9*b*tan(a/2)**8*tan(b*x/2)**2 + 3*b*tan(a/2)**8 + 12*b*tan(a/2)**6*tan(
b*x/2)**6 + 36*b*tan(a/2)**6*tan(b*x/2)**4 + 36*b*tan(a/2)**6*tan(b*x/2)**2
 + 12*b*tan(a/2)**6 + 18*b*tan(a/2)**4*tan(b*x/2)**6 + 54*b*tan(a/2)**4*tan
(b*x/2)**4 + 54*b*tan(a/2)**4*tan(b*x/2)**2 + 18*b*tan(a/2)**4 + 12*b*tan(a
/2)**2*tan(b*x/2)**6 + 36*b*tan(a/2)**2*tan(b*x/2)**4 + 36*b*tan(a/2)**2*ta
n(b*x/2)**2 + 12*b*tan(a/2)**2 + 3*b*tan(b*x/2)**6 + 9*b*tan(b*x/2)**4 + 9*
b*tan(b*x/2)**2 + 3*b) - 72*log(tan(a/2) + tan(b*x/2))*tan(a/2)**4*tan(b*x/
2)**2/(3*b*tan(a/2)**8*tan(b*x/2)**6 + 9*b*tan(a/2)**8*tan(b*x/2)**4 + 9*b*
tan(a/2)**8*tan(b*x/2)**2 + 3*b*tan(a/2)**8 + 12*b*tan(a/2)**6*tan(b*x/2)**
6 + 36*b*tan(a/2)**6*tan(b*x/2)**4 + 36*b*tan(a/2)**6*tan(b*x/2)**2 + 12*b*
tan(a/2)**6 + 18*b*tan(a/2)**4*tan(b*x/2)**6 + 54*b*tan(a/2)**4*tan(b*x/2)*
*4 + 54*b*tan(a/2)**4*tan(b*x/2)**2 + 18*b*tan(a/2)**4 + 12*b*tan(a/2)**2*t
an(b*x/2)**6 + 36*b*tan(a/2)**2*tan(b*x/2)**4 + 36*b*tan(a/2)**2*tan(b*x/2)
**2 + 12*b*tan(a/2)**2 + 3*b*tan(b*x/2)**6 + 9*b*tan(b*x/2)**4 + 9*b*tan(b*
x/2)**2 + 3*b) - 24*log(tan(a/2) + tan(b*x/2))*tan(a/2)**4/(3*b*tan(a/2)**8
*tan(b*x/2)**6 + 9*b*tan(a/2)**8*tan(b*x/2)**4 + 9*b*tan(a/2)**8*tan(b*x/2)
**2 + 3*b*tan(a/2)**8 + 12*b*tan(a/2)**6*tan(b*x/2)**6 + 36*b*tan(a/2)**6*t
an(b*x/2)**4 + 36*b*tan(a/2)**6*tan(b*x/2)**2 + 12*b*tan(a/2)**6 + 18*b*tan
(a/2)**4*tan(b*x/2)**6 + 54*b*tan(a/2)**4*tan(b*x/2)**4 + 54*b*tan(a/2)**4*
tan(b*x/2)**2 + 18*b*tan(a/2)**4 + 12*b*tan(a/2)**2*tan(b*x/2)**6 + 36*b*ta
n(a/2)**2*tan(b*x/2)**4 + 36*b*tan(a/2)**2*tan(b*x/2)**2 + 12*b*tan(a/2)**2
 + 3*b*tan(b*x/2)**6 + 9*b*tan(b*x/2)**4 + 9*b*tan(b*x/2)**2 + 3*b) + 12*lo
g(tan(a/2) + tan(b*x/2))*tan(a/2)**2*tan(b*x/2)**6/(3*b*tan(a/2)**8*tan(b*x
/2)**6 + 9*b*tan(a/2)**8*tan(b*x/2)**4 + 9*b*tan(a/2)**8*tan(b*x/2)**2 + 3*
b*tan(a/2)**8 + 12*b*tan(a/2)**6*tan(b*x/2)**6 ...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(13) = 26$.

time = 0.45, size = 52, normalized size = 3.47

$$\frac{8\left(\frac{3\,(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+1\right)}{3\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 8/3*(3*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 1)/(b*((cos(b*x + a) - 1
)/(cos(b*x + a) + 1) - 1)^3)
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.87

$$-\frac{4\cos\left(a+b\,x\right)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^2/sin(a + b*x),x)
```

```
[Out] -(4*cos(a + b*x)^3)/(3*b)
```

3.40 $\int \csc(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=11

$$\frac{2\sin(a+bx)}{b}$$

[Out] 2*sin(b*x+a)/b

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4373, 2717}

$$\frac{2\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x],x]

[Out] (2*Sin[a + b*x])/b

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sin(2a+2bx)\,dx &= 2\int \cos(a+bx)\,dx\\ &= \frac{2\sin(a+bx)}{b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(11) = 22.
time = 0.01, size = 23, normalized size = 2.09

$$2\left(\frac{\cos(bx)\sin(a)}{b}+\frac{\cos(a)\sin(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x],x]
[Out] 2*((Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b)
```

Maple [A]
time = 0.04, size = 12, normalized size = 1.09

method	result	size
default	$\frac{2\sin(xb+a)}{b}$	12
risch	$\frac{2\sin(xb+a)}{b}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
[Out] 2*sin(b*x+a)/b
```

Maxima [A]
time = 0.27, size = 11, normalized size = 1.00

$$\frac{2\,\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
[Out] 2*sin(b*x + a)/b
```

Fricas [A]
time = 2.06, size = 11, normalized size = 1.00

$$\frac{2\,\sin\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
[Out] 2*sin(b*x + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(8) = 16$.
time = 10.96, size = 3636, normalized size = 330.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] 4*Piecewise((x, Eq(a, 0) & Eq(b, 0)), (sin(b*x)/b, Eq(a, 0)), (0, Eq(b, 0))
, (2*log(tan(a/2) + tan(b*x/2))*tan(a/2)**3*tan(b*x/2)**2/(b*tan(a/2)**4*ta
n(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)*
*2 + b*tan(b*x/2)**2 + b) + 2*log(tan(a/2) + tan(b*x/2))*tan(a/2)**3/(b*tan
(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*
b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(a/2) + tan(b*x/2))*tan(a/2
)*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)
**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(a/2)
 + tan(b*x/2))*tan(a/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*
tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(
tan(b*x/2) - 1/tan(a/2))*tan(a/2)**3*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2
)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*
tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)**3/(b*tan(a/2)
**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan
(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)*t
an(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2
*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2)
 - 1/tan(a/2))*tan(a/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*t
an(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*tan(a
/2)**4*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/
2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*tan(a/2)**
3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)
**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 4*tan(a/2)/(b*tan(a/2)**4*ta
n(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)*
*2 + b*tan(b*x/2)**2 + b) + 2*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*t
an(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)
**2 + b), True))*cos(a)**2 - 2*Piecewise((x, Eq(a, 0) & Eq(b, 0)), (sin(b*x
)/b, Eq(a, 0)), (0, Eq(b, 0)), (2*log(tan(a/2) + tan(b*x/2))*tan(a/2)**3*ta
n(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*
tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(a/2) + t
an(b*x/2))*tan(a/2)**3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*t
an(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(t
an(a/2) + tan(b*x/2))*tan(a/2)*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 +
 b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*
x/2)**2 + b) - 2*log(tan(a/2) + tan(b*x/2))*tan(a/2)/(b*tan(a/2)**4*tan(b*x
/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 +
b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)**3*tan(b*x/2
)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x
/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(
a/2))*tan(a/2)**3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/
2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*log(tan(b*
x/2) - 1/tan(a/2))*tan(a/2)*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*
tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2
```

```
)**2 + b) + 2*log(tan(b*x/2) - 1/tan(a/2))*tan(a/2)/(b*tan(a/2)**4*tan(b*x/
2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b
*tan(b*x/2)**2 + b) - 2*tan(a/2)**4*tan(b*x/2)/(b*tan(a/2)**4*tan(b*x/2)**2
 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(
b*x/2)**2 + b) - 4*tan(a/2)**3/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4
 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) -
 4*tan(a/2)/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*
tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + 2*tan(b*x/2)/(b*ta
n(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2
*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b), True)) - 2*Piecewise((zoo*x, Eq(a, 0
) & Eq(b, 0)), (x/sin(a), Eq(b, 0)), (log(tan(b*x/2))/b, Eq(a, 0)), (log(ta
n(a/2) + tan(b*x/2))/b - log(tan(b*x/2) - 1/tan(a/2))/b, True))*sin(a)*cos(
a) + 4*Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (log(tan(b*x/2))*tan(b*x/2)*
*2/(b*tan(b*x/2)**2 + b) + log(tan(b*x/2))/(b*tan(b*x/2)**2 + b) + 2/(b*tan
(b*x/2)**2 + b), Eq(a, 0)), (x/sin(a), Eq(b, 0)), (log(tan(a/2) + tan(b*x/2
))*tan(a/2)**4*tan(b*x/2)**2/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 +
 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + l
og(tan(a/2) + tan(b*x/2))*tan(a/2)**4/(b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(
a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2
 + b) - 2*log(tan(a/2) + tan(b*x/2))*tan(a/2)**2*tan(b*x/2)**2/(b*tan(a/2)*
*4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2 + 2*b*tan(
a/2)**2 + b*tan(b*x/2)**2 + b) - 2*log(tan(a/2) + tan(b*x/2))*tan(a/2)**2/(
b*tan(a/2)**4*tan(b*x/2)**2 + b*tan(a/2)**4 + 2*b*tan(a/2)**2*tan(b*x/2)**2
 + 2*b*tan(a/2)**2 + b*tan(b*x/2)**2 + b) + log...
```

Giac [A]
time = 0.40, size = 11, normalized size = 1.00

$$\frac{2 \sin\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 2*sin(b*x + a)/b
```

Mupad [B]
time = 0.02, size = 11, normalized size = 1.00

$$\frac{2 \sin\left(a + b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)/sin(a + b*x),x)
```

```
[Out] (2*sin(a + b*x))/b
```

3.41 $\int \csc(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=28

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\csc(a+bx)}{2b}$$

```
[Out] 1/2*arctanh(sin(b*x+a))/b-1/2*csc(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4373, 2701, 327, 213}

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\csc(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[2*a + 2*b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) - Csc[a + b*x]/(2*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
```

```
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc(a+bx)\csc(2a+2bx)\,dx &= \frac{1}{2}\int \csc^2(a+bx)\sec(a+bx)\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}\\
&= -\frac{\csc(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}\\
&= \frac{\tanh^{-1}(\sin(a+bx))}{2b} - \frac{\csc(a+bx)}{2b}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 29, normalized size = 1.04

$$-\frac{\csc(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x],x]
[Out] -1/2*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b
```

Maple [A]
time = 0.07, size = 31, normalized size = 1.11

method	result	size
default	$\frac{-\frac{1}{\sin(xb+a)}+\ln(\sec(xb+a)+\tan(xb+a))}{2b}$	31
risch	$-\frac{i e^{i(xb+a)}}{b\left(e^{2i(xb+a)}-1\right)}+\frac{\ln\left(i+e^{i(xb+a)}\right)}{2b}-\frac{\ln\left(e^{i(xb+a)}-i\right)}{2b}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(2*b*x+2*a),x,method=_RETURNVERBOSE)
[Out] 1/2/b*(-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(24) = 48.
time = 0.51, size = 233, normalized size = 8.32

$$-\frac{\left(\cos\left(2\,bx+2\,a\right)^2+\sin\left(2\,bx+2\,a\right)^2-2\,\cos\left(2\,bx+2\,a\right)+1\right)\log\left(\frac{\cos(bx+2\,a)^2+\cos(a)^2-2\,\cos(a)\sin(bx+2\,a)+\sin(bx+2\,a)^2+2\,\cos(bx+2\,a)\sin(a)+\sin(a)^2}{\cos(bx+2\,a)^2+\cos(a)^2+2\,\cos(a)\sin(bx+2\,a)+\sin(bx+2\,a)^2-2\,\cos(bx+2\,a)\sin(a)+\sin(a)^2}\right)+4\,\cos\left(bx+a\right)\sin\left(2\,bx+2\,a\right)-4\,\cos\left(2\,bx+2\,a\right)\sin\left(bx+a\right)+4\,\sin\left(bx+a\right)}{4\,\left(b\cos\left(2\,bx+2\,a\right)^2+b\sin\left(2\,bx+2\,a\right)^2-2\,b\cos\left(2\,bx+2\,a\right)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/4*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*lo
g((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2
 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos
(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)
) + 4*cos(b*x + a)*sin(2*b*x + 2*a) - 4*cos(2*b*x + 2*a)*sin(b*x + a) + 4*s
in(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x +
 2*a) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(24) = 48$.
time = 2.63, size = 50, normalized size = 1.79

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-2}{4\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/4*(log(sin(b*x + a) + 1)*sin(b*x + a) - log(-sin(b*x + a) + 1)*sin(b*x +
a) - 2)/(b*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\csc\left(a+bx\right)\csc\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a),x)
```

```
[Out] Integral(csc(a + b*x)*csc(2*a + 2*b*x), x)
```

Giac [A]
time = 0.42, size = 38, normalized size = 1.36

$$-\frac{\frac{2}{\sin\left(bx+a\right)}-\log\left(\sin\left(bx+a\right)+1\right)+\log\left(-\sin\left(bx+a\right)+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/4*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b
```

Mupad [B]
time = 0.11, size = 26, normalized size = 0.93

$$\frac{\operatorname{atanh}(\sin(a+b\,x))}{2\,b}-\frac{1}{2\,b\,\sin(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)),x)
[Out] atanh(sin(a + b*x))/(2*b) - 1/(2*b*sin(a + b*x))
```

3.42 $\int \csc(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=49

$$-\frac{3\tanh^{-1}(\cos(a+bx))}{8b}+\frac{3\sec(a+bx)}{8b}-\frac{\csc^2(a+bx)\sec(a+bx)}{8b}$$

```
[Out] -3/8*arctanh(cos(b*x+a))/b+3/8*sec(b*x+a)/b-1/8*csc(b*x+a)^2*sec(b*x+a)/b
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {4373, 2702, 294, 327, 213}

$$\frac{3\sec(a+bx)}{8b}-\frac{3\tanh^{-1}(\cos(a+bx))}{8b}-\frac{\csc^2(a+bx)\sec(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(8*b) + (3*Sec[a + b*x])/(8*b) - (Csc[a + b*x]^2
*Sec[a + b*x])/(8*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc(a+bx)\csc^2(2a+2bx)\,dx &= \frac{1}{4}\int \csc^3(a+bx)\sec^2(a+bx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{4b} \\
&= -\frac{\csc^2(a+bx)\sec(a+bx)}{8b} + \frac{3\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{8b} \\
&= \frac{3\sec(a+bx)}{8b} - \frac{\csc^2(a+bx)\sec(a+bx)}{8b} + \frac{3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{8b} \\
&= -\frac{3\tanh^{-1}(\cos(a+bx))}{8b} + \frac{3\sec(a+bx)}{8b} - \frac{\csc^2(a+bx)\sec(a+bx)}{8b}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 143 vs. 2(49) = 98.
time = 0.28, size = 143, normalized size = 2.92

$$
\frac{\csc^4(a+bx)\left(2-6\cos(2(a+bx))+2\cos(3(a+bx))+3\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-3\cos(3(a+bx))\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)+\cos(a+bx)\left(-2-3\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)+3\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)\right)\right)}{8b\left(\csc^2\left(\frac{1}{2}(a+bx)\right)-\sec^2\left(\frac{1}{2}(a+bx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] (Csc[a + b*x]^4*(2 - 6*Cos[2*(a + b*x)] + 2*Cos[3*(a + b*x)] + 3*Cos[3*(a +
 b*x)]*Log[Cos[(a + b*x)/2]] - 3*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + C
os[a + b*x]*(-2 - 3*Log[Cos[(a + b*x)/2]] + 3*Log[Sin[(a + b*x)/2]])))/(8*b
*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2))
```

Maple [A]
time = 0.10, size = 53, normalized size = 1.08

method	result	size
default	$\frac{-\frac{1}{2\sin(xb+a)^2\cos(xb+a)}+\frac{3}{2\cos(xb+a)}+\frac{3\ln(\csc(xb+a)-\cot(xb+a))}{2}}{4b}$	53
risch	$\frac{3\,\mathrm{e}^{5i(xb+a)}-2\,\mathrm{e}^{3i(xb+a)}+3\,\mathrm{e}^{i(xb+a)}}{4b\left(\mathrm{e}^{2i(xb+a)}-1\right)^2\left(\mathrm{e}^{2i(xb+a)}+1\right)}-\frac{3\ln\left(\mathrm{e}^{i(xb+a)}+1\right)}{8b}+\frac{3\ln\left(\mathrm{e}^{i(xb+a)}-1\right)}{8b}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*(-1/2/sin(b*x+a)^2/cos(b*x+a)+3/2/cos(b*x+a)+3/2*ln(csc(b*x+a)-cot(b*
x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(43) = 86$.
time = 0.29, size = 974, normalized size = 19.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(3*cos(5*b*x + 5*a) - 2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*b*
x + 6*a) - 12*(cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) +
4*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 8*(cos(2*b*x + 2
*a) - 1)*cos(3*b*x + 3*a) - 12*cos(2*b*x + 2*a)*cos(b*x + a) + 3*(2*(cos(4*
b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 -
2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(2*b*x
+ 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - sin(6
*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*(2*(cos(4*
b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 -
2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(2*b*x
+ 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - sin(6
*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(5*b
*x + 5*a) - 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(6*b*x + 6*a) - 12*(sin
(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 4*(2*sin(3*b*x + 3*a)
- 3*sin(b*x + a))*sin(4*b*x + 4*a) + 8*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) -
12*sin(2*b*x + 2*a)*sin(b*x + a) + 12*cos(b*x + a))/(b*cos(6*b*x + 6*a)^2 +
 b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(6*b*x + 6*a)^2 + b*sin
(4*b*x + 4*a)^2 + 2*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a
)^2 - 2*(b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 2*
```

```
(b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin
(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(43) = 86$.
time = 2.99, size = 96, normalized size = 1.96

$$\frac{6 \cos \left(bx+a\right)^{2}-3 \left(\cos \left(bx+a\right)^{3}-\cos \left(bx+a\right)\right) \log \left(\frac{1}{2} \cos \left(bx+a\right)+\frac{1}{2}\right)+3 \left(\cos \left(bx+a\right)^{3}-\cos \left(bx+a\right)\right) \log \left(-\frac{1}{2} \cos \left(bx+a\right)+\frac{1}{2}\right)-4}{16 \left(b \cos \left(bx+a\right)^{3}-b \cos \left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(6*cos(b*x + a)^2 - 3*(cos(b*x + a)^3 - cos(b*x + a))*log(1/2*cos(b*x
+ a) + 1/2) + 3*(cos(b*x + a)^3 - cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2
) - 4)/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc \left(a+bx\right) \csc^{2}\left(2a+2bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)**2,x)
```

```
[Out] Integral(csc(a + b*x)*csc(2*a + 2*b*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(43) = 86$.
time = 0.45, size = 137, normalized size = 2.80

$$\frac{\frac{\frac{14 \left(\cos(bx+a)-1\right)}{\cos(bx+a)+1}-\frac{3 \left(\cos(bx+a)-1\right)^{2}}{\left(\cos(bx+a)+1\right)^{2}}+1}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+\frac{\left(\cos(bx+a)-1\right)^{2}}{\left(\cos(bx+a)+1\right)^{2}}}-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+6 \log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{32\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 1/32*((14*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 3*(cos(b*x + a) - 1)^2/(c
os(b*x + a) + 1)^2 + 1)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x +
 a) - 1)^2/(cos(b*x + a) + 1)^2) - (cos(b*x + a) - 1)/(cos(b*x + a) + 1) +
6*log(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

Mupad [B]
time = 0.13, size = 49, normalized size = 1.00

$$-\frac{3\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{8\,b}-\frac{\frac{3\,{\cos\left(a+b\,x\right)}^2}{8}-\frac{1}{4}}{b\,\left(\cos\left(a+b\,x\right)-{\cos\left(a+b\,x\right)}^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^2),x)
```

```
[Out] - (3*atanh(cos(a + b*x)))/(8*b) - ((3*cos(a + b*x)^2)/8 - 1/4)/(b*(cos(a +
b*x) - cos(a + b*x)^3))
```

3.43 $\int \csc(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=66

$$\frac{5\tanh^{-1}(\sin(a+bx))}{16b}-\frac{5\csc(a+bx)}{16b}-\frac{5\csc^3(a+bx)}{48b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{16b}$$

```
[Out] 5/16*arctanh(sin(b*x+a))/b-5/16*csc(b*x+a)/b-5/48*csc(b*x+a)^3/b+1/16*csc(b
*x+a)^3*sec(b*x+a)^2/b
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.278$, Rules used = {4373, 2701, 294, 308, 213}

$$-\frac{5\csc^3(a+bx)}{48b}-\frac{5\csc(a+bx)}{16b}+\frac{5\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(16*b) - (5*Csc[a + b*x])/(16*b) - (5*Csc[a + b*x
]^3)/(48*b) + (Csc[a + b*x]^3*Sec[a + b*x]^2)/(16*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
```

```
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\csc^3(2a+2bx)\,dx &= \frac{1}{8}\int \csc^4(a+bx)\sec^3(a+bx)\,dx\\ &= -\frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{8b}\\ &= \frac{\csc^3(a+bx)\sec^2(a+bx)}{16b} - \frac{5\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b}\\ &= \frac{\csc^3(a+bx)\sec^2(a+bx)}{16b} - \frac{5\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \csc(a+bx\right.}{16b}\\ &= -\frac{5\csc(a+bx)}{16b} - \frac{5\csc^3(a+bx)}{48b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{16b} - \frac{5\text{Subst}\left(\int\right.}{}\\ &= \frac{5\tanh^{-1}(\sin(a+bx))}{16b} - \frac{5\csc(a+bx)}{16b} - \frac{5\csc^3(a+bx)}{48b} + \frac{\csc^3(a+bx)\,\text{se}}{16b}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 31, normalized size = 0.47

$$-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\sin^2(a+bx)\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] -1/24*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, Sin[a + b*x]^2])/b
```

Maple [A]
time = 0.09, size = 69, normalized size = 1.05

method	result	size
default	$\frac{-\frac{1}{3\sin(xb+a)^3\cos(xb+a)^2}+\frac{5}{6\sin(xb+a)\cos(xb+a)^2}-\frac{5}{2\sin(xb+a)}+\frac{5\ln(\sec(xb+a)+\tan(xb+a))}{2}}{8b}$	69
risch	$-\frac{i\left(15\,\mathrm{e}^{9i(xb+a)}-20\,\mathrm{e}^{7i(xb+a)}-22\,\mathrm{e}^{5i(xb+a)}-20\,\mathrm{e}^{3i(xb+a)}+15\,\mathrm{e}^{i(xb+a)}\right)}{24b\left(\mathrm{e}^{2i(xb+a)}-1\right)^3\left(\mathrm{e}^{2i(xb+a)}+1\right)^2}-\frac{5\ln\left(\mathrm{e}^{i(xb+a)}-i\right)}{16b}+\frac{5\ln\left(i+\mathrm{e}^{i(xb+a)}\right)}{16b}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(-1/3/sin(b*x+a)^3/cos(b*x+a)^2+5/6/sin(b*x+a)/cos(b*x+a)^2-5/2/sin(b
*x+a)+5/2*ln(sec(b*x+a)+tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1780 vs. $2(58) = 116$.
time = 0.55, size = 1780, normalized size = 26.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(4*(15*sin(9*b*x + 9*a) - 20*sin(7*b*x + 7*a) - 22*sin(5*b*x + 5*a) -
20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) + 60*(sin(8*b*x +
 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(9*b
*x + 9*a) + 4*(20*sin(7*b*x + 7*a) + 22*sin(5*b*x + 5*a) + 20*sin(3*b*x + 3
*a) - 15*sin(b*x + a))*cos(8*b*x + 8*a) - 80*(2*sin(6*b*x + 6*a) - 2*sin(4*
b*x + 4*a) - sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(22*sin(5*b*x + 5*a) +
20*sin(3*b*x + 3*a) - 15*sin(b*x + a))*cos(6*b*x + 6*a) + 88*(2*sin(4*b*x +
 4*a) + sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(4*sin(3*b*x + 3*a) - 3*sin
(b*x + a))*cos(4*b*x + 4*a) + 15*(2*(cos(8*b*x + 8*a) + 2*cos(6*b*x + 6*a)
- 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) - cos(10*b*
x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a)
 + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 4*(2*cos(4*b*x + 4*a) + cos(2
*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*a)^2 - 4*(cos(2*b*x + 2
*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 - cos(2*b*x + 2*a)^2 + 2*(
sin(8*b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^2 - 2*(2*sin(6*b*x + 6*a) - 2*s
in(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - sin(8*b*x + 8*a)^2 +
 4*(2*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x +
 6*a)^2 - 4*sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 -
 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin
(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2
*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(15*cos(9*b*x + 9*a) - 20*
```

```
cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a) - 20*cos(3*b*x + 3*a) + 15*cos(b*x +
 a))*sin(10*b*x + 10*a) - 60*(cos(8*b*x + 8*a) + 2*cos(6*b*x + 6*a) - 2*cos
(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*sin(9*b*x + 9*a) - 4*(20*cos(7*b*x +
7*a) + 22*cos(5*b*x + 5*a) + 20*cos(3*b*x + 3*a) - 15*cos(b*x + a))*sin(8*b
*x + 8*a) + 80*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a)
+ 1)*sin(7*b*x + 7*a) - 8*(22*cos(5*b*x + 5*a) + 20*cos(3*b*x + 3*a) - 15*c
os(b*x + a))*sin(6*b*x + 6*a) - 88*(2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) -
 1)*sin(5*b*x + 5*a) + 40*(4*cos(3*b*x + 3*a) - 3*cos(b*x + a))*sin(4*b*x +
 4*a) - 80*(cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a) + 80*cos(3*b*x + 3*a)*si
n(2*b*x + 2*a) - 60*cos(b*x + a)*sin(2*b*x + 2*a) + 60*cos(2*b*x + 2*a)*sin
(b*x + a) - 60*sin(b*x + a))/(b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2
 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 +
 b*sin(10*b*x + 10*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4
*b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x
 + 2*a)^2 - 2*(b*cos(8*b*x + 8*a) + 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x +
4*a) - b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a) + 2*(2*b*cos(6*b*x + 6*a)
 - 2*b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*b
*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 4*(b*cos(2*b
*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin(8*b*x + 8
*a) + 2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin
(10*b*x + 10*a) + 2*(2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - b*sin(2*
b*x + 2*a))*sin(8*b*x + 8*a) - 4*(2*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a)
)*sin(6*b*x + 6*a) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(58) = 116$.
time = 3.45, size = 130, normalized size = 1.97

$$-\frac{30\cos\left(bx+a\right)^{4}-15\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)+15\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}\right)\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-40\cos\left(bx+a\right)^{2}+6}{96\left(b\cos\left(bx+a\right)^{4}-b\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -1/96*(30*cos(b*x + a)^4 - 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(sin(b*x
 + a) + 1)*sin(b*x + a) + 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-sin(b*x
 + a) + 1)*sin(b*x + a) - 40*cos(b*x + a)^2 + 6)/((b*cos(b*x + a)^4 - b*cos
(b*x + a)^2)*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\csc\left(a+bx\right)\csc^{3}\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)**3,x)
```

```
[Out] Integral(csc(a + b*x)*csc(2*a + 2*b*x)**3, x)
```

Giac [A]
time = 0.50, size = 72, normalized size = 1.09

$$-\frac{\frac{6\,\sin(bx+a)}{\sin(bx+a)^2-1}+\frac{4\left(6\,\sin(bx+a)^2+1\right)}{\sin(bx+a)^3}-15\,\log\left(\sin\left(bx+a\right)+1\right)+15\,\log\left(-\sin\left(bx+a\right)+1\right)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -1/96*(6*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 4*(6*sin(b*x + a)^2 + 1)/sin(b
*x + a)^3 - 15*log(sin(b*x + a) + 1) + 15*log(-sin(b*x + a) + 1))/b
```

Mupad [B]
time = 0.19, size = 61, normalized size = 0.92

$$\frac{5\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{16\,b}-\frac{-\frac{5\,\sin(a+b\,x)^4}{16}+\frac{5\,\sin(a+b\,x)^2}{24}+\frac{1}{24}}{b\,\left(\sin\left(a+b\,x\right)^3-\sin\left(a+b\,x\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^3),x)
```

```
[Out] (5*atanh(sin(a + b*x)))/(16*b) - ((5*sin(a + b*x)^2)/24 - (5*sin(a + b*x)^4
)/16 + 1/24)/(b*(sin(a + b*x)^3 - sin(a + b*x)^5))
```

3.44 $\int \csc(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=89

$$-\frac{35\tanh^{-1}(\cos(a+bx))}{128b}+\frac{35\sec(a+bx)}{128b}+\frac{35\sec^3(a+bx)}{384b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{64b}$$

```
[Out] -35/128*arctanh(cos(b*x+a))/b+35/128*sec(b*x+a)/b+35/384*sec(b*x+a)^3/b-7/1
28*csc(b*x+a)^2*sec(b*x+a)^3/b-1/64*csc(b*x+a)^4*sec(b*x+a)^3/b
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {4373, 2702, 294, 308, 213}

$$\frac{35\sec^3(a+bx)}{384b}+\frac{35\sec(a+bx)}{128b}-\frac{35\tanh^{-1}(\cos(a+bx))}{128b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{64b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(128*b) + (35*Sec[a + b*x])/(128*b) + (35*Sec[a
 + b*x]^3)/(384*b) - (7*Csc[a + b*x]^2*Sec[a + b*x]^3)/(128*b) - (Csc[a + b
*x]^4*Sec[a + b*x]^3)/(64*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc(a+bx)\csc^4(2a+2bx)\,dx = \frac{1}{16}\int \csc^5(a+bx)\sec^4(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, \sec(a+bx)\right)}{16b}$$
$$= -\frac{\csc^4(a+bx)\sec^3(a+bx)}{64b} + \frac{7\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{64b}$$
$$= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{64b} + \frac{35\text{Subst}\left(\int \right.}{}$$
$$= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{64b} + \frac{35\text{Subst}\left(\int\right.}{}$$
$$= \frac{35\sec(a+bx)}{128b} + \frac{35\sec^3(a+bx)}{384b} - \frac{7\csc^2(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^4(a}{}$$
$$= -\frac{35\tanh^{-1}(\cos(a+bx))}{128b} + \frac{35\sec(a+bx)}{128b} + \frac{35\sec^3(a+bx)}{384b} - \frac{7\csc^2(a}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 268 vs. 2(89) = 178.
time = 0.53, size = 268, normalized size = 3.01

$$-\frac{\csc^{10}(a+bx)\left(-204+658\cos(2(a+bx))-228\cos(3(a+bx))+140\cos(4(a+bx))-76\cos(5(a+bx))-210\cos(6(a+bx))+76\cos(7(a+bx))-315\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-105\cos(5(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)+105\cos(7(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)+3\cos(a+bx)\left(76+105\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-105\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)\right)+315\cos(3(a+bx))\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)+105\cos(5(a+bx))\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)-105\cos(7(a+bx))\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)\right)}{384b\left(\csc^2\left(\frac{1}{2}(a+bx)\right)-\sec^2\left(\frac{1}{2}(a+bx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] -1/384*(Csc[a + b*x]^10*(-204 + 658*Cos[2*(a + b*x)] - 228*Cos[3*(a + b*x)]
 + 140*Cos[4*(a + b*x)] - 76*Cos[5*(a + b*x)] - 210*Cos[6*(a + b*x)] + 76*C
```

```
os[7*(a + b*x)] - 315*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 105*Cos[5*(a
 + b*x)]*Log[Cos[(a + b*x)/2]] + 105*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/2]]
 + 3*Cos[a + b*x]*(76 + 105*Log[Cos[(a + b*x)/2]] - 105*Log[Sin[(a + b*x)/2
]]) + 315*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 105*Cos[5*(a + b*x)]*Log
[Sin[(a + b*x)/2]] - 105*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]]))/(b*(Csc[(
a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Maple [A]
time = 0.13, size = 89, normalized size = 1.00

method	result
default	$\frac{-\frac{1}{4\sin(xb+a)^4\cos(xb+a)^3}+\frac{7}{12\sin(xb+a)^2\cos(xb+a)^3}-\frac{35}{24\sin(xb+a)^2\cos(xb+a)}+\frac{35}{8\cos(xb+a)}+\frac{35\ln(\csc(xb+a)-\cot(xb+a))}{8}}{16b}$
risch	$\frac{105\,e^{13i(xb+a)}-70\,e^{11i(xb+a)}-329\,e^{9i(xb+a)}+204\,e^{7i(xb+a)}-329\,e^{5i(xb+a)}-70\,e^{3i(xb+a)}+105\,e^{i(xb+a)}}{192b\left(e^{2i(xb+a)}-1\right)^4\left(e^{2i(xb+a)}+1\right)^3}-\frac{35\ln\left(e^{i(xb+a)}+1\right)}{128b}+\frac{35\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*csc(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)^3+7/12/sin(b*x+a)^2/cos(b*x+a)^3-35/24
/sin(b*x+a)^2/cos(b*x+a)+35/8/cos(b*x+a)+35/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3846 vs. $2(79) = 158$.
time = 0.42, size = 3846, normalized size = 43.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/768*(4*(105*cos(13*b*x + 13*a) - 70*cos(11*b*x + 11*a) - 329*cos(9*b*x +
9*a) + 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) +
105*cos(b*x + a))*cos(14*b*x + 14*a) - 420*(cos(12*b*x + 12*a) + 3*cos(10*b
*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) +
 cos(2*b*x + 2*a) - 1)*cos(13*b*x + 13*a) + 4*(70*cos(11*b*x + 11*a) + 329*
cos(9*b*x + 9*a) - 204*cos(7*b*x + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*b
*x + 3*a) - 105*cos(b*x + a))*cos(12*b*x + 12*a) + 280*(3*cos(10*b*x + 10*a
) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*
x + 2*a) - 1)*cos(11*b*x + 11*a) + 12*(329*cos(9*b*x + 9*a) - 204*cos(7*b*x
 + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*b*x + 3*a) - 105*cos(b*x + a))*co
s(10*b*x + 10*a) - 1316*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*
b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(9*b*x + 9*a) + 12*(204*cos(7*b*x + 7
*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) + 105*cos(b*x + a))*cos(8*
b*x + 8*a) + 816*(3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a
) + 1)*cos(7*b*x + 7*a) - 84*(47*cos(5*b*x + 5*a) + 10*cos(3*b*x + 3*a) - 1
```

```
5*cos(b*x + a))*cos(6*b*x + 6*a) + 1316*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2
*a) - 1)*cos(5*b*x + 5*a) + 420*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4
*b*x + 4*a) + 280*(cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a) - 420*cos(2*b*x +
 2*a)*cos(b*x + a) + 105*(2*(cos(12*b*x + 12*a) + 3*cos(10*b*x + 10*a) - 3*
cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*x + 2*
a) - 1)*cos(14*b*x + 14*a) - cos(14*b*x + 14*a)^2 - 2*(3*cos(10*b*x + 10*a)
 - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*x
 + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 + 6*(3*cos(8*b*x + 8
*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(1
0*b*x + 10*a) - 9*cos(10*b*x + 10*a)^2 - 6*(3*cos(6*b*x + 6*a) - 3*cos(4*b*
x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x + 8*a)^2 +
6*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 9*cos(6*b*
x + 6*a)^2 - 6*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)
^2 - cos(2*b*x + 2*a)^2 + 2*(sin(12*b*x + 12*a) + 3*sin(10*b*x + 10*a) - 3*
sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*
a))*sin(14*b*x + 14*a) - sin(14*b*x + 14*a)^2 - 2*(3*sin(10*b*x + 10*a) - 3
*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2
*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 + 6*(3*sin(8*b*x + 8*a) + 3*
sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(10*b*x + 10*a
) - 9*sin(10*b*x + 10*a)^2 - 6*(3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - s
in(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 + 6*(3*sin(4*b*x +
 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 9*sin(6*b*x + 6*a)^2 - 9*sin(4
*b*x + 4*a)^2 - 6*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 +
2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin
(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 105*(2*(cos(12*b*x + 12*a) + 3*co
s(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x +
4*a) + cos(2*b*x + 2*a) - 1)*cos(14*b*x + 14*a) - cos(14*b*x + 14*a)^2 - 2*
(3*cos(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b
*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2
 + 6*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*
b*x + 2*a) + 1)*cos(10*b*x + 10*a) - 9*cos(10*b*x + 10*a)^2 - 6*(3*cos(6*b*
x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*
cos(8*b*x + 8*a)^2 + 6*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*
x + 6*a) - 9*cos(6*b*x + 6*a)^2 - 6*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a)
 - 9*cos(4*b*x + 4*a)^2 - cos(2*b*x + 2*a)^2 + 2*(sin(12*b*x + 12*a) + 3*si
n(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x +
4*a) + sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - sin(14*b*x + 14*a)^2 - 2*(3*s
in(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x +
 4*a) + sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 + 6*(3*
sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) - 9*sin(10*b*x + 10*a)^2 - 6*(3*sin(6*b*x + 6*a) - 3
*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)
^2 + 6*(3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 9*sin(6*b
*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 - 6*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*co
```

```
s(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(105*sin(1
3*b*x + 13*a) - 70*sin(11*b*x + 11*a) - 329*sin(9*b*x + 9*a) + 204*sin(7*b*
x + 7*a) - 329*sin(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) + 105*sin(b*x + a))*s
in(14*b*x + 14*a) - 420*(sin(12*b*x + 12*a) + 3*sin(10*b*x + 10*a) - 3*sin(
8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*
sin(13*b*x + 13*a) + 4*(70*sin(11*b*x + 11*a) + 329*sin(9*b*x + 9*a) - 204*
sin(7*b*x + 7*a) + 329*sin(5*b*x + 5*a) + 70*si...
```

Fricas [A]
time = 3.35, size = 148, normalized size = 1.66

$$\frac{210 \cos(bx+a)^6 - 350 \cos(bx+a)^4 + 112 \cos(bx+a)^2 - 105 \left(\cos(bx+a)^7 - 2 \cos(bx+a)^5 + \cos(bx+a)^3\right) \log\left(\frac{1}{2} \cos(bx+a) + \frac{1}{2}\right) + 105 \left(\cos(bx+a)^7 - 2 \cos(bx+a)^5 + \cos(bx+a)^3\right) \log\left(-\frac{1}{2} \cos(bx+a) + \frac{1}{2}\right) + 16}{768 \left(b \cos(bx+a)^7 - 2 b \cos(bx+a)^5 + b \cos(bx+a)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/768*(210*cos(b*x + a)^6 - 350*cos(b*x + a)^4 + 112*cos(b*x + a)^2 - 105*(
cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(1/2*cos(b*x + a) +
1/2) + 105*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(-1/2*co
s(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x
+ a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc(a + bx) \csc^4(2a + 2bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)**4,x)
```

```
[Out] Integral(csc(a + b*x)*csc(2*a + 2*b*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(79) = 158.
time = 0.44, size = 206, normalized size = 2.31

$$\frac{\frac{3\left(\frac{24(\cos(bx+a)-1)}{\cos(bx+a)+1} - \frac{210(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} - 1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2} - \frac{72(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{3(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + \frac{256\left(\frac{9(\cos(bx+a)-1)}{\cos(bx+a)+1} + \frac{6(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2} + 5\right)}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1} + 1\right)^3} + 420 \log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{3072\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 1/3072*(3*(24*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 210*(cos(b*x + a) - 1
)^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 7
2*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 3*(cos(b*x + a) - 1)^2/(cos(b*x +
```

```
 a) + 1)^2 + 256*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 6*(cos(b*x + a)
 - 1)^2/(cos(b*x + a) + 1)^2 + 5)/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) +
1)^3 + 420*log(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

Mupad [B]
time = 0.12, size = 78, normalized size = 0.88

$$\frac{\frac{35\cos(a+b\,x)^6}{128}-\frac{175\cos(a+b\,x)^4}{384}+\frac{7\cos(a+b\,x)^2}{48}+\frac{1}{48}}{b\,\left(\cos\left(a+b\,x\right)^7-2\cos\left(a+b\,x\right)^5+\cos\left(a+b\,x\right)^3\right)}-\frac{35\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^4),x)
```

```
[Out] ((7*cos(a + b*x)^2)/48 - (175*cos(a + b*x)^4)/384 + (35*cos(a + b*x)^6)/128
 + 1/48)/(b*(cos(a + b*x)^3 - 2*cos(a + b*x)^5 + cos(a + b*x)^7)) - (35*ata
nh(cos(a + b*x)))/(128*b)
```

3.45 $\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx$

Optimal. Leaf size=155

$$\frac{5x}{8}+\frac{5\cos(a+bx)\sin(a+bx)}{8b}+\frac{5\cos^3(a+bx)\sin(a+bx)}{12b}+\frac{\cos^5(a+bx)\sin(a+bx)}{3b}+\frac{2\cos^7(a+bx)\sin(a+}{7b}$$

```
[Out] 5/8*x+5/8*cos(b*x+a)*sin(b*x+a)/b+5/12*cos(b*x+a)^3*sin(b*x+a)/b+1/3*cos(b*
x+a)^5*sin(b*x+a)/b+2/7*cos(b*x+a)^7*sin(b*x+a)/b-16/7*cos(b*x+a)^9*sin(b*x
+a)/b-160/21*cos(b*x+a)^9*sin(b*x+a)^3/b-128/7*cos(b*x+a)^9*sin(b*x+a)^5/b
```

Rubi [A]
time = 0.13, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4373, 2648, 2715, 8}

$$-\frac{128\sin^5(a+bx)\cos^9(a+bx)}{7b}-\frac{160\sin^3(a+bx)\cos^9(a+bx)}{21b}-\frac{16\sin(a+bx)\cos^9(a+bx)}{7b}+\frac{2\sin(a+bx)\cos^7(a+bx)}{7b}+\frac{\sin(a+bx)\cos^5(a+bx)}{3b}+\frac{5\sin(a+bx)\cos^3(a+bx)}{12b}+\frac{5\sin(a+bx)\cos(a+bx)}{8b}+\frac{5x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (5*x)/8 + (5*Cos[a + b*x]*Sin[a + b*x])/(8*b) + (5*Cos[a + b*x]^3*Sin[a + b
*x])/(12*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(3*b) + (2*Cos[a + b*x]^7*Sin[a
 + b*x])/(7*b) - (16*Cos[a + b*x]^9*Sin[a + b*x])/(7*b) - (160*Cos[a + b*x]
^9*Sin[a + b*x]^3)/(21*b) - (128*Cos[a + b*x]^9*Sin[a + b*x]^5)/(7*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^8(2a+2bx)\,dx = 256\int \cos^8(a+bx)\sin^6(a+bx)\,dx$$
$$= -\frac{128\cos^9(a+bx)\sin^5(a+bx)}{7b} + \frac{640}{7}\int \cos^8(a+bx)\sin^4(a+bx)\,dx$$
$$= -\frac{160\cos^9(a+bx)\sin^3(a+bx)}{21b} - \frac{128\cos^9(a+bx)\sin^5(a+bx)}{7b} + \frac{160}{7}$$
$$= -\frac{16\cos^9(a+bx)\sin(a+bx)}{7b} - \frac{160\cos^9(a+bx)\sin^3(a+bx)}{21b} - \frac{128\cos}{}$$
$$= \frac{2\cos^7(a+bx)\sin(a+bx)}{7b} - \frac{16\cos^9(a+bx)\sin(a+bx)}{7b} - \frac{160\cos^9(a+}{}$$
$$= \frac{\cos^5(a+bx)\sin(a+bx)}{3b} + \frac{2\cos^7(a+bx)\sin(a+bx)}{7b} - \frac{16\cos^9(a+bx}{7b}$$
$$= \frac{5\cos^3(a+bx)\sin(a+bx)}{12b} + \frac{\cos^5(a+bx)\sin(a+bx)}{3b} + \frac{2\cos^7(a+bx)}{7b}$$
$$= \frac{5\cos(a+bx)\sin(a+bx)}{8b} + \frac{5\cos^3(a+bx)\sin(a+bx)}{12b} + \frac{\cos^5(a+bx)\,\mathrm{s}}{3b}$$
$$= \frac{5x}{8} + \frac{5\cos(a+bx)\sin(a+bx)}{8b} + \frac{5\cos^3(a+bx)\sin(a+bx)}{12b} + \frac{\cos^5(a+}{}$$

Mathematica [A]
time = 0.27, size = 85, normalized size = 0.55

$$\frac{840a+840bx+105\sin(2(a+bx))-315\sin(4(a+bx))-63\sin(6(a+bx))+63\sin(8(a+bx))+21\sin(10(a+bx))-7\sin(12(a+bx))-3\sin(14(a+bx))}{1344b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^8,x]
[Out] (840*a + 840*b*x + 105*Sin[2*(a + b*x)] - 315*Sin[4*(a + b*x)] - 63*Sin[6*(
a + b*x)] + 63*Sin[8*(a + b*x)] + 21*Sin[10*(a + b*x)] - 7*Sin[12*(a + b*x)
] - 3*Sin[14*(a + b*x)])/(1344*b)
```

Maple [A]
time = 0.12, size = 111, normalized size = 0.72

method	result
risch	$\frac{5x}{8} - \frac{\sin(14xb+14a)}{448b} - \frac{\sin(12xb+12a)}{192b} + \frac{\sin(10xb+10a)}{64b} + \frac{3\sin(8xb+8a)}{64b} - \frac{3\sin(6xb+6a)}{64b} - \frac{15\sin(4xb+4a)}{64b} + \frac{5\sin(2xb+}{64b}$
default	$\frac{-\frac{128\left(\sin^5(xb+a)\right)\left(\cos^9(xb+a)\right)}{7} - \frac{160\left(\sin^3(xb+a)\right)\left(\cos^9(xb+a)\right)}{21} - \frac{16\sin(xb+a)\left(\cos^9(xb+a)\right)}{7} + \frac{2\left(\cos^7(xb+a) + \frac{7\left(\cos^5(xb+a)\right)}{6} + \frac{35\left(\cos^3(xb+a)\right)}{24}\right.}{7}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 256/b*(-1/14*sin(b*x+a)^5*cos(b*x+a)^9-5/168*sin(b*x+a)^3*cos(b*x+a)^9-1/11
2*sin(b*x+a)*cos(b*x+a)^9+1/896*(cos(b*x+a)^7+7/6*cos(b*x+a)^5+35/24*cos(b*
x+a)^3+35/16*cos(b*x+a))*sin(b*x+a)+5/2048*x*b+5/2048*a)
```

Maxima [A]
time = 0.28, size = 87, normalized size = 0.56

$$\frac{840\,bx - 3\,\sin\left(14\,bx + 14\,a\right) - 7\,\sin\left(12\,bx + 12\,a\right) + 21\,\sin\left(10\,bx + 10\,a\right) + 63\,\sin\left(8\,bx + 8\,a\right) - 63\,\sin\left(6\,bx + 6\,a\right) - 315\,\sin\left(4\,bx + 4\,a\right) + 105\,\sin\left(2\,bx + 2\,a\right)}{1344\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

```
[Out] 1/1344*(840*b*x - 3*sin(14*b*x + 14*a) - 7*sin(12*b*x + 12*a) + 21*sin(10*b
*x + 10*a) + 63*sin(8*b*x + 8*a) - 63*sin(6*b*x + 6*a) - 315*sin(4*b*x + 4*
a) + 105*sin(2*b*x + 2*a))/b
```

Fricas [A]
time = 3.40, size = 87, normalized size = 0.56

$$\frac{105\,bx - \left(3072\,\cos\left(bx + a\right)^{13} - 7424\,\cos\left(bx + a\right)^{11} + 4736\,\cos\left(bx + a\right)^{9} - 48\,\cos\left(bx + a\right)^{7} - 56\,\cos\left(bx + a\right)^{5} - 70\,\cos\left(bx + a\right)^{3} - 105\,\cos\left(bx + a\right)\right)\sin\left(bx + a\right)}{168\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

```
[Out] 1/168*(105*b*x - (3072*cos(b*x + a)^13 - 7424*cos(b*x + a)^11 + 4736*cos(b*
x + a)^9 - 48*cos(b*x + a)^7 - 56*cos(b*x + a)^5 - 70*cos(b*x + a)^3 - 105*
cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.42, size = 95, normalized size = 0.61

$$\frac{105\,bx+105\,a+\frac{105\,\tan\left(bx+a\right)^{13}+700\,\tan\left(bx+a\right)^{11}+1981\,\tan\left(bx+a\right)^{9}+3072\,\tan\left(bx+a\right)^{7}-1981\,\tan\left(bx+a\right)^{5}-700\,\tan\left(bx+a\right)^{3}-105\,\tan\left(bx+a\right)}{\left(\tan\left(bx+a\right)^{2}+1\right)^{7}}}{168\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

```
[Out] 1/168*(105*b*x + 105*a + (105*tan(b*x + a)^13 + 700*tan(b*x + a)^11 + 1981*
tan(b*x + a)^9 + 3072*tan(b*x + a)^7 - 1981*tan(b*x + a)^5 - 700*tan(b*x +
a)^3 - 105*tan(b*x + a))/(tan(b*x + a)^2 + 1)^7)/b
```

Mupad [B]
time = 2.27, size = 149, normalized size = 0.96

$$\frac{5\,x}{8}+\frac{\frac{5\,\tan\left(a+b\,x\right)^{13}}{8}+\frac{25\,\tan\left(a+b\,x\right)^{11}}{6}+\frac{283\,\tan\left(a+b\,x\right)^{9}}{24}+\frac{128\,\tan\left(a+b\,x\right)^{7}}{7}-\frac{283\,\tan\left(a+b\,x\right)^{5}}{24}-\frac{25\,\tan\left(a+b\,x\right)^{3}}{6}-\frac{5\,\tan\left(a+b\,x\right)}{8}}{b\,\left({\tan\left(a+b\,x\right)}^{14}+7\,{\tan\left(a+b\,x\right)}^{12}+21\,{\tan\left(a+b\,x\right)}^{10}+35\,{\tan\left(a+b\,x\right)}^{8}+35\,{\tan\left(a+b\,x\right)}^{6}+21\,{\tan\left(a+b\,x\right)}^{4}+7\,{\tan\left(a+b\,x\right)}^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^8/sin(a + b*x)^2,x)
```

```
[Out] (5*x)/8 + ((128*tan(a + b*x)^7)/7 - (25*tan(a + b*x)^3)/6 - (283*tan(a + b*
x)^5)/24 - (5*tan(a + b*x))/8 + (283*tan(a + b*x)^9)/24 + (25*tan(a + b*x)^
11)/6 + (5*tan(a + b*x)^13)/8)/(b*(7*tan(a + b*x)^2 + 21*tan(a + b*x)^4 + 3
5*tan(a + b*x)^6 + 35*tan(a + b*x)^8 + 21*tan(a + b*x)^10 + 7*tan(a + b*x)^
12 + tan(a + b*x)^14 + 1))
```

3.46 $\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=44

$$-\frac{16\cos^8(a+bx)}{b}+\frac{128\cos^{10}(a+bx)}{5b}-\frac{32\cos^{12}(a+bx)}{3b}$$

```
[Out] -16*cos(b*x+a)^8/b+128/5*cos(b*x+a)^10/b-32/3*cos(b*x+a)^12/b
```

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4373, 2645, 272, 45}

$$-\frac{32\cos^{12}(a+bx)}{3b}+\frac{128\cos^{10}(a+bx)}{5b}-\frac{16\cos^8(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (-16*Cos[a + b*x]^8)/b + (128*Cos[a + b*x]^10)/(5*b) - (32*Cos[a + b*x]^12)
/(3*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
```

```
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(a+bx)\sin^7(2a+2bx)\,dx &= 128\int \cos^7(a+bx)\sin^5(a+bx)\,dx\\
&= -\frac{128\text{Subst}\left(\int x^7\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\
&= -\frac{64\text{Subst}\left(\int (1-x)^2x^3\,dx,x,\cos^2(a+bx)\right)}{b}\\
&= -\frac{64\text{Subst}\left(\int \left(x^3-2x^4+x^5\right)\,dx,x,\cos^2(a+bx)\right)}{b}\\
&= -\frac{16\cos^8(a+bx)}{b}+\frac{128\cos^{10}(a+bx)}{5b}-\frac{32\cos^{12}(a+bx)}{3b}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 48, normalized size = 1.09

$$\frac{16\left(20\sin^6(a+bx)-45\sin^8(a+bx)+36\sin^{10}(a+bx)-10\sin^{12}(a+bx)\right)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (16*(20*Sin[a + b*x]^6 - 45*Sin[a + b*x]^8 + 36*Sin[a + b*x]^10 - 10*Sin[a
+ b*x]^12))/(15*b)
```

Maple [A]
time = 0.07, size = 53, normalized size = 1.20

method	result	size
default	$\frac{-\frac{32\left(\sin^4(xb+a)\right)\left(\cos^8(xb+a)\right)}{3}-\frac{64\left(\sin^2(xb+a)\right)\left(\cos^8(xb+a)\right)}{15}-\frac{16\left(\cos^8(xb+a)\right)}{15}}{b}$	53
risch	$-\frac{\cos(12xb+12a)}{192b}-\frac{\cos(10xb+10a)}{80b}+\frac{\cos(8xb+8a)}{32b}+\frac{5\cos(6xb+6a)}{48b}-\frac{5\cos(4xb+4a)}{64b}-\frac{5\cos(2xb+2a)}{8b}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 128/b*(-1/12*sin(b*x+a)^4*cos(b*x+a)^8-1/30*sin(b*x+a)^2*cos(b*x+a)^8-1/120
*cos(b*x+a)^8)
```

Maxima [A]
time = 0.27, size = 72, normalized size = 1.64

$$-\frac{5\cos(12bx+12a)+12\cos(10bx+10a)-30\cos(8bx+8a)-100\cos(6bx+6a)+75\cos(4bx+4a)+600\cos(2bx+2a)}{960b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="maxima")
[Out] -1/960*(5*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a)
- 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) + 600*cos(2*b*x + 2*a))/b
```

Fricas [A]
time = 4.33, size = 36, normalized size = 0.82

$$-\frac{16\left(10\cos(bx+a)^{12}-24\cos(bx+a)^{10}+15\cos(bx+a)^{8}\right)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="fricas")
[Out] -16/15*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**7,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [A]
time = 0.43, size = 46, normalized size = 1.05

$$-\frac{16\left(10\sin(bx+a)^{12}-36\sin(bx+a)^{10}+45\sin(bx+a)^{8}-20\sin(bx+a)^{6}\right)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^7,x, algorithm="giac")
[Out] -16/15*(10*sin(b*x + a)^12 - 36*sin(b*x + a)^10 + 45*sin(b*x + a)^8 - 20*si
n(b*x + a)^6)/b
```

Mupad [B]
time = 0.15, size = 35, normalized size = 0.80

$$-\frac{16\cos(a+b\,x)^8\left(10\cos(a+b\,x)^4-24\cos(a+b\,x)^2+15\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^7/sin(a + b*x)^2,x)
[Out] -(16*cos(a + b*x)^8*(10*cos(a + b*x)^4 - 24*cos(a + b*x)^2 + 15))/(15*b)
```

3.47 $\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=111

$$\frac{3x}{4}+\frac{3\cos(a+bx)\sin(a+bx)}{4b}+\frac{\cos^3(a+bx)\sin(a+bx)}{2b}+\frac{2\cos^5(a+bx)\sin(a+bx)}{5b}-\frac{12\cos^7(a+bx)\sin(a+$$

```
[Out] 3/4*x+3/4*cos(b*x+a)*sin(b*x+a)/b+1/2*cos(b*x+a)^3*sin(b*x+a)/b+2/5*cos(b*x
+a)^5*sin(b*x+a)/b-12/5*cos(b*x+a)^7*sin(b*x+a)/b-32/5*cos(b*x+a)^7*sin(b*x
+a)^3/b
```

Rubi [A]
time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4373, 2648, 2715, 8}

$$-\frac{32\sin^3(a+bx)\cos^7(a+bx)}{5b}-\frac{12\sin(a+bx)\cos^7(a+bx)}{5b}+\frac{2\sin(a+bx)\cos^5(a+bx)}{5b}+\frac{\sin(a+bx)\cos^3(a+bx)}{2b}+\frac{3\sin(a+bx)\cos(a+bx)}{4b}+\frac{3x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (3*x)/4 + (3*Cos[a + b*x]*Sin[a + b*x])/(4*b) + (Cos[a + b*x]^3*Sin[a + b*x
])/(2*b) + (2*Cos[a + b*x]^5*Sin[a + b*x])/(5*b) - (12*Cos[a + b*x]^7*Sin[a
 + b*x])/(5*b) - (32*Cos[a + b*x]^7*Sin[a + b*x]^3)/(5*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
```

```
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^6(2a+2bx)\,dx = 64\int \cos^6(a+bx)\sin^4(a+bx)\,dx$$
$$= -\frac{32\cos^7(a+bx)\sin^3(a+bx)}{5b} + \frac{96}{5}\int \cos^6(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{12\cos^7(a+bx)\sin(a+bx)}{5b} - \frac{32\cos^7(a+bx)\sin^3(a+bx)}{5b} + \frac{12}{5}\int \mathrm{co}$$
$$= \frac{2\cos^5(a+bx)\sin(a+bx)}{5b} - \frac{12\cos^7(a+bx)\sin(a+bx)}{5b} - \frac{32\cos^7(a+}{}$$
$$= \frac{\cos^3(a+bx)\sin(a+bx)}{2b} + \frac{2\cos^5(a+bx)\sin(a+bx)}{5b} - \frac{12\cos^7(a+bx}{5b}$$
$$= \frac{3\cos(a+bx)\sin(a+bx)}{4b} + \frac{\cos^3(a+bx)\sin(a+bx)}{2b} + \frac{2\cos^5(a+bx)\,\mathrm{s}}{5b}$$
$$= \frac{3x}{4} + \frac{3\cos(a+bx)\sin(a+bx)}{4b} + \frac{\cos^3(a+bx)\sin(a+bx)}{2b} + \frac{2\cos^5(a+}{}$$

Mathematica [A]
time = 0.22, size = 62, normalized size = 0.56

$$\frac{120bx + 20\sin(2(a+bx)) - 40\sin(4(a+bx)) - 10\sin(6(a+bx)) + 5\sin(8(a+bx)) + 2\sin(10(a+bx))}{160b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^6,x]
[Out] (120*b*x + 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] - 10*Sin[6*(a + b*x)]
+ 5*Sin[8*(a + b*x)] + 2*Sin[10*(a + b*x)])/(160*b)
```

Maple [A]
time = 0.10, size = 83, normalized size = 0.75

method	result	size
risch	$\frac{3x}{4}+\frac{\sin(10xb+10a)}{80b}+\frac{\sin(8xb+8a)}{32b}-\frac{\sin(6xb+6a)}{16b}-\frac{\sin(4xb+4a)}{4b}+\frac{\sin(2xb+2a)}{8b}$	75
default	$\frac{-\frac{32\left(\sin^3(xb+a)\right)\left(\cos^7(xb+a)\right)}{5}-\frac{12\sin(xb+a)\left(\cos^7(xb+a)\right)}{5}+\frac{2\left(\cos^5(xb+a)+\frac{5\left(\cos^3(xb+a)\right)}{4}+\frac{15\cos(xb+a)}{8}\right)\sin(xb+a)}{5}+\frac{3xb}{4}+\frac{3a}{4}}{b}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 64/b*(-1/10*sin(b*x+a)^3*cos(b*x+a)^7-3/80*sin(b*x+a)*cos(b*x+a)^7+1/160*(c
os(b*x+a)^5+5/4*cos(b*x+a)^3+15/8*cos(b*x+a))*sin(b*x+a)+3/256*x*b+3/256*a)
```

Maxima [A]
time = 0.28, size = 65, normalized size = 0.59

$$\frac{120\,bx+2\,\sin\left(10\,bx+10\,a\right)+5\,\sin\left(8\,bx+8\,a\right)-10\,\sin\left(6\,bx+6\,a\right)-40\,\sin\left(4\,bx+4\,a\right)+20\,\sin\left(2\,bx+2\,a\right)}{160\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] 1/160*(120*b*x + 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) - 10*sin(6*b*x +
 6*a) - 40*sin(4*b*x + 4*a) + 20*sin(2*b*x + 2*a))/b
```

Fricas [A]
time = 2.89, size = 66, normalized size = 0.59

$$\frac{15\,bx+\left(128\,\cos\left(bx+a\right)^{9}-176\,\cos\left(bx+a\right)^{7}+8\,\cos\left(bx+a\right)^{5}+10\,\cos\left(bx+a\right)^{3}+15\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{20\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] 1/20*(15*b*x + (128*cos(b*x + a)^9 - 176*cos(b*x + a)^7 + 8*cos(b*x + a)^5
+ 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**6,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 75, normalized size = 0.68

$$\frac{15\,bx+15\,a+\frac{15\,\tan\left(bx+a\right)^{9}+70\,\tan\left(bx+a\right)^{7}+128\,\tan\left(bx+a\right)^{5}-70\,\tan\left(bx+a\right)^{3}-15\,\tan\left(bx+a\right)}{\left(\tan\left(bx+a\right)^{2}+1\right)^{5}}}{20\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] 1/20*(15*b*x + 15*a + (15*tan(b*x + a)^9 + 70*tan(b*x + a)^7 + 128*tan(b*x
+ a)^5 - 70*tan(b*x + a)^3 - 15*tan(b*x + a))/(tan(b*x + a)^2 + 1)^5)/b
```

Mupad [B]
time = 1.69, size = 109, normalized size = 0.98

$$\frac{3\,x}{4}+\frac{\frac{3\,\tan\left(a+b\,x\right)^9}{4}+\frac{7\,\tan\left(a+b\,x\right)^7}{2}+\frac{32\,\tan\left(a+b\,x\right)^5}{5}-\frac{7\,\tan\left(a+b\,x\right)^3}{2}-\frac{3\,\tan\left(a+b\,x\right)}{4}}{b\,\left({\tan\left(a+b\,x\right)}^{10}+5\,{\tan\left(a+b\,x\right)}^8+10\,{\tan\left(a+b\,x\right)}^6+10\,{\tan\left(a+b\,x\right)}^4+5\,{\tan\left(a+b\,x\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^6/sin(a + b*x)^2,x)
```

```
[Out] (3*x)/4 + ((32*tan(a + b*x)^5)/5 - (7*tan(a + b*x)^3)/2 - (3*tan(a + b*x))/
4 + (7*tan(a + b*x)^7)/2 + (3*tan(a + b*x)^9)/4)/(b*(5*tan(a + b*x)^2 + 10*
tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*x)^10 + 1
))
```

3.48 $\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=29

$$-\frac{16\cos^6(a+bx)}{3b}+\frac{4\cos^8(a+bx)}{b}$$

```
[Out] -16/3*cos(b*x+a)^6/b+4*cos(b*x+a)^8/b
```

Rubi [A]
time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {4373, 2645, 14}

$$\frac{4\cos^8(a+bx)}{b}-\frac{16\cos^6(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-16*Cos[a + b*x]^6)/(3*b) + (4*Cos[a + b*x]^8)/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^5(2a+2bx)\,dx = 32\int \cos^5(a+bx)\sin^3(a+bx)\,dx$$
$$= -\frac{32\text{Subst}\left(\int x^5(1-x^2)\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{32\text{Subst}\left(\int (x^5-x^7)\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{16\cos^6(a+bx)}{3b} + \frac{4\cos^8(a+bx)}{b}$$

Mathematica [A]
time = 0.15, size = 48, normalized size = 1.66

$$\frac{-72\cos(2(a+bx)) - 12\cos(4(a+bx)) + 8\cos(6(a+bx)) + 3\cos(8(a+bx))}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
[Out] (-72*Cos[2*(a + b*x)] - 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] + 3*Cos[8*
(a + b*x)])/(96*b)
```

Maple [A]
time = 0.06, size = 35, normalized size = 1.21

method	result	size
default	$\frac{-4\left(\sin^2(xb+a)\right)\left(\cos^6(xb+a)\right)-\frac{4\left(\cos^6(xb+a)\right)}{3}}{b}$	35
risch	$\frac{\cos(8xb+8a)}{32b}+\frac{\cos(6xb+6a)}{12b}-\frac{\cos(4xb+4a)}{8b}-\frac{3\cos(2xb+2a)}{4b}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
[Out] 32/b*(-1/8*sin(b*x+a)^2*cos(b*x+a)^6-1/24*cos(b*x+a)^6)
```

Maxima [A]
time = 0.31, size = 50, normalized size = 1.72

$$\frac{3\,\cos\left(8\,bx+8\,a\right)+8\,\cos\left(6\,bx+6\,a\right)-12\,\cos\left(4\,bx+4\,a\right)-72\,\cos\left(2\,bx+2\,a\right)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

[Out] 1/96*(3*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) - 72*co
s(2*b*x + 2*a))/b

Fricas [A]
time = 3.71, size = 26, normalized size = 0.90

$$\frac{4\left(3\cos\left(bx+a\right)^{8}-4\cos\left(bx+a\right)^{6}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")

[Out] 4/3*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**5,x)

[Out] Timed out

Giac [A]
time = 0.40, size = 36, normalized size = 1.24

$$\frac{4\left(3\sin\left(bx+a\right)^{8}-8\sin\left(bx+a\right)^{6}+6\sin\left(bx+a\right)^{4}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")

[Out] 4/3*(3*sin(b*x + a)^8 - 8*sin(b*x + a)^6 + 6*sin(b*x + a)^4)/b

Mupad [B]
time = 0.05, size = 25, normalized size = 0.86

$$\frac{4\cos\left(a+b\,x\right)^6\,\left(3\cos\left(a+b\,x\right)^2-4\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(2*a + 2*b*x)^5/sin(a + b*x)^2,x)

[Out] (4*cos(a + b*x)^6*(3*cos(a + b*x)^2 - 4))/(3*b)

3.49 $\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=60

$$x+\frac{\cos(a+bx)\sin(a+bx)}{b}+\frac{2\cos^3(a+bx)\sin(a+bx)}{3b}-\frac{8\cos^5(a+bx)\sin(a+bx)}{3b}$$

```
[Out] x+cos(b*x+a)*sin(b*x+a)/b+2/3*cos(b*x+a)^3*sin(b*x+a)/b-8/3*cos(b*x+a)^5*si
n(b*x+a)/b
```

Rubi [A]
time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {4373, 2648, 2715, 8}

$$-\frac{8\sin(a+bx)\cos^5(a+bx)}{3b}+\frac{2\sin(a+bx)\cos^3(a+bx)}{3b}+\frac{\sin(a+bx)\cos(a+bx)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] x + (Cos[a + b*x]*Sin[a + b*x])/b + (2*Cos[a + b*x]^3*Sin[a + b*x])/(3*b) -
 (8*Cos[a + b*x]^5*Sin[a + b*x])/(3*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
```

```
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sin^4(2a+2bx)\,dx &= 16\int \cos^4(a+bx)\sin^2(a+bx)\,dx\\ &= -\frac{8\cos^5(a+bx)\sin(a+bx)}{3b}+\frac{8}{3}\int \cos^4(a+bx)\,dx\\ &= \frac{2\cos^3(a+bx)\sin(a+bx)}{3b}-\frac{8\cos^5(a+bx)\sin(a+bx)}{3b}+2\int \cos^2(a+b\\ &= \frac{\cos(a+bx)\sin(a+bx)}{b}+\frac{2\cos^3(a+bx)\sin(a+bx)}{3b}-\frac{8\cos^5(a+bx)\sin}{3b}\\ &= x+\frac{\cos(a+bx)\sin(a+bx)}{b}+\frac{2\cos^3(a+bx)\sin(a+bx)}{3b}-\frac{8\cos^5(a+bx}{3}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 40, normalized size = 0.67

$$-\frac{-12bx-3\sin(2(a+bx))+3\sin(4(a+bx))+\sin(6(a+bx))}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] -1/12*(-12*b*x - 3*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)]
)/b
```

Maple [A]
time = 0.11, size = 55, normalized size = 0.92

method	result	size
risch	$x-\frac{\sin(6xb+6a)}{12b}-\frac{\sin(4xb+4a)}{4b}+\frac{\sin(2xb+2a)}{4b}$	45
default	$\frac{-\frac{8\sin(xb+a)\left(\cos^5(xb+a)\right)}{3}+\frac{2\left(\cos^3(xb+a)+\frac{3\cos(xb+a)}{2}\right)\sin(xb+a)}{3}+xb+a}{b}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 16/b*(-1/6*sin(b*x+a)*cos(b*x+a)^5+1/24*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b
*x+a)+1/16*x*b+1/16*a)
```

Maxima [A]
time = 0.28, size = 43, normalized size = 0.72

$$\frac{12\,bx-\sin\left(6\,bx+6\,a\right)-3\,\sin\left(4\,bx+4\,a\right)+3\,\sin\left(2\,bx+2\,a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(12*b*x - sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))/
b
```

Fricas [A]
time = 1.92, size = 47, normalized size = 0.78

$$\frac{3\,bx-\left(8\,\cos\left(bx+a\right)^{5}-2\,\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x - (8*cos(b*x + a)^5 - 2*cos(b*x + a)^3 - 3*cos(b*x + a))*sin(b*x
 + a))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 55, normalized size = 0.92

$$\frac{3\,bx+3\,a+\frac{3\,\tan\left(bx+a\right)^{5}+8\,\tan\left(bx+a\right)^{3}-3\,\tan\left(bx+a\right)}{\left(\tan\left(bx+a\right)^{2}+1\right)^{3}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*b*x + 3*a + (3*tan(b*x + a)^5 + 8*tan(b*x + a)^3 - 3*tan(b*x + a))/(
tan(b*x + a)^2 + 1)^3)/b
```

Mupad [B]
time = 0.49, size = 65, normalized size = 1.08

$$x+\frac{\tan(a+b\,x)^5+\frac{8\tan(a+b\,x)^3}{3}-\tan(a+b\,x)}{b\,\left(\tan\left(a+b\,x\right)^6+3\tan\left(a+b\,x\right)^4+3\tan\left(a+b\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^4/sin(a + b*x)^2,x)
```

```
[Out] x + ((8*tan(a + b*x)^3)/3 - tan(a + b*x) + tan(a + b*x)^5)/(b*(3*tan(a + b*
x)^2 + 3*tan(a + b*x)^4 + tan(a + b*x)^6 + 1))
```

3.50 $\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=13

$$-\frac{2\cos^4(a+bx)}{b}$$

[Out] -2*cos(b*x+a)^4/b

Rubi [A]
time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2645, 30}

$$-\frac{2\cos^4(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]

[Out] (-2*Cos[a + b*x]^4)/b

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^3(2a+2bx)\,dx = 8\int \cos^3(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{8\text{Subst}\left(\int x^3\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{2\cos^4(a+bx)}{b}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$-\frac{2\cos^4(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
[Out] (-2*Cos[a + b*x]^4)/b
```

Maple [A]
time = 0.05, size = 14, normalized size = 1.08

method	result	size
default	$-\frac{2(\cos^4(xb+a))}{b}$	14
risch	$-\frac{\cos(4xb+4a)}{4b}-\frac{\cos(2xb+2a)}{b}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
[Out] -2*cos(b*x+a)^4/b
```

Maxima [A]
time = 0.27, size = 26, normalized size = 2.00

$$-\frac{\cos(4\,bx+4\,a)+4\,\cos(2\,bx+2\,a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
[Out] -1/4*(cos(4*b*x + 4*a) + 4*cos(2*b*x + 2*a))/b
```

Fricas [**A**]
time = 3.53, size = 13, normalized size = 1.00

$$-\frac{2\cos(bx+a)^4}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
[Out] -2*cos(b*x + a)^4/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**3,x)
[Out] Timed out
```

Giac [**A**]
time = 0.55, size = 13, normalized size = 1.00

$$-\frac{2\cos(bx+a)^4}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
[Out] -2*cos(b*x + a)^4/b
```

Mupad [**B**]
time = 0.11, size = 13, normalized size = 1.00

$$-\frac{2\cos(a+b\,x)^4}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^3/sin(a + b*x)^2,x)
[Out] -(2*cos(a + b*x)^4)/b
```

3.51 $\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=21

$$2x+\frac{2\cos(a+bx)\sin(a+bx)}{b}$$

```
[Out] 2*x+2*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A]

time = 0.03, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4373, 2715, 8}

$$\frac{2\sin(a+bx)\cos(a+bx)}{b}+2x$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] 2*x + (2*Cos[a + b*x]*Sin[a + b*x])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^2(a+bx)\,dx\\ &= \frac{2\cos(a+bx)\sin(a+bx)}{b}+2\int 1\,dx\\ &= 2x+\frac{2\cos(a+bx)\sin(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 20, normalized size = 0.95

$$\frac{2(a+bx)+\sin(2(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
[Out] (2*(a + b*x) + Sin[2*(a + b*x)])/b
```

Maple [A]
time = 0.06, size = 28, normalized size = 1.33

method	result	size
risch	$2x+\frac{\sin(2xb+2a)}{b}$	18
default	$\frac{2\cos(xb+a)\sin(xb+a)+2xb+2a}{b}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
[Out] 4/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*x*b+1/2*a)
```

Maxima [A]
time = 0.26, size = 18, normalized size = 0.86

$$\frac{2\,bx+\sin\left(2\,bx+2\,a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
[Out] (2*b*x + sin(2*b*x + 2*a))/b
```

Fricas [A]
time = 2.53, size = 22, normalized size = 1.05

$$\frac{2\left(bx+\cos\left(bx+a\right)\sin\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
[Out] 2*(b*x + cos(b*x + a)*sin(b*x + a))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.49, size = 29, normalized size = 1.38

$$\frac{2\left(bx+a+\frac{\tan(bx+a)}{\tan(bx+a)^2+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 2*(b*x + a + tan(b*x + a)/(tan(b*x + a)^2 + 1))/b
```

Mupad [B]
time = 0.15, size = 17, normalized size = 0.81

$$2\,x+\frac{\sin\left(2\,a+2\,b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^2/sin(a + b*x)^2,x)
```

```
[Out] 2*x + sin(2*a + 2*b*x)/b
```

3.52 $\int \csc^2(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=12

$$\frac{2\log(\sin(a+bx))}{b}$$

```
[Out] 2*ln(sin(b*x+a))/b
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{4373, 3556\}$

$$\frac{2\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*Log[Sin[a + b*x]])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sin(2a+2bx)\,dx &= 2\int \cot(a+bx)\,dx\\ &= \frac{2\log(\sin(a+bx))}{b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 1.67

$$\frac{2(\log(\cos(a+bx))+\log(\tan(a+bx)))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

```
[Out] (2*(Log[Cos[a + b*x]] + Log[Tan[a + b*x]]))/b
```

Maple [A]
time = 0.06, size = 13, normalized size = 1.08

method	result	size
default	$\frac{2\ln(\sin(xb+a))}{b}$	13
risch	$-2ix-\frac{4ia}{b}+\frac{2\ln\left(e^{2i(xb+a)}-1\right)}{b}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*ln(sin(b*x+a))/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(12) = 24$.
time = 0.27, size = 81, normalized size = 6.75

$$\frac{\log\left(\cos(bx)^2+2\cos(bx)\cos(a)+\cos(a)^2+\sin(bx)^2-2\sin(bx)\sin(a)+\sin(a)^2\right)+\log\left(\cos(bx)^2-2\cos(bx)\cos(a)+\cos(a)^2+\sin(bx)^2+2\sin(bx)\sin(a)+\sin(a)^2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] (log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*si
n(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)
^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Fricas [A]
time = 3.92, size = 14, normalized size = 1.17

$$\frac{2\log\left(\frac{1}{2}\sin(bx+a)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 2*log(1/2*sin(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 6307 vs. $2(10) = 20$.
time = 155.40, size = 18894, normalized size = 1574.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a),x)
```

```
[Out] 4*Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (log(sin(b*x))/b, Eq(a, 0)), (0,
Eq(b, 0)), (-4*b*x*tan(a/2)**5*tan(b*x/2)/(b*tan(a/2)**6*tan(b*x/2) + b*tan
(a/2)**5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*tan
(a/2)**3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b*tan
(a/2)*tan(b*x/2)**2 - b*tan(a/2) - b*tan(b*x/2)) - 4*b*x*tan(a/2)**4*tan(b*
x/2)**2/(b*tan(a/2)**6*tan(b*x/2) + b*tan(a/2)**5*tan(b*x/2)**2 - b*tan(a/2
)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*tan(a/2)**3*tan(b*x/2)**2 - 2*b*tan(a
/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b*tan(a/2)*tan(b*x/2)**2 - b*tan(a/2) -
 b*tan(b*x/2)) + 4*b*x*tan(a/2)**4/(b*tan(a/2)**6*tan(b*x/2) + b*tan(a/2)**
5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*tan(a/2)**
3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b*tan(a/2)*t
an(b*x/2)**2 - b*tan(a/2) - b*tan(b*x/2)) + 8*b*x*tan(a/2)**3*tan(b*x/2)/(b
*tan(a/2)**6*tan(b*x/2) + b*tan(a/2)**5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*t
an(a/2)**4*tan(b*x/2) + 2*b*tan(a/2)**3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b
*tan(a/2)**2*tan(b*x/2) + b*tan(a/2)*tan(b*x/2)**2 - b*tan(a/2) - b*tan(b*x
/2)) + 4*b*x*tan(a/2)**2*tan(b*x/2)**2/(b*tan(a/2)**6*tan(b*x/2) + b*tan(a/
2)**5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*tan(a/
2)**3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b*tan(a/
2)*tan(b*x/2)**2 - b*tan(a/2) - b*tan(b*x/2)) - 4*b*x*tan(a/2)**2/(b*tan(a/
2)**6*tan(b*x/2) + b*tan(a/2)**5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*tan(a/2)
**4*tan(b*x/2) + 2*b*tan(a/2)**3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b*tan(a/
2)**2*tan(b*x/2) + b*tan(a/2)*tan(b*x/2)**2 - b*tan(a/2) - b*tan(b*x/2)) -
4*b*x*tan(a/2)*tan(b*x/2)/(b*tan(a/2)**6*tan(b*x/2) + b*tan(a/2)**5*tan(b*x
/2)**2 - b*tan(a/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*tan(a/2)**3*tan(b*x
/2)**2 - 2*b*tan(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b*tan(a/2)*tan(b*x/2)
**2 - b*tan(a/2) - b*tan(b*x/2)) + log(tan(a/2) + tan(b*x/2))*tan(a/2)**6*t
an(b*x/2)/(b*tan(a/2)**6*tan(b*x/2) + b*tan(a/2)**5*tan(b*x/2)**2 - b*tan(a
/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*tan(a/2)**3*tan(b*x/2)**2 - 2*b*tan
(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b*tan(a/2)*tan(b*x/2)**2 - b*tan(a/2)
 - b*tan(b*x/2)) + log(tan(a/2) + tan(b*x/2))*tan(a/2)**5*tan(b*x/2)**2/(b*
tan(a/2)**6*tan(b*x/2) + b*tan(a/2)**5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*ta
n(a/2)**4*tan(b*x/2) + 2*b*tan(a/2)**3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b*
tan(a/2)**2*tan(b*x/2) + b*tan(a/2)*tan(b*x/2)**2 - b*tan(a/2) - b*tan(b*x/
2)) - log(tan(a/2) + tan(b*x/2))*tan(a/2)**5/(b*tan(a/2)**6*tan(b*x/2) + b*
tan(a/2)**5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*
tan(a/2)**3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b*
tan(a/2)*tan(b*x/2)**2 - b*tan(a/2) - b*tan(b*x/2)) - 7*log(tan(a/2) + tan(
b*x/2))*tan(a/2)**4*tan(b*x/2)/(b*tan(a/2)**6*tan(b*x/2) + b*tan(a/2)**5*ta
n(b*x/2)**2 - b*tan(a/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*tan(a/2)**3*ta
n(b*x/2)**2 - 2*b*tan(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b*tan(a/2)*tan(b
*x/2)**2 - b*tan(a/2) - b*tan(b*x/2)) - 6*log(tan(a/2) + tan(b*x/2))*tan(a/
```

```
2)**3*tan(b*x/2)**2/(b*tan(a/2)**6*tan(b*x/2) + b*tan(a/2)**5*tan(b*x/2)**2
 - b*tan(a/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*tan(a/2)**3*tan(b*x/2)**2
 - 2*b*tan(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b*tan(a/2)*tan(b*x/2)**2 -
b*tan(a/2) - b*tan(b*x/2)) + 6*log(tan(a/2) + tan(b*x/2))*tan(a/2)**3/(b*ta
n(a/2)**6*tan(b*x/2) + b*tan(a/2)**5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*tan(
a/2)**4*tan(b*x/2) + 2*b*tan(a/2)**3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b*ta
n(a/2)**2*tan(b*x/2) + b*tan(a/2)*tan(b*x/2)**2 - b*tan(a/2) - b*tan(b*x/2)
) + 7*log(tan(a/2) + tan(b*x/2))*tan(a/2)**2*tan(b*x/2)/(b*tan(a/2)**6*tan(
b*x/2) + b*tan(a/2)**5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*tan(a/2)**4*tan(b*
x/2) + 2*b*tan(a/2)**3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b*tan(a/2)**2*tan(
b*x/2) + b*tan(a/2)*tan(b*x/2)**2 - b*tan(a/2) - b*tan(b*x/2)) + log(tan(a/
2) + tan(b*x/2))*tan(a/2)*tan(b*x/2)**2/(b*tan(a/2)**6*tan(b*x/2) + b*tan(a
/2)**5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*tan(a
/2)**3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b*tan(a
/2)*tan(b*x/2)**2 - b*tan(a/2) - b*tan(b*x/2)) - log(tan(a/2) + tan(b*x/2))
*tan(a/2)/(b*tan(a/2)**6*tan(b*x/2) + b*tan(a/2)**5*tan(b*x/2)**2 - b*tan(a
/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*tan(a/2)**3*tan(b*x/2)**2 - 2*b*tan
(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b*tan(a/2)*tan(b*x/2)**2 - b*tan(a/2)
 - b*tan(b*x/2)) - log(tan(a/2) + tan(b*x/2))*tan(b*x/2)/(b*tan(a/2)**6*tan
(b*x/2) + b*tan(a/2)**5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*tan(a/2)**4*tan(b
*x/2) + 2*b*tan(a/2)**3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b*tan(a/2)**2*tan
(b*x/2) + b*tan(a/2)*tan(b*x/2)**2 - b*tan(a/2) - b*tan(b*x/2)) + log(tan(b
*x/2) - 1/tan(a/2))*tan(a/2)**6*tan(b*x/2)/(b*tan(a/2)**6*tan(b*x/2) + b*ta
n(a/2)**5*tan(b*x/2)**2 - b*tan(a/2)**5 + b*tan(a/2)**4*tan(b*x/2) + 2*b*ta
n(a/2)**3*tan(b*x/2)**2 - 2*b*tan(a/2)**3 - b*tan(a/2)**2*tan(b*x/2) + b*ta
n(a/2)*tan(b*x/2)**2 - b*tan(a/2) - b*tan(b*x/2...
```

Giac [A]
time = 0.42, size = 13, normalized size = 1.08

$$\frac{2 \log\left(\left|\sin\left(bx + a\right)\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
[Out] 2*log(abs(sin(b*x + a)))/b
```

Mupad [B]
time = 0.13, size = 13, normalized size = 1.08

$$\frac{\ln\left({\sin\left(a+b\,x\right)}^2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)/sin(a + b*x)^2,x)
[Out] log(sin(a + b*x)^2)/b
```

3.53 $\int \csc^2(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=30

$$-\frac{\cot^2(a+bx)}{4b}+\frac{\log(\tan(a+bx))}{2b}$$

```
[Out] -1/4*cot(b*x+a)^2/b+1/2*ln(tan(b*x+a))/b
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4373, 2700, 14}

$$\frac{\log(\tan(a+bx))}{2b}-\frac{\cot^2(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

```
[Out] -1/4*Cot[a + b*x]^2/b + Log[Tan[a + b*x]]/(2*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^2(a+bx)\csc(2a+2bx)\,dx = \frac{1}{2}\int \csc^3(a+bx)\sec(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^3}\,dx, x, \tan(a+bx)\right)}{2b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)\,dx, x, \tan(a+bx)\right)}{2b}$$
$$= -\frac{\cot^2(a+bx)}{4b} + \frac{\log(\tan(a+bx))}{2b}$$

Mathematica [**A**]
time = 0.06, size = 34, normalized size = 1.13

$$-\frac{\csc^2(a+bx)+2\log(\cos(a+bx))-2\log(\sin(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

```
[Out] -1/4*(Csc[a + b*x]^2 + 2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]])/b
```

Maple [**A**]
time = 0.08, size = 24, normalized size = 0.80

method	result	size
default	$\frac{-\frac{1}{2\sin(xb+a)^2}+\ln(\tan(xb+a))}{2b}$	24
risch	$\frac{e^{2i(xb+a)}}{b\left(e^{2i(xb+a)}-1\right)^2}+\frac{\ln\left(e^{2i(xb+a)}-1\right)}{2b}-\frac{\ln\left(e^{2i(xb+a)}+1\right)}{2b}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*(-1/2/sin(b*x+a)^2+ln(tan(b*x+a)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(26) = 52.
time = 0.29, size = 656, normalized size = 21.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/4*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) - 8*cos(2*b*x + 2*a)^2 + (2*(2*cos
(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*
a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b
*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2
*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) - (2*
(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*
x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*s
in(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos
(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*(2*cos(2*b
*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2
 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x +
 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(
a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b*x + 4*a)*sin(
2*b*x + 2*a) - 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b*cos(4*b*x + 4*
a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*sin(4*b*x + 4*a)
*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*b*x + 2*a) - b)*c
os(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) + b)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.
time = 4.32, size = 65, normalized size = 2.17

$$-\frac{\left(\cos\left(bx+a\right)^2-1\right)\log\left(\cos\left(bx+a\right)^2\right)-\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\frac{1}{4}\cos\left(bx+a\right)^2+\frac{1}{4}\right)-1}{4\left(b\cos\left(bx+a\right)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(b*x + a)^2 - 1)*log(cos(b*x + a)^2) - (cos(b*x + a)^2 - 1)*log(-
1/4*cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^2\left(a+bx\right)\csc\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a),x)
```

```
[Out] Integral(csc(a + b*x)**2*csc(2*a + 2*b*x), x)
```

Giac [**A**]
time = 0.43, size = 37, normalized size = 1.23

$$-\frac{\frac{1}{\sin(bx+a)^2}+\log\left(-\sin\left(bx+a\right)^2+1\right)-2\log\left(\left|\sin\left(bx+a\right)\right|\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/4*(1/sin(b*x + a)^2 + log(-sin(b*x + a)^2 + 1) - 2*log(abs(sin(b*x + a))
))/b
```

Mupad [B]
time = 0.14, size = 36, normalized size = 1.20

$$-\frac{\frac{\ln(\cos(a+b\,x))}{2}-\frac{\ln\left(\sin(a+b\,x)^2\right)}{4}+\frac{1}{4\,\sin(a+b\,x)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)),x)
```

```
[Out] -(log(cos(a + b*x))/2 - log(sin(a + b*x)^2)/4 + 1/(4*sin(a + b*x)^2))/b
```

3.54 $\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=42

$$-\frac{\cot(a+bx)}{2b}-\frac{\cot^3(a+bx)}{12b}+\frac{\tan(a+bx)}{4b}$$

```
[Out] -1/2*cot(b*x+a)/b-1/12*cot(b*x+a)^3/b+1/4*tan(b*x+a)/b
```

Rubi [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4373, 2700, 276}

$$\frac{\tan(a+bx)}{4b}-\frac{\cot^3(a+bx)}{12b}-\frac{\cot(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -1/2*Cot[a + b*x]/b - Cot[a + b*x]^3/(12*b) + Tan[a + b*x]/(4*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^2(a+bx)\csc^2(2a+2bx)\,dx = \frac{1}{4}\int \csc^4(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4}\,dx, x, \tan(a+bx)\right)}{4b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)\,dx, x, \tan(a+bx)\right)}{4b}$$
$$= -\frac{\cot(a+bx)}{2b} - \frac{\cot^3(a+bx)}{12b} + \frac{\tan(a+bx)}{4b}$$

Mathematica [A]
time = 0.09, size = 48, normalized size = 1.14

$$-\frac{5\cot(a+bx)}{12b} - \frac{\cot(a+bx)\csc^2(a+bx)}{12b} + \frac{\tan(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] (-5*Cot[a + b*x])/(12*b) - (Cot[a + b*x]*Csc[a + b*x]^2)/(12*b) + Tan[a + b
*x]/(4*b)
```

Maple [A]
time = 0.10, size = 51, normalized size = 1.21

method	result	size
risch	$\frac{4i\left(2\,{\mathrm e}^{2i(xb+a)}-1\right)}{3b\left({\mathrm e}^{2i(xb+a)}-1\right)^{3}\left({\mathrm e}^{2i(xb+a)}+1\right)}$	46
default	$\frac{-\frac{1}{3\sin(xb+a)^{3}\cos(xb+a)}+\frac{4}{3\sin(xb+a)\cos(xb+a)}-\frac{8\cot(xb+a)}{3}}{4b}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*(-1/3/sin(b*x+a)^3/cos(b*x+a)+4/3/sin(b*x+a)/cos(b*x+a)-8/3*cot(b*x+a
))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(36) = 72$.
time = 0.28, size = 308, normalized size = 7.33

$$\frac{4\left((2\cos(2bx+2a)-1)\sin(8bx+8a)-2(2\cos(2bx+2a)-1)\sin(6bx+6a)-2\cos(8bx+8a)\sin(2bx+2a)+4\cos(6bx+6a)\sin(2bx+2a)\right)}{3\left(b\cos(8bx+8a)^2+4b\cos(6bx+6a)^2+4b\cos(2bx+2a)^2+b\sin(8bx+8a)^2+4b\sin(6bx+6a)^2-8b\sin(6bx+6a)\sin(2bx+2a)+4b\sin(2bx+2a)^2-2(2b\cos(6bx+6a)-2b\cos(2bx+2a)+b)\cos(8bx+8a)-4(2b\cos(2bx+2a)-b)\cos(6bx+6a)-4b\cos(2bx+2a)-4(b\sin(6bx+6a)-b\sin(2bx+2a))\sin(8bx+8a)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 4/3*((2*cos(2*b*x + 2*a) - 1)*sin(8*b*x + 8*a) - 2*(2*cos(2*b*x + 2*a) - 1)
*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) + 4*cos(6*b*x + 6*a
)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*co
s(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b*sin(
6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(6*b*x +
 6*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*x + 2*a
) - b)*cos(6*b*x + 6*a) - 4*b*cos(2*b*x + 2*a) - 4*(b*sin(6*b*x + 6*a) - b*
sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

Fricas [A]
time = 2.80, size = 54, normalized size = 1.29

$$-\frac{8\cos(bx+a)^4-12\cos(bx+a)^2+3}{12\left(b\cos(bx+a)^3-b\cos(bx+a)\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(8*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 3)/((b*cos(b*x + a)^3 - b*cos
(b*x + a))*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\csc^2(a+bx)\csc^2(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**2,x)
```

```
[Out] Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**2, x)
```

Giac [A]
time = 0.45, size = 35, normalized size = 0.83

$$-\frac{\frac{6\tan(bx+a)^2+1}{\tan(bx+a)^3}-3\tan(bx+a)}{12b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/12*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

Mupad [B]
time = 0.14, size = 37, normalized size = 0.88

$$\frac{\tan(a+b\,x)}{4\,b}-\frac{\frac{\tan(a+b\,x)^2}{2}+\frac{1}{12}}{b\tan\left(a+b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^2),x)
[Out] tan(a + b*x)/(4*b) - (tan(a + b*x)^2/2 + 1/12)/(b*tan(a + b*x)^3)
```

3.55 $\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=60

$$-\frac{3\cot^2(a+bx)}{16b}-\frac{\cot^4(a+bx)}{32b}+\frac{3\log(\tan(a+bx))}{8b}+\frac{\tan^2(a+bx)}{16b}$$

```
[Out] -3/16*cot(b*x+a)^2/b-1/32*cot(b*x+a)^4/b+3/8*ln(tan(b*x+a))/b+1/16*tan(b*x+
a)^2/b
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {4373, 2700, 272, 45}

$$\frac{\tan^2(a+bx)}{16b}-\frac{\cot^4(a+bx)}{32b}-\frac{3\cot^2(a+bx)}{16b}+\frac{3\log(\tan(a+bx))}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (-3*Cot[a + b*x]^2)/(16*b) - Cot[a + b*x]^4/(32*b) + (3*Log[Tan[a + b*x]])/
(8*b) + Tan[a + b*x]^2/(16*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
```

```
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx &= \frac{1}{8}\int \csc^5(a+bx)\sec^3(a+bx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^5}\,dx, x, \tan(a+bx)\right)}{8b} \\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^3}\,dx, x, \tan^2(a+bx)\right)}{16b} \\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^3}+\frac{3}{x^2}+\frac{3}{x}\right)\,dx, x, \tan^2(a+bx)\right)}{16b} \\
&= -\frac{3\cot^2(a+bx)}{16b}-\frac{\cot^4(a+bx)}{32b}+\frac{3\log(\tan(a+bx))}{8b}+\frac{\tan^2(a+bx)}{16b}
\end{aligned}
$$

Mathematica [A]
time = 0.38, size = 54, normalized size = 0.90

$$
-\frac{4\csc^2(a+bx)+\csc^4(a+bx)+12\log(\cos(a+bx))-12\log(\sin(a+bx))-2\sec^2(a+bx)}{32b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] -1/32*(4*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 12*Log[Cos[a + b*x]] - 12*Log[Si
n[a + b*x]] - 2*Sec[a + b*x]^2)/b
```

Maple [A]
time = 0.10, size = 62, normalized size = 1.03

method	result	size
default	$\frac{-\frac{1}{4\sin(xb+a)^4\cos(xb+a)^2}+\frac{3}{4\sin(xb+a)^2\cos(xb+a)^2}-\frac{3}{2\sin(xb+a)^2}+3\ln(\tan(xb+a))}{8b}$	62
risch	$\frac{3\,e^{10i(xb+a)}-6\,e^{8i(xb+a)}-2\,e^{6i(xb+a)}-6\,e^{4i(xb+a)}+3\,e^{2i(xb+a)}}{4b\left(e^{2i(xb+a)}-1\right)^4\left(e^{2i(xb+a)}+1\right)^2}+\frac{3\ln\left(e^{2i(xb+a)}-1\right)}{8b}-\frac{3\ln\left(e^{2i(xb+a)}+1\right)}{8b}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)^2+3/4/sin(b*x+a)^2/cos(b*x+a)^2-3/2/sin
(b*x+a)^2+3*ln(tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3188 vs. $2(52) = 104$.
time = 0.35, size = 3188, normalized size = 53.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(3*cos(10*b*x + 10*a) - 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 6
*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b*x
 + 8*a) + 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) - 12*cos(2*b*x + 2*a) +
3)*cos(10*b*x + 10*a) - 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a) -
12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) + 6)*cos(8*b*x + 8*a) + 24*cos(8*b
*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) - 8*cos(2*b*x + 2*a) + 1)*cos(6*b*x +
6*a) - 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) - 2)*cos(4*b*x + 4*a)
 + 24*cos(4*b*x + 4*a)^2 - 24*cos(2*b*x + 2*a)^2 + 3*(2*(2*cos(10*b*x + 10*
a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x
 + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a
) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(10*
b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) - cos(4*b*x +
4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 8*(c
os(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 16*cos(6*b*x +
 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2
- 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*x + 8*a) - 4*sin
(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) -
 sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*
x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*sin(10*b*x + 10*a)^2
+ 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x
+ 8*a) - sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(6*b*x + 6*a) - 16*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 4*sin(4*b*x +
4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(
cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*
b*x)*sin(2*a) + sin(2*a)^2) - 3*(2*(2*cos(10*b*x + 10*a) + cos(8*b*x + 8*a)
 - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b
*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a
) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - 4*cos(1
0*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) - 2*cos(2*b*x +
2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*c
os(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*
b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^
2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4
*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2
 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*
x + 2*a))*sin(10*b*x + 10*a) - 4*sin(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*
```

```
a) - sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - sin(8*b*x +
8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 16*si
n(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
 - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*(2*(2
*cos(10*b*x + 10*a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4
*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4
*(cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x +
2*a) - 1)*cos(10*b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*
a) - cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*
x + 8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a)
 - 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - co
s(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b
*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x
+ 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*sin(
10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2*b*x +
 2*a))*sin(8*b*x + 8*a) - sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(
2*b*x + 2*a))*sin(6*b*x + 6*a) - 16*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2
 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x
 + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(10*b*x + 10*a) - 6*sin(8*b*x + 8*a)
 - 2*sin(6*b*x + 6*a) - 6*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x
 + 12*a) + 4*(9*sin(8*b*x + 8*a) + 16*sin(6*b*x + 6*a) + 9*sin(4*b*x + 4*a)
 - 12*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 24*sin(10*b*x + 10*a)^2 - 4*(2
2*sin(6*b*x + 6*a) - 12*sin(4*b*x + 4*a) - 9*sin(2*b*x + 2*a))*sin(8*b*x +
8*a) + 24*sin(8*b*x + 8*a)^2 - 8*(11*sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))
*sin(6*b*x + 6*a) - 32*sin(6*b*x + 6*a)^2 + 24*sin(4*b*x + 4*a)^2 + 36*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) - 24*sin(2*b*x + 2*a)^2 + 12*cos(2*b*x + 2*a)
)/(b*cos(12*b*x + 12*a)^2 + 4*b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2
 + 16*b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x + 4*a)...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(52) = 104$.
time = 3.32, size = 138, normalized size = 2.30

$$\frac{6 \cos(bx+a)^4 - 9 \cos(bx+a)^2 - 6 \left(\cos(bx+a)^6 - 2 \cos(bx+a)^4 + \cos(bx+a)^2\right) \log\left(\cos(bx+a)^2\right) + 6 \left(\cos(bx+a)^6 - 2 \cos(bx+a)^4 + \cos(bx+a)^2\right) \log\left(-\frac{1}{4} \cos(bx+a)^2 + \frac{1}{4}\right) + 2}{32 \left(b \cos(bx+a)^6 - 2 b \cos(bx+a)^4 + b \cos(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - 2*cos(b*x +
 a)^4 + cos(b*x + a)^2)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - 2*cos(b*x
 + a)^4 + cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2)/(b*cos(b*x +
a)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^2(a+bx)\csc^3(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**3,x)
```

```
[Out] Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**3, x)
```

Giac [A]
time = 0.50, size = 74, normalized size = 1.23

$$-\frac{\frac{6\,\sin(bx+a)^4-3\,\sin(bx+a)^2-1}{\left(\sin(bx+a)^2-1\right)\sin(bx+a)^4}+6\,\log\left(-\sin\left(bx+a\right)^2+1\right)-12\,\log\left(\left|\sin\left(bx+a\right)\right|\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -1/32*((6*sin(b*x + a)^4 - 3*sin(b*x + a)^2 - 1)/((sin(b*x + a)^2 - 1)*sin(
b*x + a)^4) + 6*log(-sin(b*x + a)^2 + 1) - 12*log(abs(sin(b*x + a))))/b
```

Mupad [B]
time = 0.22, size = 82, normalized size = 1.37

$$\frac{3\,\ln\left(\sin\left(a+b\,x\right)^2\right)}{16\,b}-\frac{3\,\ln\left(\cos\left(a+b\,x\right)\right)}{8\,b}+\frac{\frac{3\cos(a+b\,x)^4}{16}-\frac{9\cos(a+b\,x)^2}{32}+\frac{1}{16}}{b\,\left(\cos\left(a+b\,x\right)^6-2\cos\left(a+b\,x\right)^4+\cos\left(a+b\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^3),x)
```

```
[Out] (3*log(sin(a + b*x)^2))/(16*b) - (3*log(cos(a + b*x)))/(8*b) + ((3*cos(a +
b*x)^4)/16 - (9*cos(a + b*x)^2)/32 + 1/16)/(b*(cos(a + b*x)^2 - 2*cos(a + b
*x)^4 + cos(a + b*x)^6))
```

3.56 $\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=72

$$-\frac{3\cot(a+bx)}{8b}-\frac{\cot^3(a+bx)}{12b}-\frac{\cot^5(a+bx)}{80b}+\frac{\tan(a+bx)}{4b}+\frac{\tan^3(a+bx)}{48b}$$

```
[Out] -3/8*cot(b*x+a)/b-1/12*cot(b*x+a)^3/b-1/80*cot(b*x+a)^5/b+1/4*tan(b*x+a)/b+
1/48*tan(b*x+a)^3/b
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {4373, 2700, 276}

$$\frac{\tan^3(a+bx)}{48b}+\frac{\tan(a+bx)}{4b}-\frac{\cot^5(a+bx)}{80b}-\frac{\cot^3(a+bx)}{12b}-\frac{3\cot(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-3*Cot[a + b*x])/(8*b) - Cot[a + b*x]^3/(12*b) - Cot[a + b*x]^5/(80*b) + T
an[a + b*x]/(4*b) + Tan[a + b*x]^3/(48*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx &= \frac{1}{16}\int \csc^6(a+bx)\sec^4(a+bx)\,dx \\ &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^6}\,dx, x, \tan(a+bx)\right)}{16b} \\ &= \frac{\text{Subst}\left(\int \left(4+\frac{1}{x^6}+\frac{4}{x^4}+\frac{6}{x^2}+x^2\right)\,dx, x, \tan(a+bx)\right)}{16b} \\ &= -\frac{3\cot(a+bx)}{8b}-\frac{\cot^3(a+bx)}{12b}-\frac{\cot^5(a+bx)}{80b}+\frac{\tan(a+bx)}{4b}+\frac{\tan^3(}{4}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 90, normalized size = 1.25

$$-\frac{73\cot(a+bx)}{240b}-\frac{7\cot(a+bx)\csc^2(a+bx)}{120b}-\frac{\cot(a+bx)\csc^4(a+bx)}{80b}+\frac{11\tan(a+bx)}{48b}+\frac{\sec^2(a+bx)\tan(a+bx)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-73*Cot[a + b*x])/(240*b) - (7*Cot[a + b*x]*Csc[a + b*x]^2)/(120*b) - (Cot
[a + b*x]*Csc[a + b*x]^4)/(80*b) + (11*Tan[a + b*x])/(48*b) + (Sec[a + b*x]
^2*Tan[a + b*x])/(48*b)
```

Maple [A]
time = 0.12, size = 87, normalized size = 1.21

method	result	size
risch	$-\frac{16i\left(6\,e^{6i(xb+a)}-2\,e^{4i(xb+a)}-2\,e^{2i(xb+a)}+1\right)}{15b\left(e^{2i(xb+a)}+1\right)^3\left(e^{2i(xb+a)}-1\right)^5}$	68
default	$\frac{-\frac{1}{5\sin(xb+a)^5\cos(xb+a)^3}+\frac{8}{15\sin(xb+a)^3\cos(xb+a)^3}-\frac{16}{15\sin(xb+a)^3\cos(xb+a)}+\frac{64}{15\sin(xb+a)\cos(xb+a)}-\frac{128\cot(xb+a)}{15}}{16b}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/b*(-1/5/sin(b*x+a)^5/cos(b*x+a)^3+8/15/sin(b*x+a)^3/cos(b*x+a)^3-16/15
/sin(b*x+a)^3/cos(b*x+a)+64/15/sin(b*x+a)/cos(b*x+a)-128/15*cot(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. 2(62) = 124.
time = 0.34, size = 1227, normalized size = 17.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 16/15*(2*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(16*
b*x + 16*a) - 4*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*
cos(14*b*x + 14*a) - 4*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin(2*b*x +
 2*a))*cos(12*b*x + 12*a) + 12*(3*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - sin
(2*b*x + 2*a))*cos(10*b*x + 10*a) - (6*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a
) - 2*cos(2*b*x + 2*a) + 1)*sin(16*b*x + 16*a) + 2*(6*cos(6*b*x + 6*a) - 2*
cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*sin(14*b*x + 14*a) + 2*(6*cos(6*
b*x + 6*a) - 2*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*sin(12*b*x + 12*a
) - 6*(6*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*si
n(10*b*x + 10*a))/(b*cos(16*b*x + 16*a)^2 + 4*b*cos(14*b*x + 14*a)^2 + 4*b*
cos(12*b*x + 12*a)^2 + 36*b*cos(10*b*x + 10*a)^2 + 36*b*cos(6*b*x + 6*a)^2
+ 4*b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(16*b*x + 16*a)^2
+ 4*b*sin(14*b*x + 14*a)^2 + 4*b*sin(12*b*x + 12*a)^2 + 36*b*sin(10*b*x + 1
0*a)^2 + 36*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x + 4*a)^2 + 8*b*sin(4*b*x +
 4*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(14*b*x + 14*a)
 + 2*b*cos(12*b*x + 12*a) - 6*b*cos(10*b*x + 10*a) + 6*b*cos(6*b*x + 6*a) -
 2*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(16*b*x + 16*a) + 4*(2
*b*cos(12*b*x + 12*a) - 6*b*cos(10*b*x + 10*a) + 6*b*cos(6*b*x + 6*a) - 2*b
*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(14*b*x + 14*a) - 4*(6*b*c
os(10*b*x + 10*a) - 6*b*cos(6*b*x + 6*a) + 2*b*cos(4*b*x + 4*a) + 2*b*cos(2
*b*x + 2*a) - b)*cos(12*b*x + 12*a) - 12*(6*b*cos(6*b*x + 6*a) - 2*b*cos(4*
b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a) - 12*(2*b*cos(4*b
*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 4*(2*b*cos(2*b*x +
 2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) - 4*(b*sin(14*b*x + 14*a
) + b*sin(12*b*x + 12*a) - 3*b*sin(10*b*x + 10*a) + 3*b*sin(6*b*x + 6*a) -
b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) + 8*(b*sin(12*b
*x + 12*a) - 3*b*sin(10*b*x + 10*a) + 3*b*sin(6*b*x + 6*a) - b*sin(4*b*x +
4*a) - b*sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - 8*(3*b*sin(10*b*x + 10*a) -
 3*b*sin(6*b*x + 6*a) + b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(12*b*x
 + 12*a) - 24*(3*b*sin(6*b*x + 6*a) - b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*
a))*sin(10*b*x + 10*a) - 24*(b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(6
*b*x + 6*a) + b)
```

Fricas [A]
time = 2.68, size = 86, normalized size = 1.19

$$-\frac{128\,\cos\left(bx+a\right)^{8}-320\,\cos\left(bx+a\right)^{6}+240\,\cos\left(bx+a\right)^{4}-40\,\cos\left(bx+a\right)^{2}-5}{240\,\left(b\cos\left(bx+a\right)^{7}-2\,b\cos\left(bx+a\right)^{5}+b\cos\left(bx+a\right)^{3}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -1/240*(128*cos(b*x + a)^8 - 320*cos(b*x + a)^6 + 240*cos(b*x + a)^4 - 40*c
os(b*x + a)^2 - 5)/((b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x + a)
^3)*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^2(a+bx)\csc^4(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**4,x)
```

```
[Out] Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**4, x)
```

Giac [A]
time = 0.43, size = 56, normalized size = 0.78

$$\frac{5\,\tan(bx+a)^3 - \frac{90\,\tan(bx+a)^4+20\,\tan(bx+a)^2+3}{\tan(bx+a)^5} + 60\,\tan(bx+a)}{240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 1/240*(5*tan(b*x + a)^3 - (90*tan(b*x + a)^4 + 20*tan(b*x + a)^2 + 3)/tan(b
*x + a)^5 + 60*tan(b*x + a))/b
```

Mupad [B]
time = 0.38, size = 55, normalized size = 0.76

$$-\frac{-5\tan(a+b\,x)^8 - 60\tan(a+b\,x)^6 + 90\tan(a+b\,x)^4 + 20\tan(a+b\,x)^2 + 3}{240\,b\tan(a+b\,x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^4),x)
```

```
[Out] -(20*tan(a + b*x)^2 + 90*tan(a + b*x)^4 - 60*tan(a + b*x)^6 - 5*tan(a + b*x
)^8 + 3)/(240*b*tan(a + b*x)^5)
```

3.57 $\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx$

Optimal. Leaf size=90

$$-\frac{5\cot^2(a+bx)}{32b}-\frac{5\cot^4(a+bx)}{128b}-\frac{\cot^6(a+bx)}{192b}+\frac{5\log(\tan(a+bx))}{16b}+\frac{5\tan^2(a+bx)}{64b}+\frac{\tan^4(a+bx)}{128b}$$

```
[Out] -5/32*cot(b*x+a)^2/b-5/128*cot(b*x+a)^4/b-1/192*cot(b*x+a)^6/b+5/16*ln(tan(
b*x+a))/b+5/64*tan(b*x+a)^2/b+1/128*tan(b*x+a)^4/b
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4373, 2700, 272, 45}

$$\frac{\tan^4(a+bx)}{128b}+\frac{5\tan^2(a+bx)}{64b}-\frac{\cot^6(a+bx)}{192b}-\frac{5\cot^4(a+bx)}{128b}-\frac{5\cot^2(a+bx)}{32b}+\frac{5\log(\tan(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] (-5*Cot[a + b*x]^2)/(32*b) - (5*Cot[a + b*x]^4)/(128*b) - Cot[a + b*x]^6/(1
92*b) + (5*Log[Tan[a + b*x]])/(16*b) + (5*Tan[a + b*x]^2)/(64*b) + Tan[a +
b*x]^4/(128*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
```

```
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx &= \frac{1}{32}\int \csc^7(a+bx)\sec^5(a+bx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^5}{x^7}\,dx, x, \tan(a+bx)\right)}{32b} \\
&= \frac{\text{Subst}\left(\int \frac{(1+x)^5}{x^4}\,dx, x, \tan^2(a+bx)\right)}{64b} \\
&= \frac{\text{Subst}\left(\int \left(5+\frac{1}{x^4}+\frac{5}{x^3}+\frac{10}{x^2}+\frac{10}{x}+x\right)\,dx, x, \tan^2(a+bx)\right)}{64b} \\
&= -\frac{5\cot^2(a+bx)}{32b}-\frac{5\cot^4(a+bx)}{128b}-\frac{\cot^6(a+bx)}{192b}+\frac{5\log(\tan(a+bx))}{16b}
\end{aligned}$$

Mathematica [A]
time = 0.46, size = 76, normalized size = 0.84

$$-\frac{36\csc^2(a+bx)+9\csc^4(a+bx)+2\csc^6(a+bx)+120\log(\cos(a+bx))-120\log(\sin(a+bx))-24\sec^2(a+bx)-3\sec^4(a+bx)}{384b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
[Out] -1/384*(36*Csc[a + b*x]^2 + 9*Csc[a + b*x]^4 + 2*Csc[a + b*x]^6 + 120*Log[C
os[a + b*x]] - 120*Log[Sin[a + b*x]] - 24*Sec[a + b*x]^2 - 3*Sec[a + b*x]^4
)/b
```

Maple [A]
time = 0.12, size = 98, normalized size = 1.09

method	result
default	$\frac{-\frac{1}{6\sin(xb+a)^6\cos(xb+a)^4}+\frac{5}{12\sin(xb+a)^4\cos(xb+a)^4}-\frac{5}{6\sin(xb+a)^4\cos(xb+a)^2}+\frac{5}{2\sin(xb+a)^2\cos(xb+a)^2}-\frac{5}{\sin(xb+a)^2}+10\ln(\tan(xb+a))}{32b}$
risch	$\frac{15\,e^{18i(xb+a)}-30\,e^{16i(xb+a)}-40\,e^{14i(xb+a)}+110\,e^{12i(xb+a)}+18\,e^{10i(xb+a)}+110\,e^{8i(xb+a)}-40\,e^{6i(xb+a)}-30\,e^{4i(xb+a)}+15\,e^{2i(xb+a)}}{24b\left(e^{2i(xb+a)}-1\right)^6\left(e^{2i(xb+a)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/b*(-1/6/sin(b*x+a)^6/cos(b*x+a)^4+5/12/sin(b*x+a)^4/cos(b*x+a)^4-5/6/s
in(b*x+a)^4/cos(b*x+a)^2+5/2/sin(b*x+a)^2/cos(b*x+a)^2-5/sin(b*x+a)^2+10*ln
(tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 7650 vs. $2(78) = 156$.
time = 0.60, size = 7650, normalized size = 85.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/96*(4*(15*cos(18*b*x + 18*a) - 30*cos(16*b*x + 16*a) - 40*cos(14*b*x + 14
*a) + 110*cos(12*b*x + 12*a) + 18*cos(10*b*x + 10*a) + 110*cos(8*b*x + 8*a)
 - 40*cos(6*b*x + 6*a) - 30*cos(4*b*x + 4*a) + 15*cos(2*b*x + 2*a))*cos(20*
b*x + 20*a) + 4*(15*cos(16*b*x + 16*a) + 200*cos(14*b*x + 14*a) - 190*cos(1
2*b*x + 12*a) - 216*cos(10*b*x + 10*a) - 190*cos(8*b*x + 8*a) + 200*cos(6*b
*x + 6*a) + 15*cos(4*b*x + 4*a) - 60*cos(2*b*x + 2*a) + 15)*cos(18*b*x + 18
*a) - 120*cos(18*b*x + 18*a)^2 - 12*(40*cos(14*b*x + 14*a) + 130*cos(12*b*x
 + 12*a) - 102*cos(10*b*x + 10*a) + 130*cos(8*b*x + 8*a) + 40*cos(6*b*x + 6
*a) - 60*cos(4*b*x + 4*a) - 5*cos(2*b*x + 2*a) + 10)*cos(16*b*x + 16*a) + 3
60*cos(16*b*x + 16*a)^2 + 32*(100*cos(12*b*x + 12*a) + 78*cos(10*b*x + 10*a
) + 100*cos(8*b*x + 8*a) - 80*cos(6*b*x + 6*a) - 15*cos(4*b*x + 4*a) + 25*c
os(2*b*x + 2*a) - 5)*cos(14*b*x + 14*a) - 1280*cos(14*b*x + 14*a)^2 - 8*(64
2*cos(10*b*x + 10*a) - 220*cos(8*b*x + 8*a) - 400*cos(6*b*x + 6*a) + 195*co
s(4*b*x + 4*a) + 95*cos(2*b*x + 2*a) - 55)*cos(12*b*x + 12*a) + 880*cos(12*
b*x + 12*a)^2 - 24*(214*cos(8*b*x + 8*a) - 104*cos(6*b*x + 6*a) - 51*cos(4*
b*x + 4*a) + 36*cos(2*b*x + 2*a) - 3)*cos(10*b*x + 10*a) - 864*cos(10*b*x +
 10*a)^2 + 40*(80*cos(6*b*x + 6*a) - 39*cos(4*b*x + 4*a) - 19*cos(2*b*x + 2
*a) + 11)*cos(8*b*x + 8*a) + 880*cos(8*b*x + 8*a)^2 - 160*(3*cos(4*b*x + 4*
a) - 5*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - 1280*cos(6*b*x + 6*a)^2 + 6
0*(cos(2*b*x + 2*a) - 2)*cos(4*b*x + 4*a) + 360*cos(4*b*x + 4*a)^2 - 120*co
s(2*b*x + 2*a)^2 + 15*(2*(2*cos(18*b*x + 18*a) + 3*cos(16*b*x + 16*a) - 8*c
os(14*b*x + 14*a) - 2*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 2*cos(8*
b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) -
 1)*cos(20*b*x + 20*a) - cos(20*b*x + 20*a)^2 - 4*(3*cos(16*b*x + 16*a) - 8
*cos(14*b*x + 14*a) - 2*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 2*cos(
8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a)
 - 1)*cos(18*b*x + 18*a) - 4*cos(18*b*x + 18*a)^2 + 6*(8*cos(14*b*x + 14*a)
 + 2*cos(12*b*x + 12*a) - 12*cos(10*b*x + 10*a) + 2*cos(8*b*x + 8*a) + 8*co
s(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(16*b*x +
16*a) - 9*cos(16*b*x + 16*a)^2 - 16*(2*cos(12*b*x + 12*a) - 12*cos(10*b*x +
 10*a) + 2*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*c
os(2*b*x + 2*a) + 1)*cos(14*b*x + 14*a) - 64*cos(14*b*x + 14*a)^2 + 4*(12*c
```

```
os(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x +
 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - 4*cos(12*b*x + 12*a)^2
 + 24*(2*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos
(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) - 144*cos(10*b*x + 10*a)^2 - 4*(8*cos
(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*
a) - 4*cos(8*b*x + 8*a)^2 + 16*(3*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1
)*cos(6*b*x + 6*a) - 64*cos(6*b*x + 6*a)^2 - 6*(2*cos(2*b*x + 2*a) - 1)*cos
(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(18*b
*x + 18*a) + 3*sin(16*b*x + 16*a) - 8*sin(14*b*x + 14*a) - 2*sin(12*b*x + 1
2*a) + 12*sin(10*b*x + 10*a) - 2*sin(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*
sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(20*b*x + 20*a) - sin(20*b*x + 20
*a)^2 - 4*(3*sin(16*b*x + 16*a) - 8*sin(14*b*x + 14*a) - 2*sin(12*b*x + 12*
a) + 12*sin(10*b*x + 10*a) - 2*sin(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*si
n(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(18*b*x + 18*a) - 4*sin(18*b*x + 18
*a)^2 + 6*(8*sin(14*b*x + 14*a) + 2*sin(12*b*x + 12*a) - 12*sin(10*b*x + 10
*a) + 2*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(
2*b*x + 2*a))*sin(16*b*x + 16*a) - 9*sin(16*b*x + 16*a)^2 - 16*(2*sin(12*b*
x + 12*a) - 12*sin(10*b*x + 10*a) + 2*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a)
 - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - 64*sin(14*
b*x + 14*a)^2 + 4*(12*sin(10*b*x + 10*a) - 2*sin(8*b*x + 8*a) - 8*sin(6*b*x
 + 6*a) + 3*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 4*s
in(12*b*x + 12*a)^2 + 24*(2*sin(8*b*x + 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(4
*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 144*sin(10*b*x + 10*
a)^2 - 4*(8*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin
(8*b*x + 8*a) - 4*sin(8*b*x + 8*a)^2 + 16*(3*sin(4*b*x + 4*a) + 2*sin(2*b*x
 + 2*a))*sin(6*b*x + 6*a) - 64*sin(6*b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 -
12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x +
 2*a) - 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*
x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) - 15*(2*(2*cos(18*b*x + 18*a) +
3*cos(16*b*x + 16*a) - 8*cos(14*b*x + 14*a) - 2*cos(12*b*x + 12*a) + 12*cos
(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4
*a) + 2*cos(2*b*x + 2*a) - 1)*cos(20*b*x + 20*a) - cos(20*b*x + 20*a)^2 - 4
*(3*cos(16*b*x + 16*a) - 8*cos(14*b*x + 14*a) - 2*cos(12*b*x + 12*a) + 12*c
os(10*b*x + 10*a) - 2*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(4*b*x +
 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(18*b*x + 18...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(78) = 156$.
time = 3.05, size = 194, normalized size = 2.16

$$\frac{60\,\cos\left(bx+a\right)^{8}-150\,\cos\left(bx+a\right)^{6}+110\,\cos\left(bx+a\right)^{4}-15\,\cos\left(bx+a\right)^{2}-60\,\left(\cos\left(bx+a\right)^{10}-3\,\cos\left(bx+a\right)^{8}+3\,\cos\left(bx+a\right)^{6}-\cos\left(bx+a\right)^{4}\right)\log\left(\cos\left(bx+a\right)^{2}\right)+60\,\left(\cos\left(bx+a\right)^{10}-3\,\cos\left(bx+a\right)^{8}+3\,\cos\left(bx+a\right)^{6}-\cos\left(bx+a\right)^{4}\right)\log\left(-\frac{1}{4}\,\cos\left(bx+a\right)^{2}+\frac{1}{4}\right)-3}{384\,\left(b\cos\left(bx+a\right)^{10}-3\,b\cos\left(bx+a\right)^{8}+3\,b\cos\left(bx+a\right)^{6}-b\cos\left(bx+a\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 1/384*(60*cos(b*x + a)^8 - 150*cos(b*x + a)^6 + 110*cos(b*x + a)^4 - 15*cos
(b*x + a)^2 - 60*(cos(b*x + a)^10 - 3*cos(b*x + a)^8 + 3*cos(b*x + a)^6 - c
os(b*x + a)^4)*log(cos(b*x + a)^2) + 60*(cos(b*x + a)^10 - 3*cos(b*x + a)^8
 + 3*cos(b*x + a)^6 - cos(b*x + a)^4)*log(-1/4*cos(b*x + a)^2 + 1/4) - 3)/(
b*cos(b*x + a)^10 - 3*b*cos(b*x + a)^8 + 3*b*cos(b*x + a)^6 - b*cos(b*x + a
)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^2(a+bx)\csc^5(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**5,x)
```

```
[Out] Integral(csc(a + b*x)**2*csc(2*a + 2*b*x)**5, x)
```

Giac [A]
time = 0.42, size = 94, normalized size = 1.04

$$-\frac{\frac{60\,\sin(bx+a)^8-90\,\sin(bx+a)^6+20\,\sin(bx+a)^4+5\,\sin(bx+a)^2+2}{\left(\sin(bx+a)^2-1\right)^2\sin(bx+a)^6}+60\,\log\left(-\sin\left(bx+a\right)^2+1\right)-120\,\log\left(\left|\sin\left(bx+a\right)\right|\right)}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] -1/384*((60*sin(b*x + a)^8 - 90*sin(b*x + a)^6 + 20*sin(b*x + a)^4 + 5*sin(
b*x + a)^2 + 2)/((sin(b*x + a)^2 - 1)^2*sin(b*x + a)^6) + 60*log(-sin(b*x +
 a)^2 + 1) - 120*log(abs(sin(b*x + a))))/b
```

Mupad [B]
time = 0.29, size = 114, normalized size = 1.27

$$\frac{5\,\ln\left(\sin\left(a+b\,x\right)^2\right)}{32\,b}-\frac{5\,\ln\left(\cos\left(a+b\,x\right)\right)}{16\,b}+\frac{-\frac{5\cos(a+b\,x)^8}{32}+\frac{25\cos(a+b\,x)^6}{64}-\frac{55\cos(a+b\,x)^4}{192}+\frac{5\cos(a+b\,x)^2}{128}+\frac{1}{128}}{b\,\left(-\cos\left(a+b\,x\right)^{10}+3\cos\left(a+b\,x\right)^8-3\cos\left(a+b\,x\right)^6+\cos\left(a+b\,x\right)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^5),x)
```

```
[Out] (5*log(sin(a + b*x)^2))/(32*b) - (5*log(cos(a + b*x)))/(16*b) + ((5*cos(a +
 b*x)^2)/128 - (55*cos(a + b*x)^4)/192 + (25*cos(a + b*x)^6)/64 - (5*cos(a
+ b*x)^8)/32 + 1/128)/(b*(cos(a + b*x)^4 - 3*cos(a + b*x)^6 + 3*cos(a + b*x
)^8 - cos(a + b*x)^10))
```

3.58 $\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx$

Optimal. Leaf size=102

$$-\frac{5\cot(a+bx)}{16b}-\frac{5\cot^3(a+bx)}{64b}-\frac{3\cot^5(a+bx)}{160b}-\frac{\cot^7(a+bx)}{448b}+\frac{15\tan(a+bx)}{64b}+\frac{\tan^3(a+bx)}{32b}+\frac{\tan^5(a+bx)}{320b}$$

```
[Out] -5/16*cot(b*x+a)/b-5/64*cot(b*x+a)^3/b-3/160*cot(b*x+a)^5/b-1/448*cot(b*x+a
)^7/b+15/64*tan(b*x+a)/b+1/32*tan(b*x+a)^3/b+1/320*tan(b*x+a)^5/b
```

Rubi [A]
time = 0.06, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2700, 276}

$$\frac{\tan^5(a+bx)}{320b}+\frac{\tan^3(a+bx)}{32b}+\frac{15\tan(a+bx)}{64b}-\frac{\cot^7(a+bx)}{448b}-\frac{3\cot^5(a+bx)}{160b}-\frac{5\cot^3(a+bx)}{64b}-\frac{5\cot(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^6,x]
```

```
[Out] (-5*Cot[a + b*x])/(16*b) - (5*Cot[a + b*x]^3)/(64*b) - (3*Cot[a + b*x]^5)/(
160*b) - Cot[a + b*x]^7/(448*b) + (15*Tan[a + b*x])/(64*b) + Tan[a + b*x]^3
/(32*b) + Tan[a + b*x]^5/(320*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^2(a+bx)\csc^6(2a+2bx)\,dx = \frac{1}{64}\int \csc^8(a+bx)\sec^6(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^6}{x^8}\,dx, x, \tan(a+bx)\right)}{64b}$$
$$= \frac{\text{Subst}\left(\int \left(15+\frac{1}{x^8}+\frac{6}{x^6}+\frac{15}{x^4}+\frac{20}{x^2}+6x^2+x^4\right)\,dx, x, \tan(a+bx)\right)}{64b}$$
$$= -\frac{5\cot(a+bx)}{16b}-\frac{5\cot^3(a+bx)}{64b}-\frac{3\cot^5(a+bx)}{160b}-\frac{\cot^7(a+bx)}{448b}+\frac{15\,\mathrm{t}}{}$$

Mathematica [A]
time = 0.08, size = 132, normalized size = 1.29

$$-\frac{281\cot(a+bx)}{1120b}-\frac{53\cot(a+bx)\csc^2(a+bx)}{1120b}-\frac{27\cot(a+bx)\csc^4(a+bx)}{2240b}-\frac{\cot(a+bx)\csc^6(a+bx)}{448b}+\frac{33\tan(a+bx)}{160b}+\frac{\sec^2(a+bx)\tan(a+bx)}{40b}+\frac{\sec^4(a+bx)\tan(a+bx)}{320b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Csc[2*a + 2*b*x]^6,x]
```

```
[Out] (-281*Cot[a + b*x])/(1120*b) - (53*Cot[a + b*x]*Csc[a + b*x]^2)/(1120*b) -
(27*Cot[a + b*x]*Csc[a + b*x]^4)/(2240*b) - (Cot[a + b*x]*Csc[a + b*x]^6)/(
448*b) + (33*Tan[a + b*x])/(160*b) + (Sec[a + b*x]^2*Tan[a + b*x])/(40*b) +
 (Sec[a + b*x]^4*Tan[a + b*x])/(320*b)
```

Maple [A]
time = 0.12, size = 123, normalized size = 1.21

method	result
risch	$\frac{32i\left(20\,\mathrm{e}^{10i(xb+a)}-5\,\mathrm{e}^{8i(xb+a)}-10\,\mathrm{e}^{6i(xb+a)}+4\,\mathrm{e}^{4i(xb+a)}+2\,\mathrm{e}^{2i(xb+a)}-1\right)}{35b\left(\mathrm{e}^{2i(xb+a)}-1\right)^7\left(\mathrm{e}^{2i(xb+a)}+1\right)^5}$
default	$\frac{-\frac{1}{7\sin(xb+a)^7\cos(xb+a)^5}+\frac{12}{35\sin(xb+a)^5\cos(xb+a)^5}-\frac{24}{35\sin(xb+a)^5\cos(xb+a)^3}+\frac{64}{35\sin(xb+a)^3\cos(xb+a)^3}-\frac{128}{35\sin(xb+a)^3\cos(xb+a)}+\frac{}{35\sin(x}}{64b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/64/b*(-1/7/sin(b*x+a)^7/cos(b*x+a)^5+12/35/sin(b*x+a)^5/cos(b*x+a)^5-24/3
5/sin(b*x+a)^5/cos(b*x+a)^3+64/35/sin(b*x+a)^3/cos(b*x+a)^3-128/35/sin(b*x+
a)^3/cos(b*x+a)+512/35/sin(b*x+a)/cos(b*x+a)-1024/35*cot(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2710 vs. $2(88) = 176$.
time = 0.46, size = 2710, normalized size = 26.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="maxima")

[Out] -32/35*((20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(24*b*x + 24*a) - 2*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(22*b*x + 22*a) - 4*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(20*b*x + 20*a) + 10*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(18*b*x + 18*a) + 5*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(16*b*x + 16*a) - 20*(20*sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a) - 10*sin(6*b*x + 6*a) + 4*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(14*b*x + 14*a) - (20*cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(24*b*x + 24*a) + 2*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(22*b*x + 22*a) + 4*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(20*b*x + 20*a) - 10*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(18*b*x + 18*a) - 5*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(16*b*x + 16*a) + 20*(20*cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) - 10*cos(6*b*x + 6*a) + 4*cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*sin(14*b*x + 14*a))/(b*cos(24*b*x + 24*a)^2 + 4*b*cos(22*b*x + 22*a)^2 + 16*b*cos(20*b*x + 20*a)^2 + 100*b*cos(18*b*x + 18*a)^2 + 25*b*cos(16*b*x + 16*a)^2 + 400*b*cos(14*b*x + 14*a)^2 + 400*b*cos(10*b*x + 10*a)^2 + 25*b*cos(8*b*x + 8*a)^2 + 100*b*cos(6*b*x + 6*a)^2 + 16*b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(24*b*x + 24*a)^2 + 4*b*sin(22*b*x + 22*a)^2 + 16*b*sin(20*b*x + 20*a)^2 + 100*b*sin(18*b*x + 18*a)^2 + 25*b*sin(16*b*x + 16*a)^2 + 400*b*sin(14*b*x + 14*a)^2 + 400*b*sin(10*b*x + 10*a)^2 + 25*b*sin(8*b*x + 8*a)^2 + 100*b*sin(6*b*x + 6*a)^2 + 16*b*sin(4*b*x + 4*a)^2 + 16*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(22*b*x + 22*a) + 4*b*cos(20*b*x + 20*a) - 10*b*cos(18*b*x + 18*a) - 5*b*cos(16*b*x + 16*a) + 20*b*cos(14*b*x + 14*a) - 20*b*cos(10*b*x + 10*a) + 5*b*cos(8*b*x + 8*a) + 10*b*cos(6*b*x + 6*a) - 4*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(24*b*x + 24*a) + 4*(4*b*cos(20*b*x + 20*a) - 10*b*cos(18*b*x + 18*a) - 5*b*cos(16*b*x + 16*a) + 20*b*cos(14*b*x + 14*a) - 20*b*cos(10*b*x + 10*a) + 5*b*cos(8*b*x + 8*a) + 10*b*cos(6*b*x + 6*a) - 4*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(22*b*x + 22*a) - 8*(10*b*cos(18*b*x + 18*a) + 5*b*cos(16*b*x + 16*a) - 20*b*cos(14*b*x + 14*a) + 20*b*cos(10*b*x + 10*a) - 5*b*cos(8*b*x + 8*a) - 10*b*cos(6*b*x + 6*a) + 4*b*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(20*b*x + 20*a) + 20*(5*b*cos(16*b*x + 16*a) - 20*b*cos(14*b*x + 14*a) + 20*b*cos(10*b*x + 10*

```
a) - 5*b*cos(8*b*x + 8*a) - 10*b*cos(6*b*x + 6*a) + 4*b*cos(4*b*x + 4*a) +
2*b*cos(2*b*x + 2*a) - b)*cos(18*b*x + 18*a) - 10*(20*b*cos(14*b*x + 14*a)
- 20*b*cos(10*b*x + 10*a) + 5*b*cos(8*b*x + 8*a) + 10*b*cos(6*b*x + 6*a) -
4*b*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(16*b*x + 16*a) - 40*(2
0*b*cos(10*b*x + 10*a) - 5*b*cos(8*b*x + 8*a) - 10*b*cos(6*b*x + 6*a) + 4*b
*cos(4*b*x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(14*b*x + 14*a) - 40*(5*b*
cos(8*b*x + 8*a) + 10*b*cos(6*b*x + 6*a) - 4*b*cos(4*b*x + 4*a) - 2*b*cos(2
*b*x + 2*a) + b)*cos(10*b*x + 10*a) + 10*(10*b*cos(6*b*x + 6*a) - 4*b*cos(4
*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 20*(4*b*cos(4*b*
x + 4*a) + 2*b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 8*(2*b*cos(2*b*x +
2*a) - b)*cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) - 2*(2*b*sin(22*b*x + 22*
a) + 4*b*sin(20*b*x + 20*a) - 10*b*sin(18*b*x + 18*a) - 5*b*sin(16*b*x + 16
*a) + 20*b*sin(14*b*x + 14*a) - 20*b*sin(10*b*x + 10*a) + 5*b*sin(8*b*x + 8
*a) + 10*b*sin(6*b*x + 6*a) - 4*b*sin(4*b*x + 4*a) - 2*b*sin(2*b*x + 2*a))*
sin(24*b*x + 24*a) + 4*(4*b*sin(20*b*x + 20*a) - 10*b*sin(18*b*x + 18*a) -
5*b*sin(16*b*x + 16*a) + 20*b*sin(14*b*x + 14*a) - 20*b*sin(10*b*x + 10*a)
+ 5*b*sin(8*b*x + 8*a) + 10*b*sin(6*b*x + 6*a) - 4*b*sin(4*b*x + 4*a) - 2*b
*sin(2*b*x + 2*a))*sin(22*b*x + 22*a) - 8*(10*b*sin(18*b*x + 18*a) + 5*b*si
n(16*b*x + 16*a) - 20*b*sin(14*b*x + 14*a) + 20*b*sin(10*b*x + 10*a) - 5*b*
sin(8*b*x + 8*a) - 10*b*sin(6*b*x + 6*a) + 4*b*sin(4*b*x + 4*a) + 2*b*sin(2
*b*x + 2*a))*sin(20*b*x + 20*a) + 20*(5*b*sin(16*b*x + 16*a) - 20*b*sin(14*
b*x + 14*a) + 20*b*sin(10*b*x + 10*a) - 5*b*sin(8*b*x + 8*a) - 10*b*sin(6*b
*x + 6*a) + 4*b*sin(4*b*x + 4*a) + 2*b*sin(2*b*x + 2*a))*sin(18*b*x + 18*a)
 - 10*(20*b*sin(14*b*x + 14*a) - 20*b*sin(10*b*x + 10*a) + 5*b*sin(8*b*x +
8*a) + 10*b*sin(6*b*x + 6*a) - 4*b*sin(4*b*x + 4*a) - 2*b*sin(2*b*x + 2*a))
*sin(16*b*x + 16*a) - 40*(20*b*sin(10*b*x + 10*...
```

Fricas [A]
time = 2.79, size = 118, normalized size = 1.16

$$-\frac{1024 \cos\left(bx+a\right)^{12}-3584 \cos\left(bx+a\right)^{10}+4480 \cos\left(bx+a\right)^{8}-2240 \cos\left(bx+a\right)^{6}+280 \cos\left(bx+a\right)^{4}+28 \cos\left(bx+a\right)^{2}+7}{2240 \left(b\cos\left(bx+a\right)^{11}-3\,b\cos\left(bx+a\right)^{9}+3\,b\cos\left(bx+a\right)^{7}-b\cos\left(bx+a\right)^{5}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] -1/2240*(1024*cos(b*x + a)^12 - 3584*cos(b*x + a)^10 + 4480*cos(b*x + a)^8
- 2240*cos(b*x + a)^6 + 280*cos(b*x + a)^4 + 28*cos(b*x + a)^2 + 7)/((b*cos
(b*x + a)^11 - 3*b*cos(b*x + a)^9 + 3*b*cos(b*x + a)^7 - b*cos(b*x + a)^5)*
sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*csc(2*b*x+2*a)**6,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 76, normalized size = 0.75

$$\frac{7 \tan\left(bx+a\right)^{5}+70 \tan\left(bx+a\right)^{3}-\frac{700 \tan\left(bx+a\right)^{6}+175 \tan\left(bx+a\right)^{4}+42 \tan\left(bx+a\right)^{2}+5}{\tan\left(bx+a\right)^{7}}+525 \tan\left(bx+a\right)}{2240\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*csc(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] 1/2240*(7*tan(b*x + a)^5 + 70*tan(b*x + a)^3 - (700*tan(b*x + a)^6 + 175*ta
n(b*x + a)^4 + 42*tan(b*x + a)^2 + 5)/tan(b*x + a)^7 + 525*tan(b*x + a))/b
```

Mupad [B]
time = 0.28, size = 83, normalized size = 0.81

$$\frac{15\tan(a+b\,x)}{64\,b}+\frac{\tan(a+b\,x)^3}{32\,b}+\frac{\tan(a+b\,x)^5}{320\,b}-\frac{\cot(a+b\,x)^7\left(\frac{5\tan(a+b\,x)^6}{16}+\frac{5\tan(a+b\,x)^4}{64}+\frac{3\tan(a+b\,x)^2}{160}+\frac{1}{448}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^6),x)
```

```
[Out] (15*tan(a + b*x))/(64*b) + tan(a + b*x)^3/(32*b) + tan(a + b*x)^5/(320*b) -
 (cot(a + b*x)^7*((3*tan(a + b*x)^2)/160 + (5*tan(a + b*x)^4)/64 + (5*tan(a
 + b*x)^6)/16 + 1/448))/b
```

3.59 $\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{1024\cos^{11}(a+bx)}{11b}+\frac{3072\cos^{13}(a+bx)}{13b}-\frac{1024\cos^{15}(a+bx)}{5b}+\frac{1024\cos^{17}(a+bx)}{17b}$$

```
[Out] -1024/11*cos(b*x+a)^11/b+3072/13*cos(b*x+a)^13/b-1024/5*cos(b*x+a)^15/b+102
4/17*cos(b*x+a)^17/b
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2645, 276}

$$\frac{1024\cos^{17}(a+bx)}{17b}-\frac{1024\cos^{15}(a+bx)}{5b}+\frac{3072\cos^{13}(a+bx)}{13b}-\frac{1024\cos^{11}(a+bx)}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^10,x]
```

```
[Out] (-1024*Cos[a + b*x]^11)/(11*b) + (3072*Cos[a + b*x]^13)/(13*b) - (1024*Cos[
a + b*x]^15)/(5*b) + (1024*Cos[a + b*x]^17)/(17*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^{10}(2a+2bx)\,dx = 1024\int \cos^{10}(a+bx)\sin^7(a+bx)\,dx$$
$$= -\frac{1024\text{Subst}\left(\int x^{10}(1-x^2)^3\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{1024\text{Subst}\left(\int (x^{10}-3x^{12}+3x^{14}-x^{16})\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{1024\cos^{11}(a+bx)}{11b} + \frac{3072\cos^{13}(a+bx)}{13b} - \frac{1024\cos^{15}(a+bx)}{5b} + \frac{102}{}$$

Mathematica [A]
time = 0.16, size = 119, normalized size = 1.95

$$-\frac{35\cos(a+bx)}{32b} - \frac{7\cos(3(a+bx))}{16b} + \frac{7\cos(5(a+bx))}{80b} + \frac{\cos(7(a+bx))}{8b} - \frac{5\cos(11(a+bx))}{176b} - \frac{\cos(13(a+bx))}{208b} + \frac{\cos(15(a+bx))}{320b} + \frac{\cos(17(a+bx))}{1088b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^10,x]
```

```
[Out] (-35*Cos[a + b*x])/(32*b) - (7*Cos[3*(a + b*x)])/(16*b) + (7*Cos[5*(a + b*x
)])/(80*b) + Cos[7*(a + b*x)]/(8*b) - (5*Cos[11*(a + b*x)])/(176*b) - Cos[1
3*(a + b*x)]/(208*b) + Cos[15*(a + b*x)]/(320*b) + Cos[17*(a + b*x)]/(1088*
b)
```

Maple [A]
time = 0.10, size = 71, normalized size = 1.16

method	result
default	$\frac{-\frac{1024\left(\sin^6(xb+a)\right)\left(\cos^{11}(xb+a)\right)}{17}-\frac{2048\left(\sin^4(xb+a)\right)\left(\cos^{11}(xb+a)\right)}{85}-\frac{8192\left(\sin^2(xb+a)\right)\left(\cos^{11}(xb+a)\right)}{1105}-\frac{16384\left(\cos^{11}(xb+a)\right)}{12155}}{b}$
risch	$-\frac{35\cos(xb+a)}{32b}+\frac{\cos(17xb+17a)}{1088b}+\frac{\cos(15xb+15a)}{320b}-\frac{\cos(13xb+13a)}{208b}-\frac{5\cos(11xb+11a)}{176b}+\frac{\cos(7xb+7a)}{8b}+\frac{7\cos(5xb+5a}{80b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1024/b*(-1/17*sin(b*x+a)^6*cos(b*x+a)^11-2/85*sin(b*x+a)^4*cos(b*x+a)^11-8/
1105*sin(b*x+a)^2*cos(b*x+a)^11-16/12155*cos(b*x+a)^11)
```

Maxima [A]
time = 0.27, size = 91, normalized size = 1.49

$$\frac{715\cos(17bx+17a)+2431\cos(15bx+15a)-3740\cos(13bx+13a)-22100\cos(11bx+11a)+97240\cos(7bx+7a)+68068\cos(5bx+5a)-340340\cos(3bx+3a)-850850\cos(bx+a)}{777920b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="maxima")
```

```
[Out] 1/777920*(715*cos(17*b*x + 17*a) + 2431*cos(15*b*x + 15*a) - 3740*cos(13*b*
x + 13*a) - 22100*cos(11*b*x + 11*a) + 97240*cos(7*b*x + 7*a) + 68068*cos(5
*b*x + 5*a) - 340340*cos(3*b*x + 3*a) - 850850*cos(b*x + a))/b
```

Fricas [A]
time = 2.68, size = 46, normalized size = 0.75

$$\frac{1024\left(715\cos\left(bx+a\right)^{17}-2431\cos\left(bx+a\right)^{15}+2805\cos\left(bx+a\right)^{13}-1105\cos\left(bx+a\right)^{11}\right)}{12155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="fricas")
```

```
[Out] 1024/12155*(715*cos(b*x + a)^17 - 2431*cos(b*x + a)^15 + 2805*cos(b*x + a)^
13 - 1105*cos(b*x + a)^11)/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**10,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(53) = 106.
time = 0.46, size = 314, normalized size = 5.15

$$-\frac{32768\left(\frac{17\left(\cos\left(bx+a\right)-1\right)}{\cos\left(bx+a\right)+1}-\frac{136\left(\cos\left(bx+a\right)-1\right)^{2}}{\left(\cos\left(bx+a\right)+1\right)^{2}}+\frac{680\left(\cos\left(bx+a\right)-1\right)^{3}}{\left(\cos\left(bx+a\right)+1\right)^{3}}+\frac{9775\left(\cos\left(bx+a\right)-1\right)^{4}}{\left(\cos\left(bx+a\right)+1\right)^{4}}+\frac{71825\left(\cos\left(bx+a\right)-1\right)^{5}}{\left(\cos\left(bx+a\right)+1\right)^{5}}+\frac{221000\left(\cos\left(bx+a\right)-1\right)^{6}}{\left(\cos\left(bx+a\right)+1\right)^{6}}+\frac{486200\left(\cos\left(bx+a\right)-1\right)^{7}}{\left(\cos\left(bx+a\right)+1\right)^{7}}+\frac{668525\left(\cos\left(bx+a\right)-1\right)^{8}}{\left(\cos\left(bx+a\right)+1\right)^{8}}+\frac{692835\left(\cos\left(bx+a\right)-1\right)^{9}}{\left(\cos\left(bx+a\right)+1\right)^{9}}+\frac{466752\left(\cos\left(bx+a\right)-1\right)^{10}}{\left(\cos\left(bx+a\right)+1\right)^{10}}+\frac{233376\left(\cos\left(bx+a\right)-1\right)^{11}}{\left(\cos\left(bx+a\right)+1\right)^{11}}+\frac{65637\left(\cos\left(bx+a\right)-1\right)^{12}}{\left(\cos\left(bx+a\right)+1\right)^{12}}+\frac{12155\left(\cos\left(bx+a\right)-1\right)^{13}}{\left(\cos\left(bx+a\right)+1\right)^{13}}-1\right)}{12155\,b\left(\frac{\cos\left(bx+a\right)-1}{\cos\left(bx+a\right)+1}-1\right)^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^10,x, algorithm="giac")
```

```
[Out] -32768/12155*(17*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 136*(cos(b*x + a)
- 1)^2/(cos(b*x + a) + 1)^2 + 680*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3
 + 9775*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 71825*(cos(b*x + a) - 1
)^5/(cos(b*x + a) + 1)^5 + 221000*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6
 + 486200*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 + 668525*(cos(b*x + a)
- 1)^8/(cos(b*x + a) + 1)^8 + 692835*(cos(b*x + a) - 1)^9/(cos(b*x + a) + 1
)^9 + 466752*(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 + 233376*(cos(b*x
+ a) - 1)^11/(cos(b*x + a) + 1)^11 + 65637*(cos(b*x + a) - 1)^12/(cos(b*x +
 a) + 1)^12 + 12155*(cos(b*x + a) - 1)^13/(cos(b*x + a) + 1)^13 - 1)/(b*((c
os(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^17)
```

Mupad [B]
time = 0.13, size = 46, normalized size = 0.75

$$-\frac{-\frac{1024\cos(a+b\,x)^{17}}{17}+\frac{1024\cos(a+b\,x)^{15}}{5}-\frac{3072\cos(a+b\,x)^{13}}{13}+\frac{1024\cos(a+b\,x)^{11}}{11}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^10/sin(a + b*x)^3,x)
[Out] -((1024*cos(a + b*x)^11)/11 - (3072*cos(a + b*x)^13)/13 + (1024*cos(a + b*x
)^15)/5 - (1024*cos(a + b*x)^17)/17)/b
```

3.60 $\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx$

Optimal. Leaf size=76

$$\frac{512\sin^7(a+bx)}{7b}-\frac{2048\sin^9(a+bx)}{9b}+\frac{3072\sin^{11}(a+bx)}{11b}-\frac{2048\sin^{13}(a+bx)}{13b}+\frac{512\sin^{15}(a+bx)}{15b}$$

```
[Out] 512/7*sin(b*x+a)^7/b-2048/9*sin(b*x+a)^9/b+3072/11*sin(b*x+a)^11/b-2048/13*
sin(b*x+a)^13/b+512/15*sin(b*x+a)^15/b
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2644, 276}

$$\frac{512\sin^{15}(a+bx)}{15b}-\frac{2048\sin^{13}(a+bx)}{13b}+\frac{3072\sin^{11}(a+bx)}{11b}-\frac{2048\sin^9(a+bx)}{9b}+\frac{512\sin^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^9,x]
```

```
[Out] (512*Sin[a + b*x]^7)/(7*b) - (2048*Sin[a + b*x]^9)/(9*b) + (3072*Sin[a + b*
x]^11)/(11*b) - (2048*Sin[a + b*x]^13)/(13*b) + (512*Sin[a + b*x]^15)/(15*b
)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^9(2a+2bx)\,dx = 512\int \cos^9(a+bx)\sin^6(a+bx)\,dx$$
$$= \frac{512\text{Subst}\left(\int x^6\left(1-x^2\right)^4\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{512\text{Subst}\left(\int \left(x^6-4x^8+6x^{10}-4x^{12}+x^{14}\right)dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{512\sin^7(a+bx)}{7b} - \frac{2048\sin^9(a+bx)}{9b} + \frac{3072\sin^{11}(a+bx)}{11b} - \frac{2048\sin^{1}}{1}$$

Mathematica [A]
time = 0.35, size = 58, normalized size = 0.76

$$\frac{512\left(6435\sin^7(a+bx)-20020\sin^9(a+bx)+24570\sin^{11}(a+bx)-13860\sin^{13}(a+bx)+3003\sin^{15}(a+bx)\right)}{45045b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^9,x]
[Out] (512*(6435*Sin[a + b*x]^7 - 20020*Sin[a + b*x]^9 + 24570*Sin[a + b*x]^11 -
13860*Sin[a + b*x]^13 + 3003*Sin[a + b*x]^15))/(45045*b)
```

Maple [A]
time = 0.12, size = 107, normalized size = 1.41

method	result
default	$\frac{-\frac{512\left(\sin^5(xb+a)\right)\left(\cos^{10}(xb+a)\right)}{15}-\frac{512\left(\sin^3(xb+a)\right)\left(\cos^{10}(xb+a)\right)}{39}-\frac{512\sin(xb+a)\left(\cos^{10}(xb+a)\right)}{143}+\frac{512\left(\frac{128}{35}+\cos^8(xb+a)+\frac{8\left(\cos^6(xb+a)\right)}{7}+\frac{4}{}\right.}{}}{b}$
risch	$\frac{45\sin(xb+a)}{32b}-\frac{\sin(15xb+15a)}{480b}-\frac{3\sin(13xb+13a)}{416b}+\frac{3\sin(11xb+11a)}{352b}+\frac{17\sin(9xb+9a)}{288b}+\frac{3\sin(7xb+7a)}{224b}-\frac{39\sin(5xb+5a}{160b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x,method=_RETURNVERBOSE)
[Out] 512/b*(-1/15*sin(b*x+a)^5*cos(b*x+a)^10-1/39*sin(b*x+a)^3*cos(b*x+a)^10-1/1
43*sin(b*x+a)*cos(b*x+a)^10+1/1287*(128/35+cos(b*x+a)^8+8/7*cos(b*x+a)^6+48
/35*cos(b*x+a)^4+64/35*cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A]
time = 0.27, size = 91, normalized size = 1.20

$$-\frac{3003\,\sin\left(15\,bx+15\,a\right)+10395\,\sin\left(13\,bx+13\,a\right)-12285\,\sin\left(11\,bx+11\,a\right)-85085\,\sin\left(9\,bx+9\,a\right)-19305\,\sin\left(7\,bx+7\,a\right)+351351\,\sin\left(5\,bx+5\,a\right)+375375\,\sin\left(3\,bx+3\,a\right)-2027025\,\sin\left(bx+a\right)}{1441440\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="maxima")
```

```
[Out] -1/1441440*(3003*sin(15*b*x + 15*a) + 10395*sin(13*b*x + 13*a) - 12285*sin(
11*b*x + 11*a) - 85085*sin(9*b*x + 9*a) - 19305*sin(7*b*x + 7*a) + 351351*s
in(5*b*x + 5*a) + 375375*sin(3*b*x + 3*a) - 2027025*sin(b*x + a))/b
```

Fricas [A]
time = 2.81, size = 83, normalized size = 1.09

$$-\frac{512\left(3003\cos\left(bx+a\right)^{14}-7161\cos\left(bx+a\right)^{12}+4473\cos\left(bx+a\right)^{10}-35\cos\left(bx+a\right)^{8}-40\cos\left(bx+a\right)^{6}-48\cos\left(bx+a\right)^{4}-64\cos\left(bx+a\right)^{2}-128\right)\sin\left(bx+a\right)}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="fricas")
```

```
[Out] -512/45045*(3003*cos(b*x + a)^14 - 7161*cos(b*x + a)^12 + 4473*cos(b*x + a)
^10 - 35*cos(b*x + a)^8 - 40*cos(b*x + a)^6 - 48*cos(b*x + a)^4 - 64*cos(b*
x + a)^2 - 128)*sin(b*x + a)/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**9,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8568 deep
```

Giac [A]
time = 0.44, size = 56, normalized size = 0.74

$$\frac{512\left(3003\sin\left(bx+a\right)^{15}-13860\sin\left(bx+a\right)^{13}+24570\sin\left(bx+a\right)^{11}-20020\sin\left(bx+a\right)^{9}+6435\sin\left(bx+a\right)^{7}\right)}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^9,x, algorithm="giac")
```

```
[Out] 512/45045*(3003*sin(b*x + a)^15 - 13860*sin(b*x + a)^13 + 24570*sin(b*x + a
)^11 - 20020*sin(b*x + a)^9 + 6435*sin(b*x + a)^7)/b
```

Mupad [B]
time = 0.12, size = 55, normalized size = 0.72

$$\frac{\frac{512\sin(a+b\,x)^{15}}{15}-\frac{2048\sin(a+b\,x)^{13}}{13}+\frac{3072\sin(a+b\,x)^{11}}{11}-\frac{2048\sin(a+b\,x)^{9}}{9}+\frac{512\sin(a+b\,x)^{7}}{7}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^9/sin(a + b*x)^3,x)
```

```
[Out] ((512*sin(a + b*x)^7)/7 - (2048*sin(a + b*x)^9)/9 + (3072*sin(a + b*x)^11)/
11 - (2048*sin(a + b*x)^13)/13 + (512*sin(a + b*x)^15)/15)/b
```

3.61 $\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{256\cos^9(a+bx)}{9b}+\frac{512\cos^{11}(a+bx)}{11b}-\frac{256\cos^{13}(a+bx)}{13b}$$

```
[Out] -256/9*cos(b*x+a)^9/b+512/11*cos(b*x+a)^11/b-256/13*cos(b*x+a)^13/b
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2645, 276}

$$-\frac{256\cos^{13}(a+bx)}{13b}+\frac{512\cos^{11}(a+bx)}{11b}-\frac{256\cos^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (-256*Cos[a + b*x]^9)/(9*b) + (512*Cos[a + b*x]^11)/(11*b) - (256*Cos[a + b
*x]^13)/(13*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^8(2a+2bx)\,dx = 256\int \cos^8(a+bx)\sin^5(a+bx)\,dx$$
$$= -\frac{256\text{Subst}\left(\int x^8(1-x^2)^2\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{256\text{Subst}\left(\int (x^8-2x^{10}+x^{12})\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{256\cos^9(a+bx)}{9b}+\frac{512\cos^{11}(a+bx)}{11b}-\frac{256\cos^{13}(a+bx)}{13b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 104 vs. 2(46) = 92.
time = 0.11, size = 104, normalized size = 2.26

$$-\frac{5\cos(a+bx)}{4b}-\frac{25\cos(3(a+bx))}{48b}+\frac{\cos(5(a+bx))}{16b}+\frac{\cos(7(a+bx))}{8b}+\frac{\cos(9(a+bx))}{72b}-\frac{3\cos(11(a+bx))}{176b}-\frac{\cos(13(a+bx))}{208b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^8,x]
```

```
[Out] (-5*Cos[a + b*x])/(4*b) - (25*Cos[3*(a + b*x)])/(48*b) + Cos[5*(a + b*x)]/(
16*b) + Cos[7*(a + b*x)]/(8*b) + Cos[9*(a + b*x)]/(72*b) - (3*Cos[11*(a + b
*x)])/(176*b) - Cos[13*(a + b*x)]/(208*b)
```

Maple [A]
time = 0.08, size = 53, normalized size = 1.15

method	result	si
default	$\frac{-\frac{256\left(\sin^4(xb+a)\right)\left(\cos^9(xb+a)\right)}{13}-\frac{1024\left(\sin^2(xb+a)\right)\left(\cos^9(xb+a)\right)}{143}-\frac{2048\left(\cos^9(xb+a)\right)}{1287}}{b}$	5
risch	$-\frac{5\cos(xb+a)}{4b}-\frac{\cos(13xb+13a)}{208b}-\frac{3\cos(11xb+11a)}{176b}+\frac{\cos(9xb+9a)}{72b}+\frac{\cos(7xb+7a)}{8b}+\frac{\cos(5xb+5a)}{16b}-\frac{25\cos(3xb+3a)}{48b}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 256/b*(-1/13*sin(b*x+a)^4*cos(b*x+a)^9-4/143*sin(b*x+a)^2*cos(b*x+a)^9-8/12
87*cos(b*x+a)^9)
```

Maxima [A]
time = 0.27, size = 80, normalized size = 1.74

$$-\frac{99\cos(13bx+13a)+351\cos(11bx+11a)-286\cos(9bx+9a)-2574\cos(7bx+7a)-1287\cos(5bx+5a)+10725\cos(3bx+3a)+25740\cos(bx+a)}{20592b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="maxima")
```

```
[Out] -1/20592*(99*cos(13*b*x + 13*a) + 351*cos(11*b*x + 11*a) - 286*cos(9*b*x +
9*a) - 2574*cos(7*b*x + 7*a) - 1287*cos(5*b*x + 5*a) + 10725*cos(3*b*x + 3*
a) + 25740*cos(b*x + a))/b
```

Fricas [A]
time = 3.15, size = 36, normalized size = 0.78

$$-\frac{256 \left(99 \cos \left(bx+a\right)^{13}-234 \cos \left(bx+a\right)^{11}+143 \cos \left(bx+a\right)^{9}\right)}{1287 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="fricas")
```

```
[Out] -256/1287*(99*cos(b*x + a)^13 - 234*cos(b*x + a)^11 + 143*cos(b*x + a)^9)/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**8,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6188 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(40) = 80.
time = 0.44, size = 248, normalized size = 5.39

$$-\frac{4096 \left(\frac{13 \left(\cos \left(bx+a\right)-1\right)}{\cos \left(bx+a\right)+1}-\frac{78 \left(\cos \left(bx+a\right)-1\right)^{2}}{\left(\cos \left(bx+a\right)+1\right)^{2}}-\frac{572 \left(\cos \left(bx+a\right)-1\right)^{3}}{\left(\cos \left(bx+a\right)+1\right)^{3}}-\frac{3718 \left(\cos \left(bx+a\right)-1\right)^{4}}{\left(\cos \left(bx+a\right)+1\right)^{4}}-\frac{7722 \left(\cos \left(bx+a\right)-1\right)^{5}}{\left(\cos \left(bx+a\right)+1\right)^{5}}-\frac{13728 \left(\cos \left(bx+a\right)-1\right)^{6}}{\left(\cos \left(bx+a\right)+1\right)^{6}}-\frac{12012 \left(\cos \left(bx+a\right)-1\right)^{7}}{\left(\cos \left(bx+a\right)+1\right)^{7}}-\frac{9009 \left(\cos \left(bx+a\right)-1\right)^{8}}{\left(\cos \left(bx+a\right)+1\right)^{8}}-\frac{3003 \left(\cos \left(bx+a\right)-1\right)^{9}}{\left(\cos \left(bx+a\right)+1\right)^{9}}-\frac{858 \left(\cos \left(bx+a\right)-1\right)^{10}}{\left(\cos \left(bx+a\right)+1\right)^{10}}-1\right)}{1287 \, b \left(\frac{\cos \left(bx+a\right)-1}{\cos \left(bx+a\right)+1}-1\right)^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^8,x, algorithm="giac")
```

```
[Out] -4096/1287*(13*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 78*(cos(b*x + a) - 1
)^2/(cos(b*x + a) + 1)^2 - 572*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 -
3718*(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 - 7722*(cos(b*x + a) - 1)^5/
(cos(b*x + a) + 1)^5 - 13728*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 - 12
012*(cos(b*x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 9009*(cos(b*x + a) - 1)^8/(
cos(b*x + a) + 1)^8 - 3003*(cos(b*x + a) - 1)^9/(cos(b*x + a) + 1)^9 - 858*
(cos(b*x + a) - 1)^10/(cos(b*x + a) + 1)^10 - 1)/(b*((cos(b*x + a) - 1)/(co
s(b*x + a) + 1) - 1)^13)
```

Mupad [B]
time = 0.14, size = 36, normalized size = 0.78

$$-\frac{256\,\left(99\,{\cos\left(a+b\,x\right)}^{13}-234\,{\cos\left(a+b\,x\right)}^{11}+143\,{\cos\left(a+b\,x\right)}^9\right)}{1287\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^8/sin(a + b*x)^3,x)
[Out] -(256*(143*cos(a + b*x)^9 - 234*cos(a + b*x)^11 + 99*cos(a + b*x)^13))/(128
7*b)
```

3.62 $\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{128\sin^5(a+bx)}{5b}-\frac{384\sin^7(a+bx)}{7b}+\frac{128\sin^9(a+bx)}{3b}-\frac{128\sin^{11}(a+bx)}{11b}$$

```
[Out] 128/5*sin(b*x+a)^5/b-384/7*sin(b*x+a)^7/b+128/3*sin(b*x+a)^9/b-128/11*sin(b
*x+a)^11/b
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2644, 276}

$$-\frac{128\sin^{11}(a+bx)}{11b}+\frac{128\sin^9(a+bx)}{3b}-\frac{384\sin^7(a+bx)}{7b}+\frac{128\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*Sin[a + b*x]^5)/(5*b) - (384*Sin[a + b*x]^7)/(7*b) + (128*Sin[a + b*x]
^9)/(3*b) - (128*Sin[a + b*x]^11)/(11*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^7(2a+2bx)\,dx = 128\int \cos^7(a+bx)\sin^4(a+bx)\,dx$$
$$= \frac{128\text{Subst}\left(\int x^4(1-x^2)^3\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{128\text{Subst}\left(\int (x^4-3x^6+3x^8-x^{10})\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{128\sin^5(a+bx)}{5b} - \frac{384\sin^7(a+bx)}{7b} + \frac{128\sin^9(a+bx)}{3b} - \frac{128\sin^{11}(a+}{11b}$$

Mathematica [A]
time = 0.18, size = 48, normalized size = 0.79

$$\frac{128(231\sin^5(a+bx)-495\sin^7(a+bx)+385\sin^9(a+bx)-105\sin^{11}(a+bx))}{1155b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (128*(231*Sin[a + b*x]^5 - 495*Sin[a + b*x]^7 + 385*Sin[a + b*x]^9 - 105*Si
n[a + b*x]^11))/(1155*b)
```

Maple [A]
time = 0.10, size = 79, normalized size = 1.30

method	result	size
default	$\frac{-\frac{128\left(\sin^3(xb+a)\right)\left(\cos^8(xb+a)\right)}{11}-\frac{128\sin(xb+a)\left(\cos^8(xb+a)\right)}{33}+\frac{128\left(\frac{16}{5}+\cos^6(xb+a)+\frac{6\left(\cos^4(xb+a)\right)}{5}+\frac{8\left(\cos^2(xb+a)\right)}{5}\right)\sin(xb+a)}{231}}{b}$	79
risch	$\frac{7\sin(xb+a)}{4b}+\frac{\sin(11xb+11a)}{88b}+\frac{\sin(9xb+9a)}{24b}-\frac{\sin(7xb+7a)}{56b}-\frac{11\sin(5xb+5a)}{40b}-\frac{\sin(3xb+3a)}{4b}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 128/b*(-1/11*sin(b*x+a)^3*cos(b*x+a)^8-1/33*sin(b*x+a)*cos(b*x+a)^8+1/231*(
16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*cos(b*x+a)^2)*sin(b*x+a))
```

Maxima [A]
time = 0.27, size = 69, normalized size = 1.13

$$\frac{105\sin(11\,bx+11\,a)+385\sin(9\,bx+9\,a)-165\sin(7\,bx+7\,a)-2541\sin(5\,bx+5\,a)-2310\sin(3\,bx+3\,a)+16170\sin(bx+a)}{9240\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

```
[Out] 1/9240*(105*sin(11*b*x + 11*a) + 385*sin(9*b*x + 9*a) - 165*sin(7*b*x + 7*a
) - 2541*sin(5*b*x + 5*a) - 2310*sin(3*b*x + 3*a) + 16170*sin(b*x + a))/b
```

Fricas [A]
time = 3.26, size = 63, normalized size = 1.03

$$\frac{128\left(105\cos\left(bx+a\right)^{10}-140\cos\left(bx+a\right)^{8}+5\cos\left(bx+a\right)^{6}+6\cos\left(bx+a\right)^{4}+8\cos\left(bx+a\right)^{2}+16\right)\sin\left(bx+a\right)}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

```
[Out] 128/1155*(105*cos(b*x + a)^10 - 140*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*c
os(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x + a)/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**7,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4368 deep
```

Giac [A]
time = 0.43, size = 46, normalized size = 0.75

$$-\frac{128\left(105\sin\left(bx+a\right)^{11}-385\sin\left(bx+a\right)^{9}+495\sin\left(bx+a\right)^{7}-231\sin\left(bx+a\right)^{5}\right)}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

```
[Out] -128/1155*(105*sin(b*x + a)^11 - 385*sin(b*x + a)^9 + 495*sin(b*x + a)^7 -
231*sin(b*x + a)^5)/b
```

Mupad [B]
time = 0.12, size = 45, normalized size = 0.74

$$\frac{-\frac{128\sin(a+b\,x)^{11}}{11}+\frac{128\sin(a+b\,x)^{9}}{3}-\frac{384\sin(a+b\,x)^{7}}{7}+\frac{128\sin(a+b\,x)^{5}}{5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^7/sin(a + b*x)^3,x)
```

```
[Out] ((128*sin(a + b*x)^5)/5 - (384*sin(a + b*x)^7)/7 + (128*sin(a + b*x)^9)/3 -
 (128*sin(a + b*x)^11)/11)/b
```

3.63 $\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=31

$$-\frac{64\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{9b}$$

```
[Out] -64/7*cos(b*x+a)^7/b+64/9*cos(b*x+a)^9/b
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2645, 14}

$$\frac{64\cos^9(a+bx)}{9b}-\frac{64\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (-64*Cos[a + b*x]^7)/(7*b) + (64*Cos[a + b*x]^9)/(9*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^6(2a+2bx)\,dx &= 64\int \cos^6(a+bx)\sin^3(a+bx)\,dx\\ &= -\frac{64\text{Subst}\left(\int x^6(1-x^2)\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{64\text{Subst}\left(\int (x^6-x^8)\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{64\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{9b}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 27, normalized size = 0.87

$$\frac{32\cos^7(a+bx)(-11+7\cos(2(a+bx)))}{63b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^6,x]
[Out] (32*Cos[a + b*x]^7*(-11 + 7*Cos[2*(a + b*x)]))/(63*b)
```

Maple [A]
time = 0.07, size = 35, normalized size = 1.13

method	result	size
default	$\frac{-\frac{64\left(\sin^2(xb+a)\right)\left(\cos^7(xb+a)\right)}{9}-\frac{128\left(\cos^7(xb+a)\right)}{63}}{b}$	35
risch	$-\frac{3\cos(xb+a)}{2b}+\frac{\cos(9xb+9a)}{36b}+\frac{3\cos(7xb+7a)}{28b}-\frac{2\cos(3xb+3a)}{3b}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
[Out] 64/b*(-1/9*sin(b*x+a)^2*cos(b*x+a)^7-2/63*cos(b*x+a)^7)
```

Maxima [A]
time = 0.26, size = 47, normalized size = 1.52

$$\frac{7\cos(9\,bx+9\,a)+27\cos(7\,bx+7\,a)-168\cos(3\,bx+3\,a)-378\cos(bx+a)}{252\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] 1/252*(7*cos(9*b*x + 9*a) + 27*cos(7*b*x + 7*a) - 168*cos(3*b*x + 3*a) - 37
8*cos(b*x + a))/b
```

Fricas [A]
time = 3.21, size = 26, normalized size = 0.84

$$\frac{64\left(7\cos\left(bx+a\right)^{9}-9\cos\left(bx+a\right)^{7}\right)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] 64/63*(7*cos(b*x + a)^9 - 9*cos(b*x + a)^7)/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**6,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(27) = 54.
time = 0.43, size = 182, normalized size = 5.87

$$-\frac{256\left(\frac{9\left(\cos(bx+a)-1\right)}{\cos(bx+a)+1}+\frac{27\left(\cos(bx+a)-1\right)^{2}}{\left(\cos(bx+a)+1\right)^{2}}+\frac{189\left(\cos(bx+a)-1\right)^{3}}{\left(\cos(bx+a)+1\right)^{3}}+\frac{189\left(\cos(bx+a)-1\right)^{4}}{\left(\cos(bx+a)+1\right)^{4}}+\frac{315\left(\cos(bx+a)-1\right)^{5}}{\left(\cos(bx+a)+1\right)^{5}}+\frac{105\left(\cos(bx+a)-1\right)^{6}}{\left(\cos(bx+a)+1\right)^{6}}+\frac{63\left(\cos(bx+a)-1\right)^{7}}{\left(\cos(bx+a)+1\right)^{7}}-1\right)}{63\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] -256/63*(9*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 27*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 + 189*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + 189*
(cos(b*x + a) - 1)^4/(cos(b*x + a) + 1)^4 + 315*(cos(b*x + a) - 1)^5/(cos(b
*x + a) + 1)^5 + 105*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 + 63*(cos(b*
x + a) - 1)^7/(cos(b*x + a) + 1)^7 - 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a
) + 1) - 1)^9)
```

Mupad [B]
time = 0.05, size = 26, normalized size = 0.84

$$-\frac{64\left(9\cos\left(a+b\,x\right)^{7}-7\cos\left(a+b\,x\right)^{9}\right)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^6/sin(a + b*x)^3,x)
```

```
[Out] -(64*(9*cos(a + b*x)^7 - 7*cos(a + b*x)^9))/(63*b)
```

3.64 $\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{32\sin^3(a+bx)}{3b}-\frac{64\sin^5(a+bx)}{5b}+\frac{32\sin^7(a+bx)}{7b}$$

```
[Out] 32/3*sin(b*x+a)^3/b-64/5*sin(b*x+a)^5/b+32/7*sin(b*x+a)^7/b
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2644, 276}

$$\frac{32\sin^7(a+bx)}{7b}-\frac{64\sin^5(a+bx)}{5b}+\frac{32\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (32*Sin[a + b*x]^3)/(3*b) - (64*Sin[a + b*x]^5)/(5*b) + (32*Sin[a + b*x]^7)
/(7*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^5(2a+2bx)\,dx = 32\int \cos^5(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{32\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{32\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{32\sin^3(a+bx)}{3b} - \frac{64\sin^5(a+bx)}{5b} + \frac{32\sin^7(a+bx)}{7b}$$

Mathematica [A]
time = 0.13, size = 37, normalized size = 0.80

$$\frac{4(157+108\cos(2(a+bx))+15\cos(4(a+bx)))\sin^3(a+bx)}{105b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*(157 + 108*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)])*Sin[a + b*x]^3)/(105*
b)
```

Maple [A]
time = 0.09, size = 51, normalized size = 1.11

method	result	size
default	$\frac{-\frac{32\sin(xb+a)\left(\cos^6(xb+a)\right)}{7}+\frac{32\left(\frac{8}{3}+\cos^4(xb+a)+\frac{4\left(\cos^2(xb+a)\right)}{3}\right)\sin(xb+a)}{35}}{b}$	51
risch	$\frac{5\sin(xb+a)}{2b}-\frac{\sin(7xb+7a)}{14b}-\frac{3\sin(5xb+5a)}{10b}-\frac{\sin(3xb+3a)}{6b}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 32/b*(-1/7*sin(b*x+a)*cos(b*x+a)^6+1/35*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)
*sin(b*x+a))
```

Maxima [A]
time = 0.27, size = 47, normalized size = 1.02

$$-\frac{15\,\sin\left(7\,bx+7\,a\right)+63\,\sin\left(5\,bx+5\,a\right)+35\,\sin\left(3\,bx+3\,a\right)-525\,\sin\left(bx+a\right)}{210\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/210*(15*sin(7*b*x + 7*a) + 63*sin(5*b*x + 5*a) + 35*sin(3*b*x + 3*a) - 5
25*sin(b*x + a))/b
```

Fricas [**A**]
time = 4.79, size = 43, normalized size = 0.93

$$-\frac{32\left(15\cos\left(bx+a\right)^{6}-3\cos\left(bx+a\right)^{4}-4\cos\left(bx+a\right)^{2}-8\right)\sin\left(bx+a\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -32/105*(15*cos(b*x + a)^6 - 3*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b
*x + a)/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.43, size = 36, normalized size = 0.78

$$\frac{32\left(15\sin\left(bx+a\right)^{7}-42\sin\left(bx+a\right)^{5}+35\sin\left(bx+a\right)^{3}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 32/105*(15*sin(b*x + a)^7 - 42*sin(b*x + a)^5 + 35*sin(b*x + a)^3)/b
```

Mupad [**B**]
time = 0.12, size = 36, normalized size = 0.78

$$\frac{32\,\left(15\sin\left(a+b\,x\right)^7-42\sin\left(a+b\,x\right)^5+35\sin\left(a+b\,x\right)^3\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^5/sin(a + b*x)^3,x)
```

```
[Out] (32*(35*sin(a + b*x)^3 - 42*sin(a + b*x)^5 + 15*sin(a + b*x)^7))/(105*b)
```

3.65 $\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=15

$$-\frac{16\cos^5(a+bx)}{5b}$$

```
[Out] -16/5*cos(b*x+a)^5/b
```

Rubi [A]

time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4373, 2645, 30}

$$-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (-16*Cos[a + b*x]^5)/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^4(2a+2bx)\,dx = 16\int \cos^4(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{16\text{Subst}\left(\int x^4\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{16\cos^5(a+bx)}{5b}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\frac{16\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
[Out] (-16*Cos[a + b*x]^5)/(5*b)
```

Maple [A]
time = 0.06, size = 14, normalized size = 0.93

method	result	size
default	$-\frac{16(\cos^5(xb+a))}{5b}$	14
risch	$-\frac{2\cos(xb+a)}{b}-\frac{\cos(5xb+5a)}{5b}-\frac{\cos(3xb+3a)}{b}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
[Out] -16/5*cos(b*x+a)^5/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(13) = 26.
time = 0.27, size = 34, normalized size = 2.27

$$-\frac{\cos(5\,bx+5\,a)+5\,\cos(3\,bx+3\,a)+10\,\cos(bx+a)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
[Out] -1/5*(cos(5*b*x + 5*a) + 5*cos(3*b*x + 3*a) + 10*cos(b*x + a))/b
```

Fricas [A]
time = 2.79, size = 13, normalized size = 0.87

$$-\frac{16\,\cos\left(bx+a\right)^{5}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -16/5*cos(b*x + a)^5/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(13) = 26.
time = 0.42, size = 74, normalized size = 4.93

$$\frac{32\left(\frac{10\,(\cos(bx+a)-1)^{2}}{(\cos(bx+a)+1)^{2}}+\frac{5\,(\cos(bx+a)-1)^{4}}{(\cos(bx+a)+1)^{4}}+1\right)}{5\,b\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 32/5*(10*(cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2 + 5*(cos(b*x + a) - 1)^4
/(cos(b*x + a) + 1)^4 + 1)/(b*((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1)^5
)
```

Mupad [B]
time = 0.05, size = 13, normalized size = 0.87

$$-\frac{16\cos\left(a+b\,x\right)^{5}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^4/sin(a + b*x)^3,x)
```

```
[Out] -(16*cos(a + b*x)^5)/(5*b)
```

3.66 $\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=27

$$\frac{8\sin(a+bx)}{b}-\frac{8\sin^3(a+bx)}{3b}$$

```
[Out] 8*sin(b*x+a)/b-8/3*sin(b*x+a)^3/b
```

Rubi [A]
time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4373, 2713}

$$\frac{8\sin(a+bx)}{b}-\frac{8\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (8*Sin[a + b*x])/b - (8*Sin[a + b*x]^3)/(3*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^3(2a+2bx)\,dx&=8\int \cos^3(a+bx)\,dx\\&=-\frac{8\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(a+bx)\right)}{b}\\&=\frac{8\sin(a+bx)}{b}-\frac{8\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.04

$$8\left(\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] 8*(Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b))
```

Maple [A]
time = 0.09, size = 22, normalized size = 0.81

method	result	size
default	$\frac{8(2+\cos^2(xb+a))\sin(xb+a)}{3b}$	22
risch	$\frac{6\sin(xb+a)}{b}+\frac{2\sin(3xb+3a)}{3b}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 8/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)
```

Maxima [A]
time = 0.27, size = 23, normalized size = 0.85

$$\frac{2\,(\sin\left(3\,bx+3\,a\right)+9\,\sin\left(bx+a\right))}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 2/3*(sin(3*b*x + 3*a) + 9*sin(b*x + a))/b
```

Fricas [A]
time = 2.80, size = 21, normalized size = 0.78

$$\frac{8\,\left(\cos\left(bx+a\right)^2+2\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 8/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.62, size = 22, normalized size = 0.81

$$-\frac{8\left(\sin\left(bx+a\right)^{3}-3\,\sin\left(bx+a\right)\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -8/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

Mupad [**B**]
time = 0.10, size = 24, normalized size = 0.89

$$\frac{8\left(3\,\sin\left(a+b\,x\right)-\sin\left(a+b\,x\right)^3\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^3/sin(a + b*x)^3,x)
```

```
[Out] (8*(3*sin(a + b*x) - sin(a + b*x)^3))/(3*b)
```

3.67 $\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=24

$$-\frac{4\tanh^{-1}(\cos(a+bx))}{b}+\frac{4\cos(a+bx)}{b}$$

```
[Out] -4*arctanh(cos(b*x+a))/b+4*cos(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4373, 2672, 327, 212}

$$\frac{4\cos(a+bx)}{b}-\frac{4\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*ArcTanh[Cos[a + b*x]])/b + (4*Cos[a + b*x])/b
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
```

```
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos(a+bx)\cot(a+bx)\,dx\\ &= -\frac{4\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}\\ &= \frac{4\cos(a+bx)}{b}-\frac{4\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{4\tanh^{-1}(\cos(a+bx))}{b}+\frac{4\cos(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 1.83

$$4\left(\frac{\cos(a+bx)}{b}-\frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
[Out] 4*(Cos[a + b*x]/b - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b)
```

Maple [A]
time = 0.09, size = 29, normalized size = 1.21

method	result	size
default	$\frac{4\cos(xb+a)+4\ln(\csc(xb+a)-\cot(xb+a))}{b}$	29
risch	$\frac{2\,e^{i(xb+a)}}{b}+\frac{2\,e^{-i(xb+a)}}{b}-\frac{4\ln\left(e^{i(xb+a)}+1\right)}{b}+\frac{4\ln\left(e^{i(xb+a)}-1\right)}{b}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
[Out] 4/b*(cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(24) = 48$.
time = 0.27, size = 92, normalized size = 3.83

$$\frac{2\left(2\cos(bx+a)-\log\left(\cos(bx)^2+2\cos(bx)\cos(a)+\cos(a)^2+\sin(bx)^2-2\sin(bx)\sin(a)+\sin(a)^2\right)+\log\left(\cos(bx)^2-2\cos(bx)\cos(a)+\cos(a)^2+\sin(bx)^2+2\sin(bx)\sin(a)+\sin(a)^2\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 2*(2*cos(b*x + a) - log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x
)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) +
cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Fricas [A]
time = 5.11, size = 38, normalized size = 1.58

$$\frac{2\left(2\cos\left(bx+a\right)-\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)+\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 2*(2*cos(b*x + a) - log(1/2*cos(b*x + a) + 1/2) + log(-1/2*cos(b*x + a) + 1
/2))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.
time = 0.45, size = 54, normalized size = 2.25

$$-\frac{2\left(\frac{4}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}-1}-\log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -2*(4/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 1) - log(-(cos(b*x + a) - 1)
/(cos(b*x + a) + 1)))/b
```

Mupad [B]
time = 0.11, size = 22, normalized size = 0.92

$$\frac{4\cos\left(a+b\,x\right)-4\,\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^2/sin(a + b*x)^3,x)
```

```
[Out] (4*cos(a + b*x) - 4*atanh(cos(a + b*x)))/b
```

3.68 $\int \csc^3(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=11

$$-\frac{2\csc(a+bx)}{b}$$

[Out] -2*csc(b*x+a)/b

Rubi [A]

time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4373, 2686, 8}

$$-\frac{2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x],x]

[Out] (-2*Csc[a + b*x])/b

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin(2a+2bx)\,dx &= 2\int \cot(a+bx)\csc(a+bx)\,dx\\ &= -\frac{2\text{Subst}\left(\int 1\,dx, x, \csc(a+bx)\right)}{b}\\ &= -\frac{2\csc(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 1.00

$$-\frac{2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*Csc[a + b*x])/b
```

Maple [A]
time = 0.05, size = 14, normalized size = 1.27

method	result	size
default	$-\frac{2}{b\sin(xb+a)}$	14
risch	$-\frac{4i\,\mathrm{e}^{i(xb+a)}}{b\left(\mathrm{e}^{2i(xb+a)}-1\right)}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/sin(b*x+a)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(11) = 22$.
time = 0.28, size = 84, normalized size = 7.64

$$-\frac{4\left(\cos\left(bx+a\right)\sin\left(2\,bx+2\,a\right)-\cos\left(2\,bx+2\,a\right)\sin\left(bx+a\right)+\sin\left(bx+a\right)\right)}{b\cos\left(2\,bx+2\,a\right)^{2}+b\sin\left(2\,bx+2\,a\right)^{2}-2\,b\cos\left(2\,bx+2\,a\right)+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -4*(cos(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x
 + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x + 2*a)
+ b)
```

Fricas [A]
time = 2.47, size = 13, normalized size = 1.18

$$-\frac{2}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -2/(b*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a),x)
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 13, normalized size = 1.18

$$-\frac{2}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
[Out] -2/(b*sin(b*x + a))
```

Mupad [**B**]
time = 0.04, size = 13, normalized size = 1.18

$$-\frac{2}{b\,\sin\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)/sin(a + b*x)^3,x)
[Out] -2/(b*sin(a + b*x))
```

3.69 $\int \csc^3(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\csc(a+bx)}{2b}-\frac{\csc^3(a+bx)}{6b}$$

```
[Out] 1/2*arctanh(sin(b*x+a))/b-1/2*csc(b*x+a)/b-1/6*csc(b*x+a)^3/b
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4373, 2701, 308, 213}

$$-\frac{\csc^3(a+bx)}{6b}-\frac{\csc(a+bx)}{2b}+\frac{\tanh^{-1}(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(2*b) - Csc[a + b*x]/(2*b) - Csc[a + b*x]^3/(6*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc^3(a+bx)\csc(2a+2bx)\,dx &= \frac{1}{2}\int \csc^4(a+bx)\sec(a+bx)\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}\\
&= -\frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \csc(a+bx)\right)}{2b}\\
&= -\frac{\csc(a+bx)}{2b}-\frac{\csc^3(a+bx)}{6b}-\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{2b}\\
&= \frac{\tanh^{-1}(\sin(a+bx))}{2b}-\frac{\csc(a+bx)}{2b}-\frac{\csc^3(a+bx)}{6b}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 31, normalized size = 0.72

$$
-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\sin^2(a+bx)\right)}{6b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x],x]
[Out] -1/6*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[a + b*x]^2])/b
```

Maple [A]
time = 0.10, size = 41, normalized size = 0.95

method	result	size
default	$\frac{-\frac{1}{3\sin(xb+a)^3}-\frac{1}{\sin(xb+a)}+\ln(\sec(xb+a)+\tan(xb+a))}{2b}$	41
risch	$-\frac{i\left(3\,e^{5i(xb+a)}-10\,e^{3i(xb+a)}+3\,e^{i(xb+a)}\right)}{3b\left(e^{2i(xb+a)}-1\right)^3}-\frac{\ln\left(e^{i(xb+a)}-i\right)}{2b}+\frac{\ln\left(i+e^{i(xb+a)}\right)}{2b}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*csc(2*b*x+2*a),x,method=_RETURNVERBOSE)
[Out] 1/2/b*(-1/3/sin(b*x+a)^3-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 834 vs. 2(37) = 74.
time = 0.52, size = 834, normalized size = 19.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(3*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*b
*x + 6*a) + 36*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 12*
(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*cos(4*b*x + 4*a) + 3*(2*(3*cos(4*b*x
 + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 + 6
*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 9*cos(2
*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6*a) -
sin(6*b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 + 18*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) - 1)*log((cos(b*x + 2*a)^
2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a
)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a)
 + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(3*cos(5*b*x
 + 5*a) - 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a) - 12*(3*co
s(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) - 12*(10*cos(3*b*
x + 3*a) - 3*cos(b*x + a))*sin(4*b*x + 4*a) - 40*(3*cos(2*b*x + 2*a) - 1)*s
in(3*b*x + 3*a) + 120*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 36*cos(b*x + a)*s
in(2*b*x + 2*a) + 36*cos(2*b*x + 2*a)*sin(b*x + a) - 12*sin(b*x + a))/(b*co
s(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(
6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 - 18*b*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*cos(4*b*x + 4*a) - 3*b*cos(2*b*x +
2*a) + b)*cos(6*b*x + 6*a) - 6*(3*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a)
- 6*b*cos(2*b*x + 2*a) - 6*(b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(6*
b*x + 6*a) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(37) = 74$.
time = 2.51, size = 94, normalized size = 2.19

$$\frac{3\left(\cos\left(bx+a\right)^2-1\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-3\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-6\cos\left(bx+a\right)^2+8}{12\left(b\cos\left(bx+a\right)^2-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*(cos(b*x + a)^2 - 1)*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*(cos(b*
x + a)^2 - 1)*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 8)/(
(b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^3\left(a+bx\right)\csc\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*csc(2*b*x+2*a),x)
```

```
[Out] Integral(csc(a + b*x)**3*csc(2*a + 2*b*x), x)
```

Giac [A]
time = 0.43, size = 52, normalized size = 1.21

$$-\frac{\frac{2\left(3\,\sin(bx+a)^2+1\right)}{\sin(bx+a)^3}-3\,\log\left(\sin\left(bx+a\right)+1\right)+3\,\log\left(-\sin\left(bx+a\right)+1\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/12*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(sin(b*x + a) + 1) +
3*log(-sin(b*x + a) + 1))/b
```

Mupad [B]
time = 0.13, size = 38, normalized size = 0.88

$$\frac{\operatorname{atanh}(\sin\left(a+b\,x\right))}{2\,b}-\frac{\frac{\sin(a+b\,x)^2}{2}+\frac{1}{6}}{b\sin\left(a+b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)),x)
```

```
[Out] atanh(sin(a + b*x))/(2*b) - (sin(a + b*x)^2/2 + 1/6)/(b*sin(a + b*x)^3)
```

3.70 $\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=70

$$-\frac{15\tanh^{-1}(\cos(a+bx))}{32b}+\frac{15\sec(a+bx)}{32b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{32b}-\frac{\csc^4(a+bx)\sec(a+bx)}{16b}$$

```
[Out] -15/32*arctanh(cos(b*x+a))/b+15/32*sec(b*x+a)/b-5/32*csc(b*x+a)^2*sec(b*x+a
)/b-1/16*csc(b*x+a)^4*sec(b*x+a)/b
```

Rubi [A]
time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4373, 2702, 294, 327, 213}

$$\frac{15\sec(a+bx)}{32b}-\frac{15\tanh^{-1}(\cos(a+bx))}{32b}-\frac{\csc^4(a+bx)\sec(a+bx)}{16b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(32*b) + (15*Sec[a + b*x])/(32*b) - (5*Csc[a +
b*x]^2*Sec[a + b*x])/(32*b) - (Csc[a + b*x]^4*Sec[a + b*x])/(16*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx = \frac{1}{4}\int \csc^5(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \sec(a+bx)\right)}{4b}$$
$$= -\frac{\csc^4(a+bx)\sec(a+bx)}{16b} + \frac{5\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{16b}$$
$$= -\frac{5\csc^2(a+bx)\sec(a+bx)}{32b} - \frac{\csc^4(a+bx)\sec(a+bx)}{16b} + \frac{15\text{Subst}\left(\int \right.}{}$$
$$= \frac{15\sec(a+bx)}{32b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{32b} - \frac{\csc^4(a+bx)\sec(a+bx)}{16b}$$
$$= -\frac{15\tanh^{-1}(\cos(a+bx))}{32b} + \frac{15\sec(a+bx)}{32b} - \frac{5\csc^2(a+bx)\sec(a+bx}{32b}$$

Mathematica [A]
time = 4.66, size = 129, normalized size = 1.84

$$-\frac{14\csc^2\left(\frac{1}{2}(a+bx)\right)+\csc^4\left(\frac{1}{2}(a+bx)\right)+\frac{\sec^2\left(\frac{1}{2}(a+bx)\right)\left(78+\cos(a+bx)\left(-8\left(8+15\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-15\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)\right)+\sec^4\left(\frac{1}{2}(a+bx)\right)\right)-14\tan^2\left(\frac{1}{2}(a+bx)\right)\right)}{-1+\tan^2\left(\frac{1}{2}(a+bx)\right)}}{256b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -1/256*(14*Csc[(a + b*x)/2]^2 + Csc[(a + b*x)/2]^4 + (Sec[(a + b*x)/2]^2*(7
8 + Cos[a + b*x]*(-8*(8 + 15*Log[Cos[(a + b*x)/2]] - 15*Log[Sin[(a + b*x)/2
]]) + Sec[(a + b*x)/2]^4) - 14*Tan[(a + b*x)/2]^2))/(-1 + Tan[(a + b*x)/2]^
2))/b
```

Maple [A]
time = 0.10, size = 71, normalized size = 1.01

method	result	size
default	$\frac{-\frac{1}{4\sin(xb+a)^{4}\cos(xb+a)}-\frac{5}{8\sin(xb+a)^{2}\cos(xb+a)}+\frac{15}{8\cos(xb+a)}+\frac{15\ln(\csc(xb+a)-\cot(xb+a))}{8}}{4b}$	71
risch	$\frac{15\,\mathrm{e}^{9i(xb+a)}-40\,\mathrm{e}^{7i(xb+a)}+18\,\mathrm{e}^{5i(xb+a)}-40\,\mathrm{e}^{3i(xb+a)}+15\,\mathrm{e}^{i(xb+a)}}{16b\left(\mathrm{e}^{2i(xb+a)}-1\right)^{4}\left(\mathrm{e}^{2i(xb+a)}+1\right)}+\frac{15\ln\left(\mathrm{e}^{i(xb+a)}-1\right)}{32b}-\frac{15\ln\left(\mathrm{e}^{i(xb+a)}+1\right)}{32b}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)-5/8/sin(b*x+a)^2/cos(b*x+a)+15/8/cos(b*
x+a)+15/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2237 vs. $2(62) = 124$.
time = 0.33, size = 2237, normalized size = 31.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(4*(15*cos(9*b*x + 9*a) - 40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a) -
40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) - 60*(3*cos(8*b*x
 + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)
*cos(9*b*x + 9*a) + 12*(40*cos(7*b*x + 7*a) - 18*cos(5*b*x + 5*a) + 40*cos(
3*b*x + 3*a) - 15*cos(b*x + a))*cos(8*b*x + 8*a) - 160*(2*cos(6*b*x + 6*a)
+ 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(7*b*x + 7*a) + 8*(18*cos
(5*b*x + 5*a) - 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(6*b*x + 6*a) + 7
2*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x + 5*a) - 40*(8*co
s(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 160*(3*cos(2*b*x + 2*a)
 - 1)*cos(3*b*x + 3*a) - 180*cos(2*b*x + 2*a)*cos(b*x + a) + 15*(2*(3*cos(8
*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a)
- 1)*cos(10*b*x + 10*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) + 2*
cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x +
 8*a)^2 - 4*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a)
- 4*cos(6*b*x + 6*a)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*co
s(4*b*x + 4*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) - 2*sin(6*b
*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - s
in(10*b*x + 10*a)^2 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*
b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a)
 - 3*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b*
x + 4*a)^2 + 12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 +
```

```
6*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin
(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 15*(2*(3*cos(8*b*x + 8*a) - 2*cos
(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 1
0*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) + 2*cos(4*b*x + 4*a) -
3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x + 8*a)^2 - 4*(2*cos(
4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*a
)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 -
9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4
*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^2
 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(8*b
*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*
a))*sin(6*b*x + 6*a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b*x + 4*a)^2 + 12*sin
(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a)
- 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x
)*sin(a) + sin(a)^2) + 4*(15*sin(9*b*x + 9*a) - 40*sin(7*b*x + 7*a) + 18*si
n(5*b*x + 5*a) - 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(10*b*x + 10*a)
- 60*(3*sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin(
2*b*x + 2*a))*sin(9*b*x + 9*a) + 12*(40*sin(7*b*x + 7*a) - 18*sin(5*b*x + 5
*a) + 40*sin(3*b*x + 3*a) - 15*sin(b*x + a))*sin(8*b*x + 8*a) - 160*(2*sin(
6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(7*b*x + 7*a) +
8*(18*sin(5*b*x + 5*a) - 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(6*b*x +
 6*a) + 72*(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(5*b*x + 5*a) - 40*
(8*sin(3*b*x + 3*a) - 3*sin(b*x + a))*sin(4*b*x + 4*a) + 480*sin(3*b*x + 3*
a)*sin(2*b*x + 2*a) - 180*sin(2*b*x + 2*a)*sin(b*x + a) + 60*cos(b*x + a))/
(b*cos(10*b*x + 10*a)^2 + 9*b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 +
 4*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(10*b*x + 10*a)^2 +
 9*b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x + 4*a)^2 -
 12*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*c
os(8*b*x + 8*a) - 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) + 3*b*cos(2*b
*x + 2*a) - b)*cos(10*b*x + 10*a) - 6*(2*b*cos(6*b*x + 6*a) + 2*b*cos(4*b*x
 + 4*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) + 4*(2*b*cos(4*b*x + 4
*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) - 4*(3*b*cos(2*b*x + 2*a)
- b)*cos(4*b*x + 4*a) - 6*b*cos(2*b*x + 2*a) - 2*(3*b*sin(8*b*x + 8*a) - 2*
b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) + 3*b*sin(2*b*x + 2*a))*sin(10*b*
x + 10*a) - 6*(2*b*sin(6*b*x + 6*a) + 2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x
+ 2*a))*sin(8*b*x + 8*a) + 4*(2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x + 2*a))*
sin(6*b*x + 6*a) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(62) = 124$.
time = 2.25, size = 132, normalized size = 1.89

$$\frac{30 \cos(bx+a)^4 - 50 \cos(bx+a)^2 - 15 \left(\cos(bx+a)^5 - 2 \cos(bx+a)^3 + \cos(bx+a)\right) \log\left(\frac{1}{2} \cos(bx+a) + \frac{1}{2}\right) + 15 \left(\cos(bx+a)^5 - 2 \cos(bx+a)^3 + \cos(bx+a)\right) \log\left(-\frac{1}{2} \cos(bx+a) + \frac{1}{2}\right) + 16}{64 \left(b \cos(bx+a)^5 - 2 b \cos(bx+a)^3 + b \cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(30*cos(b*x + a)^4 - 50*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - 2*cos(b*
x + a)^3 + cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5 -
 2*cos(b*x + a)^3 + cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) + 16)/(b*cos
(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^3(a+bx)\csc^2(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**2,x)
```

```
[Out] Integral(csc(a + b*x)**3*csc(2*a + 2*b*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(62) = 124.
time = 0.50, size = 160, normalized size = 2.29

$$\frac{\frac{\left(\frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1}-\frac{90(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)^2}{(\cos(bx+a)-1)^2}-\frac{16(\cos(bx+a)-1)}{\cos(bx+a)+1}+\frac{(\cos(bx+a)-1)^2}{(\cos(bx+a)+1)^2}+\frac{128}{\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1}+60\log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 1/256*((16*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 90*(cos(b*x + a) - 1)^2/
(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)^2/(cos(b*x + a) - 1)^2 - 16*(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) +
1)^2 + 128/((cos(b*x + a) - 1)/(cos(b*x + a) + 1) + 1) + 60*log(-(cos(b*x +
 a) - 1)/(cos(b*x + a) + 1)))/b
```

Mupad [B]
time = 0.15, size = 66, normalized size = 0.94

$$\frac{\frac{15\cos(a+b\,x)^4}{32}-\frac{25\cos(a+b\,x)^2}{32}+\frac{1}{4}}{b\,\left(\cos\left(a+b\,x\right)^5-2\cos\left(a+b\,x\right)^3+\cos\left(a+b\,x\right)\right)}-\frac{15\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^2),x)
```

```
[Out] ((15*cos(a + b*x)^4)/32 - (25*cos(a + b*x)^2)/32 + 1/4)/(b*(cos(a + b*x) -
2*cos(a + b*x)^3 + cos(a + b*x)^5)) - (15*atanh(cos(a + b*x)))/(32*b)
```

3.71 $\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=81

$$\frac{7\tanh^{-1}(\sin(a+bx))}{16b}-\frac{7\csc(a+bx)}{16b}-\frac{7\csc^3(a+bx)}{48b}-\frac{7\csc^5(a+bx)}{80b}+\frac{\csc^5(a+bx)\sec^2(a+bx)}{16b}$$

```
[Out] 7/16*arctanh(sin(b*x+a))/b-7/16*csc(b*x+a)/b-7/48*csc(b*x+a)^3/b-7/80*csc(b
*x+a)^5/b+1/16*csc(b*x+a)^5*sec(b*x+a)^2/b
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4373, 2701, 294, 308, 213}

$$-\frac{7\csc^5(a+bx)}{80b}-\frac{7\csc^3(a+bx)}{48b}-\frac{7\csc(a+bx)}{16b}+\frac{7\tanh^{-1}(\sin(a+bx))}{16b}+\frac{\csc^5(a+bx)\sec^2(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (7*ArcTanh[Sin[a + b*x]])/(16*b) - (7*Csc[a + b*x])/(16*b) - (7*Csc[a + b*x
]^3)/(48*b) - (7*Csc[a + b*x]^5)/(80*b) + (Csc[a + b*x]^5*Sec[a + b*x]^2)/(
16*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\csc^3(2a+2bx)\,dx &= \frac{1}{8}\int \csc^6(a+bx)\sec^3(a+bx)\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{8b}\\
&= \frac{\csc^5(a+bx)\sec^2(a+bx)}{16b} - \frac{7\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b}\\
&= \frac{\csc^5(a+bx)\sec^2(a+bx)}{16b} - \frac{7\text{Subst}\left(\int \left(1+x^2+x^4+\frac{1}{-1+x^2}\right)\,dx, x, \csc(\right.}{16b}\\
&= -\frac{7\csc(a+bx)}{16b} - \frac{7\csc^3(a+bx)}{48b} - \frac{7\csc^5(a+bx)}{80b} + \frac{\csc^5(a+bx)\sec^2(a}{16b}\\
&= \frac{7\tanh^{-1}(\sin(a+bx))}{16b} - \frac{7\csc(a+bx)}{16b} - \frac{7\csc^3(a+bx)}{48b} - \frac{7\csc^5(a+bx)}{80b}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.04, size = 31, normalized size = 0.38

$$-\frac{\csc^5(a+bx)\,{}_2F_1\left(-\frac{5}{2},2;-\frac{3}{2};\sin^2(a+bx)\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] -1/40*(Csc[a + b*x]^5*Hypergeometric2F1[-5/2, 2, -3/2, Sin[a + b*x]^2])/b
```

Maple [A]
time = 0.10, size = 87, normalized size = 1.07

method	result
default	$\frac{-\frac{1}{5\sin(xb+a)^5\cos(xb+a)^2}-\frac{7}{15\sin(xb+a)^3\cos(xb+a)^2}+\frac{7}{6\sin(xb+a)\cos(xb+a)^2}-\frac{7}{2\sin(xb+a)}+\frac{7\ln(\sec(xb+a)+\tan(xb+a))}{2}}{8b}$
risch	$-\frac{i(105\,e^{13i(xb+a)}-350\,e^{11i(xb+a)}+231\,e^{9i(xb+a)}+412\,e^{7i(xb+a)}+231\,e^{5i(xb+a)}-350\,e^{3i(xb+a)}+105\,e^{i(xb+a)})}{120b(e^{2i(xb+a)}-1)^5(e^{2i(xb+a)}+1)^2}+\frac{7\ln(i+e^{i(xb+a)})}{16b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(-1/5/sin(b*x+a)^5/cos(b*x+a)^2-7/15/sin(b*x+a)^3/cos(b*x+a)^2+7/6/si
n(b*x+a)/cos(b*x+a)^2-7/2/sin(b*x+a)+7/2*ln(sec(b*x+a)+tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3095 vs. $2(71) = 142$.
time = 0.65, size = 3095, normalized size = 38.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/480*(4*(105*sin(13*b*x + 13*a) - 350*sin(11*b*x + 11*a) + 231*sin(9*b*x +
 9*a) + 412*sin(7*b*x + 7*a) + 231*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a)
+ 105*sin(b*x + a))*cos(14*b*x + 14*a) + 420*(3*sin(12*b*x + 12*a) - sin(10
*b*x + 10*a) - 5*sin(8*b*x + 8*a) + 5*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) -
 3*sin(2*b*x + 2*a))*cos(13*b*x + 13*a) + 12*(350*sin(11*b*x + 11*a) - 231*
sin(9*b*x + 9*a) - 412*sin(7*b*x + 7*a) - 231*sin(5*b*x + 5*a) + 350*sin(3*
b*x + 3*a) - 105*sin(b*x + a))*cos(12*b*x + 12*a) + 1400*(sin(10*b*x + 10*a
) + 5*sin(8*b*x + 8*a) - 5*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) + 3*sin(2*b*
x + 2*a))*cos(11*b*x + 11*a) + 4*(231*sin(9*b*x + 9*a) + 412*sin(7*b*x + 7*
a) + 231*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a) + 105*sin(b*x + a))*cos(10
*b*x + 10*a) - 924*(5*sin(8*b*x + 8*a) - 5*sin(6*b*x + 6*a) - sin(4*b*x + 4
*a) + 3*sin(2*b*x + 2*a))*cos(9*b*x + 9*a) + 20*(412*sin(7*b*x + 7*a) + 231
*sin(5*b*x + 5*a) - 350*sin(3*b*x + 3*a) + 105*sin(b*x + a))*cos(8*b*x + 8*
a) + 1648*(5*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*cos(
7*b*x + 7*a) - 140*(33*sin(5*b*x + 5*a) - 50*sin(3*b*x + 3*a) + 15*sin(b*x
+ a))*cos(6*b*x + 6*a) + 924*(sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*cos(5*
b*x + 5*a) + 140*(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*cos(4*b*x + 4*a) +
105*(2*(3*cos(12*b*x + 12*a) - cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) + 5*
cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(14*b*x +
14*a) - cos(14*b*x + 14*a)^2 + 6*(cos(10*b*x + 10*a) + 5*cos(8*b*x + 8*a) -
 5*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*cos(12*b*x
 + 12*a) - 9*cos(12*b*x + 12*a)^2 - 2*(5*cos(8*b*x + 8*a) - 5*cos(6*b*x + 6
*a) - cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - cos(1
```

```
0*b*x + 10*a)^2 + 10*(5*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 3*cos(2*b*x +
 2*a) + 1)*cos(8*b*x + 8*a) - 25*cos(8*b*x + 8*a)^2 - 10*(cos(4*b*x + 4*a)
- 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - 25*cos(6*b*x + 6*a)^2 + 2*(3*c
os(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 9*cos(2*b*x +
2*a)^2 + 2*(3*sin(12*b*x + 12*a) - sin(10*b*x + 10*a) - 5*sin(8*b*x + 8*a)
+ 5*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(14*b*x +
14*a) - sin(14*b*x + 14*a)^2 + 6*(sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) -
 5*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x + 1
2*a) - 9*sin(12*b*x + 12*a)^2 - 2*(5*sin(8*b*x + 8*a) - 5*sin(6*b*x + 6*a)
- sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - sin(10*b*x +
10*a)^2 + 10*(5*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*s
in(8*b*x + 8*a) - 25*sin(8*b*x + 8*a)^2 - 10*(sin(4*b*x + 4*a) - 3*sin(2*b*
x + 2*a))*sin(6*b*x + 6*a) - 25*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 + 6
*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2
*a) - 1)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b
*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a
)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a)
+ sin(a)^2)) - 4*(105*cos(13*b*x + 13*a) - 350*cos(11*b*x + 11*a) + 231*cos
(9*b*x + 9*a) + 412*cos(7*b*x + 7*a) + 231*cos(5*b*x + 5*a) - 350*cos(3*b*x
 + 3*a) + 105*cos(b*x + a))*sin(14*b*x + 14*a) - 420*(3*cos(12*b*x + 12*a)
- cos(10*b*x + 10*a) - 5*cos(8*b*x + 8*a) + 5*cos(6*b*x + 6*a) + cos(4*b*x
+ 4*a) - 3*cos(2*b*x + 2*a) + 1)*sin(13*b*x + 13*a) - 12*(350*cos(11*b*x +
11*a) - 231*cos(9*b*x + 9*a) - 412*cos(7*b*x + 7*a) - 231*cos(5*b*x + 5*a)
+ 350*cos(3*b*x + 3*a) - 105*cos(b*x + a))*sin(12*b*x + 12*a) - 1400*(cos(1
0*b*x + 10*a) + 5*cos(8*b*x + 8*a) - 5*cos(6*b*x + 6*a) - cos(4*b*x + 4*a)
+ 3*cos(2*b*x + 2*a) - 1)*sin(11*b*x + 11*a) - 4*(231*cos(9*b*x + 9*a) + 41
2*cos(7*b*x + 7*a) + 231*cos(5*b*x + 5*a) - 350*cos(3*b*x + 3*a) + 105*cos(
b*x + a))*sin(10*b*x + 10*a) + 924*(5*cos(8*b*x + 8*a) - 5*cos(6*b*x + 6*a)
 - cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*sin(9*b*x + 9*a) - 20*(412*co
s(7*b*x + 7*a) + 231*cos(5*b*x + 5*a) - 350*cos(3*b*x + 3*a) + 105*cos(b*x
+ a))*sin(8*b*x + 8*a) - 1648*(5*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) - 3*co
s(2*b*x + 2*a) + 1)*sin(7*b*x + 7*a) + 140*(33*cos(5*b*x + 5*a) - 50*cos(3*
b*x + 3*a) + 15*cos(b*x + a))*sin(6*b*x + 6*a) - 924*(cos(4*b*x + 4*a) - 3*
cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) - 140*(10*cos(3*b*x + 3*a) - 3*cos(b
*x + a))*sin(4*b*x + 4*a) - 1400*(3*cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)
+ 4200*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 1260*cos(b*x + a)*sin(2*b*x + 2*
a) + 1260*cos(2*b*x + 2*a)*sin(b*x + a) - 420*sin(b*x + a))/(b*cos(14*b*x +
 14*a)^2 + 9*b*cos(12*b*x + 12*a)^2 + b*cos(10*b*x + 10*a)^2 + 25*b*cos(8*b
*x + 8*a)^2 + 25*b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*
x + 2*a)^2 + b*sin(14*b*x + 14*a)^2 + 9*b*sin(12*b*x + 12*a)^2 + b*sin(10*b
*x + 10*a)^2 + 25*b*sin(8*b*x + 8*a)^2 + 25*b*sin(6*b*x + 6*a)^2 + b*sin(4*
b*x + 4*a)^2 - 6*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)
^2 - 2*(3*b*cos(12*b*x + 12*a) - b*cos(10*b*x + 10*a) - 5*b*cos(8*b*x + 8*a
) + 5*b*cos(6*b*x + 6*a) + b*cos(4*b*x + 4*a) -...
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(71) = 142$.
time = 2.83, size = 166, normalized size = 2.05

$$-\frac{210 \cos\left(bx+a\right)^{6} - 490 \cos\left(bx+a\right)^{4} - 105 \left(\cos\left(bx+a\right)^{6} - 2 \cos\left(bx+a\right)^{4} + \cos\left(bx+a\right)^{2}\right) \log\left(\sin\left(bx+a\right)+1\right) \sin\left(bx+a\right) + 105 \left(\cos\left(bx+a\right)^{6} - 2 \cos\left(bx+a\right)^{4} + \cos\left(bx+a\right)^{2}\right) \log\left(-\sin\left(bx+a\right)+1\right) \sin\left(bx+a\right) + 322 \cos\left(bx+a\right)^{2} - 30}{480 \left(b \cos\left(bx+a\right)^{6} - 2 b \cos\left(bx+a\right)^{4} + b \cos\left(bx+a\right)^{2}\right) \sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -1/480*(210*cos(b*x + a)^6 - 490*cos(b*x + a)^4 - 105*(cos(b*x + a)^6 - 2*c
os(b*x + a)^4 + cos(b*x + a)^2)*log(sin(b*x + a) + 1)*sin(b*x + a) + 105*(c
os(b*x + a)^6 - 2*cos(b*x + a)^4 + cos(b*x + a)^2)*log(-sin(b*x + a) + 1)*s
in(b*x + a) + 322*cos(b*x + a)^2 - 30)/((b*cos(b*x + a)^6 - 2*b*cos(b*x + a
)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^{3}\left(a + bx\right) \csc^{3}\left(2a + 2bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**3,x)
```

```
[Out] Integral(csc(a + b*x)**3*csc(2*a + 2*b*x)**3, x)
```

Giac [A]
time = 0.48, size = 82, normalized size = 1.01

$$-\frac{\frac{30 \sin\left(bx+a\right)}{\sin\left(bx+a\right)^{2}-1} + \frac{4 \left(45 \sin\left(bx+a\right)^{4} + 10 \sin\left(bx+a\right)^{2} + 3\right)}{\sin\left(bx+a\right)^{5}} - 105 \log\left(\sin\left(bx+a\right)+1\right) + 105 \log\left(-\sin\left(bx+a\right)+1\right)}{480\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] -1/480*(30*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 4*(45*sin(b*x + a)^4 + 10*si
n(b*x + a)^2 + 3)/sin(b*x + a)^5 - 105*log(sin(b*x + a) + 1) + 105*log(-sin
(b*x + a) + 1))/b
```

Mupad [B]
time = 0.18, size = 71, normalized size = 0.88

$$\frac{7\,\mathrm{atanh}\left(\sin\left(a+b\,x\right)\right)}{16\,b} - \frac{-\frac{7\,{\sin\left(a+b\,x\right)}^6}{16} + \frac{7\,{\sin\left(a+b\,x\right)}^4}{24} + \frac{7\,{\sin\left(a+b\,x\right)}^2}{120} + \frac{1}{40}}{b\,\left({\sin\left(a+b\,x\right)}^5 - {\sin\left(a+b\,x\right)}^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^3),x)
```

```
[Out] (7*atanh(sin(a + b*x)))/(16*b) - ((7*sin(a + b*x)^2)/120 + (7*sin(a + b*x)^
4)/24 - (7*sin(a + b*x)^6)/16 + 1/40)/(b*(sin(a + b*x)^5 - sin(a + b*x)^7))
```

3.72 $\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=112

$$-\frac{105\tanh^{-1}(\cos(a+bx))}{256b}+\frac{105\sec(a+bx)}{256b}+\frac{35\sec^3(a+bx)}{256b}-\frac{21\csc^2(a+bx)\sec^3(a+bx)}{256b}-\frac{3\csc^4(a+bx)\sec^3(a+bx)}{128b}$$

```
[Out] -105/256*arctanh(cos(b*x+a))/b+105/256*sec(b*x+a)/b+35/256*sec(b*x+a)^3/b-2
1/256*csc(b*x+a)^2*sec(b*x+a)^3/b-3/128*csc(b*x+a)^4*sec(b*x+a)^3/b-1/96*cs
c(b*x+a)^6*sec(b*x+a)^3/b
```

Rubi [A]
time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4373, 2702, 294, 308, 213}

$$\frac{35\sec^3(a+bx)}{256b}+\frac{105\sec(a+bx)}{256b}-\frac{105\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^6(a+bx)\sec^3(a+bx)}{96b}-\frac{3\csc^4(a+bx)\sec^3(a+bx)}{128b}-\frac{21\csc^2(a+bx)\sec^3(a+bx)}{256b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-105*ArcTanh[Cos[a + b*x]])/(256*b) + (105*Sec[a + b*x])/(256*b) + (35*Sec
[a + b*x]^3)/(256*b) - (21*Csc[a + b*x]^2*Sec[a + b*x]^3)/(256*b) - (3*Csc[
a + b*x]^4*Sec[a + b*x]^3)/(128*b) - (Csc[a + b*x]^6*Sec[a + b*x]^3)/(96*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4373

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \csc^3(a+bx)\csc^4(2a+2bx)\,dx &= \frac{1}{16}\int \csc^7(a+bx)\sec^4(a+bx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^{10}}{(-1+x^2)^4}\,dx, x, \sec(a+bx)\right)}{16b} \\
&= -\frac{\csc^6(a+bx)\sec^3(a+bx)}{96b} + \frac{3\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, \sec(a+bx)\right)}{32b} \\
&= -\frac{3\csc^4(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^6(a+bx)\sec^3(a+bx)}{96b} + \frac{21\text{Subst}\left(\int \right.}{} \\
&= -\frac{21\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{3\csc^4(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^6(a+}{} \\
&= -\frac{21\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{3\csc^4(a+bx)\sec^3(a+bx)}{128b} - \frac{\csc^6(a+}{} \\
&= \frac{105\sec(a+bx)}{256b} + \frac{35\sec^3(a+bx)}{256b} - \frac{21\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{3\,\mathrm{c}}{} \\
&= -\frac{105\tanh^{-1}(\cos(a+bx))}{256b} + \frac{105\sec(a+bx)}{256b} + \frac{35\sec^3(a+bx)}{256b} - \frac{21\,\mathrm{cs}}{}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 278 vs. 2(112) = 224.
time = 0.87, size = 278, normalized size = 2.48

$$
\frac{\csc^{12}(a+bx)\left(1150-4752\cos(2(a+bx))+1600\cos(3(a+bx))+504\cos(4(a+bx))+1680\cos(6(a+bx))-600\cos(7(a+bx))-630\cos(8(a+bx))+200\cos(9(a+bx))+2520\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-945\cos(7(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)+315\cos(9(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-30\cos(a+bx)\left(40+63\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-63\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)\right)-2520\cos(3(a+bx))\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)+945\cos(7(a+bx))\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)-315\cos(9(a+bx))\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)\right)}{3072b\left(\csc^2\left(\frac{1}{2}(a+bx)\right)-\sec^2\left(\frac{1}{2}(a+bx)\right)\right)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (Csc[a + b*x]^12*(1150 - 4752*Cos[2*(a + b*x)] + 1600*Cos[3*(a + b*x)] + 50
4*Cos[4*(a + b*x)] + 1680*Cos[6*(a + b*x)] - 600*Cos[7*(a + b*x)] - 630*Cos
[8*(a + b*x)] + 200*Cos[9*(a + b*x)] + 2520*Cos[3*(a + b*x)]*Log[Cos[(a + b
*x)/2]] - 945*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/2]] + 315*Cos[9*(a + b*x)]
*Log[Cos[(a + b*x)/2]] - 30*Cos[a + b*x]*(40 + 63*Log[Cos[(a + b*x)/2]] - 6
3*Log[Sin[(a + b*x)/2]]) - 2520*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 94
5*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]] - 315*Cos[9*(a + b*x)]*Log[Sin[(a
+ b*x)/2]]))/(3072*b*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Maple [A]
time = 0.14, size = 107, normalized size = 0.96

method	result
default	$\frac{-\frac{1}{6\sin(xb+a)^6\cos(xb+a)^3}-\frac{3}{8\sin(xb+a)^4\cos(xb+a)^3}+\frac{7}{8\sin(xb+a)^2\cos(xb+a)^3}-\frac{35}{16\sin(xb+a)^2\cos(xb+a)}+\frac{105}{16\cos(xb+a)}+\frac{105\ln(\csc(xb+a)-\cot(}{16}}{16b}$
risch	$\frac{315\,e^{17i(xb+a)}-840\,e^{15i(xb+a)}-252\,e^{13i(xb+a)}+2376\,e^{11i(xb+a)}-1150\,e^{9i(xb+a)}+2376\,e^{7i(xb+a)}-252\,e^{5i(xb+a)}-840\,e^{3i(xb+a)}+315\,e^{i}}{384b\left(e^{2i(xb+a)}-1\right)^6\left(e^{2i(xb+a)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/b*(-1/6/sin(b*x+a)^6/cos(b*x+a)^3-3/8/sin(b*x+a)^4/cos(b*x+a)^3+7/8/si
n(b*x+a)^2/cos(b*x+a)^3-35/16/sin(b*x+a)^2/cos(b*x+a)+105/16/cos(b*x+a)+105
/16*ln(csc(b*x+a)-cot(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4268 vs. $2(100) = 200$.
time = 0.47, size = 4268, normalized size = 38.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/1536*(4*(315*cos(17*b*x + 17*a) - 840*cos(15*b*x + 15*a) - 252*cos(13*b*x
 + 13*a) + 2376*cos(11*b*x + 11*a) - 1150*cos(9*b*x + 9*a) + 2376*cos(7*b*x
 + 7*a) - 252*cos(5*b*x + 5*a) - 840*cos(3*b*x + 3*a) + 315*cos(b*x + a))*c
os(18*b*x + 18*a) - 1260*(3*cos(16*b*x + 16*a) - 8*cos(12*b*x + 12*a) + 6*c
os(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x +
 2*a) - 1)*cos(17*b*x + 17*a) + 12*(840*cos(15*b*x + 15*a) + 252*cos(13*b*x
 + 13*a) - 2376*cos(11*b*x + 11*a) + 1150*cos(9*b*x + 9*a) - 2376*cos(7*b*x
 + 7*a) + 252*cos(5*b*x + 5*a) + 840*cos(3*b*x + 3*a) - 315*cos(b*x + a))*c
os(16*b*x + 16*a) - 3360*(8*cos(12*b*x + 12*a) - 6*cos(10*b*x + 10*a) - 6*c
os(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*cos(15*b*x +
 15*a) - 1008*(8*cos(12*b*x + 12*a) - 6*cos(10*b*x + 10*a) - 6*cos(8*b*x +
8*a) + 8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*cos(13*b*x + 13*a) + 32
```

*(2376*cos(11*b*x + 11*a) - 1150*cos(9*b*x + 9*a) + 2376*cos(7*b*x + 7*a) -
252*cos(5*b*x + 5*a) - 840*cos(3*b*x + 3*a) + 315*cos(b*x + a))*cos(12*b*x
+ 12*a) - 9504*(6*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x +
6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(11*b*x + 11*a) + 24*(1150*cos(9*b*x + 9*
a) - 2376*cos(7*b*x + 7*a) + 252*cos(5*b*x + 5*a) + 840*cos(3*b*x + 3*a) -
315*cos(b*x + a))*cos(10*b*x + 10*a) + 4600*(6*cos(8*b*x + 8*a) - 8*cos(6*b
*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(9*b*x + 9*a) - 72*(792*cos(7*b*x +
7*a) - 84*cos(5*b*x + 5*a) - 280*cos(3*b*x + 3*a) + 105*cos(b*x + a))*cos(8
*b*x + 8*a) + 9504*(8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*cos(7*b*x
+ 7*a) - 672*(12*cos(5*b*x + 5*a) + 40*cos(3*b*x + 3*a) - 15*cos(b*x + a))*
cos(6*b*x + 6*a) + 1008*(3*cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) + 3360*(3
*cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a) - 3780*cos(2*b*x + 2*a)*cos(b*x + a
) + 315*(2*(3*cos(16*b*x + 16*a) - 8*cos(12*b*x + 12*a) + 6*cos(10*b*x + 10
*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos
(18*b*x + 18*a) - cos(18*b*x + 18*a)^2 + 6*(8*cos(12*b*x + 12*a) - 6*cos(10
*b*x + 10*a) - 6*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a)
+ 1)*cos(16*b*x + 16*a) - 9*cos(16*b*x + 16*a)^2 + 16*(6*cos(10*b*x + 10*a
) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(1
2*b*x + 12*a) - 64*cos(12*b*x + 12*a)^2 - 12*(6*cos(8*b*x + 8*a) - 8*cos(6*
b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - 36*cos(10*b*x + 1
0*a)^2 + 12*(8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a)
- 36*cos(8*b*x + 8*a)^2 + 16*(3*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 64
*cos(6*b*x + 6*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(16*b*x + 16*a) - 8*si
n(12*b*x + 12*a) + 6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a) - 8*sin(6*b*x
+ 6*a) + 3*sin(2*b*x + 2*a))*sin(18*b*x + 18*a) - sin(18*b*x + 18*a)^2 + 6*
(8*sin(12*b*x + 12*a) - 6*sin(10*b*x + 10*a) - 6*sin(8*b*x + 8*a) + 8*sin(6
*b*x + 6*a) - 3*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) - 9*sin(16*b*x + 16*a)
^2 + 16*(6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3
*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 64*sin(12*b*x + 12*a)^2 - 12*(6*sin
(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a)
- 36*sin(10*b*x + 10*a)^2 + 12*(8*sin(6*b*x + 6*a) - 3*sin(2*b*x + 2*a))*s
in(8*b*x + 8*a) - 36*sin(8*b*x + 8*a)^2 - 64*sin(6*b*x + 6*a)^2 + 48*sin(6*
b*x + 6*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) - 1
)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*s
in(a) + sin(a)^2) - 315*(2*(3*cos(16*b*x + 16*a) - 8*cos(12*b*x + 12*a) + 6
*cos(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x
+ 2*a) - 1)*cos(18*b*x + 18*a) - cos(18*b*x + 18*a)^2 + 6*(8*cos(12*b*x +
12*a) - 6*cos(10*b*x + 10*a) - 6*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 3*
cos(2*b*x + 2*a) + 1)*cos(16*b*x + 16*a) - 9*cos(16*b*x + 16*a)^2 + 16*(6*c
os(10*b*x + 10*a) + 6*cos(8*b*x + 8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x +
2*a) - 1)*cos(12*b*x + 12*a) - 64*cos(12*b*x + 12*a)^2 - 12*(6*cos(8*b*x +
8*a) - 8*cos(6*b*x + 6*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - 3
6*cos(10*b*x + 10*a)^2 + 12*(8*cos(6*b*x + 6*a) - 3*cos(2*b*x + 2*a) + 1)*c
os(8*b*x + 8*a) - 36*cos(8*b*x + 8*a)^2 + 16*(3*cos(2*b*x + 2*a) - 1)*cos(6
*b*x + 6*a) - 64*cos(6*b*x + 6*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(16*b*

```
x + 16*a) - 8*sin(12*b*x + 12*a) + 6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a
) - 8*sin(6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(18*b*x + 18*a) - sin(18*b*
x + 18*a)^2 + 6*(8*sin(12*b*x + 12*a) - 6*sin(10*b*x + 10*a) - 6*sin(8*b*x
+ 8*a) + 8*sin(6*b*x + 6*a) - 3*sin(2*b*x + 2*a))*sin(16*b*x + 16*a) - 9*si
n(16*b*x + 16*a)^2 + 16*(6*sin(10*b*x + 10*a) + 6*sin(8*b*x + 8*a) - 8*sin(
6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - 64*sin(12*b*x + 12*
a)^2 - 12*(6*sin(8*b*x + 8*a) - 8*sin(6*b*x + 6*a) + 3*sin(2*b*x + 2*a))*si
n(10*b*x + 10*a) - 36*sin(10*b*x + 10*a)^2 + 12*(8*sin(6*b*x + 6*a) - 3*sin
(2*b*x + 2*a))*sin(8*b*x + 8*a) - 36*sin(8*b*x + 8*a)^2 - 64*sin(6*b*x + 6*
a)^2 + 48*sin(6*b*x + 6*a)*sin(2*b*x + 2*a) - 9...
```

Fricas [A]
time = 2.21, size = 194, normalized size = 1.73

$$\frac{630 \cos\left(bx+a\right)^{8} - 1680 \cos\left(bx+a\right)^{6} + 1386 \cos\left(bx+a\right)^{4} - 288 \cos\left(bx+a\right)^{2} - 315 \left(\cos\left(bx+a\right)^{9} - 3 \cos\left(bx+a\right)^{7} + 3 \cos\left(bx+a\right)^{5} - \cos\left(bx+a\right)^{3}\right) \log\left(\frac{1}{2} \cos\left(bx+a\right) + \frac{1}{2}\right) + 315 \left(\cos\left(bx+a\right)^{9} - 3 \cos\left(bx+a\right)^{7} + 3 \cos\left(bx+a\right)^{5} - \cos\left(bx+a\right)^{3}\right) \log\left(-\frac{1}{2} \cos\left(bx+a\right) + \frac{1}{2}\right) - 32}{1536 \left(b \cos\left(bx+a\right)^{9} - 3 b \cos\left(bx+a\right)^{7} + 3 b \cos\left(bx+a\right)^{5} - b \cos\left(bx+a\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/1536*(630*cos(b*x + a)^8 - 1680*cos(b*x + a)^6 + 1386*cos(b*x + a)^4 - 28
8*cos(b*x + a)^2 - 315*(cos(b*x + a)^9 - 3*cos(b*x + a)^7 + 3*cos(b*x + a)^
5 - cos(b*x + a)^3)*log(1/2*cos(b*x + a) + 1/2) + 315*(cos(b*x + a)^9 - 3*c
os(b*x + a)^7 + 3*cos(b*x + a)^5 - cos(b*x + a)^3)*log(-1/2*cos(b*x + a) +
1/2) - 32)/(b*cos(b*x + a)^9 - 3*b*cos(b*x + a)^7 + 3*b*cos(b*x + a)^5 - b*
cos(b*x + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \csc^{3}\left(a + bx\right) \csc^{4}\left(2a + 2bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*csc(2*b*x+2*a)**4,x)
```

```
[Out] Integral(csc(a + b*x)**3*csc(2*a + 2*b*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(100) = 200.
time = 0.46, size = 268, normalized size = 2.39

$$-\frac{\frac{285 \left(\cos(bx+a)-1\right)}{\cos(bx+a)+1} - \frac{21 \left(\cos(bx+a)-1\right)^{2}}{\left(\cos(bx+a)+1\right)^{2}} + \frac{\left(\cos(bx+a)-1\right)^{3}}{\left(\cos(bx+a)+1\right)^{3}} + \frac{\frac{18 \left(\cos(bx+a)-1\right)}{\cos(bx+a)+1} - \frac{225 \left(\cos(bx+a)-1\right)^{2}}{\left(\cos(bx+a)+1\right)^{2}} - \frac{2966 \left(\cos(bx+a)-1\right)^{3}}{\left(\cos(bx+a)+1\right)^{3}} - \frac{3513 \left(\cos(bx+a)-1\right)^{4}}{\left(\cos(bx+a)+1\right)^{4}} - \frac{660 \left(\cos(bx+a)-1\right)^{5}}{\left(\cos(bx+a)+1\right)^{5}} + \frac{1155 \left(\cos(bx+a)-1\right)^{6}}{\left(\cos(bx+a)+1\right)^{6}} - 1}{\left(\frac{\cos(bx+a)-1}{\cos(bx+a)+1} + \frac{\left(\cos(bx+a)-1\right)^{2}}{\left(\cos(bx+a)+1\right)^{2}}\right)^{3}} - 1260 \log\left(-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}\right)}{6144\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*csc(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/6144*(285*(cos(b*x + a) - 1)/(cos(b*x + a) + 1) - 21*(cos(b*x + a) - 1)^
2/(cos(b*x + a) + 1)^2 + (cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 + (18*(c
os(b*x + a) - 1)/(cos(b*x + a) + 1) - 225*(cos(b*x + a) - 1)^2/(cos(b*x + a
) + 1)^2 - 2966*(cos(b*x + a) - 1)^3/(cos(b*x + a) + 1)^3 - 3513*(cos(b*x +
 a) - 1)^4/(cos(b*x + a) + 1)^4 - 660*(cos(b*x + a) - 1)^5/(cos(b*x + a) +
1)^5 + 1155*(cos(b*x + a) - 1)^6/(cos(b*x + a) + 1)^6 - 1)/((cos(b*x + a) -
 1)/(cos(b*x + a) + 1) + (cos(b*x + a) - 1)^2/(cos(b*x + a) + 1)^2)^3 - 126
0*log(-(cos(b*x + a) - 1)/(cos(b*x + a) + 1)))/b
```

Mupad [B]
time = 0.23, size = 100, normalized size = 0.89

$$\frac{-\frac{105\cos(a+b\,x)^8}{256}+\frac{35\cos(a+b\,x)^6}{32}-\frac{231\cos(a+b\,x)^4}{256}+\frac{3\cos(a+b\,x)^2}{16}+\frac{1}{48}}{b\,\left(-\cos\left(a+b\,x\right)^9+3\cos\left(a+b\,x\right)^7-3\cos\left(a+b\,x\right)^5+\cos\left(a+b\,x\right)^3\right)}-\frac{105\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^4),x)
```

```
[Out] ((3*cos(a + b*x)^2)/16 - (231*cos(a + b*x)^4)/256 + (35*cos(a + b*x)^6)/32
- (105*cos(a + b*x)^8)/256 + 1/48)/(b*(cos(a + b*x)^3 - 3*cos(a + b*x)^5 +
3*cos(a + b*x)^7 - cos(a + b*x)^9)) - (105*atanh(cos(a + b*x)))/(256*b)
```

3.73 $\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$-\frac{5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{32b}+\frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{32b}-\frac{5\cos(a+bx)}{}$$

```
[Out] -5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b+5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b+5/24*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-1/6*cos(b*x+a)*sin(
2*b*x+2*a)^(5/2)/b-5/16*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4387, 4386, 4390}

$$-\frac{5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{32b}+\frac{5\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{6b}-\frac{5\sqrt{\sin(2a+2bx)}\cos(a+bx)}{16b}+\frac{5\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) + (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) - (5*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) + (5*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(24*b) - (Cos[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(6*b)
```

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
```

```
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = -\frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}+\frac{5}{6}\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= \frac{5\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b}-\frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}+\frac{5}{8}\int \sin(a$$
$$= -\frac{5\cos(a+bx)\sqrt{\sin(2a+2bx)}}{16b}+\frac{5\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b}-\cos($$
$$= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}+\frac{5\log\left(\cos(a+bx)+\sin(a+bx)\right.}{32b}$$

Mathematica [**A**]
time = 0.32, size = 98, normalized size = 0.72

$$\frac{15\left(-\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)-2(14\cos(a+bx)+3\cos(3(a+bx))-2\cos(5(a+bx)))\sqrt{\sin(2(a+bx))}}{96b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
[Out] (15*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]]) - 2*(14*Cos[a + b*x] + 3*Cos[3*(a + b*x)] - 2*C
os[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/(96*b)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 52.96, size = 183652438, normalized size = 1350385.57

method	result	size
default	Expression too large to display	183652438

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(118) = 236$.
time = 3.62, size = 291, normalized size = 2.14

$$\frac{8\sqrt{2}\left(32\cos\left(bx+a\right)^{5}-52\cos\left(bx+a\right)^{3}+5\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+30\arctan\left(-\frac{\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\left(\cos(bx+a)-\sin(bx+a)\right)+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^{2}+2\cos(bx+a)\sin(bx+a)-1}\right)-30\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)-15\log\left(-32\cos\left(bx+a\right)^{4}+4\sqrt{2}\left(4\cos\left(bx+a\right)^{3}-\left(4\cos\left(bx+a\right)^{2}+1\right)\sin\left(bx+a\right)-5\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\cos\left(bx+a\right)^{2}+16\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/384*(8*sqrt(2)*(32*cos(b*x + a)^5 - 52*cos(b*x + a)^3 + 5*cos(b*x + a))*s
qrt(cos(b*x + a)*sin(b*x + a)) + 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(
b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b
*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) - 30*arctan(-(2*sqrt(2)*sqrt(
cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - s
in(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (
4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(
b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,{\sin\left(2\,a+2\,b\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^(5/2),x)
[Out] int(sin(a + b*x)*sin(2*a + 2*b*x)^(5/2), x)
```

3.74 $\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=110

$$-\frac{3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{16b}-\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{16b}+\frac{3\sin(a+bx)}{}$$

[Out] -3/16*arcsin(cos(b*x+a)-sin(b*x+a))/b-3/16*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*a)^(1/2))/b-1/4*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+3/8*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4387, 4386, 4391}

$$-\frac{3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{16b}+\frac{3\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{4b}-\frac{3\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{16b}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]

[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(16*b) - (3*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(16*b) + (3*Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(8*b) - (Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
```

```
a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}+\frac{3}{4}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{3\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b}-\frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}+\frac{3}{8}\int \frac{\text{s}}{\sqrt{\text{s}}}$$
$$= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}-\frac{3\log\left(\cos(a+bx)+\sin(a+bx)\right)}{16b}$$

Mathematica [A]
time = 0.22, size = 86, normalized size = 0.78

$$\frac{-3\left(\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+2\sqrt{\sin(2(a+bx))}\,(2\sin(a+bx)-\sin(3(a+bx)))}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-3*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(2*Sin[a + b*x] - Sin
[3*(a + b*x)]))/(16*b)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 28.84, size = 73720488, normalized size = 670186.25

method	result	size
default	Expression too large to display	73720488

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(96) = 192.
time = 6.04, size = 280, normalized size = 2.55

$$-\frac{8\sqrt{2}\left(4\cos\left(bx+a\right)^{2}-3\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\sin\left(bx+a\right)-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\left(\cos(bx+a)-\sin(bx+a)\right)+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^{2}+2\cos(bx+a)\sin(bx+a)-1}\right)+6\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)-3\log\left(-32\cos\left(bx+a\right)^{4}+4\sqrt{2}\left(4\cos\left(bx+a\right)^{3}-\left(4\cos\left(bx+a\right)^{2}+1\right)\sin\left(bx+a\right)-5\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\cos\left(bx+a\right)^{2}+16\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
[Out] -1/64*(8*sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a))*sin
(b*x + a) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a
) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
+ a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x +
a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-
32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*si
n(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x +
 a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,{\sin\left(2\,a+2\,b\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^(3/2),x)
[Out] int(sin(a + b*x)*sin(2*a + 2*b*x)^(3/2), x)
```

3.75 $\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=84

$$-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{4b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b}-\frac{\cos(a+bx)\sqrt{\sin(2a+2bx)}}{2b}$$

```
[Out] -1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b+1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b-1/2*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4387, 4390}

$$-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{2b}+\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/4*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) - (Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
2*b)
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= -\frac{\cos(a+bx)\sqrt{\sin(2a+2bx)}}{2b}+\frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\right.}{4b}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 72, normalized size = 0.86

$$\frac{-\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-2\cos(a+bx)\sqrt{\sin(2(a+bx))}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + S
qrt[Sin[2*(a + b*x)]]] - 2*Cos[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.44, size = 6806586, normalized size = 81030.79

method	result	size
default	Expression too large to display	6806586

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(74) = 148.
time = 3.64, size = 266, normalized size = 3.17

$$-\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a)-2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)+2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)+\log\left(-32\cos(bx+a)^4+4\sqrt{2}\left(4\cos(bx+a)^3-\left(4\cos(bx+a)^2+1\right)\sin(bx+a)-5\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}+32\cos(bx+a)^2+16\cos(bx+a)\sin(bx+a)+1\right)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) - 2*arctan(-(
sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos
(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1))
 + 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - si
```

```
n(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 + 4*sqr
t(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x +
a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*s
in(b*x + a) + 1))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(sin(a + b*x)*sin(2*a + 2*b*x)^(1/2), x)
```

3.76 $\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{2b}$$

```
[Out] -1/2*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{4391\}$

$$-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx=-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{2b}$$

Mathematica [A]
time = 0.06, size = 50, normalized size = 0.86

$$-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/2*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]])/b
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 1.56, size = 18282335, normalized size = 315212.67

method	result	size
default	Expression too large to display	18282335

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(52) = 104.
time = 3.01, size = 240, normalized size = 4.14

$$\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\,(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)+\log\left(-32\cos(bx+a)^4+4\sqrt{2}\left(4\cos(bx+a)^3-\left(4\cos(bx+a)^2+1\right)\sin(bx+a)-5\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}+32\cos(bx+a)^2+16\cos(bx+a)\sin(bx+a)+1\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin
(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*si
n(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - c
os(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*
x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a
) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 1
6*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(a+b\,x\right)}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(sin(a + b*x)/sin(2*a + 2*b*x)^(1/2), x)
```

3.77 $\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=23

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{4377\}$

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] Sin[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.96

$$\frac{\sin(a+bx)}{b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] Sin[a + b*x]/(b*Sqrt[Sin[2*(a + b*x)]])
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 10.37, size = 67736131, normalized size = 2945049.17

method	result	size
default	Expression too large to display	67736131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [A]
time = 2.33, size = 39, normalized size = 1.70

$$\frac{\sqrt{2}\,\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+\cos\left(bx+a\right)}{2\,b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + cos(b*x + a))/(b*cos(b*x + a
))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4852 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2029 vs. $2(21) = 42$.
time = 12.61, size = 2029, normalized size = 88.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)^
4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2*b
*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(1/2
*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1/2*b
*x) + tan(1/2*a))*((2*(sqrt(2)*tan(1/2*a)^25 + 10*sqrt(2)*tan(1/2*a)^23 + 4
4*sqrt(2)*tan(1/2*a)^21 + 110*sqrt(2)*tan(1/2*a)^19 + 165*sqrt(2)*tan(1/2*a
)^17 + 132*sqrt(2)*tan(1/2*a)^15 - 132*sqrt(2)*tan(1/2*a)^11 - 165*sqrt(2)*
tan(1/2*a)^9 - 110*sqrt(2)*tan(1/2*a)^7 - 44*sqrt(2)*tan(1/2*a)^5 - 10*sqrt
(2)*tan(1/2*a)^3 - sqrt(2)*tan(1/2*a))*tan(1/2*b*x)/(tan(1/2*a)^24 + 12*tan
(1/2*a)^22 + 66*tan(1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan(1/2*a)^16 + 792
*tan(1/2*a)^14 + 924*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 495*tan(1/2*a)^8 +
 220*tan(1/2*a)^6 + 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 + 1) + (sqrt(2)*tan(1
/2*a)^26 + 5*sqrt(2)*tan(1/2*a)^24 - 10*sqrt(2)*tan(1/2*a)^22 - 154*sqrt(2)
*tan(1/2*a)^20 - 605*sqrt(2)*tan(1/2*a)^18 - 1353*sqrt(2)*tan(1/2*a)^16 - 1
980*sqrt(2)*tan(1/2*a)^14 - 1980*sqrt(2)*tan(1/2*a)^12 - 1353*sqrt(2)*tan(1
/2*a)^10 - 605*sqrt(2)*tan(1/2*a)^8 - 154*sqrt(2)*tan(1/2*a)^6 - 10*sqrt(2)
*tan(1/2*a)^4 + 5*sqrt(2)*tan(1/2*a)^2 + sqrt(2))/(tan(1/2*a)^24 + 12*tan(1
/2*a)^22 + 66*tan(1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan(1/2*a)^16 + 792*t
an(1/2*a)^14 + 924*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 495*tan(1/2*a)^8 + 2
20*tan(1/2*a)^6 + 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 + 1))*tan(1/2*b*x) - 2*
(sqrt(2)*tan(1/2*a)^25 + 10*sqrt(2)*tan(1/2*a)^23 + 44*sqrt(2)*tan(1/2*a)^2
1 + 110*sqrt(2)*tan(1/2*a)^19 + 165*sqrt(2)*tan(1/2*a)^17 + 132*sqrt(2)*tan
(1/2*a)^15 - 132*sqrt(2)*tan(1/2*a)^11 - 165*sqrt(2)*tan(1/2*a)^9 - 110*sqr
t(2)*tan(1/2*a)^7 - 44*sqrt(2)*tan(1/2*a)^5 - 10*sqrt(2)*tan(1/2*a)^3 - sqr
t(2)*tan(1/2*a))/(tan(1/2*a)^24 + 12*tan(1/2*a)^22 + 66*tan(1/2*a)^20 + 220
*tan(1/2*a)^18 + 495*tan(1/2*a)^16 + 792*tan(1/2*a)^14 + 924*tan(1/2*a)^12
+ 792*tan(1/2*a)^10 + 495*tan(1/2*a)^8 + 220*tan(1/2*a)^6 + 66*tan(1/2*a)^4
 + 12*tan(1/2*a)^2 + 1))*cos(a)/((tan(1/2*b*x)^4*tan(1/2*a)^3 + tan(1/2*b*x
)^3*tan(1/2*a)^4 - tan(1/2*b*x)^4*tan(1/2*a) - 6*tan(1/2*b*x)^3*tan(1/2*a)^
2 - 6*tan(1/2*b*x)^2*tan(1/2*a)^3 - tan(1/2*b*x)*tan(1/2*a)^4 + tan(1/2*b*x
)^3 + 6*tan(1/2*b*x)^2*tan(1/2*a) + 6*tan(1/2*b*x)*tan(1/2*a)^2 + tan(1/2*a
)^3 - tan(1/2*b*x) - tan(1/2*a))*b) - 1/4*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(
1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(
1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(
1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*ta
n(1/2*a)^2 - tan(1/2*a)^3 + tan(1/2*b*x) + tan(1/2*a))*(((sqrt(2)*tan(1/2*a
```

```
)^26 + 5*sqrt(2)*tan(1/2*a)^24 - 10*sqrt(2)*tan(1/2*a)^22 - 154*sqrt(2)*tan
(1/2*a)^20 - 605*sqrt(2)*tan(1/2*a)^18 - 1353*sqrt(2)*tan(1/2*a)^16 - 1980*
sqrt(2)*tan(1/2*a)^14 - 1980*sqrt(2)*tan(1/2*a)^12 - 1353*sqrt(2)*tan(1/2*a
)^10 - 605*sqrt(2)*tan(1/2*a)^8 - 154*sqrt(2)*tan(1/2*a)^6 - 10*sqrt(2)*tan
(1/2*a)^4 + 5*sqrt(2)*tan(1/2*a)^2 + sqrt(2))*tan(1/2*b*x)/(tan(1/2*a)^24 +
 12*tan(1/2*a)^22 + 66*tan(1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan(1/2*a)^1
6 + 792*tan(1/2*a)^14 + 924*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 495*tan(1/2
*a)^8 + 220*tan(1/2*a)^6 + 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 + 1) - 8*(sqrt
(2)*tan(1/2*a)^25 + 10*sqrt(2)*tan(1/2*a)^23 + 44*sqrt(2)*tan(1/2*a)^21 + 1
10*sqrt(2)*tan(1/2*a)^19 + 165*sqrt(2)*tan(1/2*a)^17 + 132*sqrt(2)*tan(1/2*
a)^15 - 132*sqrt(2)*tan(1/2*a)^11 - 165*sqrt(2)*tan(1/2*a)^9 - 110*sqrt(2)*
tan(1/2*a)^7 - 44*sqrt(2)*tan(1/2*a)^5 - 10*sqrt(2)*tan(1/2*a)^3 - sqrt(2)*
tan(1/2*a))/(tan(1/2*a)^24 + 12*tan(1/2*a)^22 + 66*tan(1/2*a)^20 + 220*tan(
1/2*a)^18 + 495*tan(1/2*a)^16 + 792*tan(1/2*a)^14 + 924*tan(1/2*a)^12 + 792
*tan(1/2*a)^10 + 495*tan(1/2*a)^8 + 220*tan(1/2*a)^6 + 66*tan(1/2*a)^4 + 12
*tan(1/2*a)^2 + 1))*tan(1/2*b*x) - (sqrt(2)*tan(1/2*a)^26 + 5*sqrt(2)*tan(1
/2*a)^24 - 10*sqrt(2)*tan(1/2*a)^22 - 154*sqrt(2)*tan(1/2*a)^20 - 605*sqrt(
2)*tan(1/2*a)^18 - 1353*sqrt(2)*tan(1/2*a)^16 - 1980*sqrt(2)*tan(1/2*a)^14
- 1980*sqrt(2)*tan(1/2*a)^12 - 1353*sqrt(2)*tan(1/2*a)^10 - 605*sqrt(2)*tan
(1/2*a)^8 - 154*sqrt(2)*tan(1/2*a)^6 - 10*sqrt(2)*tan(1/2*a)^4 + 5*sqrt(2)*
tan(1/2*a)^2 + sqrt(2))/(tan(1/2*a)^24 + 12*tan(1/2*a)^22 + 66*tan(1/2*a)^2
0 + 220*tan(1/2*a)^18 + 495*tan(1/2*a)^16 + 792*tan(1/2*a)^14 + 924*tan(1/2
*a)^12 + 792*tan(1/2*a)^10 + 495*tan(1/2*a)^8 + 220*tan(1/2*a)^6 + 66*tan(1
/2*a)^4 + 12*tan(1/2*a)^2 + 1))*sin(a)/((tan(1/2*b*x)^4*tan(1/2*a)^3 + tan(
1/2*b*x)^3*tan(1/2*a)^4 - tan(1/2*b*x)^4*tan(1/2*a) - 6*tan(1/2*b*x)^3*tan(
1/2*a)^2 - 6*tan(1/2*b*x)^2*tan(1/2*a)^3 - tan(1/2*b*x)*tan(1/2*a)^4 + tan(
1/2*b*x)^3 + 6*tan(1/2*b*x)^2*tan(1/2*a) + 6*tan(1/2*b*x)*tan(1/2*a)^2 + ta
n(1/2*a)^3 - tan(1/2*b*x) - tan(1/2*a))*b)
```

Mupad [**B**]
time = 0.28, size = 34, normalized size = 1.48

$$\frac{\cos\left(a+b\,x\right)\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}}{b\,\left(\cos\left(2\,a+2\,b\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^(3/2),x)
[Out] (cos(a + b*x)*sin(2*a + 2*b*x)^(1/2))/(b*(cos(2*a + 2*b*x) + 1))
```

3.78 $\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=53

$$\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/3*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-2/3*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4389, 4376}

$$\frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] Sin[a + b*x]/(3*b*Sin[2*a + 2*b*x]^(3/2)) - (2*Cos[a + b*x])/(3*b*Sqrt[Sin[
2*a + 2*b*x]])
```

Rule 4376

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4389

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + D
ist[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x],
 x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !I
ntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= \frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{2}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= \frac{\sin(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 43, normalized size = 0.81

$$\frac{\sqrt{\sin(2(a+bx))}\left(-\frac{1}{4}\csc(a+bx)+\frac{1}{12}\sec(a+bx)\tan(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (Sqrt[Sin[2*(a + b*x)]]*(-1/4*Csc[a + b*x] + (Sec[a + b*x]*Tan[a + b*x])/12
))/b
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 59.40, size = 597, normalized size = 11.26

method	result
default	$\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\left(6\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\mathrm{EllipticE}\left(\sqrt{\tan\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(6*(tan(1/2*a+1/
2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)
*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*a+1/2*x*b)*(t
an(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^2-3*(tan(1/2*a+1/2*x*b)+1)
^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*Elliptic
F((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+
1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^2+6*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-
2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/
2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^
2-1))^(1/2)-3*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*
(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1
/2))*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)+2*(tan(1/2*a+1/2*x
*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4+2*(tan(1/2*a+1/2*x*b)*
(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^4-2*(tan(1/2*a+1/2*x*b)*
(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^2-2*(tan(1/2*a+1/2*x*b)^
3-tan(1/2*a+1/2*x*b))^(1/2))/tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^3-tan(1
/2*a+1/2*x*b))^(1/2)/(1+tan(1/2*a+1/2*x*b)^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [A]
time = 2.69, size = 69, normalized size = 1.30

$$-\frac{4\cos\left(bx+a\right)^{2}\sin\left(bx+a\right)+\sqrt{2}\left(4\cos\left(bx+a\right)^{2}-1\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}}{12\,b\cos\left(bx+a\right)^{2}\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(4*cos(b*x + a)^2*sin(b*x + a) + sqrt(2)*(4*cos(b*x + a)^2 - 1)*sqrt(
cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^2*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 7875 vs. $2(45) = 90$.
time = 48.01, size = 7875, normalized size = 148.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)
^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2*
b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(1/
2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1/2*
b*x) + tan(1/2*a))*((((((2*(sqrt(2)*tan(1/2*a)^56 + 23*sqrt(2)*tan(1/2*a)^5
4 + 251*sqrt(2)*tan(1/2*a)^52 + 1725*sqrt(2)*tan(1/2*a)^50 + 8350*sqrt(2)*t
an(1/2*a)^48 + 30130*sqrt(2)*tan(1/2*a)^46 + 83490*sqrt(2)*tan(1/2*a)^44 +
179630*sqrt(2)*tan(1/2*a)^42 + 297275*sqrt(2)*tan(1/2*a)^40 + 360525*sqrt(2
)*tan(1/2*a)^38 + 264385*sqrt(2)*tan(1/2*a)^36 - 37145*sqrt(2)*tan(1/2*a)^3
```

4 - 445740*sqrt(2)*tan(1/2*a)^32 - 742900*sqrt(2)*tan(1/2*a)^30 - 742900*sq
rt(2)*tan(1/2*a)^28 - 445740*sqrt(2)*tan(1/2*a)^26 - 37145*sqrt(2)*tan(1/2*
a)^24 + 264385*sqrt(2)*tan(1/2*a)^22 + 360525*sqrt(2)*tan(1/2*a)^20 + 29727
5*sqrt(2)*tan(1/2*a)^18 + 179630*sqrt(2)*tan(1/2*a)^16 + 83490*sqrt(2)*tan(
1/2*a)^14 + 30130*sqrt(2)*tan(1/2*a)^12 + 8350*sqrt(2)*tan(1/2*a)^10 + 1725
*sqrt(2)*tan(1/2*a)^8 + 251*sqrt(2)*tan(1/2*a)^6 + 23*sqrt(2)*tan(1/2*a)^4
+ sqrt(2)*tan(1/2*a)^2)*tan(1/2*b*x)/(tan(1/2*a)^51 + 23*tan(1/2*a)^49 + 25
2*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/2*a)^43 + 31878*tan(1/2*a
)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37 + 389367*tan(1/2*a)^35 +
572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 534888*tan(1/2*a)^29 + 208012
*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 534888*tan(1/2*a)^23 - 653752*tan(1
/2*a)^21 - 572033*tan(1/2*a)^19 - 389367*tan(1/2*a)^17 - 211508*tan(1/2*a)^
15 - 92092*tan(1/2*a)^13 - 31878*tan(1/2*a)^11 - 8602*tan(1/2*a)^9 - 1748*t
an(1/2*a)^7 - 252*tan(1/2*a)^5 - 23*tan(1/2*a)^3 - tan(1/2*a)) + 3*(sqrt(2)
*tan(1/2*a)^57 + 18*sqrt(2)*tan(1/2*a)^55 + 132*sqrt(2)*tan(1/2*a)^53 + 374
*sqrt(2)*tan(1/2*a)^51 - 1375*sqrt(2)*tan(1/2*a)^49 - 19620*sqrt(2)*tan(1/2
*a)^47 - 108560*sqrt(2)*tan(1/2*a)^45 - 399740*sqrt(2)*tan(1/2*a)^43 - 1096
755*sqrt(2)*tan(1/2*a)^41 - 2340250*sqrt(2)*tan(1/2*a)^39 - 3941740*sqrt(2)
*tan(1/2*a)^37 - 5204670*sqrt(2)*tan(1/2*a)^35 - 5163155*sqrt(2)*tan(1/2*a)
^33 - 3268760*sqrt(2)*tan(1/2*a)^31 + 3268760*sqrt(2)*tan(1/2*a)^27 + 51631
55*sqrt(2)*tan(1/2*a)^25 + 5204670*sqrt(2)*tan(1/2*a)^23 + 3941740*sqrt(2)*
tan(1/2*a)^21 + 2340250*sqrt(2)*tan(1/2*a)^19 + 1096755*sqrt(2)*tan(1/2*a)^
17 + 399740*sqrt(2)*tan(1/2*a)^15 + 108560*sqrt(2)*tan(1/2*a)^13 + 19620*sq
rt(2)*tan(1/2*a)^11 + 1375*sqrt(2)*tan(1/2*a)^9 - 374*sqrt(2)*tan(1/2*a)^7
- 132*sqrt(2)*tan(1/2*a)^5 - 18*sqrt(2)*tan(1/2*a)^3 - sqrt(2)*tan(1/2*a))/
(tan(1/2*a)^51 + 23*tan(1/2*a)^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45
+ 8602*tan(1/2*a)^43 + 31878*tan(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*t
an(1/2*a)^37 + 389367*tan(1/2*a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2
*a)^31 + 534888*tan(1/2*a)^29 + 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25
- 534888*tan(1/2*a)^23 - 653752*tan(1/2*a)^21 - 572033*tan(1/2*a)^19 - 389
367*tan(1/2*a)^17 - 211508*tan(1/2*a)^15 - 92092*tan(1/2*a)^13 - 31878*tan(
1/2*a)^11 - 8602*tan(1/2*a)^9 - 1748*tan(1/2*a)^7 - 252*tan(1/2*a)^5 - 23*t
an(1/2*a)^3 - tan(1/2*a)))*tan(1/2*b*x) - 30*(sqrt(2)*tan(1/2*a)^56 + 23*sq
rt(2)*tan(1/2*a)^54 + 251*sqrt(2)*tan(1/2*a)^52 + 1725*sqrt(2)*tan(1/2*a)^5
0 + 8350*sqrt(2)*tan(1/2*a)^48 + 30130*sqrt(2)*tan(1/2*a)^46 + 83490*sqrt(2
)*tan(1/2*a)^44 + 179630*sqrt(2)*tan(1/2*a)^42 + 297275*sqrt(2)*tan(1/2*a)^
40 + 360525*sqrt(2)*tan(1/2*a)^38 + 264385*sqrt(2)*tan(1/2*a)^36 - 37145*sq
rt(2)*tan(1/2*a)^34 - 445740*sqrt(2)*tan(1/2*a)^32 - 742900*sqrt(2)*tan(1/2
*a)^30 - 742900*sqrt(2)*tan(1/2*a)^28 - 445740*sqrt(2)*tan(1/2*a)^26 - 3714
5*sqrt(2)*tan(1/2*a)^24 + 264385*sqrt(2)*tan(1/2*a)^22 + 360525*sqrt(2)*tan
(1/2*a)^20 + 297275*sqrt(2)*tan(1/2*a)^18 + 179630*sqrt(2)*tan(1/2*a)^16 +
83490*sqrt(2)*tan(1/2*a)^14 + 30130*sqrt(2)*tan(1/2*a)^12 + 8350*sqrt(2)*ta
n(1/2*a)^10 + 1725*sqrt(2)*tan(1/2*a)^8 + 251*sqrt(2)*tan(1/2*a)^6 + 23*sqr
t(2)*tan(1/2*a)^4 + sqrt(2)*tan(1/2*a)^2)/(tan(1/2*a)^51 + 23*tan(1/2*a)^49
+ 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/2*a)^43 + 31878*tan(

```
1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37 + 389367*tan(1/2*a)^
35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 534888*tan(1/2*a)^29 + 2
08012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 534888*tan(1/2*a)^23 - 653752*
tan(1/2*a)^21 - 572033*tan(1/2*a)^19 - 389367*tan(1/2*a)^17 - 211508*tan(1/
2*a)^15 - 92092*tan(1/2*a)^13 - 31878*tan(1/2*a)^11 - 8602*tan(1/2*a)^9 - 1
748*tan(1/2*a)^7 - 252*tan(1/2*a)^5 - 23*tan(1/2*a)^3 - tan(1/2*a)))*tan(1/
2*b*x) - 10*(sqrt(2)*tan(1/2*a)^57 + 18*sqrt(2)*tan(1/2*a)^55 + 132*sqrt(2)
*tan(1/2*a)^53 + 374*sqrt(2)*tan(1/2*a)^51 - 1375*sqrt(2)*tan(1/2*a)^49 - 1
9620*sqrt(2)*tan(1/2*a)^47 - 108560*sqrt(2)*tan(1/2*a)^45 - 399740*sqrt(2)*
tan(1/2*a)^43 - 1096755*sqrt(2)*tan(1/2*a)^41 - 2340250*sqrt(2)*tan(1/2*a)^
39 - 3941740*sqrt(2)*tan(1/2*a)^37 - 5204670*sqrt(2)*tan(1/2*a)^35 - 516315
5*sqrt(2)*tan(1/2*a)^33 - 3268760*sqrt(2)*tan(1/2*a)^31 + 3268760*sqrt(2)*t
an(1/2*a)^27 + 5163155*sqrt(2)*tan(1/2*a)^25 + ...
```

Mupad [B]
time = 3.08, size = 108, normalized size = 2.04

$$-\frac{2\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{3\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^(5/2),x)
```

```
[Out] -(2*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2)*(exp(a*2i + b*x*2i)*1i + exp(a*4i + b*x*4i)*1i + 1i))/(3*b*(exp
(a*2i + b*x*2i) - 1)*(exp(a*2i + b*x*2i) + 1)^2)
```

3.79 $\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=79

$$\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/5*sin(b*x+a)/b/sin(2*b*x+2*a)^(5/2)-4/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2
)+8/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4389, 4388, 4377}

$$\frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sqrt{\sin(2a+2bx)}}-\frac{4\cos(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] Sin[a + b*x]/(5*b*Sin[2*a + 2*b*x]^(5/2)) - (4*Cos[a + b*x])/(15*b*Sin[2*a
+ 2*b*x]^(3/2)) + (8*Sin[a + b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4388

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4389

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + D
ist[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x],
 x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !I
```

```
ntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx &= \frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= \frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8}{15}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= \frac{\sin(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 52, normalized size = 0.66

$$\frac{\left(-5\cot(a+bx)\csc(a+bx)+3\sec(a+bx)\left(9+\sec^2(a+bx)\right)\right)\sqrt{\sin(2(a+bx))}}{120b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
[Out] ((-5*Cot[a + b*x]*Csc[a + b*x] + 3*Sec[a + b*x]*(9 + Sec[a + b*x]^2))*Sqrt[
Sin[2*(a + b*x)]])/(120*b)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(xb+a\right)}{\sin\left(2xb+2a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
[Out] int(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

[Out] integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)

Fricas [**A**]
time = 4.58, size = 88, normalized size = 1.11

$$\frac{32\,\cos\left(bx+a\right)^{5}-32\,\cos\left(bx+a\right)^{3}+\sqrt{2}\left(32\,\cos\left(bx+a\right)^{4}-24\,\cos\left(bx+a\right)^{2}-3\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}}{120\left(b\cos\left(bx+a\right)^{5}-b\cos\left(bx+a\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")

[Out] 1/120*(32*cos(b*x + a)^5 - 32*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4 - 24*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^5 - b*cos(b*x + a)^3)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(7/2),x)

[Out] Timed out

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 18022 vs. 2(67) = 134.
time = 162.48, size = 18022, normalized size = 228.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")

[Out] 1/480*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1/2*b*x) + tan(1/2*a))*(((((((((2*(sqrt(2)*tan(1/2*a)^87 - 92*sqrt(2)*tan(1/2*a)^85 - 3213*sqrt(2)*tan(1/2*a)^83 - 48008*sqrt(2)*tan(1/2*a)^81 - 443462*sqrt(2)*tan(1/2*a)^79 - 2875040*sqrt(2)*tan(1/2*a)^77 - 13907802*sqrt(2)*tan(1/2*a)^75 - 51781432*sqrt(2)*tan(1/2*a)^73 - 149943911*sqrt(2)*tan(1/2*a)^71 - 333456564*sqrt(2)*tan(1/2*a)^69 - 536442973*sqrt(2)*tan(1/2*a)^67 - 482080288*sqrt(2)*tan(1/2*a)^65 + 316221080*sqrt(2)*tan(1/2*a)^63 + 2190937152*sqrt(2)*tan(1/2*a)^61 + 4607763368*sqrt(2)*tan(1/2*a)^59 + 5742984608*sqrt(2)*tan(1/2*a)^57 + 3316624962*sqrt(2)*tan(1/2*a)^55 - 3241815576*sqrt(2)*

tan(1/2*a)^53 - 11030972730*sqrt(2)*tan(1/2*a)^51 - 14712027120*sqrt(2)*tan
(1/2*a)^49 - 10524179460*sqrt(2)*tan(1/2*a)^47 + 10524179460*sqrt(2)*tan(1/
2*a)^43 + 14712027120*sqrt(2)*tan(1/2*a)^41 + 11030972730*sqrt(2)*tan(1/2*a
)^39 + 3241815576*sqrt(2)*tan(1/2*a)^37 - 3316624962*sqrt(2)*tan(1/2*a)^35
- 5742984608*sqrt(2)*tan(1/2*a)^33 - 4607763368*sqrt(2)*tan(1/2*a)^31 - 219
0937152*sqrt(2)*tan(1/2*a)^29 - 316221080*sqrt(2)*tan(1/2*a)^27 + 482080288
*sqrt(2)*tan(1/2*a)^25 + 536442973*sqrt(2)*tan(1/2*a)^23 + 333456564*sqrt(2
)*tan(1/2*a)^21 + 149943911*sqrt(2)*tan(1/2*a)^19 + 51781432*sqrt(2)*tan(1/
2*a)^17 + 13907802*sqrt(2)*tan(1/2*a)^15 + 2875040*sqrt(2)*tan(1/2*a)^13 +
443462*sqrt(2)*tan(1/2*a)^11 + 48008*sqrt(2)*tan(1/2*a)^9 + 3213*sqrt(2)*ta
n(1/2*a)^7 + 92*sqrt(2)*tan(1/2*a)^5 - sqrt(2)*tan(1/2*a)^3)*tan(1/2*b*x)/(
tan(1/2*a)^78 + 34*tan(1/2*a)^76 + 559*tan(1/2*a)^74 + 5916*tan(1/2*a)^72 +
45255*tan(1/2*a)^70 + 266322*tan(1/2*a)^68 + 1252713*tan(1/2*a)^66 + 48290
88*tan(1/2*a)^64 + 15512772*tan(1/2*a)^62 + 41970280*tan(1/2*a)^60 + 961606
36*tan(1/2*a)^58 + 186574864*tan(1/2*a)^56 + 304253964*tan(1/2*a)^54 + 4082
39496*tan(1/2*a)^52 + 426395700*tan(1/2*a)^50 + 286097760*tan(1/2*a)^48 - 3
1635810*tan(1/2*a)^46 - 450345060*tan(1/2*a)^44 - 811985790*tan(1/2*a)^42 -
955277400*tan(1/2*a)^40 - 811985790*tan(1/2*a)^38 - 450345060*tan(1/2*a)^3
6 - 31635810*tan(1/2*a)^34 + 286097760*tan(1/2*a)^32 + 426395700*tan(1/2*a)
^30 + 408239496*tan(1/2*a)^28 + 304253964*tan(1/2*a)^26 + 186574864*tan(1/2
*a)^24 + 96160636*tan(1/2*a)^22 + 41970280*tan(1/2*a)^20 + 15512772*tan(1/2
*a)^18 + 4829088*tan(1/2*a)^16 + 1252713*tan(1/2*a)^14 + 266322*tan(1/2*a)^
12 + 45255*tan(1/2*a)^10 + 5916*tan(1/2*a)^8 + 559*tan(1/2*a)^6 + 34*tan(1/
2*a)^4 + tan(1/2*a)^2) + 5*(sqrt(2)*tan(1/2*a)^88 - 97*sqrt(2)*tan(1/2*a)^8
6 - 2757*sqrt(2)*tan(1/2*a)^84 - 31579*sqrt(2)*tan(1/2*a)^82 - 190206*sqrt(
2)*tan(1/2*a)^80 - 452482*sqrt(2)*tan(1/2*a)^78 + 2446494*sqrt(2)*tan(1/2*a
)^76 + 31236834*sqrt(2)*tan(1/2*a)^74 + 178073713*sqrt(2)*tan(1/2*a)^72 + 6
92499183*sqrt(2)*tan(1/2*a)^70 + 2006851683*sqrt(2)*tan(1/2*a)^68 + 4409972
669*sqrt(2)*tan(1/2*a)^66 + 7082232504*sqrt(2)*tan(1/2*a)^64 + 6893762888*s
qrt(2)*tan(1/2*a)^62 - 1327875576*sqrt(2)*tan(1/2*a)^60 - 21040534024*sqrt(
2)*tan(1/2*a)^58 - 47574053342*sqrt(2)*tan(1/2*a)^56 - 64972634082*sqrt(2)*
tan(1/2*a)^54 - 53236088394*sqrt(2)*tan(1/2*a)^52 - 5004094710*sqrt(2)*tan(
1/2*a)^50 + 61712915820*sqrt(2)*tan(1/2*a)^48 + 110145965460*sqrt(2)*tan(1/
2*a)^46 + 110145965460*sqrt(2)*tan(1/2*a)^44 + 61712915820*sqrt(2)*tan(1/2*
a)^42 - 5004094710*sqrt(2)*tan(1/2*a)^40 - 53236088394*sqrt(2)*tan(1/2*a)^3
8 - 64972634082*sqrt(2)*tan(1/2*a)^36 - 47574053342*sqrt(2)*tan(1/2*a)^34 -
21040534024*sqrt(2)*tan(1/2*a)^32 - 1327875576*sqrt(2)*tan(1/2*a)^30 + 689
3762888*sqrt(2)*tan(1/2*a)^28 + 7082232504*sqrt(2)*tan(1/2*a)^26 + 44099726
69*sqrt(2)*tan(1/2*a)^24 + 2006851683*sqrt(2)*tan(1/2*a)^22 + 692499183*sqr
t(2)*tan(1/2*a)^20 + 178073713*sqrt(2)*tan(1/2*a)^18 + 31236834*sqrt(2)*tan
(1/2*a)^16 + 2446494*sqrt(2)*tan(1/2*a)^14 - 452482*sqrt(2)*tan(1/2*a)^12 -
190206*sqrt(2)*tan(1/2*a)^10 - 31579*sqrt(2)*tan(1/2*a)^8 - 2757*sqrt(2)*t
an(1/2*a)^6 - 97*sqrt(2)*tan(1/2*a)^4 + sqrt(2)*tan(1/2*a)^2)/(tan(1/2*a)^7
8 + 34*tan(1/2*a)^76 + 559*tan(1/2*a)^74 + 5916*tan(1/2*a)^72 + 45255*tan(1
/2*a)^70 + 266322*tan(1/2*a)^68 + 1252713*tan(1/2*a)^66 + 4829088*tan(1/2*a

```
)^64 + 15512772*tan(1/2*a)^62 + 41970280*tan(1/2*a)^60 + 96160636*tan(1/2*a
)^58 + 186574864*tan(1/2*a)^56 + 304253964*tan(1/2*a)^54 + 408239496*tan(1/
2*a)^52 + 426395700*tan(1/2*a)^50 + 286097760*tan(1/2*a)^48 - 31635810*tan(
1/2*a)^46 - 450345060*tan(1/2*a)^44 - 811985790*tan(1/2*a)^42 - 955277400*t
an(1/2*a)^40 - 811985790*tan(1/2*a)^38 - 450345060*tan(1/2*a)^36 - 31635810
*tan(1/2*a)^34 + 286097760*tan(1/2*a)^32 + 426395700*tan(1/2*a)^30 + 408239
496*tan(1/2*a)^28 + 304253964*tan(1/2*a)^26 + 186574864*tan(1/2*a)^24 + 961
60636*tan(1/2*a)^22 + 41970280*tan(1/2*a)^20 + 15512772*tan(1/2*a)^18 + 482
9088*tan(1/2*a)^16 + 1252713*tan(1/2*a)^14 + 266322*tan(1/2*a)^12 + 45255*t
an(1/2*a)^10 + 5916*tan(1/2*a)^8 + 559*tan(1/2*...
```

Mupad [B]
time = 3.39, size = 131, normalized size = 1.66

$$\frac{4\,{\mathrm{e}}^{a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,\sqrt{\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}}\,\left(2\,{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}-3\,{\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}+2\,{\mathrm{e}}^{a\,6{}\mathrm{i}+b\,x\,6{}\mathrm{i}}+2\,{\mathrm{e}}^{a\,8{}\mathrm{i}+b\,x\,8{}\mathrm{i}}+2\right)}{15\,b\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}-1\right)}^2\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^(7/2),x)
```

```
[Out] (4*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2)*(2*exp(a*2i + b*x*2i) - 3*exp(a*4i + b*x*4i) + 2*exp(a*6i + b*x*
6i) + 2*exp(a*8i + b*x*8i) + 2))/(15*b*(exp(a*2i + b*x*2i) - 1)^2*(exp(a*2i
 + b*x*2i) + 1)^3)
```

3.80 $\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=105

$$\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/7*sin(b*x+a)/b/sin(2*b*x+2*a)^(7/2)-6/35*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/2
)+8/35*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-16/35*cos(b*x+a)/b/sin(2*b*x+2*a)^
(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4389, 4388, 4376}

$$\frac{8\sin(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] Sin[a + b*x]/(7*b*Sin[2*a + 2*b*x]^(7/2)) - (6*Cos[a + b*x])/(35*b*Sin[2*a
+ 2*b*x]^(5/2)) + (8*Sin[a + b*x])/(35*b*Sin[2*a + 2*b*x]^(3/2)) - (16*Cos[
a + b*x])/(35*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4376

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4388

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4389

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + D
```

```
ist[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x],
 x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !I
ntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx &= \frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{6}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx\\ &= \frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{24}{35}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= \frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{16}{35}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\\ &= \frac{\sin(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{6\cos(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{35b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 67, normalized size = 0.64

$$\frac{(-5-10\cos(2(a+bx))+4\cos(4(a+bx))+4\cos(6(a+bx)))\csc^3(a+bx)\sec^4(a+bx)\sqrt{\sin(2(a+bx))}}{560b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
[Out] ((-5 - 10*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[a
 + b*x]^3*Sec[a + b*x]^4*Sqrt[Sin[2*(a + b*x)]])/(560*b)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(xb+a)}{\sin(2xb+2a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
[Out] int(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)/sin(2*b*x + 2*a)^(9/2), x)
```

Fricas [A]
time = 5.99, size = 113, normalized size = 1.08

$$-\frac{\sqrt{2}\left(128\cos\left(bx+a\right)^{6}-160\cos\left(bx+a\right)^{4}+20\cos\left(bx+a\right)^{2}+5\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+128\left(\cos\left(bx+a\right)^{6}-\cos\left(bx+a\right)^{4}\right)\sin\left(bx+a\right)}{560\left(b\cos\left(bx+a\right)^{6}-b\cos\left(bx+a\right)^{4}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/560*(sqrt(2)*(128*cos(b*x + a)^6 - 160*cos(b*x + a)^4 + 20*cos(b*x + a)^
2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - cos(b*x + a)
^4)*sin(b*x + a))/((b*cos(b*x + a)^6 - b*cos(b*x + a)^4)*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.34, size = 351, normalized size = 3.34

$$-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,1\mathrm{i}}{7\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}^4}+\frac{16\,\mathrm{e}^{a\,3\mathrm{i}+b\,x\,3\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{35\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{1\mathrm{i}}{7\,b}+\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,8\mathrm{i}}{35\,b}\right)\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}^2}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{16}{35\,b}-\frac{44\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}}{35\,b}\right)\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(2*a + 2*b*x)^(9/2),x)
```

```
[Out] (16*exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2))/(35*b*(exp(a*2i + b*x*2i) - 1)*(exp(a*2i + b*x*2i)*1i + 1i)) -
 (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*1i)/(7*b*(exp(a*2i + b*x*2i)*1i + 1i)^4) - (exp(a*1i + b*x*1i)*(1
i/(7*b) + (exp(a*2i + b*x*2i)*8i)/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (e
xp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*x*2i) - 1)^2*(exp(a*2i + b*x
*2i)*1i + 1i)^2) - (exp(a*1i + b*x*1i)*(16/(35*b) - (44*exp(a*2i + b*x*2i))
/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(
(exp(a*2i + b*x*2i) - 1)^3*(exp(a*2i + b*x*2i)*1i + 1i)^3)
```

3.81 $\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=98

$$\frac{5F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{42b}-\frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b}-\frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b}-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

```
[Out] -5/42*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+
b*x),2^(1/2))/b-1/14*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(5/2)/b-1/18*sin(2*b*x+2
*a)^(9/2)/b-5/42*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4383, 2715, 2720}

$$-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}+\frac{5F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{42b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{14b}-\frac{5\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{42b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (5*EllipticF[a - Pi/4 + b*x, 2])/(42*b) - (5*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(42*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(5/2))/(14*b) - Sin[
2*a + 2*b*x]^(9/2)/(18*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4383

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Sin[
a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && N
eQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \sin^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = -\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}+\frac{1}{2}\int \sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= -\frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b}-\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}+\frac{5}{14}\int \sin^{\frac{3}{2}}(2a+2bx$$
$$= -\frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b}-\frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b}-\frac{\sin}{}$$
$$= \frac{5F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{42b}-\frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b}-\frac{\cos(2a+2bx)}{1}$$

Mathematica [A]
time = 3.15, size = 96, normalized size = 0.98

$$\frac{240F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2(a+bx))}-70\sin(2(a+bx))-156\sin(4(a+bx))+35\sin(6(a+bx))+18\sin(8(a+bx))-7\sin(10(a+bx))}{2016b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (240*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] - 70*Sin[2*(a + b*
x)] - 156*Sin[4*(a + b*x)] + 35*Sin[6*(a + b*x)] + 18*Sin[8*(a + b*x)] - 7*
Sin[10*(a + b*x)])/(2016*b*Sqrt[Sin[2*(a + b*x)]])
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 233.51, size = 519395265, normalized size = 5299951.68

method	result	size
default	Expression too large to display	519395265

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(7/2)*sin(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(((cos(b*x + a)^2 - 1)*cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 + 1)*sin
(2*b*x + 2*a)^(3/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(7/2)*sin(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2),x)
```

```
[Out] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2), x)
```

3.82 $\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=69

$$\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b}-\frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b}-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

```
[Out] -3/10*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))/b-1/10*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(3/2)/b-1/14*sin(2*b*x+2
*a)^(7/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4383, 2715, 2719}

$$-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x
]^(3/2))/(10*b) - Sin[2*a + 2*b*x]^(7/2)/(14*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4383

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Sin[
a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && N
eQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \sin^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = -\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{1}{2}\int \sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$= -\frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b}-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3}{10}\int \sqrt{\sin(2a+2$$
$$= \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b}-\frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b}-\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

Mathematica [A]
time = 0.24, size = 66, normalized size = 0.96

$$\frac{84E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sqrt{\sin(2(a+bx))}\,(-15\sin(2(a+bx))-14\sin(4(a+bx))+5\sin(6(a+bx)))}{280b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
[Out] (84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(-15*Sin[2*(a + b
*x)] - 14*Sin[4*(a + b*x)] + 5*Sin[6*(a + b*x)]))/(280*b)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 130.06, size = 306311267, normalized size = 4439293.72

method	result	size
default	Expression too large to display	306311267

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
[Out] integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(((cos(b*x + a)^2 - 1)*cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 + 1)*sqr
t(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(5/2)*sin(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2),x)
```

```
[Out] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2), x)
```

3.83 $\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=69

$$\frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b}-\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

```
[Out] -1/6*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b-1/10*sin(2*b*x+2*a)^(5/2)/b-1/6*cos(2*b*x+2*a)*sin(2*b*x+2*a
)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {4383, 2715, 2720}

$$-\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b*x
]])/(6*b) - Sin[2*a + 2*b*x]^(5/2)/(10*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4383

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Sin[
a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && N
eQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \sin^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = -\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b} + \frac{1}{2}\int \sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= -\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b} - \frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b} + \frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2}$$
$$= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b} - \frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b} - \frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

Mathematica [A]
time = 0.33, size = 76, normalized size = 1.10

$$\frac{20F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2(a+bx))} - 9\sin(2(a+bx)) - 10\sin(4(a+bx)) + 3\sin(6(a+bx))}{120b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] - 9*Sin[2*(a + b*x)
] - 10*Sin[4*(a + b*x)] + 3*Sin[6*(a + b*x)])/(120*b*Sqrt[Sin[2*(a + b*x)]]
)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 64.30, size = 183042750, normalized size = 2652793.48

method	result	size
default	Expression too large to display	183042750

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(2*b*x + 2*a)^(3/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2),x)
[Out] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2), x)
```

3.84 $\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=40

$$\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

```
[Out] -1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))/b-1/6*sin(2*b*x+2*a)^(3/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4383, 2719}

$$\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] EllipticE[a - Pi/4 + b*x, 2]/(2*b) - Sin[2*a + 2*b*x]^(3/2)/(6*b)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4383

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Sin[
a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && N
eQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx&=-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}+\frac{1}{2}\int \sqrt{\sin(2a+2bx)}\,dx\\&=\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 34, normalized size = 0.85

$$-\frac{-3E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sin^{\frac{3}{2}}(2(a+bx))}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/6*(-3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2))/b
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 3.29, size = 17203919, normalized size = 430097.98

method	result	size
default	Expression too large to display	17203919

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sqrt(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sin\left(a+b\,x\right)^2\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2), x)
```

3.85 $\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=40

$$\frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

```
[Out] -1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b-1/2*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4383, 2720}

$$\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(2*b) - Sqrt[Sin[2*a + 2*b*x]]/(2*b)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4383

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Sin[
a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && N
eQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= -\frac{\sqrt{\sin(2a+2bx)}}{2b}+\frac{1}{2}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\sqrt{\sin(2a+2bx)}}{2b}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 75, normalized size = 1.88

$$-\frac{2\sqrt{\sin(2(a+bx))}+\frac{\sqrt{2}\,F\left(\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\middle|\frac{1}{2}\right)(\cos(a+bx)+\sin(a+bx))}{\sqrt{1+\sin(2(a+bx))}}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/4*(2*Sqrt[Sin[2*(a + b*x)]] + (Sqrt[2]*EllipticF[ArcSin[Cos[a + b*x] - S
in[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(a + b*x)]
])/b
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 7.41, size = 58561095, normalized size = 1464027.38

method	result	size
default	Expression too large to display	58561095

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)/sqrt(sin(2*b*x + 2*a)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(a+b\,x\right)^2}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(1/2), x)
```

3.86 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=45

$$-\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}+\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*
x),2^(1/2))/b+sin(b*x+a)^2/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4381, 2719}

$$\frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -1/2*EllipticE[a - Pi/4 + b*x, 2]/b + Sin[a + b*x]^2/(b*Sqrt[Sin[2*a + 2*b*
x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4381

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Dist[e^2*((m + 2*p + 2)/(4*g^2*(p + 1))), Int[(e*Sin[a +
b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g},
 x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && Lt
Q[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}} - \frac{1}{2}\int \sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)}{2b} + \frac{\sin^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A]
time = 0.11, size = 41, normalized size = 0.91

$$\frac{-E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)+\sqrt{\sin(2(a+bx))}\,\tan(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*Tan[a + b*x])/(2*b)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 12.41, size = 96383272, normalized size = 2141850.49

method	result	size
default	Expression too large to display	96383272

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(a+b\,x\right)^2}{\sin\left(2\,a+2\,b\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(3/2),x)
```

```
[Out] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(3/2), x)
```

3.87 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=48

$$\frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b}+\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] -1/6*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b+1/3*sin(b*x+a)^2/b/sin(2*b*x+2*a)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4381, 2720}

$$\frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) + Sin[a + b*x]^2/(3*b*Sin[2*a + 2*b*x]^(
3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4381

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Dist[e^2*((m + 2*p + 2)/(4*g^2*(p + 1))), Int[(e*Sin[a +
b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g},
 x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && Lt
Q[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx$$
$$= \frac{F\left(a-\frac{\pi}{4}+bx\middle|\,2\right)}{6b} + \frac{\sin^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Mathematica [A]
time = 0.20, size = 83, normalized size = 1.73

$$\frac{\sec^2(a+bx)\sqrt{\sin(2(a+bx))} - \frac{\sqrt{2}\,F\left(\mathrm{ArcSin}(\cos(a+bx)-\sin(a+bx))\middle|\frac{1}{2}\right)(\cos(a+bx)+\sin(a+bx))}{\sqrt{1+\sin(2(a+bx))}}}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (Sec[a + b*x]^2*Sqrt[Sin[2*(a + b*x)]] - (Sqrt[2]*EllipticF[ArcSin[Cos[a +
b*x] - Sin[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(a
 + b*x)]])/(12*b)
```

Maple [A]
time = 53.14, size = 123, normalized size = 2.56

method	result
default	$\frac{\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,\mathrm{EllipticF}\left(\sqrt{\sin(2xb+2a)+1}\,,\right.}{12\sin(2xb+2a)^{\frac{3}{2}}\cos(2xb+2a)b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/sin(2*b*x+2*a)^(3/2)/cos(2*b*x+2*a)*((sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(
2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1
/2),1/2*2^(1/2))*sin(2*b*x+2*a)-2*cos(2*b*x+2*a)^2+2*cos(2*b*x+2*a))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.64, size = 92, normalized size = 1.92

$$-\frac{\sqrt{2i}\,\cos\left(bx+a\right)^{2}\,\text{ellipticF}(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right),-1)+\sqrt{-2i}\,\cos\left(bx+a\right)^{2}\,\text{ellipticF}(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right),-1)-\sqrt{2}\,\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}}{12\,b\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(sqrt(2*I)*cos(b*x + a)^2*ellipticF(cos(b*x + a) + I*sin(b*x + a), -1
) + sqrt(-2*I)*cos(b*x + a)^2*ellipticF(cos(b*x + a) - I*sin(b*x + a), -1)
- sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(a+b\,x\right)^2}{\sin\left(2\,a+2\,b\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(5/2),x)
```

```
[Out] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(5/2), x)
```

3.88 $\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=77

$$-\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b}+\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 3/10*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))/b+1/5*sin(b*x+a)^2/b/sin(2*b*x+2*a)^(5/2)-3/10*cos(2*b*x+2*a)/
b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4381, 2716, 2719}

$$\frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) + Sin[a + b*x]^2/(5*b*Sin[2*a + 2*
b*x]^(5/2)) - (3*Cos[2*a + 2*b*x])/(10*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4381

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Dist[e^2*((m + 2*p + 2)/(4*g^2*(p + 1))), Int[(e*Sin[a +
b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g},
 x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && Lt
Q[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2
```

```
*m, 2*p]
```

Rubi steps

$$\int \frac{\sin^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{3}{10}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}} - \frac{3}{10}\int \sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{3E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)}{10b} + \frac{\sin^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A]
time = 0.87, size = 66, normalized size = 0.86

$$-\frac{12E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)+\frac{4(1+6\cos(2(a+bx))+3\cos(4(a+bx)))\sin^2(a+bx)}{\sin^{\frac{5}{2}}(2(a+bx))}}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] -1/40*(12*EllipticE[a - Pi/4 + b*x, 2] + (4*(1 + 6*Cos[2*(a + b*x)] + 3*Cos
[4*(a + b*x)])*Sin[a + b*x]^2)/Sin[2*(a + b*x)]^(5/2))/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(92) = 184.
time = 247.57, size = 227, normalized size = 2.95

method	result
default	$\frac{\sqrt{2}\left(\frac{8\sqrt{2}}{5\sin(2xb+2a)^{\frac{5}{2}}}+\frac{4\sqrt{2}\left(6\sqrt{\sin(2xb+2a)+1}\sqrt{-2\sin(2xb+2a)+2}\sqrt{-\sin(2xb+2a)}\left(\sin^2(\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32*2^(1/2)*(8/5*2^(1/2)/sin(2*b*x+2*a)^(5/2)+4/5*2^(1/2)/sin(2*b*x+2*a)^(
5/2)*(6*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*
a))^(1/2)*sin(2*b*x+2*a)^2*EllipticE((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-
3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1
```

```
/2)*sin(2*b*x+2*a)^2*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+6*sin(
2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-2)/cos(2*b*x+2*a))/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
[Out] integrate(sin(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^2}{\sin\left(2\,a+2\,b\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(7/2),x)
[Out] int(sin(a + b*x)^2/sin(2*a + 2*b*x)^(7/2), x)
```

3.89 $\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$-\frac{7\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{64b}-\frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{64b}+\frac{7\sin(a+bx)}{}$$

```
[Out] -7/64*arcsin(cos(b*x+a)-sin(b*x+a))/b-7/64*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b-7/48*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-1/12*sin(b*x+a)*sin
(2*b*x+2*a)^(5/2)/b+7/32*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.08, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4383, 4387, 4386, 4391}

$$-\frac{7\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{64b}-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}+\frac{7\sin(a+bx)\sqrt{\sin(2a+2bx)}}{32b}-\frac{7\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{48b}-\frac{7\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{64b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(64*b) - (7*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(64*b) + (7*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(32*b) - (7*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(48*b) - (Sin[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(12*b)
```

Rule 4383

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Sin[
a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && N
eQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
```

```
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= -\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}+\frac{7}{12}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx\\ &= -\frac{7\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b}-\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b}+\frac{7}{16}\int \mathrm{c}\\ &= \frac{7\sin(a+bx)\sqrt{\sin(2a+2bx)}}{32b}-\frac{7\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b}-\frac{\sin(a\\ &= -\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{64b}-\frac{7\log\left(\cos(a+bx)+\sin(a+bx\right.}{64b}\end{aligned}$$

Mathematica [A]
time = 0.37, size = 98, normalized size = 0.72

$$\frac{-7\left(\mathrm{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+\frac{2}{3}\sqrt{\sin(2(a+bx))}\,(10\sin(a+bx)-9\sin(3(a+bx))+2\sin(5(a+bx)))}{64b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-7*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(10*Sin[a + b*x] - 9
*Sin[3*(a + b*x)] + 2*Sin[5*(a + b*x)]))/3)/(64*b)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 149.50, size = 370168303, normalized size = 2721825.76

method	result	size
default	Expression too large to display	370168303

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(118) = 236.
time = 3.07, size = 290, normalized size = 2.13

$$\frac{8 \sqrt{2} \left(32 \cos\left(bx+a\right)^{4} - 60 \cos\left(bx+a\right)^{2} + 21\right) \sqrt{\cos\left(bx+a\right) \sin\left(bx+a\right)} \sin\left(bx+a\right) + 42 \arctan\left(-\frac{\sqrt{2} \sqrt{\cos\left(bx+a\right) \sin\left(bx+a\right)} \left(\cos(bx+a)-\sin(bx+a)\right)+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^{2}+2\cos(bx+a)\sin(bx+a)-1}\right) - 42 \arctan\left(-\frac{2\sqrt{2} \sqrt{\cos\left(bx+a\right) \sin\left(bx+a\right)} - \cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right) + 21 \log\left(-32 \cos\left(bx+a\right)^{4} + 4\sqrt{2} \left(4 \cos\left(bx+a\right)^{3} - \left(4\cos\left(bx+a\right)^{2}+1\right) \sin\left(bx+a\right) - 5\cos\left(bx+a\right)\right) \sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)} + 32\cos\left(bx+a\right)^{2} + 16\cos\left(bx+a\right)\sin\left(bx+a\right) + 1\right)}{768\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/768*(8*sqrt(2)*(32*cos(b*x + a)^4 - 60*cos(b*x + a)^2 + 21)*sqrt(cos(b*x
+ a)*sin(b*x + a))*sin(b*x + a) + 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin
(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(
b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) - 42*arctan(-(2*sqrt(2)*sqrt
(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) -
sin(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 -
(4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin
(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
[Out] integrate(sin(2*b*x + 2*a)^(3/2)*sin(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)^3\,\sin\left(2\,a+2\,b\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(3/2),x)
[Out] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(3/2), x)
```

3.90 $\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=110

$$-\frac{5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{32b}+\frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{32b}-\frac{5\cos(a+bx)}{}$$

```
[Out] -5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b+5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b-1/8*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-5/16*cos(b*x+a)*sin(
2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4383, 4387, 4390}

$$-\frac{5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}-\frac{5\sqrt{\sin(2a+2bx)}\cos(a+bx)}{16b}+\frac{5\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) + (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) - (5*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) - (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(8*b)
```

Rule 4383

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Sin[
a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && N
eQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
```

```
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= -\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}+\frac{5}{8}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx\\ &= -\frac{5\cos(a+bx)\sqrt{\sin(2a+2bx)}}{16b}-\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}+\frac{5}{16}\\ &= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}+\frac{5\log\left(\cos(a+bx)+\sin(a+b\right.}{32b}\end{aligned}$$

Mathematica [**A**]
time = 0.24, size = 86, normalized size = 0.78

$$\frac{5\left(-\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+2(-6\cos(a+bx)+\cos(3(a+bx)))\sqrt{\sin(2(a+bx))}}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (5*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + 2*(-6*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[Si
n[2*(a + b*x)]])/(32*b)
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 17.74, size = 57690707, normalized size = 524460.97

method	result	size
default	Expression too large to display	57690707

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(2*b*x + 2*a))*sin(b*x + a)^3, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(96) = 192$.
time = 4.01, size = 281, normalized size = 2.55

$$\frac{8\sqrt{2}\left(4\cos\left(bx+a\right)^{3}-9\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\left(\cos(bx+a)-\sin(bx+a)\right)+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^{2}+2\cos(bx+a)\sin(bx+a)-1}\right)-10\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)-5\log\left(-32\cos\left(bx+a\right)^{4}+4\sqrt{2}\left(4\cos\left(bx+a\right)^{3}-\left(4\cos\left(bx+a\right)^{2}+1\right)\sin\left(bx+a\right)-5\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\cos\left(bx+a\right)^{2}+16\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/128*(8*sqrt(2)*(4*cos(b*x + a)^3 - 9*cos(b*x + a))*sqrt(cos(b*x + a)*sin(
b*x + a)) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x +
a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
 + a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x
+ a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 5*log
(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*
sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x
 + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)^3\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(1/2),x)
[Out] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^(1/2), x)
```

3.91 $\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=84

$$-\frac{3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{8b}-\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{8b}-\sin(a+bx)\sqrt{}$$

```
[Out] -3/8*arcsin(cos(b*x+a)-sin(b*x+a))/b-3/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b-1/4*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4383, 4391}

$$-\frac{3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{8b}-\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b}-\frac{3\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) - (3*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) - (Sin[a + b*x]*Sqrt[Sin[2*a + 2*
b*x]])/(4*b)
```

Rule 4383

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Sin[
a + b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}
, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && N
eQ[m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{4b} + \frac{3}{4}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$$
$$= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b} - \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(\right.}{8b}$$

Mathematica [**A**]
time = 0.16, size = 74, normalized size = 0.88

$$-\frac{3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)+2\sin(a+bx)\sqrt{\sin(2(a+bx))}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/8*(3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a +
b*x] + Sqrt[Sin[2*(a + b*x)]]] + 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 49.99, size = 174944974, normalized size = 2082678.26

method	result	size
default	Expression too large to display	174944974

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(74) = 148.
time = 3.53, size = 268, normalized size = 3.19

$$-\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a) - 6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right) + 6\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right) - 3\log\left(-32\cos(bx+a)^4+4\sqrt{2}\left(4\cos(bx+a)^3-\left(4\cos(bx+a)^2+1\right)\sin(bx+a)-5\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}+32\cos(bx+a)^2+16\cos(bx+a)\sin(bx+a)+1\right)}{32b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/32*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) - 6*arctan(-(
sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos
(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1))
 + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - si
n(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4 + 4*s
qrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x
+ a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)
*sin(b*x + a) + 1))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^3}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(1/2), x)
```

3.92 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=81

$$\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b}+\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+
2*a)^(1/2))/b+sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4379, 4393, 4390}

$$\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{4b}+\frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(4*b) - Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) + Sin[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4379

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-e^2)*(e*Sin[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p
+ 1)/(2*b*g*(p + 1))), x] + Dist[e^4*((m + p - 1)/(4*g^2*(p + 1))), Int[(e*
Sin[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d
, e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m,
2] && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
```

```
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = \frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}} - \frac{1}{4}\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{\sin(a+bx)}{b\sqrt{\sin(2a+2bx)}} - \frac{1}{2}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$$
$$= \frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b} - \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b}$$

Mathematica [A]
time = 0.11, size = 72, normalized size = 0.89

$$\frac{\operatorname{ArcSin}(\cos(a+bx)-\sin(a+bx)) - \log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right) + 2\sec(a+bx)\sqrt{\sin(2(a+bx))}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (ArcSin[Cos[a + b*x] - Sin[a + b*x]] - Log[Cos[a + b*x] + Sin[a + b*x] + Sq
rt[Sin[2*(a + b*x)]]] + 2*Sec[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 38.64, size = 172831627, normalized size = 2133723.79

method	result	size
default	Expression too large to display	172831627

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(73) = 146$.
time = 2.42, size = 296, normalized size = 3.65

$$-\frac{2 \arctan\left(-\frac{\sqrt{2}\,\sqrt{\cos(bx+a)\sin(bx+a)}\,(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\,\cos(bx+a)\sin(bx+a)-1}\right)\cos(bx+a)-2\arctan\left(-\frac{2\sqrt{2}\,\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)\cos(bx+a)-\cos(bx+a)\log\left(-32\,\cos(bx+a)^4+4\sqrt{2}\,(4\,\cos(bx+a)^3-(4\,\cos(bx+a)^2+1)\sin(bx+a)-5\,\cos(bx+a))\sqrt{\cos(bx+a)\sin(bx+a)}+32\,\cos(bx+a)^2+16\,\cos(bx+a)\sin(bx+a)+1\right)-8\sqrt{2}\,\sqrt{\cos(bx+a)\sin(bx+a)}-8\,\cos(bx+a)}{16\,b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/16*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - s
in(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*
sin(b*x + a) - 1))*cos(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a)))*c
os(b*x + a) - cos(b*x + a)*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x +
a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x +
 a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1) -
 8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - 8*cos(b*x + a))/(b*cos(b*x + a
))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^3}{\sin(2\,a+2\,b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(3/2),x)
[Out] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(3/2), x)
```

3.93 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=28

$$\frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] 1/3*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{4377\}$

$$\frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] Sin[a + b*x]^3/(3*b*Sin[2*a + 2*b*x]^(3/2))
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = \frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Mathematica [A]
time = 0.06, size = 27, normalized size = 0.96

$$\frac{\sin^3(a+bx)}{3b\sin^{\frac{3}{2}}(2(a+bx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] Sin[a + b*x]^3/(3*b*Sin[2*(a + b*x)]^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 170.08, size = 727, normalized size = 25.96

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\,\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)\left(6\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x
*b)^2-1)*(6*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-
tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2
))*tan(1/2*a+1/2*x*b)^6-3*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*
b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1
/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^6+18*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*t
an(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a
+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^4-9*(tan(1/2*a+1/2*x*b)+
1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*Ellipt
icF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^4+6*tan(1/
2*a+1/2*x*b)^8+18*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1
/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*
2^(1/2))*tan(1/2*a+1/2*x*b)^2-9*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+
1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)
+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2-2*tan(1/2*a+1/2*x*b)^6+6*(tan(1
/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b)
)^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))-3*(tan(1/2*a+1/
2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)
*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))+10*tan(1/2*a+1/2*x*b)^
4-14*tan(1/2*a+1/2*x*b)^2)/(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1
/2)/(1+tan(1/2*a+1/2*x*b)^2)^3/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1
/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)

Fricas [A]
time = 4.32, size = 48, normalized size = 1.71

$$-\frac{\cos(bx+a)^2-\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)}{12\,b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(cos(b*x + a)^2 - sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a
))/(b*cos(b*x + a)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 15648 vs. 2(24) = 48.
time = 88.46, size = 15648, normalized size = 558.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)^
4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2*b
*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(1/2
*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1/2*b
*x) + tan(1/2*a))*((((2*(4*(2*(sqrt(2)*tan(1/2*a)^54 + 17*sqrt(2)*tan(1/2*a
)^52 + 132*sqrt(2)*tan(1/2*a)^50 + 612*sqrt(2)*tan(1/2*a)^48 + 1842*sqrt(2)
*tan(1/2*a)^46 + 3570*sqrt(2)*tan(1/2*a)^44 + 3668*sqrt(2)*tan(1/2*a)^42 -
1292*sqrt(2)*tan(1/2*a)^40 - 11457*sqrt(2)*tan(1/2*a)^38 - 19057*sqrt(2)*ta
n(1/2*a)^36 - 12920*sqrt(2)*tan(1/2*a)^34 + 7752*sqrt(2)*tan(1/2*a)^32 + 27
132*sqrt(2)*tan(1/2*a)^30 + 27132*sqrt(2)*tan(1/2*a)^28 + 7752*sqrt(2)*tan(
1/2*a)^26 - 12920*sqrt(2)*tan(1/2*a)^24 - 19057*sqrt(2)*tan(1/2*a)^22 - 114
57*sqrt(2)*tan(1/2*a)^20 - 1292*sqrt(2)*tan(1/2*a)^18 + 3668*sqrt(2)*tan(1/
2*a)^16 + 3570*sqrt(2)*tan(1/2*a)^14 + 1842*sqrt(2)*tan(1/2*a)^12 + 612*sqr
```

t(2)*tan(1/2*a)^10 + 132*sqrt(2)*tan(1/2*a)^8 + 17*sqrt(2)*tan(1/2*a)^6 + s
qrt(2)*tan(1/2*a)^4)*tan(1/2*b*x)/(tan(1/2*a)^51 + 23*tan(1/2*a)^49 + 252*t
an(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/2*a)^43 + 31878*tan(1/2*a)^4
1 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37 + 389367*tan(1/2*a)^35 + 572
033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 534888*tan(1/2*a)^29 + 208012*ta
n(1/2*a)^27 - 208012*tan(1/2*a)^25 - 534888*tan(1/2*a)^23 - 653752*tan(1/2*
a)^21 - 572033*tan(1/2*a)^19 - 389367*tan(1/2*a)^17 - 211508*tan(1/2*a)^15
- 92092*tan(1/2*a)^13 - 31878*tan(1/2*a)^11 - 8602*tan(1/2*a)^9 - 1748*tan(
1/2*a)^7 - 252*tan(1/2*a)^5 - 23*tan(1/2*a)^3 - tan(1/2*a)) + 3*(sqrt(2)*ta
n(1/2*a)^55 + 12*sqrt(2)*tan(1/2*a)^53 + 43*sqrt(2)*tan(1/2*a)^51 - 120*sqr
t(2)*tan(1/2*a)^49 - 1818*sqrt(2)*tan(1/2*a)^47 - 8688*sqrt(2)*tan(1/2*a)^4
5 - 24598*sqrt(2)*tan(1/2*a)^43 - 44328*sqrt(2)*tan(1/2*a)^41 - 44365*sqrt(
2)*tan(1/2*a)^39 + 4028*sqrt(2)*tan(1/2*a)^37 + 93993*sqrt(2)*tan(1/2*a)^35
 + 160208*sqrt(2)*tan(1/2*a)^33 + 127908*sqrt(2)*tan(1/2*a)^31 - 127908*sqr
t(2)*tan(1/2*a)^27 - 160208*sqrt(2)*tan(1/2*a)^25 - 93993*sqrt(2)*tan(1/2*a
)^23 - 4028*sqrt(2)*tan(1/2*a)^21 + 44365*sqrt(2)*tan(1/2*a)^19 + 44328*sqr
t(2)*tan(1/2*a)^17 + 24598*sqrt(2)*tan(1/2*a)^15 + 8688*sqrt(2)*tan(1/2*a)^
13 + 1818*sqrt(2)*tan(1/2*a)^11 + 120*sqrt(2)*tan(1/2*a)^9 - 43*sqrt(2)*tan
(1/2*a)^7 - 12*sqrt(2)*tan(1/2*a)^5 - sqrt(2)*tan(1/2*a)^3)/(tan(1/2*a)^51
+ 23*tan(1/2*a)^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/2*
a)^43 + 31878*tan(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37 +
389367*tan(1/2*a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 534888
*tan(1/2*a)^29 + 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 534888*tan(1
/2*a)^23 - 653752*tan(1/2*a)^21 - 572033*tan(1/2*a)^19 - 389367*tan(1/2*a)^
17 - 211508*tan(1/2*a)^15 - 92092*tan(1/2*a)^13 - 31878*tan(1/2*a)^11 - 860
2*tan(1/2*a)^9 - 1748*tan(1/2*a)^7 - 252*tan(1/2*a)^5 - 23*tan(1/2*a)^3 - t
an(1/2*a)))*tan(1/2*b*x) + 3*(sqrt(2)*tan(1/2*a)^56 - 17*sqrt(2)*tan(1/2*a)
^54 - 429*sqrt(2)*tan(1/2*a)^52 - 3555*sqrt(2)*tan(1/2*a)^50 - 16130*sqrt(2
)*tan(1/2*a)^48 - 43550*sqrt(2)*tan(1/2*a)^46 - 59310*sqrt(2)*tan(1/2*a)^44
 + 32910*sqrt(2)*tan(1/2*a)^42 + 348955*sqrt(2)*tan(1/2*a)^40 + 818805*sqrt
(2)*tan(1/2*a)^38 + 1026665*sqrt(2)*tan(1/2*a)^36 + 479655*sqrt(2)*tan(1/2*
a)^34 - 755820*sqrt(2)*tan(1/2*a)^32 - 1828180*sqrt(2)*tan(1/2*a)^30 - 1828
180*sqrt(2)*tan(1/2*a)^28 - 755820*sqrt(2)*tan(1/2*a)^26 + 479655*sqrt(2)*t
an(1/2*a)^24 + 1026665*sqrt(2)*tan(1/2*a)^22 + 818805*sqrt(2)*tan(1/2*a)^20
 + 348955*sqrt(2)*tan(1/2*a)^18 + 32910*sqrt(2)*tan(1/2*a)^16 - 59310*sqrt(
2)*tan(1/2*a)^14 - 43550*sqrt(2)*tan(1/2*a)^12 - 16130*sqrt(2)*tan(1/2*a)^1
0 - 3555*sqrt(2)*tan(1/2*a)^8 - 429*sqrt(2)*tan(1/2*a)^6 - 17*sqrt(2)*tan(1
/2*a)^4 + sqrt(2)*tan(1/2*a)^2)/(tan(1/2*a)^51 + 23*tan(1/2*a)^49 + 252*tan
(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/2*a)^43 + 31878*tan(1/2*a)^41
+ 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37 + 389367*tan(1/2*a)^35 + 57203
3*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 534888*tan(1/2*a)^29 + 208012*tan(
1/2*a)^27 - 208012*tan(1/2*a)^25 - 534888*tan(1/2*a)^23 - 653752*tan(1/2*a)
^21 - 572033*tan(1/2*a)^19 - 389367*tan(1/2*a)^17 - 211508*tan(1/2*a)^15 -
92092*tan(1/2*a)^13 - 31878*tan(1/2*a)^11 - 8602*tan(1/2*a)^9 - 1748*tan(1/
2*a)^7 - 252*tan(1/2*a)^5 - 23*tan(1/2*a)^3 - tan(1/2*a)))*tan(1/2*b*x) - (

```
sqrt(2)*tan(1/2*a)^57 + 98*sqrt(2)*tan(1/2*a)^55 + 1092*sqrt(2)*tan(1/2*a)^
53 + 3814*sqrt(2)*tan(1/2*a)^51 - 10975*sqrt(2)*tan(1/2*a)^49 - 165060*sqrt
(2)*tan(1/2*a)^47 - 803600*sqrt(2)*tan(1/2*a)^45 - 2367580*sqrt(2)*tan(1/2*
a)^43 - 4642995*sqrt(2)*tan(1/2*a)^41 - 5889450*sqrt(2)*tan(1/2*a)^39 - 361
9500*sqrt(2)*tan(1/2*a)^37 + 2314770*sqrt(2)*tan(1/2*a)^35 + 7653485*sqrt(2
)*tan(1/2*a)^33 + 6963880*sqrt(2)*tan(1/2*a)^31 - 6963880*sqrt(2)*tan(1/2*a
)^27 - 7653485*sqrt(2)*tan(1/2*a)^25 - 2314770*sqrt(2)*tan(1/2*a)^23 + 3619
500*sqrt(2)*tan(1/2*a)^21 + 5889450*sqrt(2)*tan(1/2*a)^19 + 4642995*sqrt(2)
*tan(1/2*a)^17 + 2367580*sqrt(2)*tan(1/2*a)^15 ...
```

Mupad [B]
time = 2.19, size = 85, normalized size = 3.04

$$-\frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,\left(2\,\sin\left(a+b\,x\right)+3\,\sin\left(3\,a+3\,b\,x\right)+\sin\left(5\,a+5\,b\,x\right)\right)}{6\,b\,\left(30\,\sin\left(a+b\,x\right)^2+12\,\sin\left(2\,a+2\,b\,x\right)^2+2\,\sin\left(3\,a+3\,b\,x\right)^2-32\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(5/2),x)
```

```
[Out] -(sin(2*a + 2*b*x)^(1/2)*(2*sin(a + b*x) + 3*sin(3*a + 3*b*x) + sin(5*a + 5
*b*x)))/(6*b*(12*sin(2*a + 2*b*x)^2 + 2*sin(3*a + 3*b*x)^2 + 30*sin(a + b*x
)^2 - 32))
```

3.94 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/5*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(5/2)+1/5*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/
2)
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4381, 4377}

$$\frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] Sin[a + b*x]^3/(5*b*Sin[2*a + 2*b*x]^(5/2)) + Sin[a + b*x]/(5*b*Sqrt[Sin[2*
a + 2*b*x]])
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4381

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Dist[e^2*((m + 2*p + 2)/(4*g^2*(p + 1))), Int[(e*Sin[a +
b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g},
 x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && Lt
Q[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2
*m, 2*p]
```

Rubi steps

$$\int \frac{\sin^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = \frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{1}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= \frac{\sin^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{\sin(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A]
time = 0.11, size = 35, normalized size = 0.64

$$\frac{\sec(a+bx)\left(4+\sec^2(a+bx)\right)\sqrt{\sin(2(a+bx))}}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
[Out] (Sec[a + b*x]*(4 + Sec[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/(40*b)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(xb+a)}{\sin(2xb+2a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x)
[Out] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [A]
time = 5.52, size = 55, normalized size = 1.00

$$\frac{4\cos(bx+a)^3+\sqrt{2}\left(4\cos(bx+a)^2+1\right)\sqrt{\cos(bx+a)\sin(bx+a)}}{40\,b\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/40*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 + 1)*sqrt(cos(b*x + a)*s
in(b*x + a)))/(b*cos(b*x + a)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.26, size = 88, normalized size = 1.60

$$\frac{e^{a\,1i+b\,x\,1i}\,\sqrt{\frac{e^{-a\,2i-b\,x\,2i}\,1i}{2}-\frac{e^{a\,2i+b\,x\,2i}\,1i}{2}}\,\left(3\,e^{a\,2i+b\,x\,2i}+e^{a\,4i+b\,x\,4i}+1\right)}{5\,b\,\left(e^{a\,2i+b\,x\,2i}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(7/2),x)
```

```
[Out] (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/
2)^(1/2)*(3*exp(a*2i + b*x*2i) + exp(a*4i + b*x*4i) + 1))/(5*b*(exp(a*2i +
b*x*2i) + 1)^3)
```

3.95 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=81

$$\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{2\sin(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/7*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(7/2)+2/21*sin(b*x+a)/b/sin(2*b*x+2*a)^(3
/2)-4/21*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4381, 4389, 4376}

$$\frac{2\sin(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] Sin[a + b*x]^3/(7*b*Sin[2*a + 2*b*x]^(7/2)) + (2*Sin[a + b*x])/(21*b*Sin[2*
a + 2*b*x]^(3/2)) - (4*Cos[a + b*x])/(21*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4376

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4381

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Dist[e^2*((m + 2*p + 2)/(4*g^2*(p + 1))), Int[(e*Sin[a +
b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g},
 x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && Lt
Q[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2
*m, 2*p]
```

Rule 4389

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + D
```

```
ist[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x],
 x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !I
ntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx &= \frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{2}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= \frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{2\sin(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{4}{21}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= \frac{\sin^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{2\sin(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{21b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 55, normalized size = 0.68

$$-\frac{(5+12\cos(2(a+bx))+4\cos(4(a+bx)))\csc(a+bx)\sec^4(a+bx)\sqrt{\sin(2(a+bx))}}{336b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
[Out] -1/336*((5 + 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]*Sec[a +
 b*x]^4*Sqrt[Sin[2*(a + b*x)]])/b
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(xb+a)}{\sin(2xb+2a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
[Out] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(9/2), x)
```

Fricas [**A**]
time = 3.57, size = 79, normalized size = 0.98

$$-\frac{32\cos\left(bx+a\right)^4\sin\left(bx+a\right)+\sqrt{2}\,\left(32\cos\left(bx+a\right)^4-8\cos\left(bx+a\right)^2-3\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}}{336\,b\cos\left(bx+a\right)^4\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/336*(32*cos(b*x + a)^4*sin(b*x + a) + sqrt(2)*(32*cos(b*x + a)^4 - 8*cos
(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^4*sin(b*x
 + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.71, size = 300, normalized size = 3.70

$$-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,5\mathrm{i}}{84\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}^2}+\frac{3\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{14\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}^3}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,1\mathrm{i}}{7\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}^4}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{5}{84\,b}+\frac{4\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}}{21\,b}\right)\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(9/2),x)
```

```
[Out] (3*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2))/(14*b*(exp(a*2i + b*x*2i)*1i + 1i)^3) - (exp(a*1i + b*x*1i)*((e
```

```
xp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*5i)/(84*b*(exp
(a*2i + b*x*2i)*1i + 1i)^2) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i
)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*1i)/(7*b*(exp(a*2i + b*x*2i)*1i + 1i
)^4) + (exp(a*1i + b*x*1i)*(5/(84*b) + (4*exp(a*2i + b*x*2i))/(21*b))*((exp
(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*
x*2i) - 1)*(exp(a*2i + b*x*2i)*1i + 1i))
```

3.96 $\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=107

$$\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/9*sin(b*x+a)^3/b/sin(2*b*x+2*a)^(9/2)+1/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(5
/2)-4/45*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+8/45*sin(b*x+a)/b/sin(2*b*x+2*a)
^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4381, 4389, 4388, 4377}

$$\frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{45b\sqrt{\sin(2a+2bx)}}-\frac{4\cos(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

```
[Out] Sin[a + b*x]^3/(9*b*Sin[2*a + 2*b*x]^(9/2)) + Sin[a + b*x]/(15*b*Sin[2*a +
2*b*x]^(5/2)) - (4*Cos[a + b*x])/(45*b*Sin[2*a + 2*b*x]^(3/2)) + (8*Sin[a +
 b*x])/(45*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4381

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(-(e*Sin[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(2*b*
g*(p + 1))), x] + Dist[e^2*((m + 2*p + 2)/(4*g^2*(p + 1))), Int[(e*Sin[a +
b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g},
 x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && Lt
Q[p, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2
*m, 2*p]
```

Rule 4388

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4389

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + D
ist[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x],
 x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !I
ntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx &= \frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{1}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx\\ &= \frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{15}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= \frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8}{45}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2b\ldots}\\ &= \frac{\sin^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{\sin(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{4\cos(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{45b\sqrt{\sin(2a+2bx\ldots}}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 62, normalized size = 0.58

$$\frac{(-15\cot(a+bx)\csc(a+bx)+113\sec(a+bx)+17\sec^3(a+bx)+5\sec^5(a+bx))\sqrt{\sin(2(a+bx))}}{1440b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

```
[Out] ((-15*Cot[a + b*x]*Csc[a + b*x] + 113*Sec[a + b*x] + 17*Sec[a + b*x]^3 + 5*
Sec[a + b*x]^5)*Sqrt[Sin[2*(a + b*x)]])/(1440*b)
```

Maple [F(-1)]
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

[Out] int(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)^3/sin(2*b*x + 2*a)^(11/2), x)
```

Fricas [A]
time = 3.34, size = 98, normalized size = 0.92

$$\frac{128\cos(bx+a)^7-128\cos(bx+a)^5+\sqrt{2}\left(128\cos(bx+a)^6-96\cos(bx+a)^4-12\cos(bx+a)^2-5\right)\sqrt{\cos(bx+a)\sin(bx+a)}}{1440\left(b\cos(bx+a)^7-b\cos(bx+a)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/1440*(128*cos(b*x + a)^7 - 128*cos(b*x + a)^5 + sqrt(2)*(128*cos(b*x + a)
^6 - 96*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x +
 a)))/(b*cos(b*x + a)^7 - b*cos(b*x + a)^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3/sin(2*b*x+2*a)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.16, size = 383, normalized size = 3.58

$$-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,1\mathrm{i}}{60\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)^3}-\frac{2\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{9\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)^4}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,1\mathrm{i}}{9\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)^5}+\frac{\mathrm{e}^{a\,3\mathrm{i}+b\,x\,3\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,8\mathrm{i}}{45\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{49}{180\,b}-\frac{19\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}}{180\,b}\right)\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/sin(2*a + 2*b*x)^(11/2),x)
```

```
[Out] (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/
2)^(1/2)*1i)/(9*b*(exp(a*2i + b*x*2i)*1i + 1i)^5) - (2*exp(a*1i + b*x*1i)*(
(exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(9*b*(exp(a
*2i + b*x*2i)*1i + 1i)^4) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/
2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*1i)/(60*b*(exp(a*2i + b*x*2i)*1i + 1i)
^3) + (exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i
)*1i)/2)^(1/2)*8i)/(45*b*(exp(a*2i + b*x*2i) - 1)*(exp(a*2i + b*x*2i)*1i +
1i)) - (exp(a*1i + b*x*1i)*(49/(180*b) - (19*exp(a*2i + b*x*2i))/(180*b))*(
(exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i
+ b*x*2i) - 1)^2*(exp(a*2i + b*x*2i)*1i + 1i)^2)
```

3.97 $\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$-\frac{5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{16b}-\frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{16b}+\frac{5\sin(a+bx}{}$$

```
[Out] -5/16*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/16*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b-5/12*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+1/3*sin(b*x+a)*sin(
2*b*x+2*a)^(5/2)/b+5/8*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4393, 4386, 4387, 4391}

$$-\frac{5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{16b}+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b}-\frac{5\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{12b}-\frac{5\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(16*b) - (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(16*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(8*b) - (5*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(12*b) + (Sin[a
+ b*x]*Sin[2*a + 2*b*x]^(5/2))/(3*b)
```

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
```

```
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = 2\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$= \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b} + \frac{5}{3}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= -\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{12b} + \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b} + \frac{5}{4}\int \cos(a$$
$$= \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{8b} - \frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{12b} + \frac{\sin(a+$$
$$= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b} - \frac{5\log\Big(\cos(a+bx)+\sin(a+bx)-}{16b}$$

Mathematica [A]
time = 0.34, size = 98, normalized size = 0.72

$$\frac{-5\left(\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+\frac{2}{3}\sqrt{\sin(2(a+bx))}\,(14\sin(a+bx)-3\sin(3(a+bx))-2\sin(5(a+bx)))}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(14*Sin[a + b*x] - 3
*Sin[3*(a + b*x)] - 2*Sin[5*(a + b*x)]))/3)/(16*b)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 9.66, size = 973, normalized size = 7.15

method	result	size
default	Expression too large to display	973

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)

[Out] -16/5/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(6*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^4-3*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^4+6*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^6-12*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2+6*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2-12*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4+6*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))-3*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))-4*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4+6*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^2-4*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^2)/(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)+1)/((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)/(tan(1/2*a+1/2*x*b)-1)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")

[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(7/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(118) = 236.

time = 4.23, size = 290, normalized size = 2.13

$$-\frac{8\sqrt{2}\left(32\cos\left(bx+a\right)^4-12\cos\left(bx+a\right)^2-15\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\sin\left(bx+a\right)-30\arctan\left(-\frac{\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\left(\cos(bx+a)-\sin(bx+a)\right)+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)+30\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)-15\log\left(-32\cos\left(bx+a\right)^4+4\sqrt{2}\left(4\cos\left(bx+a\right)^3-\left(4\cos\left(bx+a\right)^2+1\right)\sin\left(bx+a\right)-5\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\cos\left(bx+a\right)^2+16\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/192*(8*sqrt(2)*(32*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 15)*sqrt(cos(b*x
 + a)*sin(b*x + a))*sin(b*x + a) - 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos
(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 30*arctan(-(2*sqrt(2)*sqr
t(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) -
 sin(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 -
 (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*si
n(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{7/2}}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x),x)
```

```
[Out] int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x), x)
```

3.98 $\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=110

$$-\frac{3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{8b}+\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{8b}-\frac{3\cos(a+bx)}{}$$

```
[Out] -3/8*arcsin(cos(b*x+a)-sin(b*x+a))/b+3/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b+1/2*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-3/4*cos(b*x+a)*sin(2*b
*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4393, 4386, 4387, 4390}

$$-\frac{3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{8b}+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{2b}-\frac{3\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}+\frac{3\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) + (3*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) - (3*Cos[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(4*b) + (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(2*b)
```

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
```

```
a*d, 0] && EqQ[d/b, 2]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = 2\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{2b} + \frac{3}{2}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{3\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b} + \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{2b} + \frac{3}{4}\int \frac{}{\sqrt{}}$$
$$= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b} + \frac{3\log\left(\cos(a+bx)+\sin(a+bx)-\right.}{8b}$$

Mathematica [A]
time = 0.21, size = 86, normalized size = 0.78

$$\frac{3\left(-\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)-2(2\cos(a+bx)+\cos(3(a+bx)))\sqrt{\sin(2(a+bx))}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (3*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) - 2*(2*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[Sin
[2*(a + b*x)]])/(8*b)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 6.51, size = 243, normalized size = 2.21

method	result

default	$-\frac{8\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\,\left(\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{t\right.\right.}{3b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8/3/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*((tan(1/2*a+1/2
*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*
EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2-(t
an(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*
x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))+2*tan(1/2*a
+1/2*x*b)^3+2*tan(1/2*a+1/2*x*b))/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))
^(1/2)/(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(96) = 192$.
time = 3.90, size = 281, normalized size = 2.55

$$-\frac{8\sqrt{2}\left(4\cos\left(bx+a\right)^3-\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\left(\cos(bx+a)-\sin(bx+a)\right)+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)+6\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)+3\log\left(-32\cos\left(bx+a\right)^4+4\sqrt{2}\left(4\cos\left(bx+a\right)^3-\left(4\cos\left(bx+a\right)^2+1\right)\sin\left(bx+a\right)-5\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\cos\left(bx+a\right)^2+16\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/32*(8*sqrt(2)*(4*cos(b*x + a)^3 - cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*
x + a)) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a)
- sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x +
a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)
) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 3*log(-32
*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(
b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a
)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{5/2}}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x),x)
```

```
[Out] int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x), x)
```

3.99 $\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=81

$$-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{2b}-\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{2b}+\frac{\sin(a+bx)\sqrt{\text{s}}}{b}$$

```
[Out] -1/2*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b+sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4393, 4386, 4391}

$$-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{2b}+\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b) + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/b
```

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = 2\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$$
$$= \frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b} + \int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$$
$$= -\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b} - \frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\right.}}{2b}$$

Mathematica [A]
time = 0.09, size = 70, normalized size = 0.86

$$-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-2\sin(a+bx)\sqrt{\sin(2(a+bx))}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -1/2*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]] - 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 4.28, size = 362, normalized size = 4.47

method	result
default	$\frac{4\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)\left(2\sqrt{\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\sqrt{\tan\left(\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x*b
)^2-1)*(2*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b)
)^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(
1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))-(
(tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(ta
n(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x
*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))+2*(tan(1/2*a
+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^2)/(tan(1/2*a+1/2*
```

```
x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*
b))^(1/2)/((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b)
)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(73) = 146.
time = 3.64, size = 266, normalized size = 3.28

$$\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\,\sin(bx+a)+2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\,(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)+\log\left(-32\cos(bx+a)^4+4\sqrt{2}\,(4\cos(bx+a)^3-(4\cos(bx+a)^2+1)\sin(bx+a)-5\cos(bx+a))\sqrt{\cos(bx+a)\sin(bx+a)}+32\cos(bx+a)^2+16\cos(bx+a)\sin(bx+a)+1\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + 2*arctan(-(sq
rt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b
*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) -
 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(
b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 + 4*sqrt(
2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a)
)*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin
(b*x + a) + 1))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{3/2}}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x),x)
```

```
[Out] int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x), x)
```

3.100 $\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=53

$$-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{b}$$

```
[Out] -arcsin(cos(b*x+a)-sin(b*x+a))/b+ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*a)^(1
/2))/b
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4393, 4390}

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{b}-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b) + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*a + 2*b*x]]]/b
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \csc(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= 2\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\right.}{b}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 52, normalized size = 0.98

$$-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -(ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b) + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]/b
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 2.82, size = 157, normalized size = 2.96

method	result
default	$\frac{2\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\ \text{El}}{b\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{a}{2}+\frac{xb}{2}\right)-\tan\left(\frac{a}{2}+\frac{xb}{2}\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x*b
)^2-1)/(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x
*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)/(t
an(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)
+1)^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(51) = 102.
time = 2.40, size = 242, normalized size = 4.57

$$\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)-\log\left(-32\cos(bx+a)^4+4\sqrt{2}\left(4\cos(bx+a)^3-\left(4\cos(bx+a)^2+1\right)\sin(bx+a)-5\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}+32\cos(bx+a)^2+16\cos(bx+a)\sin(bx+a)+1\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin
(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*si
n(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - c
os(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - log(-32*cos(b*
x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a
) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 1
6*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x),x)
```

```
[Out] int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x), x)
```

3.101 $\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=24

$$-\frac{\csc(a+bx)\sqrt{\sin(2a+2bx)}}{b}$$

```
[Out] -csc(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{4377\}$

$$-\frac{\sqrt{\sin(2a+2bx)}\,\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -((Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/b)
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\csc(a+bx)\sqrt{\sin(2a+2bx)}}{b}$$

Mathematica [A]
time = 0.05, size = 23, normalized size = 0.96

$$-\frac{\csc(a+bx)\sqrt{\sin(2(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

[Out] -((Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b)

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 3.86, size = 308, normalized size = 12.83

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\left(2\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\mathrm{EllipticE}\left(\sqrt{\tan\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(2*(tan(1/2*a+1/2*
x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*E
llipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*a+1/2*x*b)*(tan
(1/2*a+1/2*x*b)^2-1))^(1/2)-(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*
x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^
(1/2),1/2*2^(1/2))*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)+(tan
(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^2-(tan(1/2*a
+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2))/tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*
b)^3-tan(1/2*a+1/2*x*b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [A]
time = 2.56, size = 39, normalized size = 1.62

$$-\frac{\sqrt{2}\,\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+\sin\left(bx+a\right)}{b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + sin(b*x + a))/(b*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [B]
time = 0.31, size = 24, normalized size = 1.00

$$-\frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}{b\,\sin\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(1/2)),x)
```

```
[Out] -sin(2*a + 2*b*x)^(1/2)/(b*sin(a + b*x))
```

3.102 $\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=53

$$-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -2/3*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+4/3*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2
)
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4393, 4388, 4377}

$$\frac{4\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(3*b*Sin[2*a + 2*b*x]^(3/2)) + (4*Sin[a + b*x])/(3*b*Sqrt
[Sin[2*a + 2*b*x]])
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4388

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx &= 2\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\
&= -\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{4}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\
&= -\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 43, normalized size = 0.81

$$\frac{\left(-\frac{1}{6}\cot(a+bx)\csc(a+bx)+\frac{1}{2}\sec(a+bx)\right)\sqrt{\sin(2(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
[Out] ((-1/6*(Cot[a + b*x]*Csc[a + b*x]) + Sec[a + b*x]/2)*Sqrt[Sin[2*(a + b*x)]]
)/b
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 7.46, size = 194, normalized size = 3.66

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)\left(2\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\right.}{12b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)}\sqrt{\tan^3(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/12/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2
*x*b)^2-1)/tan(1/2*a+1/2*x*b)*(2*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a
+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b
)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)-tan(1/2*a+1/2*x*b)^4+1)/(tan(1/2
*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a
+1/2*x*b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [A]
time = 3.76, size = 74, normalized size = 1.40

$$\frac{4 \cos\left(bx+a\right)^{3}+\sqrt{2}\,\left(4 \cos\left(bx+a\right)^{2}-3\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-4 \cos\left(bx+a\right)}{6\left(b\cos\left(bx+a\right)^{3}-b\cos\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*si
n(b*x + a)) - 4*cos(b*x + a))/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [B]
time = 2.96, size = 103, normalized size = 1.94

$$\frac{4\,{\mathrm{e}}^{a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,\sqrt{\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}}\,\left(1+{\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}-{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\right)}{3\,b\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}-1\right)}^2\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(3/2)),x)
```

```
[Out] (4*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2)*(exp(a*4i + b*x*4i) - exp(a*2i + b*x*2i) + 1))/(3*b*(exp(a*2i +
b*x*2i) - 1)^2*(exp(a*2i + b*x*2i) + 1))
```

3.103 $\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=79

$$-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -2/5*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/2)+8/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/
2)-16/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4393, 4388, 4389, 4376}

$$\frac{8\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(5*b*Sin[2*a + 2*b*x]^(5/2)) + (8*Sin[a + b*x])/(15*b*Sin
[2*a + 2*b*x]^(3/2)) - (16*Cos[a + b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4376

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4388

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4389

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + D
ist[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x],
 x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !I
```

```
ntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= 2\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx\\ &= -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{16}{15}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{8\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 52, normalized size = 0.66

$$-\frac{\sqrt{\sin(2(a+bx))}\,(27\csc(a+bx)+3\csc^3(a+bx)-5\sec(a+bx)\tan(a+bx))}{60b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -1/60*(Sqrt[Sin[2*(a + b*x)]]*(27*Csc[a + b*x] + 3*Csc[a + b*x]^3 - 5*Sec[a
 + b*x]*Tan[a + b*x]))/b
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 31.79, size = 481, normalized size = 6.09

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\left(24\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\mathrm{EllipticE}\left(\sqrt{\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/80/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/tan(1/2*a+1/2*
x*b)^3*(24*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-t
an(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2)
)*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^2-
12*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a
+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*(tan(1
/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^2+(tan(1/2
*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^6-(tan(1/2*a
+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^4+12*(tan(1/2*
a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4-(tan(1/2*a+1/2*
x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*tan(1/2*a+1/2*x*b)^2-12*(tan(1/2*a+1/2
*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^2+(tan(1/2*a+1/2*x*b)*
(tan(1/2*a+1/2*x*b)^2-1))^(1/2))/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^
(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [A]
time = 3.41, size = 103, normalized size = 1.30

$$-\frac{\sqrt{2}\left(32\cos\left(bx+a\right)^4-40\cos\left(bx+a\right)^2+5\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\left(\cos\left(bx+a\right)^4-\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)}{60\left(b\cos\left(bx+a\right)^4-b\cos\left(bx+a\right)^2\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/60*(sqrt(2)*(32*cos(b*x + a)^4 - 40*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a
)*sin(b*x + a)) + 32*(cos(b*x + a)^4 - cos(b*x + a)^2)*sin(b*x + a))/((b*co
s(b*x + a)^4 - b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [B]
time = 3.36, size = 136, normalized size = 1.72

$$\frac{8\,{\mathrm{e}}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{{\mathrm{e}}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,2\mathrm{i}+{\mathrm{e}}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,3\mathrm{i}+{\mathrm{e}}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,2\mathrm{i}-{\mathrm{e}}^{a\,8\mathrm{i}+b\,x\,8\mathrm{i}}\,2\mathrm{i}-2\mathrm{i}\right)}{15\,b\,{\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3\,{\left({\mathrm{e}}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(5/2)),x)
```

```
[Out] (8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2)*(exp(a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*x*
6i)*2i - exp(a*8i + b*x*8i)*2i - 2i))/(15*b*(exp(a*2i + b*x*2i) - 1)^3*(exp
(a*2i + b*x*2i) + 1)^2)
```

3.104 $\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=105

$$-\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -2/7*cos(b*x+a)/b/sin(2*b*x+2*a)^(7/2)+12/35*sin(b*x+a)/b/sin(2*b*x+2*a)^(5
/2)-16/35*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+32/35*sin(b*x+a)/b/sin(2*b*x+2*
a)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4393, 4388, 4389, 4377}

$$\frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}-\frac{16\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-2*Cos[a + b*x])/(7*b*Sin[2*a + 2*b*x]^(7/2)) + (12*Sin[a + b*x])/(35*b*Si
n[2*a + 2*b*x]^(5/2)) - (16*Cos[a + b*x])/(35*b*Sin[2*a + 2*b*x]^(3/2)) + (
32*Sin[a + b*x])/(35*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4388

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4389

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + D
```

```
ist[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x],
 x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !I
ntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx &= 2\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx\\ &= -\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{12}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx\\ &= -\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{48}{35}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32}{35}\int \frac{\sin(a+}{\sin^{\frac{3}{2}}(2a+}\\ &= -\frac{2\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{12\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{16\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32\sin(a+bx}{35b\sqrt{\sin(2a+2}}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 67, normalized size = 0.64

$$\frac{(5-10\cos(2(a+bx))-4\cos(4(a+bx))+4\cos(6(a+bx)))\csc^4(a+bx)\sec^3(a+bx)\sqrt{\sin(2(a+bx))}}{280b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] ((5 - 10*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[a
+ b*x]^4*Sec[a + b*x]^3*Sqrt[Sin[2*(a + b*x)]])/(280*b)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 143.65, size = 222, normalized size = 2.11

$$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\,\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)\left(3\left(\tan^8\left(\frac{a}{2}+\frac{xb}{2}\right)\right)+40\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\right.\right.}{1344b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x)

[Out] 1/1344/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x*b)^2-1)/tan(1/2*a+1/2*x*b)^3*(3*tan(1/2*a+1/2*x*b)^8+40*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^3-26*tan(1/2*a+1/2*x*b)^6+26*tan(1/2*a+1/2*x*b)^2-3)/(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")

[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)

Fricas [A]
time = 2.73, size = 118, normalized size = 1.12

$$\frac{128\cos(bx+a)^7-256\cos(bx+a)^5+128\cos(bx+a)^3+\sqrt{2}\left(128\cos(bx+a)^6-224\cos(bx+a)^4+84\cos(bx+a)^2+7\right)\sqrt{\cos(bx+a)\sin(bx+a)}}{280\left(b\cos(bx+a)^7-2b\cos(bx+a)^5+b\cos(bx+a)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")

[Out] 1/280*(128*cos(b*x + a)^7 - 256*cos(b*x + a)^5 + 128*cos(b*x + a)^3 + sqrt(2)*(128*cos(b*x + a)^6 - 224*cos(b*x + a)^4 + 84*cos(b*x + a)^2 + 7)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x + a)^3)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)**(7/2),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

Mupad [B]
time = 4.13, size = 350, normalized size = 3.33

$$-\frac{2\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{7\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^4}+\frac{\mathrm{e}^{a\,3\mathrm{i}+b\,x\,3\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,32\mathrm{i}}{35\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{2}{7\,b}-\frac{16\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}}{35\,b}\right)\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{32\mathrm{i}}{35\,b}+\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,88\mathrm{i}}{35\,b}\right)\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^3\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)*sin(2*a + 2*b*x)^(7/2)),x)
```

```
[Out] (exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/
2)^(1/2)*32i)/(35*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i))
- (2*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*
1i)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i - 1i)^4) - (exp(a*1i + b*x*1i)*(2
/(7*b) - (16*exp(a*2i + b*x*2i))/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (ex
p(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*x*2i) + 1)^2*(exp(a*2i + b*x*
2i)*1i - 1i)^2) + (exp(a*1i + b*x*1i)*(32i/(35*b) + (exp(a*2i + b*x*2i)*88i
)/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/
((exp(a*2i + b*x*2i) + 1)^3*(exp(a*2i + b*x*2i)*1i - 1i)^3)
```

3.105 $\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=106

$$\frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5b}-\frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{5b}-\frac{2\cos(2a+2bx)\sin^{\frac{7}{2}}(2a+2bx)}{7b}+\frac{\csc^2(a+bx)\sin^{\frac{11}{2}}(2a$$

```
[Out] -6/5*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))/b-2/5*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(3/2)/b-2/7*cos(2*b*x+2*a)
*sin(2*b*x+2*a)^(7/2)/b+1/7*csc(b*x+a)^2*sin(2*b*x+2*a)^(11/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4385, 2715, 2719}

$$\frac{6E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{2\sin^{\frac{7}{2}}(2a+2bx)\cos(2a+2bx)}{7b}-\frac{2\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{5b}+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^2(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (6*EllipticE[a - Pi/4 + b*x, 2])/(5*b) - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*
x]^(3/2))/(5*b) - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(7/2))/(7*b) + (Csc[
a + b*x]^2*Sin[2*a + 2*b*x]^(11/2))/(7*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx = \frac{\csc^2(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{7b} + \frac{18}{7}\int \sin^{\frac{9}{2}}(2a+2bx)\,dx$$
$$= -\frac{2\cos(2a+2bx)\sin^{\frac{7}{2}}(2a+2bx)}{7b} + \frac{\csc^2(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{7b} + 2\int$$
$$= -\frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{5b} - \frac{2\cos(2a+2bx)\sin^{\frac{7}{2}}(2a+2bx)}{7b} + \frac{c}{}$$
$$= \frac{6E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)}{5b} - \frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{5b} - \frac{2\cos(2a+2bx}{}$$

Mathematica [A]
time = 0.33, size = 66, normalized size = 0.62

$$\frac{84E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)+\sqrt{\sin(2(a+bx))}\,(15\sin(2(a+bx))-14\sin(4(a+bx))-5\sin(6(a+bx)))}{70b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(15*Sin[2*(a + b*
x)] - 14*Sin[4*(a + b*x)] - 5*Sin[6*(a + b*x)]))/(70*b)
```

Maple [A]
time = 20.42, size = 204, normalized size = 1.92

method	result
default	$\frac{8\sqrt{2}\left(\frac{\sqrt{2}\left(\sin^{\frac{7}{2}}(2xb+2a)\right)}{56}-\frac{\sqrt{2}\left(6\sqrt{\sin(2xb+2a)+1}\sqrt{-2\sin(2xb+2a)+2}\sqrt{-\sin(2xb+2a)}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 8*2^(1/2)*(1/56*2^(1/2)*sin(2*b*x+2*a)^(7/2)-1/80*2^(1/2)*(6*(sin(2*b*x+2*a
)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticE((s
in(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*
x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1/2),
1/2*2^(1/2))-2*sin(2*b*x+2*a)^4+2*sin(2*b*x+2*a)^2)/cos(2*b*x+2*a)/sin(2*b*
x+2*a)^(1/2))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(9/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
[Out] integral((cos(2*b*x + 2*a)^4 - 2*cos(2*b*x + 2*a)^2 + 1)*csc(b*x + a)^2*sqr
t(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(9/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{9/2}}{\sin\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^2,x)
[Out] int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^2, x)
```

3.106 $\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=106

$$\frac{2F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{3b}-\frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{3b}-\frac{2\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{5b}+\frac{\csc^2(a+bx)\sin^{\frac{9}{2}}(\ldots}{5b}$$

```
[Out] -2/3*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b-2/5*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(5/2)/b+1/5*csc(b*x+a)^2*s
in(2*b*x+2*a)^(9/2)/b-2/3*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]

time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4385, 2715, 2720}

$$\frac{2F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3b}-\frac{2\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{5b}-\frac{2\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{3b}+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^2(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (2*EllipticF[a - Pi/4 + b*x, 2])/(3*b) - (2*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a +
 2*b*x]])/(3*b) - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(5/2))/(5*b) + (Csc[
a + b*x]^2*Sin[2*a + 2*b*x]^(9/2))/(5*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx = \frac{\csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{5b} + \frac{14}{5}\int \sin^{\frac{7}{2}}(2a+2bx)\,dx$$
$$= -\frac{2\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{5b} + \frac{\csc^2(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{5b} + 2\int \mathrm{s}$$
$$= -\frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{3b} - \frac{2\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{5b} +$$
$$= \frac{2F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{3b} - \frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{3b} - \frac{2\cos(2a+2bx}{}$$

Mathematica [A]
time = 0.19, size = 76, normalized size = 0.72

$$\frac{20F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2(a+bx))}+9\sin(2(a+bx))-10\sin(4(a+bx))-3\sin(6(a+bx))}{30b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
[Out] (20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 9*Sin[2*(a + b*x)
] - 10*Sin[4*(a + b*x)] - 3*Sin[6*(a + b*x)])/(30*b*Sqrt[Sin[2*(a + b*x)]])
```

Maple [A]
time = 14.15, size = 139, normalized size = 1.31

method	result
default	$\frac{4\sqrt{2}\left(\frac{\sqrt{2}\left(\sin^{\frac{5}{2}}(2xb+2a)\right)}{20}+\frac{\sqrt{2}\left(\sqrt{\sin(2xb+2a)+1}\sqrt{-2\sin(2xb+2a)+2}\sqrt{-\sin(2xb+2a)}\right.}{24\cos(2xb+2a)\sqrt{\sin(2xb}}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)
[Out] 4*2^(1/2)*(1/20*2^(1/2)*sin(2*b*x+2*a)^(5/2)+1/24*2^(1/2)*((sin(2*b*x+2*a)+
1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin
(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+2*sin(2*b*x+2*a)^3-2*sin(2*b*x+2*a))/cos(
2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
[Out] integral(-(cos(2*b*x + 2*a)^2 - 1)*csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x
)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{7/2}}{\sin\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^2,x)
[Out] int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^2, x)
```

3.107 $\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=75

$$\frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b}-\frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}+\frac{\csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{3b}$$

```
[Out] -2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x
),2^(1/2))/b-2/3*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(3/2)/b+1/3*csc(b*x+a)^2*sin
(2*b*x+2*a)^(7/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.136$, Rules used = {4385, 2715, 2719}

$$\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{2\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{3b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (2*EllipticE[a - Pi/4 + b*x, 2])/b - (2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(
3/2))/(3*b) + (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2))/(3*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{\csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{3b} + \frac{10}{3}\int \sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$= -\frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b} + \frac{\csc^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{3b} + 2\int$$
$$= \frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b} - \frac{2\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b} + \frac{\csc^2(a+bx)\,\text{si}}{3b}$$

Mathematica [A]
time = 0.09, size = 34, normalized size = 0.45

$$\frac{2\left(3E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sin^{\frac{3}{2}}(2(a+bx))\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (2*(3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2)))/(3*b)
```

Maple [A]
time = 10.84, size = 137, normalized size = 1.83

method	result
default	$\frac{2\sqrt{2}\left(\frac{\sqrt{2}\left(\sin^{\frac{3}{2}}(2xb+2a)\right)}{6}-\frac{\sqrt{2}\sqrt{\sin(2xb+2a)+1}\sqrt{-2\sin(2xb+2a)+2}\sqrt{-\sin(2xb+2a)}}{4\cos(2xb+2a)\sqrt{\text{S}}}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*2^(1/2)*(1/6*2^(1/2)*sin(2*b*x+2*a)^(3/2)-1/4*2^(1/2)*(sin(2*b*x+2*a)+1)^
(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*(2*EllipticE((sin
(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^
(1/2)))/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(2*b*x + 2*a)^2 - 1)*csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{5/2}}{\sin\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^2,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^2, x)
```

3.108 $\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=70

$$\frac{2F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b}-\frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{b}+\frac{\csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b}$$

```
[Out] -2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b*x
),2^(1/2))/b+csc(b*x+a)^2*sin(2*b*x+2*a)^(5/2)/b-2*cos(2*b*x+2*a)*sin(2*b*x
+2*a)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4385, 2715, 2720}

$$\frac{2F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{2\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (2*EllipticF[a - Pi/4 + b*x, 2])/b - (2*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b
*x]])/b + (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2))/b
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{\csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b} + 6\int \sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= -\frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{b} + \frac{\csc^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b} + 2\int$$
$$= \frac{2F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b} - \frac{2\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{b} + \frac{\csc^2(a+bx)\,\mathrm{si}}{b}$$

Mathematica [A]
time = 0.93, size = 73, normalized size = 1.04

$$\frac{2\sqrt{\sin(2(a+bx))} - \frac{\sqrt{2}\,F\left(\mathrm{ArcSin}(\cos(a+bx)-\sin(a+bx))\middle|\frac{1}{2}\right)(\cos(a+bx)+\sin(a+bx))}{\sqrt{1+\sin(2(a+bx))}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (2*Sqrt[Sin[2*(a + b*x)]] - (Sqrt[2]*EllipticF[ArcSin[Cos[a + b*x] - Sin[a
+ b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(a + b*x)]])/b
```

Maple [A]
time = 7.87, size = 111, normalized size = 1.59

method	result
default	$\frac{\sqrt{2}\left(\sqrt{2}\left(\sqrt{\sin}\,(2xb+2a)\right)+\frac{\sqrt{2}\,\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}}{2\cos(2xb+2a)\sqrt{\sin(2xb+2a)}}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1/2)*(2^(1/2)*sin(2*b*x+2*a)^(1/2)+1/2*2^(1/2)*(sin(2*b*x+2*a)+1)^(1/2)*
(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*
a)+1)^(1/2),1/2*2^(1/2))/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{3/2}}{\sin\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^2,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^2, x)
```

3.109 $\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=44

$$-\frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b}-\frac{\csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}$$

```
[Out] 2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*x)
,2^(1/2))/b-csc(b*x+a)^2*sin(2*b*x+2*a)^(3/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4385, 2719}

$$-\frac{2E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^2(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-2*EllipticE[a - Pi/4 + b*x, 2])/b - (Csc[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2
))/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \csc^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= -\frac{\csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}-2\int \sqrt{\sin(2a+2bx)}\,dx\\ &= -\frac{2E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{b}-\frac{\csc^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 37, normalized size = 0.84

$$-\frac{2\left(E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\cot(a+bx)\sqrt{\sin(2(a+bx))}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-2*(EllipticE[a - Pi/4 + b*x, 2] + Cot[a + b*x]*Sqrt[Sin[2*(a + b*x)]]))/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(67) = 134.
time = 10.64, size = 176, normalized size = 4.00

method	result
default	$\frac{2\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,\text{EllipticE}\left(\sqrt{\sin(2xb+2a)+1}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/cos(2*b*x+2*a)/sin(2*b*x+2*a)^(1/2)*(2*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2
*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticE((sin(2*b*x+2*a)+1)^(1/
2),1/2*2^(1/2))-(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(
2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-2*cos(2*b
*x+2*a)^2-2*cos(2*b*x+2*a))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}{{\sin\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x)^2,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x)^2, x)
```

3.110 $\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}} \, dx$

Optimal. Leaf size=48

$$\frac{2F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{3b}-\frac{\csc^2(a+bx)\sqrt{\sin(2a+2bx)}}{3b}$$

```
[Out] -2/3*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b-1/3*csc(b*x+a)^2*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4385, 2720}

$$\frac{2F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{3b}-\frac{\sqrt{\sin(2a+2bx)}\csc^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (2*EllipticF[a - Pi/4 + b*x, 2])/(3*b) - (Csc[a + b*x]^2*Sqrt[Sin[2*a + 2*b
*x]])/(3*b)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= -\frac{\csc^2(a+bx)\sqrt{\sin(2a+2bx)}}{3b}+\frac{2}{3}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{2F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{3b}-\frac{\csc^2(a+bx)\sqrt{\sin(2a+2bx)}}{3b}\end{aligned}$$

Mathematica [A]
time = 1.03, size = 82, normalized size = 1.71

$$-\frac{\csc^2(a+bx)\sqrt{\sin(2(a+bx))}+\frac{\sqrt{2}F\left(\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\middle|\frac{1}{2}\right)(\cos(a+bx)+\sin(a+bx))}{\sqrt{1+\sin(2(a+bx))}}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
[Out] -1/3*(Csc[a + b*x]^2*Sqrt[Sin[2*(a + b*x)]] + (Sqrt[2]*EllipticF[ArcSin[Cos
[a + b*x] - Sin[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin
[2*(a + b*x)]])/b
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(xb+a)}{\sqrt{\sin(2xb+2a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x)
[Out] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 1.22, size = 101, normalized size = 2.10

$$-\frac{\sqrt{2i}\left(\cos(bx+a)^2-1\right)\text{ellipticF}(\cos(bx+a)+i\sin(bx+a),-1)+\sqrt{-2i}\left(\cos(bx+a)^2-1\right)\text{ellipticF}(\cos(bx+a)-i\sin(bx+a),-1)-\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}}{3\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
[Out] -1/3*(sqrt(2*I)*(cos(b*x + a)^2 - 1)*ellipticF(cos(b*x + a) + I*sin(b*x + a
), -1) + sqrt(-2*I)*(cos(b*x + a)^2 - 1)*ellipticF(cos(b*x + a) - I*sin(b*x
```

```
 + a), -1) - sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^2 - b
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(a+b\,x\right)^2\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(1/2)), x)
```

3.111 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=77

$$-\frac{6E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{5b}-\frac{6\cos(2a+2bx)}{5b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 6/5*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*
x),2^(1/2))/b-6/5*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(1/2)-1/5*csc(b*x+a)^2/b/
sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4385, 2716, 2719}

$$-\frac{6E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{5b}-\frac{6\cos(2a+2bx)}{5b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-6*EllipticE[a - Pi/4 + b*x, 2])/(5*b) - (6*Cos[2*a + 2*b*x])/(5*b*Sqrt[Si
n[2*a + 2*b*x]]) - Csc[a + b*x]^2/(5*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx &= -\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}+\frac{6}{5}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{6\cos(2a+2bx)}{5b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}-\frac{6}{5}\int \sqrt{\sin(2a+2bx)}\,dx\\ &= -\frac{6E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)}{5b}-\frac{6\cos(2a+2bx)}{5b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{5b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [**A**]
time = 0.61, size = 64, normalized size = 0.83

$$\frac{-12E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)+\frac{2(1-6\cos(2(a+bx))+3\cos(4(a+bx)))\cot(a+bx)}{\sin^{\frac{3}{2}}(2(a+bx))}}{10b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-12*EllipticE[a - Pi/4 + b*x, 2] + (2*(1 - 6*Cos[2*(a + b*x)] + 3*Cos[4*(a
 + b*x)])*Cot[a + b*x])/Sin[2*(a + b*x)]^(3/2))/(10*b)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(92) = 184$.
time = 42.56, size = 227, normalized size = 2.95

method	result
default	$\sqrt{2}\left(-\frac{8\sqrt{2}}{5\sin(2xb+2a)^{\frac{5}{2}}}+\frac{4\sqrt{2}\left(6\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,(\sin\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*2^(1/2)*(-8/5*2^(1/2)/sin(2*b*x+2*a)^(5/2)+4/5*2^(1/2)/sin(2*b*x+2*a)^(
5/2)*(6*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*
a))^(1/2)*sin(2*b*x+2*a)^2*EllipticE((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))-
3*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1
/2)*sin(2*b*x+2*a)^2*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))+6*sin(
2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-2)/cos(2*b*x+2*a))/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2)),x)
[Out] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(3/2)), x)
```

3.112 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=77

$$\frac{10F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{21b}-\frac{10\cos(2a+2bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] -10/21*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi
+b*x),2^(1/2))/b-10/21*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(3/2)-1/7*csc(b*x+a)
^2/b/sin(2*b*x+2*a)^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4385, 2716, 2720}

$$\frac{10F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{21b}-\frac{10\cos(2a+2bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (10*EllipticF[a - Pi/4 + b*x, 2])/(21*b) - (10*Cos[2*a + 2*b*x])/(21*b*Sin[
2*a + 2*b*x]^(3/2)) - Csc[a + b*x]^2/(7*b*Sin[2*a + 2*b*x]^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= -\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{10}{7}\int \frac{1}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{10\cos(2a+2bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{10}{21}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{10F\left(a-\frac{\pi}{4}+bx\middle|\,2\right)}{21b}-\frac{10\cos(2a+2bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{7b\sin^{\frac{3}{2}}(2a+2bx)}\end{aligned}$$

Mathematica [A]
time = 0.47, size = 66, normalized size = 0.86

$$\frac{40F\left(a-\frac{\pi}{4}+bx\middle|\,2\right)+\left(-13\csc^2(a+bx)-3\csc^4(a+bx)+7\sec^2(a+bx)\right)\sqrt{\sin(2(a+bx))}}{84b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (40*EllipticF[a - Pi/4 + b*x, 2] + (-13*Csc[a + b*x]^2 - 3*Csc[a + b*x]^4 +
 7*Sec[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/(84*b)
```

Maple [A]
time = 224.97, size = 154, normalized size = 2.00

method	result
default	$\frac{\sqrt{2}\left(-\frac{16\sqrt{2}}{7\sin(2xb+2a)^{\frac{7}{2}}}+\frac{8\sqrt{2}\left(5\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,\text{Ellipti}\right.}{21\sin(2xb+2a)^{\frac{7}{2}}\cos(}\right.}{16b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*2^(1/2)*(-16/7*2^(1/2)/sin(2*b*x+2*a)^(7/2)+8/21*2^(1/2)/sin(2*b*x+2*a
)^(7/2)*(5*(sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x
+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1/2),1/2*2^(1/2))*sin(2*b*x+2*a)
^3+10*sin(2*b*x+2*a)^4-4*sin(2*b*x+2*a)^2-6)/cos(2*b*x+2*a))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.55, size = 177, normalized size = 2.30

$$-\frac{20\sqrt{2i}\left(\cos\left(bx+a\right)^{6}-2\cos\left(bx+a\right)^{4}+\cos\left(bx+a\right)^{2}\right)\mathrm{ellipticF}\left(\cos\left(bx+a\right)+i\sin\left(bx+a\right),-1\right)+20\sqrt{-2i}\left(\cos\left(bx+a\right)^{6}-2\cos\left(bx+a\right)^{4}+\cos\left(bx+a\right)^{2}\right)\mathrm{ellipticF}\left(\cos\left(bx+a\right)-i\sin\left(bx+a\right),-1\right)-\sqrt{2}\left(20\cos\left(bx+a\right)^{4}-30\cos\left(bx+a\right)^{2}+7\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}}{84\left(b\cos\left(bx+a\right)^{6}-2b\cos\left(bx+a\right)^{4}+b\cos\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/84*(20*sqrt(2*I)*(cos(b*x + a)^6 - 2*cos(b*x + a)^4 + cos(b*x + a)^2)*el
lipticF(cos(b*x + a) + I*sin(b*x + a), -1) + 20*sqrt(-2*I)*(cos(b*x + a)^6
- 2*cos(b*x + a)^4 + cos(b*x + a)^2)*ellipticF(cos(b*x + a) - I*sin(b*x + a
), -1) - sqrt(2)*(20*cos(b*x + a)^4 - 30*cos(b*x + a)^2 + 7)*sqrt(cos(b*x +
 a)*sin(b*x + a)))/(b*cos(b*x + a)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^
2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(5/2)), x)
```

3.113 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=106

$$-\frac{14E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{15b}-\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{14\cos(2a+2bx)}{15b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 14/15*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))/b-14/45*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(5/2)-1/9*csc(b*x+a)^
2/b/sin(2*b*x+2*a)^(5/2)-14/15*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4385, 2716, 2719}

$$-\frac{14E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{15b}-\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{14\cos(2a+2bx)}{15b\sqrt{\sin(2a+2bx)}}-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-14*EllipticE[a - Pi/4 + b*x, 2])/(15*b) - (14*Cos[2*a + 2*b*x])/(45*b*Sin
[2*a + 2*b*x]^(5/2)) - Csc[a + b*x]^2/(9*b*Sin[2*a + 2*b*x]^(5/2)) - (14*Co
s[2*a + 2*b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
```

```
2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx &= -\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{14}{9}\int \frac{1}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx\\ &= -\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{14}{15}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{14\cos(2a+2bx)}{15b\sqrt{\sin(2a+2bx)}}-\frac{14}{15}\int \sqrt{\sin(2a\\ &= -\frac{14E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{15b}-\frac{14\cos(2a+2bx)}{45b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{9b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{14\cos(2a+2}{15b\sqrt{\sin(2a+}\end{aligned}$$

Mathematica [A]
time = 0.85, size = 85, normalized size = 0.80

$$-\frac{336E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\frac{(-9+98\cos(2(a+bx))-28\cos(4(a+bx))-42\cos(6(a+bx))+21\cos(8(a+bx)))\csc^2(a+bx)}{\sin^{\frac{5}{2}}(2(a+bx))}}{360b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
[Out] -1/360*(336*EllipticE[a - Pi/4 + b*x, 2] + ((-9 + 98*Cos[2*(a + b*x)] - 28*
Cos[4*(a + b*x)] - 42*Cos[6*(a + b*x)] + 21*Cos[8*(a + b*x)])*Csc[a + b*x]^
2)/Sin[2*(a + b*x)]^(5/2))/b
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(xb+a)}{\sin(2xb+2a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
[Out] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(7/2)), x)
```

3.114 $\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=106

$$\frac{30F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{77b}-\frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{30\cos(2a+2bx)}{77b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] -30/77*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi
+b*x),2^(1/2))/b-18/77*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(7/2)-1/11*csc(b*x+a
)^2/b/sin(2*b*x+2*a)^(7/2)-30/77*cos(2*b*x+2*a)/b/sin(2*b*x+2*a)^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4385, 2716, 2720}

$$\frac{30F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{77b}-\frac{30\cos(2a+2bx)}{77b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (30*EllipticF[a - Pi/4 + b*x, 2])/(77*b) - (18*Cos[2*a + 2*b*x])/(77*b*Sin[
2*a + 2*b*x]^(7/2)) - Csc[a + b*x]^2/(11*b*Sin[2*a + 2*b*x]^(7/2)) - (30*Co
s[2*a + 2*b*x])/(77*b*Sin[2*a + 2*b*x]^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
```

```
2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx &= -\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{18}{11}\int \frac{1}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx\\ &= -\frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{90}{77}\int \frac{1}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{30\cos(2a+2bx)}{77b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{30}{77}\int \frac{1}{\sqrt{\sin(2a-}}\\ &= \frac{30F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{77b}-\frac{18\cos(2a+2bx)}{77b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{\csc^2(a+bx)}{11b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{30\cos(2a+2bx}{77b\sin^{\frac{3}{2}}(2a+2b}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 86, normalized size = 0.81

$$\frac{480F\left(a-\frac{\pi}{4}+bx\middle|2\right)+\left(-141\csc^2(a+bx)-32\csc^4(a+bx)-7\csc^6(a+bx)+11\sec^2(a+bx)\left(9+\sec^2(a+bx)\right)\right)\sqrt{\sin(2(a+bx))}}{1232b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^2/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (480*EllipticF[a - Pi/4 + b*x, 2] + (-141*Csc[a + b*x]^2 - 32*Csc[a + b*x]^
4 - 7*Csc[a + b*x]^6 + 11*Sec[a + b*x]^2*(9 + Sec[a + b*x]^2))*Sqrt[Sin[2*(
a + b*x)]])/(1232*b)
```

Maple [F(-1)]
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] int(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(9/2), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.94, size = 233, normalized size = 2.20

$$-\frac{240\sqrt{2i}\left(\cos\left(bx+a\right)^{10}-3\cos\left(bx+a\right)^{8}+3\cos\left(bx+a\right)^{6}-\cos\left(bx+a\right)^{4}\right)\mathrm{ellipticF}\left(\cos\left(bx+a\right)+i\sin\left(bx+a\right),-1\right)+240\sqrt{-2i}\left(\cos\left(bx+a\right)^{10}-3\cos\left(bx+a\right)^{8}+3\cos\left(bx+a\right)^{6}-\cos\left(bx+a\right)^{4}\right)\mathrm{ellipticF}\left(\cos\left(bx+a\right)-i\sin\left(bx+a\right),-1\right)-\sqrt{2}\left(240\cos\left(bx+a\right)^{8}-600\cos\left(bx+a\right)^{6}+444\cos\left(bx+a\right)^{4}-66\cos\left(bx+a\right)^{2}-11\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}}{1232\left(b\cos\left(bx+a\right)^{10}-3b\cos\left(bx+a\right)^{8}+3b\cos\left(bx+a\right)^{6}-b\cos\left(bx+a\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/1232*(240*sqrt(2*I)*(cos(b*x + a)^10 - 3*cos(b*x + a)^8 + 3*cos(b*x + a)
^6 - cos(b*x + a)^4)*ellipticF(cos(b*x + a) + I*sin(b*x + a), -1) + 240*sqr
t(-2*I)*(cos(b*x + a)^10 - 3*cos(b*x + a)^8 + 3*cos(b*x + a)^6 - cos(b*x +
a)^4)*ellipticF(cos(b*x + a) - I*sin(b*x + a), -1) - sqrt(2)*(240*cos(b*x +
 a)^8 - 600*cos(b*x + a)^6 + 444*cos(b*x + a)^4 - 66*cos(b*x + a)^2 - 11)*s
qrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^10 - 3*b*cos(b*x + a)^8 + 3
*b*cos(b*x + a)^6 - b*cos(b*x + a)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2/sin(2*b*x + 2*a)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\sin\left(a+b\,x\right)}^2\,{\sin\left(2\,a+2\,b\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(9/2)),x)
```

```
[Out] int(1/(sin(a + b*x)^2*sin(2*a + 2*b*x)^(9/2)), x)
```

3.115 $\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=190

$$-\frac{7\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{8b}+\frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{8b}-\frac{7\cos(a+bx)}{}$$

```
[Out] -7/8*arcsin(cos(b*x+a)-sin(b*x+a))/b+7/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b+7/6*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-14/15*cos(b*x+a)*sin(2
*b*x+2*a)^(5/2)/b+4/5*sin(b*x+a)*sin(2*b*x+2*a)^(7/2)/b+1/5*csc(b*x+a)^3*si
n(2*b*x+2*a)^(11/2)/b-7/4*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.14, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4385, 4393, 4386, 4387, 4390}

$$-\frac{7\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{8b}+\frac{4\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{5b}+\frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{6b}-\frac{14\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{15b}-\frac{7\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}+\frac{\sin^{\frac{11}{2}}(2a+2bx)\csc^3(a+bx)}{5b}+\frac{7\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) + (7*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) - (7*Cos[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(4*b) + (7*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(6*b) - (14*Cos[a
+ b*x]*Sin[2*a + 2*b*x]^(5/2))/(15*b) + (4*Sin[a + b*x]*Sin[2*a + 2*b*x]^(7
/2))/(5*b) + (Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(11/2))/(5*b)
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx &= \frac{\csc^3(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{5b}+\frac{16}{5}\int \csc(a+bx)\sin^{\frac{9}{2}}(2a+2bx)\,dx\\
&= \frac{\csc^3(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{5b}+\frac{32}{5}\int \cos(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx\\
&= \frac{4\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{5b}+\frac{\csc^3(a+bx)\sin^{\frac{11}{2}}(2a+2bx)}{5b}+\frac{28}{5}\int \mathrm{si}\\
&= -\frac{14\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{15b}+\frac{4\sin(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{5b}+\frac{\csc^3(}{}\\
&= \frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{6b}-\frac{14\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{15b}+\frac{4\sin(a}{}\\
&= -\frac{7\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b}+\frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{6b}-\frac{14\,\mathrm{c}}{}\\
&= -\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b}+\frac{7\log\left(\cos(a+bx)+\sin(a+bx\right.}{8b}
\end{aligned}$$

Mathematica [A]
time = 0.37, size = 100, normalized size = 0.53

$$\frac{7\left(-\mathrm{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)-\frac{2}{3}(10\cos(a+bx)+9\cos(3(a+bx))+2\cos(5(a+bx)))\sqrt{\sin(2(a+bx))}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] (7*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) - (2*(10*Cos[a + b*x] + 9*Cos[3*(a + b*x)] + 2*C
os[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/3)/(8*b)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 286.11, size = 441, normalized size = 2.32

method	result
default	$-\frac{64\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\,\left(\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\mathrm{ta}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -64/21*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*((tan(1/2*a+1/2
*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*
EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^6-3*
(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/
2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a
+1/2*x*b)^4+2*tan(1/2*a+1/2*x*b)^7+3*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1
/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2
*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2+10*tan(1/2*a+1/2*x*b)^5-(t
an(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*
x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))+10*tan(1/2*
a+1/2*x*b)^3+2*tan(1/2*a+1/2*x*b))/(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^
2-1))^(1/2)/(tan(1/2*a+1/2*x*b)+1)^2/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*
b))^(1/2)/(tan(1/2*a+1/2*x*b)-1)^2/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(9/2), x)
```

Fricas [A]
time = 2.63, size = 291, normalized size = 1.53

$$-\frac{8\sqrt{2}\left(32\cos\left(bx+a\right)^5-4\cos\left(bx+a\right)^3-7\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-42\arctan\left(-\frac{\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\left(\cos(bx+a)-\sin(bx+a)\right)+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)+42\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)+21\log\left(-32\cos\left(bx+a\right)^4+4\sqrt{2}\left(4\cos\left(bx+a\right)^3-\left(4\cos\left(bx+a\right)^2+1\right)\sin\left(bx+a\right)-5\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\cos\left(bx+a\right)^2+16\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/96*(8*sqrt(2)*(32*cos(b*x + a)^5 - 4*cos(b*x + a)^3 - 7*cos(b*x + a))*sq
rt(cos(b*x + a)*sin(b*x + a)) - 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b
*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*
x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 42*arctan(-(2*sqrt(2)*sqrt(c
os(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - si
n(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4
*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b
*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{9/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^3,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(9/2)/sin(a + b*x)^3, x)
```

3.116 $\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=164

$$-\frac{5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b}+\frac{5\sin(a+bx)}{}$$

```
[Out] -5/4*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b-5/3*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+4/3*sin(b*x+a)*sin(2*b
*x+2*a)^(5/2)/b+1/3*csc(b*x+a)^3*sin(2*b*x+2*a)^(9/2)/b+5/2*sin(b*x+a)*sin(
2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.12, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4385, 4393, 4386, 4387, 4391}

$$-\frac{5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{4b}+\frac{4\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}-\frac{5\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{3b}+\frac{\sin^{\frac{9}{2}}(2a+2bx)\csc^3(a+bx)}{3b}-\frac{5\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(4*b) - (5*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(4*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a +
2*b*x]])/(2*b) - (5*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(3*b) + (4*Sin[a +
 b*x]*Sin[2*a + 2*b*x]^(5/2))/(3*b) + (Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(9/2
))/(3*b)
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx &= \frac{\csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{3b} + 4\int \csc(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx \\
&= \frac{\csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{3b} + 8\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx \\
&= \frac{4\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b} + \frac{\csc^3(a+bx)\sin^{\frac{9}{2}}(2a+2bx)}{3b} + \frac{20}{3}\int \sin \\
&= -\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b} + \frac{4\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{3b} + \csc^3(\\
&= \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b} - \frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b} + 4\sin(\\
&= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b} - \frac{5\log\left(\cos(a+bx)+\sin(a+bx\right.}{4b}
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 84, normalized size = 0.51

$$\frac{-5\left(\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+2\sqrt{\sin(2(a+bx))}\,(6\sin(a+bx)+\sin(3(a+bx)))}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(7/2),x]

[Out] (-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(6*Sin[a + b*x] + Sin[3*(a + b*x)]))/(4*b)

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 129.38, size = 973, normalized size = 5.93

method	result	size
default	Expression too large to display	973

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x,method=_RETURNVERBOSE)

[Out] 32/5*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(2*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^4-((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^4-4*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2+2*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2+2*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^6+2*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))-((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))+2*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4-4*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^4+2*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^2+2*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^2)/(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)/(tan(1/2*a+1/2*x*b)-1)/(tan(1/2*a+1/2*x*b)+1)/b

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [A]
time = 2.19, size = 280, normalized size = 1.71

$$\frac{8\sqrt{2}\left(4\cos\left(bx+a\right)^2+5\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\sin\left(bx+a\right)+10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-10\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)+5\log\left(-32\cos\left(bx+a\right)^4+4\sqrt{2}\left(4\cos\left(bx+a\right)^3-\left(4\cos\left(bx+a\right)^2+1\right)\sin\left(bx+a\right)-5\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\cos\left(bx+a\right)^2+16\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*sqrt(2)*(4*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(
b*x + a) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a
) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
+ a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x +
 a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 5*log(
-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*s
in(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x
+ a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{7/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^3,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(7/2)/sin(a + b*x)^3, x)
```

3.117 $\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=127

$$-\frac{3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{b}+\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{b}-\frac{6\cos(a+bx)}{}$$

```
[Out] -3*arcsin(cos(b*x+a)-sin(b*x+a))/b+3*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*a
)^(1/2))/b+4*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+csc(b*x+a)^3*sin(2*b*x+2*a)^
(7/2)/b-6*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4385, 4393, 4386, 4387, 4390}

$$-\frac{3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{b}+\frac{4\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}-\frac{6\sqrt{\sin(2a+2bx)}\cos(a+bx)}{b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)\csc^3(a+bx)}{b}+\frac{3\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/b + (3*Log[Cos[a + b*x] + Sin[a +
b*x] + Sqrt[Sin[2*a + 2*b*x]]])/b - (6*Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])
/b + (4*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/b + (Csc[a + b*x]^3*Sin[2*a +
2*b*x]^(7/2))/b
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
```

```
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx &= \frac{\csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{b}+8\int \csc(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx\\ &= \frac{\csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{b}+16\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx\\ &= \frac{4\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}+\frac{\csc^3(a+bx)\sin^{\frac{7}{2}}(2a+2bx)}{b}+12\int \sin(\\ &= -\frac{6\cos(a+bx)\sqrt{\sin(2a+2bx)}}{b}+\frac{4\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{b}+\frac{\csc^3(\\ &= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b}+\frac{3\log\left(\cos(a+bx)+\sin(a+bx)\right.}{b}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 70, normalized size = 0.55

$$\frac{-3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)+\csc(a+bx)\sin^{\frac{3}{2}}(2(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2),x]
```

[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*(a + b*x)]]] + Csc[a + b*x]*Sin[2*(a + b*x)]^(3/2))/b

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 33.22, size = 243, normalized size = 1.91

method	result
default	$\frac{16\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\left(\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\mathrm{ta}\right.\right.}{3\sqrt{\mathrm{t}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)

[Out] 16/3*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*((tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^2-(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))-tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))/(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)/b

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")

[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(5/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(119) = 238.
time = 2.46, size = 268, normalized size = 2.11

$$\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a)+6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-6\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)-3\log\left(-32\cos(bx+a)^4+4\sqrt{2}\left(4\cos(bx+a)^3-\left(4\cos(bx+a)^2+1\right)\sin(bx+a)-5\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}+32\cos(bx+a)^2+16\cos(bx+a)\sin(bx+a)+1\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")

[Out] 1/4*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) -

```
6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(
b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4 + 4*sqr
t(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x +
a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*s
in(b*x + a) + 1))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{5/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^3,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(5/2)/sin(a + b*x)^3, x)
```

3.118 $\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=104

$$\frac{2\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{b}+\frac{2\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{b}-\frac{4\sin(a+bx)\cdot}{}$$

```
[Out] 2*arcsin(cos(b*x+a)-sin(b*x+a))/b+2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*a)
^(1/2))/b-csc(b*x+a)^3*sin(2*b*x+2*a)^(5/2)/b-4*sin(b*x+a)*sin(2*b*x+2*a)^(
1/2)/b
```

Rubi [A]
time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4385, 4393, 4386, 4391}

$$\frac{2\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{b}-\frac{4\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\csc^3(a+bx)}{b}+\frac{2\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (2*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/b + (2*Log[Cos[a + b*x] + Sin[a + b
*x] + Sqrt[Sin[2*a + 2*b*x]]])/b - (4*Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/
b - (Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(5/2))/b
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
```

```
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= -\frac{\csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b} - 4\int \csc(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx \\ &= -\frac{\csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b} - 8\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx \\ &= -\frac{4\sin(a+bx)\sqrt{\sin(2a+2bx)}}{b} - \frac{\csc^3(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{b} - 4\int - \\ &= \frac{2\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{b} + \frac{2\log\left(\cos(a+bx)+\sin(a+bx)+\right.}{b}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 68, normalized size = 0.65

$$\frac{2\left(\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-2\csc(a+bx)\sqrt{\sin(2(a+bx))}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
[Out] (2*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] +
 Sqrt[Sin[2*(a + b*x)]]] - 2*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]]))/b
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 83.90, size = 542, normalized size = 5.21

method	result

default	$\frac{4\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\,\left(4\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\,\sqrt{\left(\tan\left(\frac{a}{2}+\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)

[Out] 4*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(4*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*EllipticE((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))-2*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))+((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*tan(1/2*a+1/2*x*b)^2+2*tan(1/2*a+1/2*x*b)^2*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)-((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)*(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2))/tan(1/2*a+1/2*x*b)/((tan(1/2*a+1/2*x*b)+1)*(tan(1/2*a+1/2*x*b)-1)*tan(1/2*a+1/2*x*b))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)/b

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")

[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(98) = 196.
time = 2.10, size = 295, normalized size = 2.84

$$-\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)\sin(bx+a)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)\sin(bx+a)+\log\left(-32\cos(bx+a)^4+4\sqrt{2}\left(4\cos(bx+a)^3-\left(4\cos(bx+a)^2+1\right)\sin(bx+a)-5\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}+32\cos(bx+a)^2+16\cos(bx+a)\sin(bx+a)+1\right)\sin(bx+a)+8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}+8\sin(bx+a)}{2b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")

```
[Out] -1/2*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - si
n(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*s
in(b*x + a) - 1))*sin(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin
(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a)))*si
n(b*x + a) + log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(
b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x +
a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1)*sin(b*x + a) +
8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + 8*sin(b*x + a))/(b*sin(b*x + a)
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^{3/2}}{\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^3,x)
```

```
[Out] int(sin(2*a + 2*b*x)^(3/2)/sin(a + b*x)^3, x)
```

3.119 $\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=28

$$-\frac{\csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}$$

```
[Out] -1/3*csc(b*x+a)^3*sin(2*b*x+2*a)^(3/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {4377}

$$-\frac{\sin^{\frac{3}{2}}(2a+2bx)\csc^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/3*(Csc[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2))/b
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \csc^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx = -\frac{\csc^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{3b}$$

Mathematica [A]
time = 0.06, size = 27, normalized size = 0.96

$$-\frac{\csc^3(a+bx)\sin^{\frac{3}{2}}(2(a+bx))}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/3*(Csc[a + b*x]^3*Sin[2*(a + b*x)]^(3/2))/b
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 25.58, size = 192, normalized size = 6.86

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\,\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)\left(4\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\,\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}\right.}{3\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{a}{2}+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*x*b
)^2-1)/tan(1/2*a+1/2*x*b)*(4*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a+1/2
*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b)+1)
^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)+tan(1/2*a+1/2*x*b)^4-1)/(tan(1/2*a+1
/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2
*x*b))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.
time = 2.25, size = 53, normalized size = 1.89

$$\frac{2\left(\sqrt{2}\,\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\,\cos\left(bx+a\right)+\cos\left(bx+a\right)^2-1\right)}{3\left(b\cos\left(bx+a\right)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + cos(b*x + a)^2
- 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [B]
time = 1.50, size = 95, normalized size = 3.39

$$\frac{4\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,\left(4\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-6\sin\left(\frac{3\,a}{2}+\frac{3\,b\,x}{2}\right)^2+2\sin\left(\frac{5\,a}{2}+\frac{5\,b\,x}{2}\right)^2\right)}{3\,b\,\left(30\sin\left(a+b\,x\right)^2-12\sin\left(2\,a+2\,b\,x\right)^2+2\sin\left(3\,a+3\,b\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^(1/2)/sin(a + b*x)^3,x)
```

```
[Out] (4*sin(2*a + 2*b*x)^(1/2)*(4*sin(a/2 + (b*x)/2)^2 - 6*sin((3*a)/2 + (3*b*x)
/2)^2 + 2*sin((5*a)/2 + (5*b*x)/2)^2))/(3*b*(2*sin(3*a + 3*b*x)^2 - 12*sin(
2*a + 2*b*x)^2 + 30*sin(a + b*x)^2))
```

3.120 $\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=55

$$-\frac{4\csc(a+bx)\sqrt{\sin(2a+2bx)}}{5b}-\frac{\csc^3(a+bx)\sqrt{\sin(2a+2bx)}}{5b}$$

```
[Out] -4/5*csc(b*x+a)*sin(2*b*x+2*a)^(1/2)/b-1/5*csc(b*x+a)^3*sin(2*b*x+2*a)^(1/2
)/b
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4385, 4377}

$$-\frac{\sqrt{\sin(2a+2bx)}\csc^3(a+bx)}{5b}-\frac{4\sqrt{\sin(2a+2bx)}\csc(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-4*Csc[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(5*b) - (Csc[a + b*x]^3*Sqrt[Sin[2
*a + 2*b*x]])/(5*b)
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = -\frac{\csc^3(a+bx)\sqrt{\sin(2a+2bx)}}{5b} + \frac{4}{5}\int \frac{\csc(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$$
$$= -\frac{4\csc(a+bx)\sqrt{\sin(2a+2bx)}}{5b} - \frac{\csc^3(a+bx)\sqrt{\sin(2a+2bx)}}{5b}$$

Mathematica [A]
time = 0.10, size = 35, normalized size = 0.64

$$-\frac{\csc(a+bx)\left(4+\csc^2(a+bx)\right)\sqrt{\sin(2(a+bx))}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/5*(Csc[a + b*x]*(4 + Csc[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/b
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(xb+a)}{\sqrt{\sin(2xb+2a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] int(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [A]
time = 2.87, size = 76, normalized size = 1.38

$$-\frac{\sqrt{2}\left(4\cos(bx+a)^2-5\right)\sqrt{\cos(bx+a)\sin(bx+a)}+4\left(\cos(bx+a)^2-1\right)\sin(bx+a)}{5\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*(sqrt(2)*(4*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 4*(c
os(b*x + a)^2 - 1)*sin(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [B]
time = 3.16, size = 93, normalized size = 1.69

$$-\frac{8\,{\mathrm{e}}^{a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,\sqrt{\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}}\,\left(-{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,3{}\mathrm{i}+{\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}\,1{}\mathrm{i}+1{}\mathrm{i}\right)}{5\,b\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^(1/2)),x)
```

```
[Out] -(8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2)*(exp(a*4i + b*x*4i)*1i - exp(a*2i + b*x*2i)*3i + 1i))/(5*b*(exp
(a*2i + b*x*2i) - 1)^3)
```

3.121 $\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)} dx$

Optimal. Leaf size=81

$$-\frac{16\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}+\frac{32\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -16/21*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-1/7*csc(b*x+a)^3/b/sin(2*b*x+2*a)^
(1/2)+32/21*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4385, 4393, 4388, 4377}

$$\frac{32\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}-\frac{16\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-16*Cos[a + b*x])/(21*b*Sin[2*a + 2*b*x]^(3/2)) - Csc[a + b*x]^3/(7*b*Sqrt
[Sin[2*a + 2*b*x]]) + (32*Sin[a + b*x])/(21*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4388

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx &= -\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}+\frac{8}{7}\int \frac{\csc(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}+\frac{16}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{16\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}+\frac{32}{21}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{16\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{7b\sqrt{\sin(2a+2bx)}}+\frac{32\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 55, normalized size = 0.68

$$\frac{(5-12\cos(2(a+bx))+4\cos(4(a+bx)))\csc^4(a+bx)\sec(a+bx)\sqrt{\sin(2(a+bx))}}{42b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] ((5 - 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4*Sec[a + b*x]
*Sqrt[Sin[2*(a + b*x)]])/(42*b)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 172.62, size = 222, normalized size = 2.74

$$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\,\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)\left(-3\left(\tan^8\left(\frac{a}{2}+\frac{xb}{2}\right)\right)+16\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{a}{2}\right.}\right.}{336\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] -1/336*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2*
x*b)^2-1)/tan(1/2*a+1/2*x*b)^3*(-3*tan(1/2*a+1/2*x*b)^8+16*(tan(1/2*a+1/2*x
*b)+1)^(1/2)*(-2*tan(1/2*a+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*El
lipticF((tan(1/2*a+1/2*x*b)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)^3-2*ta
n(1/2*a+1/2*x*b)^6+2*tan(1/2*a+1/2*x*b)^2+3)/(tan(1/2*a+1/2*x*b)*(tan(1/2*a
+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a+1/2*x*b))^(1/2)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [**A**]
time = 2.12, size = 104, normalized size = 1.28

$$\frac{32\cos(bx+a)^5-64\cos(bx+a)^3+\sqrt{2}\left(32\cos(bx+a)^4-56\cos(bx+a)^2+21\right)\sqrt{\cos(bx+a)\sin(bx+a)}+32\cos(bx+a)}{42\left(b\cos(bx+a)^5-2\,b\cos(bx+a)^3+b\cos(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/42*(32*cos(b*x + a)^5 - 64*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4 -
56*cos(b*x + a)^2 + 21)*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a))/
(b*cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [B]
time = 3.70, size = 302, normalized size = 3.73

$$-\frac{10\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{21\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,12\mathrm{i}}{7\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^3}-\frac{8\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{7\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^4}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{10\mathrm{i}}{21\,b}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,32\mathrm{i}}{21\,b}\right)\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^(3/2)),x)
```

```
[Out] (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/
2)^(1/2)*12i)/(7*b*(exp(a*2i + b*x*2i)*1i - 1i)^3) - (10*exp(a*1i + b*x*1i)
*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(21*b*(ex
p(a*2i + b*x*2i)*1i - 1i)^2) - (8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)
*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i - 1i
)^4) - (exp(a*1i + b*x*1i)*(10i/(21*b) - (exp(a*2i + b*x*2i)*32i)/(21*b))*(
(exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i
+ b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i))
```

3.122 $\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=107

$$-\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{64\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -8/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/2)-1/9*csc(b*x+a)^3/b/sin(2*b*x+2*a)^(
3/2)+32/45*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-64/45*cos(b*x+a)/b/sin(2*b*x+2
*a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4385, 4393, 4388, 4389, 4376}

$$\frac{32\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{64\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-8*Cos[a + b*x])/(15*b*Sin[2*a + 2*b*x]^(5/2)) - Csc[a + b*x]^3/(9*b*Sin[2
*a + 2*b*x]^(3/2)) + (32*Sin[a + b*x])/(45*b*Sin[2*a + 2*b*x]^(3/2)) - (64*
Cos[a + b*x])/(45*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4376

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4385

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
m + p + 1))), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Sin[a + b*x
])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &&
 EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m +
 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4388

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist
```

```
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4389

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + D
ist[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x],
 x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !I
ntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= -\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{4}{3}\int \frac{\csc(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx \\
&= -\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx \\
&= -\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32}{15}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx \\
&= -\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{64}{45}\int \frac{\cos(a+b}{\sin^{\frac{3}{2}}(2a+2} \\
&= -\frac{8\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\csc^3(a+bx)}{9b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{32\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{64\cos(a+bx)}{45b\sqrt{\sin(2a+2b}}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 62, normalized size = 0.58

$$-\frac{\sqrt{\sin(2(a+bx))}\,(113\csc(a+bx)+17\csc^3(a+bx)+5\csc^5(a+bx)-15\sec(a+bx)\tan(a+bx))}{180b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -1/180*(Sqrt[Sin[2*(a + b*x)]]*(113*Csc[a + b*x] + 17*Csc[a + b*x]^3 + 5*Cs
c[a + b*x]^5 - 15*Sec[a + b*x]*Tan[a + b*x]))/b
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(xb+a)}{\sin(2xb+2a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x)
```

```
[Out] int(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [A]
time = 2.99, size = 131, normalized size = 1.22

$$-\frac{\sqrt{2}\left(128\cos(bx+a)^6-288\cos(bx+a)^4+180\cos(bx+a)^2-15\right)\sqrt{\cos(bx+a)\sin(bx+a)}+128\left(\cos(bx+a)^6-2\cos(bx+a)^4+\cos(bx+a)^2\right)\sin(bx+a)}{180\left(b\cos(bx+a)^6-2\,b\cos(bx+a)^4+b\cos(bx+a)^2\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/180*(sqrt(2)*(128*cos(b*x + a)^6 - 288*cos(b*x + a)^4 + 180*cos(b*x + a)
^2 - 15)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - 2*cos(b*x
+ a)^4 + cos(b*x + a)^2)*sin(b*x + a))/((b*cos(b*x + a)^6 - 2*b*cos(b*x + a
)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [B]
time = 4.99, size = 383, normalized size = 3.58

$$-\frac{2\,{\mathrm{e}}^{a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,\sqrt{\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}}}{15\,b\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}^3}-\frac{{\mathrm{e}}^{a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,\sqrt{\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}}\,16{}\mathrm{i}}{9\,b\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}^4}+\frac{8\,{\mathrm{e}}^{a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,\sqrt{\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}}}{9\,b\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}^5}+\frac{64\,{\mathrm{e}}^{a\,3{}\mathrm{i}+b\,x\,3{}\mathrm{i}}\,\sqrt{\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}}}{45\,b\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}-\frac{{\mathrm{e}}^{a\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}}\,\left(\frac{98{}\mathrm{i}}{45\,b}+\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,38{}\mathrm{i}}{45\,b}\right)\,\sqrt{\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}}{2}}}{{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}+1\right)}^2\,{\left({\mathrm{e}}^{a\,2{}\mathrm{i}+b\,x\,2{}\mathrm{i}}\,1{}\mathrm{i}-\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(a + b*x)^3*sin(2*a + 2*b*x)^(5/2)),x)
```

```
[Out] (8*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2))/(9*b*(exp(a*2i + b*x*2i)*1i - 1i)^5) - (exp(a*1i + b*x*1i)*((ex
p(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*16i)/(9*b*(exp(
a*2i + b*x*2i)*1i - 1i)^4) - (2*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1
i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(15*b*(exp(a*2i + b*x*2i)*1i - 1i)
^3) + (64*exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x
*2i)*1i)/2)^(1/2))/(45*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i -
1i)) - (exp(a*1i + b*x*1i)*(98i/(45*b) + (exp(a*2i + b*x*2i)*38i)/(45*b))*(
(exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i
+ b*x*2i) + 1)^2*(exp(a*2i + b*x*2i)*1i - 1i)^2)
```

3.123 $\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=84

$$\frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{4+m}{2};\frac{6+m}{2};\sin^2(a+bx)\right)\sin^3(a+bx)\sin^m(2a+2bx)\tan(a+bx)}{b(4+m)}$$

```
[Out] (cos(b*x+a)^2)^(1/2-1/2*m)*hypergeom([2+1/2*m, 1/2-1/2*m],[3+1/2*m],sin(b*x
+a)^2)*sin(b*x+a)^3*sin(2*b*x+2*a)^m*tan(b*x+a)/b/(4+m)
```

Rubi [A]

time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4395, 2657}

$$\frac{\sin^3(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m+4}{2};\frac{m+6}{2};\sin^2(a+bx)\right)}{b(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (4 + m)/2, (6 +
m)/2, Sin[a + b*x]^2]*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*(4
 + m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 4395

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin^m(2a+2bx)\,dx &= \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{3+m}(\\ &= \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{4+m}{2};\frac{6+m}{2};\sin^2(a+bx)\right)\sin^3(a+bx)\sin^m(2a}{b(4+m)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 5.72, size = 602, normalized size = 7.17

$$\frac{32(4+m)\left(F_1\left(1+\frac{m}{2};-m,3+2m;2+\frac{m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-F_1\left(1+\frac{m}{2};-m,4+2m;2+\frac{m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\right)\cos^6\left(\frac{1}{2}(a+bx)\right)\sin^4\left(\frac{1}{2}(a+bx)\right)\sin^m(2(a+bx))}{b(2+m)\left(-2(4+m)F_1\left(1+\frac{m}{2};-m,4+2m;2+\frac{m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\cos^2\left(\frac{1}{2}(a+bx)\right)+2\left(mF_1\left(2+\frac{m}{2};1-m,3+2m;3+\frac{m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-mF_1\left(2+\frac{m}{2};1-m,4+2m;3+\frac{m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+3F_1\left(2+\frac{m}{2};-m,4+2m;3+\frac{m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+2mF_1\left(2+\frac{m}{2};-m,4+2m;3+\frac{m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-4F_1\left(2+\frac{m}{2};-m,5+2m;3+\frac{m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-2mF_1\left(2+\frac{m}{2};-m,5+2m;3+\frac{m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\right)(-1+\cos(a+bx))+(4+m)F_1\left(1+\frac{m}{2};-m,3+2m;2+\frac{m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)(1+\cos(a+bx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (32*(4 + m)*(AppellF1[1 + m/2, -m, 3 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2] - AppellF1[1 + m/2, -m, 4 + 2*m, 2 + m/2, Tan[(a + b*x)/
2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^6*Sin[(a + b*x)/2]^4*Sin[2*(a
+ b*x)]^m)/(b*(2 + m)*(-2*(4 + m)*AppellF1[1 + m/2, -m, 4 + 2*m, 2 + m/2, T
an[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 2*(m*AppellF1[
2 + m/2, 1 - m, 3 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
- m*AppellF1[2 + m/2, 1 - m, 4 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a
+ b*x)/2]^2] + 3*AppellF1[2 + m/2, -m, 4 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2
, -Tan[(a + b*x)/2]^2] + 2*m*AppellF1[2 + m/2, -m, 4 + 2*m, 3 + m/2, Tan[(a
 + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[2 + m/2, -m, 5 + 2*m, 3 + m
/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 2*m*AppellF1[2 + m/2, -m, 5
+ 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a + b*x
]) + (4 + m)*AppellF1[1 + m/2, -m, 3 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2]*(1 + Cos[a + b*x])))
```

Maple [F]
time = 0.57, size = 0, normalized size = 0.00

$$\int \left(\sin^3(xb+a)\right)\left(\sin^m(2xb+2a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{3}\left(a + bx\right)\sin^{m}\left(2a + 2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(2*b*x+2*a)**m,x)
[Out] Integral(sin(a + b*x)**3*sin(2*a + 2*b*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="giac")
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\sin\left(a+b\,x\right)}^3\,{\sin\left(2\,a+2\,b\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^m,x)
[Out] int(sin(a + b*x)^3*sin(2*a + 2*b*x)^m, x)
```

3.124 $\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=84

$$\frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{3+m}{2};\frac{5+m}{2};\sin^2(a+bx)\right)\sin^2(a+bx)\sin^m(2a+2bx)\tan(a+bx)}{b(3+m)}$$

```
[Out] (cos(b*x+a)^2)^(1/2-1/2*m)*hypergeom([1/2-1/2*m, 3/2+1/2*m],[5/2+1/2*m],sin
(b*x+a)^2)*sin(b*x+a)^2*sin(2*b*x+2*a)^m*tan(b*x+a)/b/(3+m)
```

Rubi [A]

time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4395, 2657}

$$\frac{\sin^2(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m+3}{2};\frac{m+5}{2};\sin^2(a+bx)\right)}{b(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (3 + m)/2, (5 +
m)/2, Sin[a + b*x]^2]*Sin[a + b*x]^2*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*(3
 + m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 4395

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx &= \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{2+m}(a\\ &= \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{3+m}{2};\frac{5+m}{2};\sin^2(a+bx)\right)\sin^2(a+bx)\sin^m(2a}{b(3+m)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 3.77, size = 602, normalized size = 7.17

$$\frac{16(3+m)\left(F_1\left(\frac{1+m}{2};-m,2(1+m);\frac{3+m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-F_1\left(\frac{1+m}{2};-m,3+2m;\frac{3+m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\right)\cos^5\left(\frac{1}{2}(a+bx)\right)\sin^3\left(\frac{1}{2}(a+bx)\right)\sin^m(2(a+bx))}{b(1+m)\left(-2(3+m)F_1\left(\frac{1+m}{2};-m,3+2m;\frac{3+m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\cos^2\left(\frac{1}{2}(a+bx)\right)+2\left(mF_1\left(\frac{3+m}{2};1-m,2(1+m);\frac{5+m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-mF_1\left(\frac{3+m}{2};1-m,3+2m;\frac{5+m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-3F_1\left(\frac{3+m}{2};-m,2(2+m);\frac{5+m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-2mF_1\left(\frac{3+m}{2};-m,2(2+m);\frac{5+m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+2F_1\left(\frac{3+m}{2};-m,3+2m;\frac{5+m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+2mF_1\left(\frac{3+m}{2};-m,3+2m;\frac{5+m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\right)(-1+\cos(a+bx))+(3+m)F_1\left(\frac{1+m}{2};-m,2(1+m);\frac{3+m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)(1+\cos(a+bx))\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (16*(3 + m)*(AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[
(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^5*Sin[(a + b*x)/2]^3
*Sin[2*(a + b*x)]^m)/(b*(1 + m)*(-2*(3 + m)*AppellF1[(1 + m)/2, -m, 3 + 2*m
, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 +
2*(m*AppellF1[(3 + m)/2, 1 - m, 2*(1 + m), (5 + m)/2, Tan[(a + b*x)/2]^2, -
Tan[(a + b*x)/2]^2] - m*AppellF1[(3 + m)/2, 1 - m, 3 + 2*m, (5 + m)/2, Tan[
(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 3*AppellF1[(3 + m)/2, -m, 2*(2 + m),
 (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 2*m*AppellF1[(3 + m)
/2, -m, 2*(2 + m), (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 2*
AppellF1[(3 + m)/2, -m, 3 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b
*x)/2]^2] + 2*m*AppellF1[(3 + m)/2, -m, 3 + 2*m, (5 + m)/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a + b*x]) + (3 + m)*AppellF1[(1 + m)/2
, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + C
os[a + b*x])))
```

Maple [F]
time = 0.36, size = 0, normalized size = 0.00

$$\int \left(\sin^2\left(xb+a\right)\right)\left(\sin^m\left(2xb+2a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(2*b*x + 2*a)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^2(a+bx)\sin^m(2a+2bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(2*b*x+2*a)**m,x)
```

```
[Out] Integral(sin(a + b*x)**2*sin(2*a + 2*b*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^m,x)
```

```
[Out] int(sin(a + b*x)^2*sin(2*a + 2*b*x)^m, x)
```

3.125 $\int \sin(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=82

$$\frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{2+m}{2};\frac{4+m}{2};\sin^2(a+bx)\right)\sin(a+bx)\sin^m(2a+2bx)\tan(a+bx)}{b(2+m)}$$

```
[Out] (cos(b*x+a)^2)^(1/2-1/2*m)*hypergeom([1+1/2*m, 1/2-1/2*m],[2+1/2*m],sin(b*x
+a)^2)*sin(b*x+a)*sin(2*b*x+2*a)^m*tan(b*x+a)/b/(2+m)
```

Rubi [A]

time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4395, 2657}

$$\frac{\sin(a+bx)\tan(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m+2}{2};\frac{m+4}{2};\sin^2(a+bx)\right)}{b(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, (2 + m)/2, (4 +
m)/2, Sin[a + b*x]^2]*Sin[a + b*x]*Sin[2*a + 2*b*x]^m*Tan[a + b*x])/(b*(2 +
 m))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 4395

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sin(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{1+m}($$

$$= \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{2+m}{2};\frac{4+m}{2};\sin^2(a+bx)\right)\sin(a+bx)\sin^m(2a-}{b(2+m)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.57, size = 152, normalized size = 1.85

$$-\frac{i2^{-1-m}e^{i(a+bx)}\left(-ie^{-2i(a+bx)}\left(-1+e^{4i(a+bx)}\right)\right)^{1+m}\left((1-2m)\,{}_2F_1\left(1,\frac{1}{4}(3+2m);\frac{1}{4}(3-2m);e^{4i(a+bx)}\right)+e^{2i(a+bx)}(1+2m)\,{}_2F_1\left(1,\frac{1}{4}(5+2m);\frac{1}{4}(5-2m);e^{4i(a+bx)}\right)\right)}{b\left(-1+4m^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[2*a + 2*b*x]^m,x]
[Out] ((-I)*2^(-1 - m)*E^(I*(a + b*x))*(((-I)*(-1 + E^((4*I)*(a + b*x))))/E^((2*I
)*(a + b*x)))^(1 + m)*((1 - 2*m)*Hypergeometric2F1[1, (3 + 2*m)/4, (3 - 2*m
)/4, E^((4*I)*(a + b*x))] + E^((2*I)*(a + b*x))*(1 + 2*m)*Hypergeometric2F1
[1, (5 + 2*m)/4, (5 - 2*m)/4, E^((4*I)*(a + b*x))]))/(b*(-1 + 4*m^2))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \sin\left(xb+a\right)\left(\sin^{m}\left(2xb+2a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(2*b*x+2*a)^m,x)
[Out] int(sin(b*x+a)*sin(2*b*x+2*a)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="maxima")
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="fricas")
[Out] integral(sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin\left(a+bx\right)\sin^{m}\left(2a+2bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)**m,x)
```

```
[Out] Integral(sin(a + b*x)*sin(2*a + 2*b*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,{\sin\left(2\,a+2\,b\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(2*a + 2*b*x)^m,x)
```

```
[Out] int(sin(a + b*x)*sin(2*a + 2*b*x)^m, x)
```

3.126 $\int \csc(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=72

$$\frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m}{2};\frac{2+m}{2};\sin^2(a+bx)\right)\sec(a+bx)\sin^m(2a+2bx)}{bm}$$

```
[Out] (cos(b*x+a)^2)^(1/2-1/2*m)*hypergeom([1/2*m, 1/2-1/2*m],[1+1/2*m],sin(b*x+a
)^2)*sec(b*x+a)*sin(2*b*x+2*a)^m/b/m
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4395, 2657}

$$\frac{\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m}{2};\frac{m+2}{2};\sin^2(a+bx)\right)}{bm}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] ((Cos[a + b*x]^2)^((1 - m)/2)*Hypergeometric2F1[(1 - m)/2, m/2, (2 + m)/2,
Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b*m)
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 4395

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{-1+m}($$

$$= \frac{\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m}{2};\frac{2+m}{2};\sin^2(a+bx)\right)\sec(a+bx)\sin^m(2a+2b}{bm}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.98, size = 254, normalized size = 3.53

$$\frac{2(2+m)F_1\left(\frac{m}{2};-m,2m;\frac{2+m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\cos^2\left(\frac{1}{2}(a+bx)\right)\sin^m(2(a+bx))}{bm\left((2+m)F_1\left(\frac{m}{2};-m,2m;\frac{2+m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)(1+\cos(a+bx))-4m\left(F_1\left(\frac{2+m}{2};1-m,2m;\frac{4+m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+2F_1\left(\frac{2+m}{2};-m,1+2m;\frac{4+m}{2};\tan^2\left(\frac{1}{2}(a+bx)\right),-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\right)\sin^2\left(\frac{1}{2}(a+bx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (2*(2 + m)*AppellF1[m/2, -m, 2*m, (2 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2]*Cos[(a + b*x)/2]^2*Sin[2*(a + b*x)]^m)/(b*m*((2 + m)*AppellF1[m/
2, -m, 2*m, (2 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a
+ b*x]) - 4*m*(AppellF1[(2 + m)/2, 1 - m, 2*m, (4 + m)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2] + 2*AppellF1[(2 + m)/2, -m, 1 + 2*m, (4 + m)/2, Tan
[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Sin[(a + b*x)/2]^2))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \csc\left(xb+a\right)\left(\sin^{m}\left(2xb+2a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(csc(b*x+a)*sin(2*b*x+2*a)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*csc(b*x + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^m(2a+2bx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)**m,x)
```

```
[Out] Integral(sin(2*a + 2*b*x)**m*csc(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^m}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^m/sin(a + b*x),x)
```

```
[Out] int(sin(2*a + 2*b*x)^m/sin(a + b*x), x)
```

3.127 $\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=85

$$-\frac{\cos^2(a+bx)^{\frac{1-m}{2}}\csc(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(a+bx)\right)\sec(a+bx)\sin^m(2a+2bx)}{b(1-m)}$$

```
[Out] -(cos(b*x+a)^2)^(1/2-1/2*m)*csc(b*x+a)*hypergeom([-1/2+1/2*m, 1/2-1/2*m],[1
/2+1/2*m],sin(b*x+a)^2)*sec(b*x+a)*sin(2*b*x+2*a)^m/b/(1-m)
```

Rubi [A]
time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4395, 2657}

$$-\frac{\csc(a+bx)\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m-1}{2};\frac{m+1}{2};\sin^2(a+bx)\right)}{b(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] -(((Cos[a + b*x]^2)^((1 - m)/2)*Csc[a + b*x]*Hypergeometric2F1[(1 - m)/2, (
-1 + m)/2, (1 + m)/2, Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b*(
1 - m)))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 4395

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^2(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{-2+m}$$

$$= -\frac{\cos^2(a+bx)^{\frac{1-m}{2}}\csc(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(a+bx)\right)}{b(1-m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 7.92, size = 938, normalized size = 11.04

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (2*((-1 + m)*AppellF1[(1 + m)/2, -m, 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2] + (1 + m)*AppellF1[(-1 + m)/2, -m, 2*m, (1 + m)/2, Tan[(
a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cot[(a + b*x)/2]^2)*Csc[a + b*x]^2*Sin[
2*(a + b*x)]^m*Tan[(a + b*x)/2])/(b*(m*(1 + m)*AppellF1[(-1 + m)/2, -m, 2*m
, (1 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - (1 + m)*AppellF1[(-
1 + m)/2, -m, 2*m, (1 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Csc[
(a + b*x)/2]^2 + (-1 + m)*AppellF1[(1 + m)/2, -m, 2*m, (3 + m)/2, Tan[(a +
b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2 - 2*(-1 + m)*m*(AppellF1
[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
 + 2*AppellF1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2])*Sec[(a + b*x)/2]^2 + (-1 + m)*m*AppellF1[(1 + m)/2, -m, 2*m
, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(-2 + 3*Cos[a + b*x])
*Sec[a + b*x] + m*(1 + m)*AppellF1[(-1 + m)/2, -m, 2*m, (1 + m)/2, Tan[(a +
 b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(-2 + 3*Cos[a + b*x])*Cot[(a + b*x)/2]^2*S
ec[a + b*x] + (-1 + m)*m*AppellF1[(1 + m)/2, -m, 2*m, (3 + m)/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2]*Tan[(a + b*x)/2]^2 - (2*(-1 + m)*m*(1 + m)*(
AppellF1[(3 + m)/2, 1 - m, 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*
x)/2]^2] + 2*AppellF1[(3 + m)/2, -m, 1 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2
, -Tan[(a + b*x)/2]^2])*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2]^2)/(3 + m) + 2*
m*(1 + m)*AppellF1[(-1 + m)/2, -m, 2*m, (1 + m)/2, Tan[(a + b*x)/2]^2, -Tan
[(a + b*x)/2]^2]*Cot[(a + b*x)/2]*Tan[a + b*x] + 2*(-1 + m)*m*AppellF1[(1 +
 m)/2, -m, 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Tan[(a
+ b*x)/2]*Tan[a + b*x]))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(\csc^2\left(xb+a\right)\right)\left(\sin^m\left(2xb+2a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*csc(b*x + a)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{m}\left(2a + 2bx\right)\csc^{2}\left(a + bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(2*b*x+2*a)**m,x)
```

```
[Out] Integral(sin(2*a + 2*b*x)**m*csc(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\sin\left(2\,a+2\,b\,x\right)}^m}{{\sin\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^m/sin(a + b*x)^2,x)
```

```
[Out] int(sin(2*a + 2*b*x)^m/sin(a + b*x)^2, x)
```

3.128 $\int \csc^3(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=85

$$-\frac{\cos^2(a+bx)^{\frac{1-m}{2}}\csc^2(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1}{2}(-2+m);\frac{m}{2};\sin^2(a+bx)\right)\sec(a+bx)\sin^m(2a+2bx)}{b(2-m)}$$

```
[Out] -(cos(b*x+a)^2)^(1/2-1/2*m)*csc(b*x+a)^2*hypergeom([-1+1/2*m, 1/2-1/2*m],[1
/2*m],sin(b*x+a)^2)*sec(b*x+a)*sin(2*b*x+2*a)^m/b/(2-m)
```

Rubi [A]

time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4395, 2657}

$$-\frac{\csc^2(a+bx)\sec(a+bx)\sin^m(2a+2bx)\cos^2(a+bx)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1-m}{2},\frac{m-2}{2};\frac{m}{2};\sin^2(a+bx)\right)}{b(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] -(((Cos[a + b*x]^2)^((1 - m)/2)*Csc[a + b*x]^2*Hypergeometric2F1[(1 - m)/2,
 (-2 + m)/2, m/2, Sin[a + b*x]^2]*Sec[a + b*x]*Sin[2*a + 2*b*x]^m)/(b*(2 -
m)))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 4395

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/(Cos[a + b*x]^p*(f*Sin[a + b*x])^
p), Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, f, g, n, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^3(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^m(a+bx)\sin^{-3+m}($$

$$= -\frac{\cos^2(a+bx)^{\frac{1-m}{2}}\csc^2(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1}{2}(-2+m);\frac{m}{2};\sin^2(a+bx)\right)\text{se}}{b(2-m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 18.96, size = 2308, normalized size = 27.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (AppellF1[-1 + m/2, -m, 2*m, m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*
Cos[(a + b*x)/2]^2*Cot[(a + b*x)/2]^2*Sin[2*(a + b*x)]^m)/(2*b*(-2 + m)*(2*
(AppellF1[m/2, 1 - m, 2*m, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2
] + 2*AppellF1[m/2, -m, 1 + 2*m, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x
)/2]^2])*(-1 + Cos[a + b*x]) + AppellF1[-1 + m/2, -m, 2*m, m/2, Tan[(a + b*
x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x]))) + ((4 + m)*AppellF1[1 +
m/2, -m, 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[a + b*x
]*Sin[(a + b*x)/2]^2*Sin[2*(a + b*x)]^m)/(2*b*(2 + m)*((4 + m)*AppellF1[1 +
 m/2, -m, 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Sec[a
 + b*x]) - 4*m*(AppellF1[2 + m/2, 1 - m, 2*m, 3 + m/2, Tan[(a + b*x)/2]^2,
-Tan[(a + b*x)/2]^2] + 2*AppellF1[2 + m/2, -m, 1 + 2*m, 3 + m/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2])*Sec[a + b*x]*Sin[(a + b*x)/2]^2)) + ((4 + m
)*AppellF1[1 + m/2, -m, 1 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x
)/2]^2]*Sin[a + b*x]^2*Sin[2*(a + b*x)]^m)/(4*b*(2 + m)*(2*(m*AppellF1[2 +
m/2, 1 - m, 1 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + (1
 + 2*m)*AppellF1[2 + m/2, -m, 2 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a
 + b*x)/2]^2])*(-1 + Cos[a + b*x]) + (4 + m)*AppellF1[1 + m/2, -m, 1 + 2*m,
 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*(1 + Cos[a + b*x]))) + (
2^(-3 + m)*Cot[(a + b*x)/2]*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-
Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m*Sin[2*(a + b*x)]^m*((2 + m)*App
ellF1[m/2, -m, 2*m, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - m*A
ppellF1[1 + m/2, -m, 1 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2
]^2]*Tan[(a + b*x)/2]^2))/(b*m*(2 + m)*(Cos[a + b*x]*Sec[(a + b*x)/2]^2)^m*
((2^(-1 + m)*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2
] + Sin[(3*(a + b*x))/2]))^(-1 + m)*(Cos[(a + b*x)/2]*(-1/2*Cos[(a + b*x)/2
] + (3*Cos[(3*(a + b*x))/2])/2) - (Sin[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Si
n[(3*(a + b*x))/2]))/2)*((2 + m)*AppellF1[m/2, -m, 2*m, 1 + m/2, Tan[(a + b
*x)/2]^2, -Tan[(a + b*x)/2]^2] - m*AppellF1[1 + m/2, -m, 1 + 2*m, 2 + m/2,
Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Tan[(a + b*x)/2]^2))/((2 + m)*(Cos
[a + b*x]*Sec[(a + b*x)/2]^2)^m) + (2^m*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a
+ b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m*Tan[(a + b*x)/2]*((
2 + m)*AppellF1[m/2, -m, 2*m, 1 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2
]^2] - m*AppellF1[1 + m/2, -m, 1 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2]*Tan[(a + b*x)/2]^2))/((2 + m)*(Cos[a + b*x]*Sec[(a + b*x)/2]
^2)^m) - (2^(-1 + m)*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[a + b*x]*Sec[(a + b*x)
/2]^2)^(-1 - m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]
```

```
))^m*(-(Sec[(a + b*x)/2]^2*Sin[a + b*x]) + Cos[a + b*x]*Sec[(a + b*x)/2]^2*
Tan[(a + b*x)/2])*((2 + m)*AppellF1[m/2, -m, 2*m, 1 + m/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] - m*AppellF1[1 + m/2, -m, 1 + 2*m, 2 + m/2, Tan[(a
 + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Tan[(a + b*x)/2]^2))/(2 + m) + (2^(-1 +
m)*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3
*(a + b*x))/2]))^m*(-(m*AppellF1[1 + m/2, -m, 1 + 2*m, 2 + m/2, Tan[(a + b*
x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2]) + (2 + m
)*(-1/2*(m^2*AppellF1[1 + m/2, 1 - m, 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Ta
n[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(1 + m/2) - (m^2*App
ellF1[1 + m/2, -m, 1 + 2*m, 2 + m/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^
2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(1 + m/2)) - m*Tan[(a + b*x)/2]^2*(
-(((1 + m/2)*m*AppellF1[2 + m/2, 1 - m, 1 + 2*m, 3 + m/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(2 + m/2)) - (
(1 + m/2)*(1 + 2*m)*AppellF1[2 + m/2, -m, 2 + 2*m, 3 + m/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(2 + m/2)))
/(m*(2 + m)*(Cos[a + b*x]*Sec[(a + b*x)/2]^2)^m))
```

Maple [**F**]
time = 0.24, size = 0, normalized size = 0.00

$$\int \left(\csc^3\left(xb+a\right)\right)\left(\sin^m\left(2xb+2a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x)
[Out] int(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="maxima")
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*csc(b*x + a)^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^m (2a + 2bx) \csc^3 (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sin(2*b*x+2*a)**m,x)
[Out] Integral(sin(2*a + 2*b*x)**m*csc(a + b*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="giac")
[Out] integrate(sin(2*b*x + 2*a)^m*csc(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(2\,a+2\,b\,x\right)^m}{\sin\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*a + 2*b*x)^m/sin(a + b*x)^3,x)
[Out] int(sin(2*a + 2*b*x)^m/sin(a + b*x)^3, x)
```

3.129 $\int \cos(a+bx)\sin^7(2a+2bx)\,dx$

Optimal. Leaf size=61

$$-\frac{128\cos^9(a+bx)}{9b}+\frac{384\cos^{11}(a+bx)}{11b}-\frac{384\cos^{13}(a+bx)}{13b}+\frac{128\cos^{15}(a+bx)}{15b}$$

```
[Out] -128/9*cos(b*x+a)^9/b+384/11*cos(b*x+a)^11/b-384/13*cos(b*x+a)^13/b+128/15*
cos(b*x+a)^15/b
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4372, 2645, 276}

$$\frac{128\cos^{15}(a+bx)}{15b}-\frac{384\cos^{13}(a+bx)}{13b}+\frac{384\cos^{11}(a+bx)}{11b}-\frac{128\cos^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (-128*Cos[a + b*x]^9)/(9*b) + (384*Cos[a + b*x]^11)/(11*b) - (384*Cos[a + b
*x]^13)/(13*b) + (128*Cos[a + b*x]^15)/(15*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos(a+bx)\sin^7(2a+2bx)\,dx = 128\int \cos^8(a+bx)\sin^7(a+bx)\,dx$$
$$= -\frac{128\text{Subst}\left(\int x^8(1-x^2)^3\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{128\text{Subst}\left(\int (x^8-3x^{10}+3x^{12}-x^{14})\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{128\cos^9(a+bx)}{9b}+\frac{384\cos^{11}(a+bx)}{11b}-\frac{384\cos^{13}(a+bx)}{13b}+\frac{128\cos^{15}}{15}$$

Mathematica [A]
time = 0.49, size = 47, normalized size = 0.77

$$\frac{4\cos^9(a+bx)(-8330+10755\cos(2(a+bx))-3366\cos(4(a+bx))+429\cos(6(a+bx)))}{6435b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^7,x]
```

```
[Out] (4*Cos[a + b*x]^9*(-8330 + 10755*Cos[2*(a + b*x)] - 3366*Cos[4*(a + b*x)] +
 429*Cos[6*(a + b*x)]))/(6435*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(53) = 106.
time = 0.22, size = 111, normalized size = 1.82

method	result
default	$-\frac{35\cos(xb+a)}{128b}-\frac{35\cos(3xb+3a)}{384b}+\frac{21\cos(5xb+5a)}{640b}+\frac{3\cos(7xb+7a)}{128b}-\frac{7\cos(9xb+9a)}{1152b}-\frac{7\cos(11xb+11a)}{1408b}+\frac{\cos(13xb+1}{1664b}$
risch	$-\frac{35\cos(xb+a)}{128b}-\frac{35\cos(3xb+3a)}{384b}+\frac{21\cos(5xb+5a)}{640b}+\frac{3\cos(7xb+7a)}{128b}-\frac{7\cos(9xb+9a)}{1152b}-\frac{7\cos(11xb+11a)}{1408b}+\frac{\cos(13xb+1}{1664b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -35/128*cos(b*x+a)/b-35/384*cos(3*b*x+3*a)/b+21/640*cos(5*b*x+5*a)/b+3/128*
cos(7*b*x+7*a)/b-7/1152*cos(9*b*x+9*a)/b-7/1408*cos(11*b*x+11*a)/b+1/1664*c
os(13*b*x+13*a)/b+1/1920*cos(15*b*x+15*a)/b
```

Maxima [A]
time = 0.29, size = 91, normalized size = 1.49

$$\frac{429\cos(15bx+15a)+495\cos(13bx+13a)-4095\cos(11bx+11a)-5005\cos(9bx+9a)+19305\cos(7bx+7a)+27027\cos(5bx+5a)-75075\cos(3bx+3a)-225225\cos(bx+a)}{823680b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="maxima")
```

```
[Out] 1/823680*(429*cos(15*b*x + 15*a) + 495*cos(13*b*x + 13*a) - 4095*cos(11*b*x
 + 11*a) - 5005*cos(9*b*x + 9*a) + 19305*cos(7*b*x + 7*a) + 27027*cos(5*b*x
 + 5*a) - 75075*cos(3*b*x + 3*a) - 225225*cos(b*x + a))/b
```

Fricas [A]
time = 2.85, size = 46, normalized size = 0.75

$$\frac{128\left(429\cos\left(bx+a\right)^{15}-1485\cos\left(bx+a\right)^{13}+1755\cos\left(bx+a\right)^{11}-715\cos\left(bx+a\right)^{9}\right)}{6435\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="fricas")
```

```
[Out] 128/6435*(429*cos(b*x + a)^15 - 1485*cos(b*x + a)^13 + 1755*cos(b*x + a)^11
 - 715*cos(b*x + a)^9)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(53) = 106$.
time = 33.36, size = 270, normalized size = 4.43

$$\begin{cases} -\frac{1241\sin\left(a+bx\right)\sin^{7}\left(2a+2bx\right)}{6435b}-\frac{376\sin\left(a+bx\right)\sin^{5}\left(2a+2bx\right)\cos^{2}\left(2a+2bx\right)}{715b}-\frac{640\sin\left(a+bx\right)\sin^{3}\left(2a+2bx\right)\cos^{4}\left(2a+2bx\right)}{1287b}-\frac{1024\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos^{6}\left(2a+2bx\right)}{6435b}-\frac{3838\sin^{6}\left(2a+2bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{6435b}-\frac{1648\sin^{4}\left(2a+2bx\right)\cos\left(a+bx\right)\cos^{3}\left(2a+2bx\right)}{1287b}-\frac{768\sin^{2}\left(2a+2bx\right)\cos\left(a+bx\right)\cos^{5}\left(2a+2bx\right)}{715b}-\frac{2048\cos\left(a+bx\right)\cos^{7}\left(2a+2bx\right)}{6435b} & \text{for}\: b\neq 0 \\ x\sin^{7}\left(2a\right)\cos\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**7,x)
```

```
[Out] Piecewise((-1241*sin(a + b*x)*sin(2*a + 2*b*x)**7/(6435*b) - 376*sin(a + b*
x)*sin(2*a + 2*b*x)**5*cos(2*a + 2*b*x)**2/(715*b) - 640*sin(a + b*x)*sin(2
*a + 2*b*x)**3*cos(2*a + 2*b*x)**4/(1287*b) - 1024*sin(a + b*x)*sin(2*a + 2
*b*x)*cos(2*a + 2*b*x)**6/(6435*b) - 3838*sin(2*a + 2*b*x)**6*cos(a + b*x)*
cos(2*a + 2*b*x)/(6435*b) - 1648*sin(2*a + 2*b*x)**4*cos(a + b*x)*cos(2*a +
 2*b*x)**3/(1287*b) - 768*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)
**5/(715*b) - 2048*cos(a + b*x)*cos(2*a + 2*b*x)**7/(6435*b), Ne(b, 0)), (x
*sin(2*a)**7*cos(a), True))
```

Giac [A]
time = 0.41, size = 46, normalized size = 0.75

$$\frac{128\left(429\cos\left(bx+a\right)^{15}-1485\cos\left(bx+a\right)^{13}+1755\cos\left(bx+a\right)^{11}-715\cos\left(bx+a\right)^{9}\right)}{6435\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^7,x, algorithm="giac")
```

```
[Out] 128/6435*(429*cos(b*x + a)^15 - 1485*cos(b*x + a)^13 + 1755*cos(b*x + a)^11
 - 715*cos(b*x + a)^9)/b
```

Mupad [B]
time = 0.03, size = 46, normalized size = 0.75

$$-\frac{-\frac{128\cos(a+b\,x)^{15}}{15}+\frac{384\cos(a+b\,x)^{13}}{13}-\frac{384\cos(a+b\,x)^{11}}{11}+\frac{128\cos(a+b\,x)^9}{9}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^7,x)
```

```
[Out] -((128*cos(a + b*x)^9)/9 - (384*cos(a + b*x)^11)/11 + (384*cos(a + b*x)^13)
/13 - (128*cos(a + b*x)^15)/15)/b
```

3.130 $\int \cos(a+bx)\sin^6(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{64\sin^7(a+bx)}{7b}-\frac{64\sin^9(a+bx)}{3b}+\frac{192\sin^{11}(a+bx)}{11b}-\frac{64\sin^{13}(a+bx)}{13b}$$

```
[Out] 64/7*sin(b*x+a)^7/b-64/3*sin(b*x+a)^9/b+192/11*sin(b*x+a)^11/b-64/13*sin(b*
x+a)^13/b
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4372, 2644, 276}

$$-\frac{64\sin^{13}(a+bx)}{13b}+\frac{192\sin^{11}(a+bx)}{11b}-\frac{64\sin^9(a+bx)}{3b}+\frac{64\sin^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^6,x]
```

```
[Out] (64*Sin[a + b*x]^7)/(7*b) - (64*Sin[a + b*x]^9)/(3*b) + (192*Sin[a + b*x]^1
1)/(11*b) - (64*Sin[a + b*x]^13)/(13*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos(a+bx)\sin^6(2a+2bx)\,dx = 64\int \cos^7(a+bx)\sin^6(a+bx)\,dx$$
$$= \frac{64\text{Subst}\left(\int x^6(1-x^2)^3\ dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{64\text{Subst}\left(\int (x^6-3x^8+3x^{10}-x^{12})\ dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{64\sin^7(a+bx)}{7b} - \frac{64\sin^9(a+bx)}{3b} + \frac{192\sin^{11}(a+bx)}{11b} - \frac{64\sin^{13}(a+bx}{13b}$$

Mathematica [A]
time = 0.36, size = 47, normalized size = 0.77

$$\frac{2(5230+6377\cos(2(a+bx))+1890\cos(4(a+bx))+231\cos(6(a+bx)))\sin^7(a+bx)}{3003b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^6,x]
[Out] (2*(5230 + 6377*Cos[2*(a + b*x)] + 1890*Cos[4*(a + b*x)] + 231*Cos[6*(a + b
*x)])*Sin[a + b*x]^7)/(3003*b)
```

Maple [A]
time = 0.32, size = 97, normalized size = 1.59

method	result	siz
default	$\frac{5\sin(xb+a)}{16b} - \frac{5\sin(3xb+3a)}{64b} - \frac{3\sin(5xb+5a)}{64b} + \frac{3\sin(7xb+7a)}{224b} + \frac{\sin(9xb+9a)}{96b} - \frac{\sin(11xb+11a)}{704b} - \frac{\sin(13xb+13a)}{832b}$	97
risch	$\frac{5\sin(xb+a)}{16b} - \frac{5\sin(3xb+3a)}{64b} - \frac{3\sin(5xb+5a)}{64b} + \frac{3\sin(7xb+7a)}{224b} + \frac{\sin(9xb+9a)}{96b} - \frac{\sin(11xb+11a)}{704b} - \frac{\sin(13xb+13a)}{832b}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^6,x,method=_RETURNVERBOSE)
[Out] 5/16*sin(b*x+a)/b-5/64*sin(3*b*x+3*a)/b-3/64/b*sin(5*b*x+5*a)+3/224/b*sin(7
*b*x+7*a)+1/96/b*sin(9*b*x+9*a)-1/704/b*sin(11*b*x+11*a)-1/832/b*sin(13*b*x
+13*a)
```

Maxima [A]
time = 0.27, size = 80, normalized size = 1.31

$$-\frac{231\sin(13\,bx+13\,a)+273\sin(11\,bx+11\,a)-2002\sin(9\,bx+9\,a)-2574\sin(7\,bx+7\,a)+9009\sin(5\,bx+5\,a)+15015\sin(3\,bx+3\,a)-60060\sin(bx+a)}{192192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="maxima")
```

```
[Out] -1/192192*(231*sin(13*b*x + 13*a) + 273*sin(11*b*x + 11*a) - 2002*sin(9*b*x
 + 9*a) - 2574*sin(7*b*x + 7*a) + 9009*sin(5*b*x + 5*a) + 15015*sin(3*b*x +
 3*a) - 60060*sin(b*x + a))/b
```

Fricas [A]
time = 3.98, size = 73, normalized size = 1.20

$$-\frac{64\left(231\,\cos\left(bx+a\right)^{12}-567\,\cos\left(bx+a\right)^{10}+371\,\cos\left(bx+a\right)^{8}-5\,\cos\left(bx+a\right)^{6}-6\,\cos\left(bx+a\right)^{4}-8\,\cos\left(bx+a\right)^{2}-16\right)\sin\left(bx+a\right)}{3003\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="fricas")
```

```
[Out] -64/3003*(231*cos(b*x + a)^12 - 567*cos(b*x + a)^10 + 371*cos(b*x + a)^8 -
5*cos(b*x + a)^6 - 6*cos(b*x + a)^4 - 8*cos(b*x + a)^2 - 16)*sin(b*x + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(53) = 106.
time = 14.95, size = 233, normalized size = 3.82

$$\begin{cases} \frac{835\sin\left(a+bx\right)\sin^{6}\left(2a+2bx\right)}{3003b}+\frac{2776\sin\left(a+bx\right)\sin^{4}\left(2a+2bx\right)\cos^{2}\left(2a+2bx\right)}{3003b}+\frac{2944\sin\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos^{4}\left(2a+2bx\right)}{3003b}+\frac{1024\sin\left(a+bx\right)\cos^{6}\left(2a+2bx\right)}{3003b}-\frac{1084\sin^{5}\left(2a+2bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{3003b}-\frac{64\sin^{3}\left(2a+2bx\right)\cos\left(a+bx\right)\cos^{3}\left(2a+2bx\right)}{143b}-\frac{512\sin\left(2a+2bx\right)\cos\left(a+bx\right)\cos^{5}\left(2a+2bx\right)}{3003b} & \text{for}\: b\neq 0\\ x\sin^{6}\left(2a\right)\cos\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**6,x)
```

```
[Out] Piecewise((835*sin(a + b*x)*sin(2*a + 2*b*x)**6/(3003*b) + 2776*sin(a + b*x
)*sin(2*a + 2*b*x)**4*cos(2*a + 2*b*x)**2/(3003*b) + 2944*sin(a + b*x)*sin(
2*a + 2*b*x)**2*cos(2*a + 2*b*x)**4/(3003*b) + 1024*sin(a + b*x)*cos(2*a +
2*b*x)**6/(3003*b) - 1084*sin(2*a + 2*b*x)**5*cos(a + b*x)*cos(2*a + 2*b*x)
/(3003*b) - 64*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**3/(143*b)
 - 512*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**5/(3003*b), Ne(b, 0)
), (x*sin(2*a)**6*cos(a), True))
```

Giac [A]
time = 0.40, size = 80, normalized size = 1.31

$$-\frac{231\,\sin\left(13\,bx+13\,a\right)+273\,\sin\left(11\,bx+11\,a\right)-2002\,\sin\left(9\,bx+9\,a\right)-2574\,\sin\left(7\,bx+7\,a\right)+9009\,\sin\left(5\,bx+5\,a\right)+15015\,\sin\left(3\,bx+3\,a\right)-60060\,\sin\left(bx+a\right)}{192192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^6,x, algorithm="giac")
```

```
[Out] -1/192192*(231*sin(13*b*x + 13*a) + 273*sin(11*b*x + 11*a) - 2002*sin(9*b*x
 + 9*a) - 2574*sin(7*b*x + 7*a) + 9009*sin(5*b*x + 5*a) + 15015*sin(3*b*x +
 3*a) - 60060*sin(b*x + a))/b
```

Mupad [B]
time = 0.13, size = 45, normalized size = 0.74

$$\frac{-\frac{64 \sin(a+b\,x)^{13}}{13}+\frac{192 \sin(a+b\,x)^{11}}{11}-\frac{64 \sin(a+b\,x)^{9}}{3}+\frac{64 \sin(a+b\,x)^{7}}{7}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^6,x)
```

```
[Out] ((64*sin(a + b*x)^7)/7 - (64*sin(a + b*x)^9)/3 + (192*sin(a + b*x)^11)/11 -
 (64*sin(a + b*x)^13)/13)/b
```

3.131 $\int \cos(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{32\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{9b}-\frac{32\cos^{11}(a+bx)}{11b}$$

```
[Out] -32/7*cos(b*x+a)^7/b+64/9*cos(b*x+a)^9/b-32/11*cos(b*x+a)^11/b
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4372, 2645, 276}

$$-\frac{32\cos^{11}(a+bx)}{11b}+\frac{64\cos^9(a+bx)}{9b}-\frac{32\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-32*Cos[a + b*x]^7)/(7*b) + (64*Cos[a + b*x]^9)/(9*b) - (32*Cos[a + b*x]^1
1)/(11*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^6(a+bx)\sin^5(a+bx)\,dx\\ &= -\frac{32\text{Subst}\left(\int x^6(1-x^2)^2\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{32\text{Subst}\left(\int (x^6-2x^8+x^{10})\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{32\cos^7(a+bx)}{7b}+\frac{64\cos^9(a+bx)}{9b}-\frac{32\cos^{11}(a+bx)}{11b}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 37, normalized size = 0.80

$$\frac{4\cos^7(a+bx)(-365+364\cos(2(a+bx))-63\cos(4(a+bx)))}{693b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*Cos[a + b*x]^7*(-365 + 364*Cos[2*(a + b*x)] - 63*Cos[4*(a + b*x)]))/(693
*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(40) = 80$.
time = 0.15, size = 83, normalized size = 1.80

method	result	size
default	$-\frac{5\cos(xb+a)}{16b}-\frac{5\cos(3xb+3a)}{48b}+\frac{\cos(5xb+5a)}{32b}+\frac{5\cos(7xb+7a)}{224b}-\frac{\cos(9xb+9a)}{288b}-\frac{\cos(11xb+11a)}{352b}$	83
risch	$-\frac{5\cos(xb+a)}{16b}-\frac{5\cos(3xb+3a)}{48b}+\frac{\cos(5xb+5a)}{32b}+\frac{5\cos(7xb+7a)}{224b}-\frac{\cos(9xb+9a)}{288b}-\frac{\cos(11xb+11a)}{352b}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -5/16*cos(b*x+a)/b-5/48*cos(3*b*x+3*a)/b+1/32*cos(5*b*x+5*a)/b+5/224*cos(7*
b*x+7*a)/b-1/288*cos(9*b*x+9*a)/b-1/352*cos(11*b*x+11*a)/b
```

Maxima [A]
time = 0.27, size = 69, normalized size = 1.50

$$-\frac{63\cos(11\,bx+11\,a)+77\cos(9\,bx+9\,a)-495\cos(7\,bx+7\,a)-693\cos(5\,bx+5\,a)+2310\cos(3\,bx+3\,a)+6930\cos(bx+a)}{22176\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/22176*(63*cos(11*b*x + 11*a) + 77*cos(9*b*x + 9*a) - 495*cos(7*b*x + 7*a
) - 693*cos(5*b*x + 5*a) + 2310*cos(3*b*x + 3*a) + 6930*cos(b*x + a))/b
```

Fricas [A]
time = 2.63, size = 36, normalized size = 0.78

$$-\frac{32 \left(63 \cos \left(bx+a\right)^{11}-154 \cos \left(bx+a\right)^{9}+99 \cos \left(bx+a\right)^{7}\right)}{693 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -32/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(39) = 78$.
time = 6.47, size = 199, normalized size = 4.33

$$\begin{cases} - \frac{151 \sin{\left(a+bx \right)} \sin^{5}{\left(2a+2bx \right)}}{693b} - \frac{272 \sin{\left(a+bx \right)} \sin^{3}{\left(2a+2bx \right)} \cos^{2}{\left(2a+2bx \right)}}{693b} - \frac{128 \sin{\left(a+bx \right)} \sin{\left(2a+2bx \right)} \cos^{4}{\left(2a+2bx \right)}}{693b} - \frac{422 \sin^{4}{\left(2a+2bx \right)} \cos{\left(a+bx \right)} \cos{\left(2a+2bx \right)}}{693b} - \frac{608 \sin^{2}{\left(2a+2bx \right)} \cos{\left(a+bx \right)} \cos^{3}{\left(2a+2bx \right)}}{693b} - \frac{256 \cos{\left(a+bx \right)} \cos^{5}{\left(2a+2bx \right)}}{693b} & \text{for}\: b \neq 0 \\ x \sin^{5}{\left(2a \right)} \cos{\left(a \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**5,x)
```

```
[Out] Piecewise((-151*sin(a + b*x)*sin(2*a + 2*b*x)**5/(693*b) - 272*sin(a + b*x)
*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**2/(693*b) - 128*sin(a + b*x)*sin(2*a
 + 2*b*x)*cos(2*a + 2*b*x)**4/(693*b) - 422*sin(2*a + 2*b*x)**4*cos(a + b*x
)*cos(2*a + 2*b*x)/(693*b) - 608*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a +
 2*b*x)**3/(693*b) - 256*cos(a + b*x)*cos(2*a + 2*b*x)**5/(693*b), Ne(b, 0)
), (x*sin(2*a)**5*cos(a), True))
```

Giac [A]
time = 0.40, size = 36, normalized size = 0.78

$$-\frac{32 \left(63 \cos \left(bx+a\right)^{11}-154 \cos \left(bx+a\right)^{9}+99 \cos \left(bx+a\right)^{7}\right)}{693 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] -32/693*(63*cos(b*x + a)^11 - 154*cos(b*x + a)^9 + 99*cos(b*x + a)^7)/b
```

Mupad [B]
time = 0.15, size = 36, normalized size = 0.78

$$-\frac{32\,\left(63\,{\cos\left(a+b\,x\right)}^{11}-154\,{\cos\left(a+b\,x\right)}^9+99\,{\cos\left(a+b\,x\right)}^7\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^5,x)
[Out] -(32*(99*cos(a + b*x)^7 - 154*cos(a + b*x)^9 + 63*cos(a + b*x)^11))/(693*b)
```

3.132 $\int \cos(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{16\sin^5(a+bx)}{5b}-\frac{32\sin^7(a+bx)}{7b}+\frac{16\sin^9(a+bx)}{9b}$$

```
[Out] 16/5*sin(b*x+a)^5/b-32/7*sin(b*x+a)^7/b+16/9*sin(b*x+a)^9/b
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4372, 2644, 276}

$$\frac{16\sin^9(a+bx)}{9b}-\frac{32\sin^7(a+bx)}{7b}+\frac{16\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (16*Sin[a + b*x]^5)/(5*b) - (32*Sin[a + b*x]^7)/(7*b) + (16*Sin[a + b*x]^9)
/(9*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos(a+bx)\sin^4(2a+2bx)\,dx = 16\int \cos^5(a+bx)\sin^4(a+bx)\,dx$$
$$= \frac{16\text{Subst}\left(\int x^4(1-x^2)^2\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{16\text{Subst}\left(\int (x^4-2x^6+x^8)\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{16\sin^5(a+bx)}{5b} - \frac{32\sin^7(a+bx)}{7b} + \frac{16\sin^9(a+bx)}{9b}$$

Mathematica [A]
time = 0.16, size = 37, normalized size = 0.80

$$\frac{2(249+220\cos(2(a+bx))+35\cos(4(a+bx)))\sin^5(a+bx)}{315b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^4,x]
[Out] (2*(249 + 220*Cos[2*(a + b*x)] + 35*Cos[4*(a + b*x)])*Sin[a + b*x]^5)/(315*
b)
```

Maple [A]
time = 0.19, size = 69, normalized size = 1.50

method	result	size
default	$\frac{3\sin(xb+a)}{8b}-\frac{\sin(3xb+3a)}{12b}-\frac{\sin(5xb+5a)}{20b}+\frac{\sin(7xb+7a)}{112b}+\frac{\sin(9xb+9a)}{144b}$	69
risch	$\frac{3\sin(xb+a)}{8b}-\frac{\sin(3xb+3a)}{12b}-\frac{\sin(5xb+5a)}{20b}+\frac{\sin(7xb+7a)}{112b}+\frac{\sin(9xb+9a)}{144b}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
[Out] 3/8*sin(b*x+a)/b-1/12*sin(3*b*x+3*a)/b-1/20/b*sin(5*b*x+5*a)+1/112/b*sin(7*
b*x+7*a)+1/144/b*sin(9*b*x+9*a)
```

Maxima [A]
time = 0.27, size = 58, normalized size = 1.26

$$\frac{35\sin(9\,bx+9\,a)+45\sin(7\,bx+7\,a)-252\sin(5\,bx+5\,a)-420\sin(3\,bx+3\,a)+1890\sin(bx+a)}{5040\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/5040*(35*sin(9*b*x + 9*a) + 45*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) -
420*sin(3*b*x + 3*a) + 1890*sin(b*x + a))/b
```

Fricas [A]
time = 2.81, size = 53, normalized size = 1.15

$$\frac{16\left(35\cos\left(bx+a\right)^{8}-50\cos\left(bx+a\right)^{6}+3\cos\left(bx+a\right)^{4}+4\cos\left(bx+a\right)^{2}+8\right)\sin\left(bx+a\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 16/315*(35*cos(b*x + a)^8 - 50*cos(b*x + a)^6 + 3*cos(b*x + a)^4 + 4*cos(b*
x + a)^2 + 8)*sin(b*x + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(39) = 78$.
time = 2.72, size = 162, normalized size = 3.52

$$\begin{cases} \frac{107\sin\left(a+bx\right)\sin^{4}\left(2a+2bx\right)}{315b}+\frac{16\sin\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos^{2}\left(2a+2bx\right)}{21b}+\frac{128\sin\left(a+bx\right)\cos^{4}\left(2a+2bx\right)}{315b}-\frac{104\sin^{3}\left(2a+2bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{315b}-\frac{64\sin\left(2a+2bx\right)\cos\left(a+bx\right)\cos^{3}\left(2a+2bx\right)}{315b} & \text{for}\ b\neq 0\\ x\sin^{4}\left(2a\right)\cos\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**4,x)
```

```
[Out] Piecewise((107*sin(a + b*x)*sin(2*a + 2*b*x)**4/(315*b) + 16*sin(a + b*x)*s
in(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/(21*b) + 128*sin(a + b*x)*cos(2*a +
2*b*x)**4/(315*b) - 104*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)/(
315*b) - 64*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**3/(315*b), Ne(b
, 0)), (x*sin(2*a)**4*cos(a), True))
```

Giac [A]
time = 0.40, size = 58, normalized size = 1.26

$$\frac{35\sin\left(9\,bx+9\,a\right)+45\sin\left(7\,bx+7\,a\right)-252\sin\left(5\,bx+5\,a\right)-420\sin\left(3\,bx+3\,a\right)+1890\sin\left(bx+a\right)}{5040\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 1/5040*(35*sin(9*b*x + 9*a) + 45*sin(7*b*x + 7*a) - 252*sin(5*b*x + 5*a) -
420*sin(3*b*x + 3*a) + 1890*sin(b*x + a))/b
```

Mupad [B]
time = 0.14, size = 36, normalized size = 0.78

$$\frac{16\left(35\sin\left(a+b\,x\right)^{9}-90\sin\left(a+b\,x\right)^{7}+63\sin\left(a+b\,x\right)^{5}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^4,x)
[Out] (16*(63*sin(a + b*x)^5 - 90*sin(a + b*x)^7 + 35*sin(a + b*x)^9))/(315*b)
```

3.133 $\int \cos(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$-\frac{8\cos^5(a+bx)}{5b}+\frac{8\cos^7(a+bx)}{7b}$$

```
[Out] -8/5*cos(b*x+a)^5/b+8/7*cos(b*x+a)^7/b
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4372, 2645, 14}

$$\frac{8\cos^7(a+bx)}{7b}-\frac{8\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-8*Cos[a + b*x]^5)/(5*b) + (8*Cos[a + b*x]^7)/(7*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos(a+bx)\sin^3(2a+2bx)\,dx = 8\int \cos^4(a+bx)\sin^3(a+bx)\,dx$$
$$= -\frac{8\text{Subst}\left(\int x^4(1-x^2)\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{8\text{Subst}\left(\int (x^4-x^6)\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{8\cos^5(a+bx)}{5b} + \frac{8\cos^7(a+bx)}{7b}$$

Mathematica [A]
time = 0.11, size = 27, normalized size = 0.87

$$\frac{4\cos^5(a+bx)(-9+5\cos(2(a+bx)))}{35b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^3,x]
[Out] (4*Cos[a + b*x]^5*(-9 + 5*Cos[2*(a + b*x)]))/(35*b)
```

Maple [A]
time = 0.11, size = 55, normalized size = 1.77

method	result	size
default	$-\frac{3\cos(xb+a)}{8b}-\frac{\cos(3xb+3a)}{8b}+\frac{\cos(5xb+5a)}{40b}+\frac{\cos(7xb+7a)}{56b}$	55
risch	$-\frac{3\cos(xb+a)}{8b}-\frac{\cos(3xb+3a)}{8b}+\frac{\cos(5xb+5a)}{40b}+\frac{\cos(7xb+7a)}{56b}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
[Out] -3/8*cos(b*x+a)/b-1/8*cos(3*b*x+3*a)/b+1/40*cos(5*b*x+5*a)/b+1/56*cos(7*b*x
+7*a)/b
```

Maxima [A]
time = 0.27, size = 47, normalized size = 1.52

$$\frac{5\,\cos\left(7\,bx+7\,a\right)+7\,\cos\left(5\,bx+5\,a\right)-35\,\cos\left(3\,bx+3\,a\right)-105\,\cos\left(bx+a\right)}{280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

[Out] 1/280*(5*cos(7*b*x + 7*a) + 7*cos(5*b*x + 5*a) - 35*cos(3*b*x + 3*a) - 105*cos(b*x + a))/b

Fricas [A]
time = 2.87, size = 26, normalized size = 0.84

$$\frac{8\left(5\cos\left(bx+a\right)^{7}-7\cos\left(bx+a\right)^{5}\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="fricas")

[Out] 8/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(26) = 52$.
time = 1.07, size = 128, normalized size = 4.13

$$\begin{cases} -\frac{9\sin\left(a+bx\right)\sin^{3}\left(2a+2bx\right)}{35b}-\frac{8\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos^{2}\left(2a+2bx\right)}{35b}-\frac{22\sin^{2}\left(2a+2bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{35b}-\frac{16\cos\left(a+bx\right)\cos^{3}\left(2a+2bx\right)}{35b} & \text{for}\ b\neq 0\\ x\sin^{3}\left(2a\right)\cos\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**3,x)

[Out] Piecewise((-9*sin(a + b*x)*sin(2*a + 2*b*x)**3/(35*b) - 8*sin(a + b*x)*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/(35*b) - 22*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)/(35*b) - 16*cos(a + b*x)*cos(2*a + 2*b*x)**3/(35*b), Ne(b, 0)), (x*sin(2*a)**3*cos(a), True))

Giac [A]
time = 0.52, size = 26, normalized size = 0.84

$$\frac{8\left(5\cos\left(bx+a\right)^{7}-7\cos\left(bx+a\right)^{5}\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^3,x, algorithm="giac")

[Out] 8/35*(5*cos(b*x + a)^7 - 7*cos(b*x + a)^5)/b

Mupad [B]
time = 0.03, size = 26, normalized size = 0.84

$$-\frac{8\left(7\cos\left(a+b\,x\right)^{5}-5\cos\left(a+b\,x\right)^{7}\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^3,x)

[Out] -(8*(7*cos(a + b*x)^5 - 5*cos(a + b*x)^7))/(35*b)

3.134 $\int \cos(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=31

$$\frac{4\sin^3(a+bx)}{3b}-\frac{4\sin^5(a+bx)}{5b}$$

```
[Out] 4/3*sin(b*x+a)^3/b-4/5*sin(b*x+a)^5/b
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4372, 2644, 14}

$$\frac{4\sin^3(a+bx)}{3b}-\frac{4\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (4*Sin[a + b*x]^3)/(3*b) - (4*Sin[a + b*x]^5)/(5*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \cos(a+bx)\sin^2(2a+2bx)\,dx &= 4\int \cos^3(a+bx)\sin^2(a+bx)\,dx \\
&= \frac{4\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx, x, \sin(a+bx)\right)}{b} \\
&= \frac{4\text{Subst}\left(\int \left(x^2-x^4\right)\,dx, x, \sin(a+bx)\right)}{b} \\
&= \frac{4\sin^3(a+bx)}{3b} - \frac{4\sin^5(a+bx)}{5b}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 27, normalized size = 0.87

$$\frac{2(7+3\cos(2(a+bx)))\sin^3(a+bx)}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (2*(7 + 3*Cos[2*(a + b*x)])*Sin[a + b*x]^3)/(15*b)
```

Maple [A]
time = 0.19, size = 41, normalized size = 1.32

method	result
default	$\frac{\sin(xb+a)}{2b}-\frac{\sin(3xb+3a)}{12b}-\frac{\sin(5xb+5a)}{20b}$
risch	$\frac{\sin(xb+a)}{2b}-\frac{\sin(3xb+3a)}{12b}-\frac{\sin(5xb+5a)}{20b}$
norman	$\frac{\frac{16\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{15b}-\frac{8\tan(xb+a)}{15b}+\frac{8\left(\tan^3(xb+a)\right)}{15b}+\frac{8\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan^2(xb+a)\right)}{5b}+\frac{16\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan^4(xb+a)\right)}{15b}+\frac{8\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)\right)\tan(xb+a)}{15b}-\frac{8(\text{t}}{}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)\right)\left(1+\tan^2(xb+a)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(b*x+a)/b-1/12*sin(3*b*x+3*a)/b-1/20/b*sin(5*b*x+5*a)
```

Maxima [A]
time = 0.28, size = 36, normalized size = 1.16

$$-\frac{3\,\sin\left(5\,bx+5\,a\right)+5\,\sin\left(3\,bx+3\,a\right)-30\,\sin\left(bx+a\right)}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/60*(3*sin(5*b*x + 5*a) + 5*sin(3*b*x + 3*a) - 30*sin(b*x + a))/b
```

Fricas [A]
time = 1.96, size = 33, normalized size = 1.06

$$-\frac{4\left(3\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}-2\right)\sin\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -4/15*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)*sin(b*x + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(26) = 52$.
time = 0.40, size = 90, normalized size = 2.90

$$\begin{cases} \frac{7\sin\left(a+bx\right)\sin^{2}\left(2a+2bx\right)}{15b}+\frac{8\sin\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{15b}-\frac{4\sin\left(2a+2bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{15b} & \text{for}\ b\neq 0\\ x\sin^{2}\left(2a\right)\cos\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((7*sin(a + b*x)*sin(2*a + 2*b*x)**2/(15*b) + 8*sin(a + b*x)*cos(2
*a + 2*b*x)**2/(15*b) - 4*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)/(1
5*b), Ne(b, 0)), (x*sin(2*a)**2*cos(a), True))
```

Giac [A]
time = 0.42, size = 36, normalized size = 1.16

$$-\frac{3\sin\left(5\,bx+5\,a\right)+5\sin\left(3\,bx+3\,a\right)-30\sin\left(bx+a\right)}{60\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/60*(3*sin(5*b*x + 5*a) + 5*sin(3*b*x + 3*a) - 30*sin(b*x + a))/b
```

Mupad [B]
time = 0.03, size = 26, normalized size = 0.84

$$\frac{4\left(5\sin\left(a+b\,x\right)^{3}-3\sin\left(a+b\,x\right)^{5}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^2,x)
```

```
[Out] (4*(5*sin(a + b*x)^3 - 3*sin(a + b*x)^5))/(15*b)
```

3.135 $\int \cos(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=30

$$-\frac{\cos(a+bx)}{2b}-\frac{\cos(3a+3bx)}{6b}$$

```
[Out] -1/2*cos(b*x+a)/b-1/6*cos(3*b*x+3*a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {4369}

$$-\frac{\cos(a+bx)}{2b}-\frac{\cos(3a+3bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x],x]
```

```
[Out] -1/2*Cos[a + b*x]/b - Cos[3*a + 3*b*x]/(6*b)
```

Rule 4369

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(a+bx)\sin(2a+2bx)\,dx=-\frac{\cos(a+bx)}{2b}-\frac{\cos(3a+3bx)}{6b}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 0.50

$$-\frac{2\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*Cos[a + b*x]^3)/(3*b)
```

Maple [A]
time = 0.09, size = 27, normalized size = 0.90

method	result	size
default	$-\frac{\cos(xb+a)}{2b}-\frac{\cos(3xb+3a)}{6b}$	27
risch	$-\frac{\cos(xb+a)}{2b}-\frac{\cos(3xb+3a)}{6b}$	27
norman	$\frac{-\frac{4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)}{3b}+\frac{4\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)\right)}{3b}+\frac{4\left(\tan^2(xb+a)\right)}{3b}}{\left(1+\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)\right)(1+\tan^2(xb+a))}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cos(b*x+a)/b-1/6*cos(3*b*x+3*a)/b
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.87

$$-\frac{\cos(3\,bx+3\,a)}{6\,b}-\frac{\cos(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/6*cos(3*b*x + 3*a)/b - 1/2*cos(b*x + a)/b
```

Fricas [A]
time = 2.21, size = 13, normalized size = 0.43

$$-\frac{2\,\cos(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -2/3*cos(b*x + a)^3/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(24) = 48$.
time = 0.16, size = 53, normalized size = 1.77

$$\begin{cases} -\frac{\sin(a+bx)\sin(2a+2bx)}{3b}-\frac{2\cos(a+bx)\cos(2a+2bx)}{3b} & \text{for}\ b\neq 0\\ x\sin(2a)\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-sin(a + b*x)*sin(2*a + 2*b*x)/(3*b) - 2*cos(a + b*x)*cos(2*a +
2*b*x)/(3*b), Ne(b, 0)), (x*sin(2*a)*cos(a), True))
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.43

$$-\frac{2\cos\left(bx+a\right)^{3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -2/3*cos(b*x + a)^3/b
```

Mupad [B]
time = 0.19, size = 43, normalized size = 1.43

$$\begin{cases} x\,\left(2\,\sin\left(a\right)-2\,\sin\left(a\right)^3\right) & \text{if}\ \ b=0\\ -\frac{3\,\cos\left(a+b\,x\right)+\cos\left(3\,a+3\,b\,x\right)}{6\,b} & \text{if}\ \ b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x),x)
```

```
[Out] piecewise(b == 0, x*(2*sin(a) - 2*sin(a)^3), b ~= 0, -(3*cos(a + b*x) + cos
(3*a + 3*b*x))/(6*b))
```

3.136 $\int \cos(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=14

$$-\frac{\tanh^{-1}(\cos(a+bx))}{2b}$$

```
[Out] -1/2*arctanh(cos(b*x+a))/b
```

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4372, 3855}

$$-\frac{\tanh^{-1}(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x],x]
```

```
[Out] -1/2*ArcTanh[Cos[a + b*x]]/b
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\csc(2a+2bx)\,dx &= \frac{1}{2}\int \csc(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cos(a+bx))}{2b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(14) = 28.
time = 0.02, size = 42, normalized size = 3.00

$$\frac{1}{2}\left(-\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}+\frac{\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x],x]
```

```
[Out] (-(Log[Cos[a/2 + (b*x)/2]]/b) + Log[Sin[a/2 + (b*x)/2]]/b)/2
```

Maple [A]
time = 0.14, size = 22, normalized size = 1.57

method	result	size
default	$\frac{\ln(\csc(xb+a)-\cot(xb+a))}{2b}$	22
risch	$-\frac{\ln\left(e^{i(xb+a)}+1\right)}{2b}+\frac{\ln\left(e^{i(xb+a)}-1\right)}{2b}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(12) = 24$.
time = 0.29, size = 84, normalized size = 6.00

$$-\frac{\log\left(\cos\left(bx\right)^2+2\,\cos\left(bx\right)\cos\left(a\right)+\cos\left(a\right)^2+\sin\left(bx\right)^2-2\,\sin\left(bx\right)\sin\left(a\right)+\sin\left(a\right)^2\right)-\log\left(\cos\left(bx\right)^2-2\,\cos\left(bx\right)\cos\left(a\right)+\cos\left(a\right)^2+\sin\left(bx\right)^2+2\,\sin\left(bx\right)\sin\left(a\right)+\sin\left(a\right)^2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/4*(log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*
x)*sin(a) + sin(a)^2) - log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin
(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(12) = 24$.
time = 2.64, size = 30, normalized size = 2.14

$$-\frac{\log\left(\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -1/4*(log(1/2*cos(b*x + a) + 1/2) - log(-1/2*cos(b*x + a) + 1/2))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(12) = 24$.
time = 0.43, size = 28, normalized size = 2.00

$$-\frac{\log\left(\cos\left(bx+a\right)+1\right)-\log\left(-\cos\left(bx+a\right)+1\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/4*(log(cos(b*x + a) + 1) - log(-cos(b*x + a) + 1))/b
```

Mupad [**B**]
time = 0.02, size = 12, normalized size = 0.86

$$-\frac{\mathrm{atanh}(\cos\left(a+b\,x\right))}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x),x)
```

```
[Out] -atanh(cos(a + b*x))/(2*b)
```

3.137 $\int \cos(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=28

$$\frac{\tanh^{-1}(\sin(a+bx))}{4b}-\frac{\csc(a+bx)}{4b}$$

```
[Out] 1/4*arctanh(sin(b*x+a))/b-1/4*csc(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4372, 2701, 327, 213}

$$\frac{\tanh^{-1}(\sin(a+bx))}{4b}-\frac{\csc(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(4*b) - Csc[a + b*x]/(4*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
```

```
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \cos(a+bx)\csc^2(2a+2bx)\,dx &= \frac{1}{4}\int \csc^2(a+bx)\sec(a+bx)\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{4b} \\
&= -\frac{\csc(a+bx)}{4b} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{4b} \\
&= \frac{\tanh^{-1}(\sin(a+bx))}{4b} - \frac{\csc(a+bx)}{4b}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 29, normalized size = 1.04

$$-\frac{\csc(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^2,x]
[Out] -1/4*(Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b
```

Maple [A]
time = 0.16, size = 31, normalized size = 1.11

method	result	size
default	$\frac{-\frac{1}{\sin(xb+a)}+\ln(\sec(xb+a)+\tan(xb+a))}{4b}$	31
risch	$-\frac{i e^{i(xb+a)}}{2b\left(e^{2i(xb+a)}-1\right)}+\frac{\ln\left(i+e^{i(xb+a)}\right)}{4b}-\frac{\ln\left(e^{i(xb+a)}-i\right)}{4b}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
[Out] 1/4/b*(-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(24) = 48.
time = 0.52, size = 233, normalized size = 8.32

$$-\frac{\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2-2\cos(2bx+2a)+1\right)\log\left(\frac{\cos(bx+2a)^2+\cos(a)^2-2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2+2\cos(bx+2a)\sin(a)+\sin(a)^2}{\cos(bx+2a)^2+\cos(a)^2+2\cos(a)\sin(bx+2a)+\sin(bx+2a)^2-2\cos(bx+2a)\sin(a)+\sin(a)^2}\right)+4\cos(bx+a)\sin(2bx+2a)-4\cos(2bx+2a)\sin(bx+a)+4\sin(bx+a)}{8\left(b\cos(2bx+2a)^2+b\sin(2bx+2a)^2-2b\cos(2bx+2a)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*lo
g((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2
 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos
(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)
) + 4*cos(b*x + a)*sin(2*b*x + 2*a) - 4*cos(2*b*x + 2*a)*sin(b*x + a) + 4*s
in(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x +
 2*a) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(24) = 48$.
time = 3.34, size = 50, normalized size = 1.79

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-2}{8\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(log(sin(b*x + a) + 1)*sin(b*x + a) - log(-sin(b*x + a) + 1)*sin(b*x +
a) - 2)/(b*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 38, normalized size = 1.36

$$-\frac{\frac{2}{\sin\left(bx+a\right)}-\log\left(\sin\left(bx+a\right)+1\right)+\log\left(-\sin\left(bx+a\right)+1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b
```

Mupad [B]
time = 0.02, size = 26, normalized size = 0.93

$$\frac{\operatorname{atanh}(\sin(a+b\,x))}{4\,b}-\frac{1}{4\,b\,\sin(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^2,x)
[Out] atanh(sin(a + b*x))/(4*b) - 1/(4*b*sin(a + b*x))
```

3.138 $\int \cos(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=49

$$-\frac{3\tanh^{-1}(\cos(a+bx))}{16b}+\frac{3\sec(a+bx)}{16b}-\frac{\csc^2(a+bx)\sec(a+bx)}{16b}$$

```
[Out] -3/16*arctanh(cos(b*x+a))/b+3/16*sec(b*x+a)/b-1/16*csc(b*x+a)^2*sec(b*x+a)/
b
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {4372, 2702, 294, 327, 213}

$$\frac{3\sec(a+bx)}{16b}-\frac{3\tanh^{-1}(\cos(a+bx))}{16b}-\frac{\csc^2(a+bx)\sec(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Cos[a + b*x]])/(16*b) + (3*Sec[a + b*x])/(16*b) - (Csc[a + b*x]
^2*Sec[a + b*x])/(16*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \cos(a+bx)\csc^3(2a+2bx)\,dx &= \frac{1}{8}\int \csc^3(a+bx)\sec^2(a+bx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{8b}\\
&= -\frac{\csc^2(a+bx)\sec(a+bx)}{16b}+\frac{3\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(a+bx)\right)}{16b}\\
&= \frac{3\sec(a+bx)}{16b}-\frac{\csc^2(a+bx)\sec(a+bx)}{16b}+\frac{3\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(a\right.}{16b}\\
&= -\frac{3\tanh^{-1}(\cos(a+bx))}{16b}+\frac{3\sec(a+bx)}{16b}-\frac{\csc^2(a+bx)\sec(a+bx)}{16b}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 143 vs. $2(49) = 98$.
time = 0.28, size = 143, normalized size = 2.92

$$
\frac{\csc^4(a+bx)\left(2-6\cos(2(a+bx))+2\cos(3(a+bx))+3\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-3\cos(3(a+bx))\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)+\cos(a+bx)\left(-2-3\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)+3\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)\right)\right)}{16b\left(\csc^2\left(\frac{1}{2}(a+bx)\right)-\sec^2\left(\frac{1}{2}(a+bx)\right)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (Csc[a + b*x]^4*(2 - 6*Cos[2*(a + b*x)] + 2*Cos[3*(a + b*x)] + 3*Cos[3*(a +
 b*x)]*Log[Cos[(a + b*x)/2]] - 3*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + C
os[a + b*x]*(-2 - 3*Log[Cos[(a + b*x)/2]] + 3*Log[Sin[(a + b*x)/2]])))/(16*
b*(Csc[(a + b*x)/2]^2 - Sec[(a + b*x)/2]^2))
```

Maple [A]
time = 0.18, size = 53, normalized size = 1.08

method	result	size
default	$\frac{-\frac{1}{2\sin(xb+a)^2\cos(xb+a)}+\frac{3}{2\cos(xb+a)}+\frac{3\ln(\csc(xb+a)-\cot(xb+a))}{2}}{8b}$	53
risch	$\frac{3\,e^{5i(xb+a)}-2\,e^{3i(xb+a)}+3\,e^{i(xb+a)}}{8b(e^{2i(xb+a)}-1)^2(e^{2i(xb+a)}+1)}+\frac{3\ln(e^{i(xb+a)}-1)}{16b}-\frac{3\ln(e^{i(xb+a)}+1)}{16b}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(-1/2/sin(b*x+a)^2/cos(b*x+a)+3/2/cos(b*x+a)+3/2*ln(csc(b*x+a)-cot(b*
x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 974 vs. $2(43) = 86$.
time = 0.29, size = 974, normalized size = 19.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(4*(3*cos(5*b*x + 5*a) - 2*cos(3*b*x + 3*a) + 3*cos(b*x + a))*cos(6*b*
x + 6*a) - 12*(cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) +
4*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 8*(cos(2*b*x + 2
*a) - 1)*cos(3*b*x + 3*a) - 12*cos(2*b*x + 2*a)*cos(b*x + a) + 3*(2*(cos(4*
b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 -
2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(2*b*x
+ 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - sin(6
*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*(2*(cos(4*
b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 -
2*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - cos(2*b*x
+ 2*a)^2 + 2*(sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - sin(6
*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 2*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(5*b
*x + 5*a) - 2*sin(3*b*x + 3*a) + 3*sin(b*x + a))*sin(6*b*x + 6*a) - 12*(sin
(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(5*b*x + 5*a) + 4*(2*sin(3*b*x + 3*a)
- 3*sin(b*x + a))*sin(4*b*x + 4*a) + 8*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) -
12*sin(2*b*x + 2*a)*sin(b*x + a) + 12*cos(b*x + a))/(b*cos(6*b*x + 6*a)^2 +
 b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2 + b*sin(6*b*x + 6*a)^2 + b*sin
(4*b*x + 4*a)^2 + 2*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*x + 2*a
)^2 - 2*(b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 2*
```

```
(b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin
(4*b*x + 4*a) + b*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) + b)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(43) = 86$.
time = 2.53, size = 96, normalized size = 1.96

$$\frac{6 \cos (bx+a)^2 - 3 \left(\cos (bx+a)^3 - \cos (bx+a)\right) \log \left(\frac{1}{2} \cos (bx+a) + \frac{1}{2}\right) + 3 \left(\cos (bx+a)^3 - \cos (bx+a)\right) \log \left(-\frac{1}{2} \cos (bx+a) + \frac{1}{2}\right) - 4}{32 \left(b \cos (bx+a)^3 - b \cos (bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(6*cos(b*x + a)^2 - 3*(cos(b*x + a)^3 - cos(b*x + a))*log(1/2*cos(b*x
+ a) + 1/2) + 3*(cos(b*x + a)^3 - cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2
) - 4)/(b*cos(b*x + a)^3 - b*cos(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.53, size = 63, normalized size = 1.29

$$\frac{\frac{2 \left(3 \cos (bx+a)^2 - 2\right)}{\cos (bx+a)^3 - \cos (bx+a)} - 3 \log (\cos (bx+a) + 1) + 3 \log (-\cos (bx+a) + 1)}{32 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(
b*x + a) + 1) + 3*log(-cos(b*x + a) + 1))/b
```

Mupad [**B**]
time = 0.08, size = 49, normalized size = 1.00

$$-\frac{3 \operatorname{atanh}(\cos (a+b\,x))}{16\,b} - \frac{\frac{3 \cos (a+b\,x)^2}{16} - \frac{1}{8}}{b\,\left(\cos (a+b\,x) - \cos (a+b\,x)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^3,x)
```

```
[Out] - (3*atanh(cos(a + b*x)))/(16*b) - ((3*cos(a + b*x)^2)/16 - 1/8)/(b*(cos(a
+ b*x) - cos(a + b*x)^3))
```

3.139 $\int \cos(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=66

$$\frac{5\tanh^{-1}(\sin(a+bx))}{32b}-\frac{5\csc(a+bx)}{32b}-\frac{5\csc^3(a+bx)}{96b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{32b}$$

```
[Out] 5/32*arctanh(sin(b*x+a))/b-5/32*csc(b*x+a)/b-5/96*csc(b*x+a)^3/b+1/32*csc(b
*x+a)^3*sec(b*x+a)^2/b
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4372, 2701, 294, 308, 213}

$$-\frac{5\csc^3(a+bx)}{96b}-\frac{5\csc(a+bx)}{32b}+\frac{5\tanh^{-1}(\sin(a+bx))}{32b}+\frac{\csc^3(a+bx)\sec^2(a+bx)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (5*ArcTanh[Sin[a + b*x]])/(32*b) - (5*Csc[a + b*x])/(32*b) - (5*Csc[a + b*x
]^3)/(96*b) + (Csc[a + b*x]^3*Sec[a + b*x]^2)/(32*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
```

```
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos(a+bx)\csc^4(2a+2bx)\,dx = \frac{1}{16}\int \csc^4(a+bx)\sec^3(a+bx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{16b}$$
$$= \frac{\csc^3(a+bx)\sec^2(a+bx)}{32b} - \frac{5\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{32b}$$
$$= \frac{\csc^3(a+bx)\sec^2(a+bx)}{32b} - \frac{5\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \csc(a+b\right.}{32b}$$
$$= -\frac{5\csc(a+bx)}{32b} - \frac{5\csc^3(a+bx)}{96b} + \frac{\csc^3(a+bx)\sec^2(a+bx)}{32b} - \frac{5\text{Subst}(}{}$$
$$= \frac{5\tanh^{-1}(\sin(a+bx))}{32b} - \frac{5\csc(a+bx)}{32b} - \frac{5\csc^3(a+bx)}{96b} + \frac{\csc^3(a+bx)}{32}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 31, normalized size = 0.47

$$-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\sin^2(a+bx)\right)}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^4,x]
[Out] -1/48*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 2, -1/2, Sin[a + b*x]^2])/b
```

Maple [A]
time = 0.19, size = 69, normalized size = 1.05

method	result	size
default	$\frac{-\frac{1}{3\sin(xb+a)^3\cos(xb+a)^2}+\frac{5}{6\sin(xb+a)\cos(xb+a)^2}-\frac{5}{2\sin(xb+a)}+\frac{5\ln(\sec(xb+a)+\tan(xb+a))}{2}}{16b}$	69
risch	$-\frac{i(15\,e^{9i(xb+a)}-20\,e^{7i(xb+a)}-22\,e^{5i(xb+a)}-20\,e^{3i(xb+a)}+15\,e^{i(xb+a)})}{48b(e^{2i(xb+a)}-1)^3(e^{2i(xb+a)}+1)^2}-\frac{5\ln(e^{i(xb+a)}-i)}{32b}+\frac{5\ln(i+e^{i(xb+a)})}{32b}$	126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/b*(-1/3/sin(b*x+a)^3/cos(b*x+a)^2+5/6/sin(b*x+a)/cos(b*x+a)^2-5/2/sin(
b*x+a)+5/2*ln(sec(b*x+a)+tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1780 vs. $2(58) = 116$.
time = 0.57, size = 1780, normalized size = 26.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/192*(4*(15*sin(9*b*x + 9*a) - 20*sin(7*b*x + 7*a) - 22*sin(5*b*x + 5*a) -
 20*sin(3*b*x + 3*a) + 15*sin(b*x + a))*cos(10*b*x + 10*a) + 60*(sin(8*b*x
+ 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(9*
b*x + 9*a) + 4*(20*sin(7*b*x + 7*a) + 22*sin(5*b*x + 5*a) + 20*sin(3*b*x +
3*a) - 15*sin(b*x + a))*cos(8*b*x + 8*a) - 80*(2*sin(6*b*x + 6*a) - 2*sin(4
*b*x + 4*a) - sin(2*b*x + 2*a))*cos(7*b*x + 7*a) + 8*(22*sin(5*b*x + 5*a) +
 20*sin(3*b*x + 3*a) - 15*sin(b*x + a))*cos(6*b*x + 6*a) + 88*(2*sin(4*b*x
+ 4*a) + sin(2*b*x + 2*a))*cos(5*b*x + 5*a) - 40*(4*sin(3*b*x + 3*a) - 3*si
n(b*x + a))*cos(4*b*x + 4*a) + 15*(2*(cos(8*b*x + 8*a) + 2*cos(6*b*x + 6*a)
 - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) - cos(10*b
*x + 10*a)^2 - 2*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a
) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 4*(2*cos(4*b*x + 4*a) + cos(
2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*a)^2 - 4*(cos(2*b*x +
2*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 - cos(2*b*x + 2*a)^2 + 2*
(sin(8*b*x + 8*a) + 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) - sin(2*b*x + 2
*a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^2 - 2*(2*sin(6*b*x + 6*a) - 2*
sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - sin(8*b*x + 8*a)^2
+ 4*(2*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x
+ 6*a)^2 - 4*sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log((cos(b*x + 2*a)^2 + cos(a)^2
- 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + si
n(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(15*cos(9*b*x + 9*a) - 20
```

```
*cos(7*b*x + 7*a) - 22*cos(5*b*x + 5*a) - 20*cos(3*b*x + 3*a) + 15*cos(b*x
+ a))*sin(10*b*x + 10*a) - 60*(cos(8*b*x + 8*a) + 2*cos(6*b*x + 6*a) - 2*co
s(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*sin(9*b*x + 9*a) - 4*(20*cos(7*b*x +
 7*a) + 22*cos(5*b*x + 5*a) + 20*cos(3*b*x + 3*a) - 15*cos(b*x + a))*sin(8*
b*x + 8*a) + 80*(2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) - cos(2*b*x + 2*a)
 + 1)*sin(7*b*x + 7*a) - 8*(22*cos(5*b*x + 5*a) + 20*cos(3*b*x + 3*a) - 15*
cos(b*x + a))*sin(6*b*x + 6*a) - 88*(2*cos(4*b*x + 4*a) + cos(2*b*x + 2*a)
- 1)*sin(5*b*x + 5*a) + 40*(4*cos(3*b*x + 3*a) - 3*cos(b*x + a))*sin(4*b*x
+ 4*a) - 80*(cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a) + 80*cos(3*b*x + 3*a)*s
in(2*b*x + 2*a) - 60*cos(b*x + a)*sin(2*b*x + 2*a) + 60*cos(2*b*x + 2*a)*si
n(b*x + a) - 60*sin(b*x + a))/(b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^
2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*cos(4*b*x + 4*a)^2 + b*cos(2*b*x + 2*a)^2
+ b*sin(10*b*x + 10*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 +
4*b*sin(4*b*x + 4*a)^2 + 4*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b*sin(2*b*
x + 2*a)^2 - 2*(b*cos(8*b*x + 8*a) + 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x +
 4*a) - b*cos(2*b*x + 2*a) + b)*cos(10*b*x + 10*a) + 2*(2*b*cos(6*b*x + 6*a
) - 2*b*cos(4*b*x + 4*a) - b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*
b*cos(4*b*x + 4*a) + b*cos(2*b*x + 2*a) - b)*cos(6*b*x + 6*a) + 4*(b*cos(2*
b*x + 2*a) - b)*cos(4*b*x + 4*a) - 2*b*cos(2*b*x + 2*a) - 2*(b*sin(8*b*x +
8*a) + 2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*si
n(10*b*x + 10*a) + 2*(2*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) - b*sin(2
*b*x + 2*a))*sin(8*b*x + 8*a) - 4*(2*b*sin(4*b*x + 4*a) + b*sin(2*b*x + 2*a
))*sin(6*b*x + 6*a) + b)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(58) = 116$.
time = 2.05, size = 130, normalized size = 1.97

$$-\frac{30\cos(bx+a)^4 - 15\left(\cos(bx+a)^4 - \cos(bx+a)^2\right)\log(\sin(bx+a)+1)\sin(bx+a) + 15\left(\cos(bx+a)^4 - \cos(bx+a)^2\right)\log(-\sin(bx+a)+1)\sin(bx+a) - 40\cos(bx+a)^2 + 6}{192\left(b\cos(bx+a)^4 - b\cos(bx+a)^2\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -1/192*(30*cos(b*x + a)^4 - 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(sin(b*
x + a) + 1)*sin(b*x + a) + 15*(cos(b*x + a)^4 - cos(b*x + a)^2)*log(-sin(b*
x + a) + 1)*sin(b*x + a) - 40*cos(b*x + a)^2 + 6)/((b*cos(b*x + a)^4 - b*co
s(b*x + a)^2)*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**4,x)
```

[Out] Timed out

Giac [A]
time = 0.44, size = 72, normalized size = 1.09

$$-\frac{\frac{6\,\sin(bx+a)}{\sin(bx+a)^2-1}+\frac{4\left(6\,\sin(bx+a)^2+1\right)}{\sin(bx+a)^3}-15\,\log\left(\sin\left(bx+a\right)+1\right)+15\,\log\left(-\sin\left(bx+a\right)+1\right)}{192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/192*(6*sin(b*x + a)/(sin(b*x + a)^2 - 1) + 4*(6*sin(b*x + a)^2 + 1)/sin(
b*x + a)^3 - 15*log(sin(b*x + a) + 1) + 15*log(-sin(b*x + a) + 1))/b
```

Mupad [B]
time = 0.10, size = 61, normalized size = 0.92

$$\frac{5\,\mathrm{atanh}(\sin\left(a+b\,x\right))}{32\,b}-\frac{-\frac{5\sin(a+b\,x)^4}{32}+\frac{5\sin(a+b\,x)^2}{48}+\frac{1}{48}}{b\,\left(\sin\left(a+b\,x\right)^3-\sin\left(a+b\,x\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^4,x)
```

```
[Out] (5*atanh(sin(a + b*x)))/(32*b) - ((5*sin(a + b*x)^2)/48 - (5*sin(a + b*x)^4
)/32 + 1/48)/(b*(sin(a + b*x)^3 - sin(a + b*x)^5))
```

3.140 $\int \cos(a+bx)\csc^5(2a+2bx)\,dx$

Optimal. Leaf size=89

$$-\frac{35\tanh^{-1}(\cos(a+bx))}{256b}+\frac{35\sec(a+bx)}{256b}+\frac{35\sec^3(a+bx)}{768b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{128b}$$

```
[Out] -35/256*arctanh(cos(b*x+a))/b+35/256*sec(b*x+a)/b+35/768*sec(b*x+a)^3/b-7/2
56*csc(b*x+a)^2*sec(b*x+a)^3/b-1/128*csc(b*x+a)^4*sec(b*x+a)^3/b
```

Rubi [A]

time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4372, 2702, 294, 308, 213}

$$\frac{35\sec^3(a+bx)}{768b}+\frac{35\sec(a+bx)}{256b}-\frac{35\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^4(a+bx)\sec^3(a+bx)}{128b}-\frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] (-35*ArcTanh[Cos[a + b*x]])/(256*b) + (35*Sec[a + b*x])/(256*b) + (35*Sec[a
 + b*x]^3)/(768*b) - (7*Csc[a + b*x]^2*Sec[a + b*x]^3)/(256*b) - (Csc[a + b
*x]^4*Sec[a + b*x]^3)/(128*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \cos(a+bx)\csc^5(2a+2bx)\,dx &= \frac{1}{32}\int \csc^5(a+bx)\sec^4(a+bx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x^8}{(-1+x^2)^3}\,dx, x, \sec(a+bx)\right)}{32b} \\
&= -\frac{\csc^4(a+bx)\sec^3(a+bx)}{128b} + \frac{7\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{128b} \\
&= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{128b} + \frac{35\text{Subst}\left(\int \ldots\right.}{} \\
&= -\frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{\csc^4(a+bx)\sec^3(a+bx)}{128b} + \frac{35\text{Subst}\left(\int (\ldots\right.}{} \\
&= \frac{35\sec(a+bx)}{256b} + \frac{35\sec^3(a+bx)}{768b} - \frac{7\csc^2(a+bx)\sec^3(a+bx)}{256b} - \frac{\csc^4(a\ldots}{} \\
&= -\frac{35\tanh^{-1}(\cos(a+bx))}{256b} + \frac{35\sec(a+bx)}{256b} + \frac{35\sec^3(a+bx)}{768b} - \frac{7\csc^2(a\ldots}{}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 268 vs. 2(89) = 178.
time = 0.54, size = 268, normalized size = 3.01

$$-\frac{\csc^{10}(a+bx)\left(-204+658\cos(2(a+bx))-228\cos(3(a+bx))+140\cos(4(a+bx))-76\cos(5(a+bx))-210\cos(6(a+bx))+76\cos(7(a+bx))-315\cos(3(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-105\cos(5(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)+105\cos(7(a+bx))\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)+3\cos(a+bx)\left(76+105\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-105\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)\right)+315\cos(3(a+bx))\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)+105\cos(5(a+bx))\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)-105\cos(7(a+bx))\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)\right)}{768b\left(\csc^2\left(\frac{1}{2}(a+bx)\right)-\sec^2\left(\frac{1}{2}(a+bx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] -1/768*(Csc[a + b*x]^10*(-204 + 658*Cos[2*(a + b*x)] - 228*Cos[3*(a + b*x)]
 + 140*Cos[4*(a + b*x)] - 76*Cos[5*(a + b*x)] - 210*Cos[6*(a + b*x)] + 76*C
```

```
os[7*(a + b*x)] - 315*Cos[3*(a + b*x)]*Log[Cos[(a + b*x)/2]] - 105*Cos[5*(a
 + b*x)]*Log[Cos[(a + b*x)/2]] + 105*Cos[7*(a + b*x)]*Log[Cos[(a + b*x)/2]]
 + 3*Cos[a + b*x]*(76 + 105*Log[Cos[(a + b*x)/2]] - 105*Log[Sin[(a + b*x)/2
]]) + 315*Cos[3*(a + b*x)]*Log[Sin[(a + b*x)/2]] + 105*Cos[5*(a + b*x)]*Log
[Sin[(a + b*x)/2]] - 105*Cos[7*(a + b*x)]*Log[Sin[(a + b*x)/2]]))/(b*(Csc[(
a + b*x)/2]^2 - Sec[(a + b*x)/2]^2)^3)
```

Maple [A]
time = 0.24, size = 89, normalized size = 1.00

method	result
default	$\frac{-\frac{1}{4\sin(xb+a)^4\cos(xb+a)^3}+\frac{7}{12\sin(xb+a)^2\cos(xb+a)^3}-\frac{35}{24\sin(xb+a)^2\cos(xb+a)}+\frac{35}{8\cos(xb+a)}+\frac{35\ln(\csc(xb+a)-\cot(xb+a))}{8}}{32b}$
risch	$\frac{105\,e^{13i(xb+a)}-70\,e^{11i(xb+a)}-329\,e^{9i(xb+a)}+204\,e^{7i(xb+a)}-329\,e^{5i(xb+a)}-70\,e^{3i(xb+a)}+105\,e^{i(xb+a)}}{384b\left(e^{2i(xb+a)}-1\right)^4\left(e^{2i(xb+a)}+1\right)^3}+\frac{35\ln\left(e^{i(xb+a)}-1\right)}{256b}-35$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)^3+7/12/sin(b*x+a)^2/cos(b*x+a)^3-35/24
/sin(b*x+a)^2/cos(b*x+a)+35/8/cos(b*x+a)+35/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3846 vs. $2(79) = 158$.
time = 0.42, size = 3846, normalized size = 43.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/1536*(4*(105*cos(13*b*x + 13*a) - 70*cos(11*b*x + 11*a) - 329*cos(9*b*x +
 9*a) + 204*cos(7*b*x + 7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) +
 105*cos(b*x + a))*cos(14*b*x + 14*a) - 420*(cos(12*b*x + 12*a) + 3*cos(10*
b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a)
+ cos(2*b*x + 2*a) - 1)*cos(13*b*x + 13*a) + 4*(70*cos(11*b*x + 11*a) + 329
*cos(9*b*x + 9*a) - 204*cos(7*b*x + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*
b*x + 3*a) - 105*cos(b*x + a))*cos(12*b*x + 12*a) + 280*(3*cos(10*b*x + 10*
a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b
*x + 2*a) - 1)*cos(11*b*x + 11*a) + 12*(329*cos(9*b*x + 9*a) - 204*cos(7*b*
x + 7*a) + 329*cos(5*b*x + 5*a) + 70*cos(3*b*x + 3*a) - 105*cos(b*x + a))*c
os(10*b*x + 10*a) - 1316*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4
*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(9*b*x + 9*a) + 12*(204*cos(7*b*x +
7*a) - 329*cos(5*b*x + 5*a) - 70*cos(3*b*x + 3*a) + 105*cos(b*x + a))*cos(8
*b*x + 8*a) + 816*(3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*
a) + 1)*cos(7*b*x + 7*a) - 84*(47*cos(5*b*x + 5*a) + 10*cos(3*b*x + 3*a) -
```

15*cos(b*x + a))*cos(6*b*x + 6*a) + 1316*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(5*b*x + 5*a) + 420*(2*cos(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 280*(cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a) - 420*cos(2*b*x + 2*a)*cos(b*x + a) + 105*(2*(cos(12*b*x + 12*a) + 3*cos(10*b*x + 10*a) - 3 *cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*x + 2 *a) - 1)*cos(14*b*x + 14*a) - cos(14*b*x + 14*a)^2 - 2*(3*cos(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b* x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 + 6*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(10*b*x + 10*a) - 9*cos(10*b*x + 10*a)^2 - 6*(3*cos(6*b*x + 6*a) - 3*cos(4*b *x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x + 8*a)^2 + 6*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 9*cos(6*b *x + 6*a)^2 - 6*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - cos(2*b*x + 2*a)^2 + 2*(sin(12*b*x + 12*a) + 3*sin(10*b*x + 10*a) - 3 *sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2 *a))*sin(14*b*x + 14*a) - sin(14*b*x + 14*a)^2 - 2*(3*sin(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 + 6*(3*sin(8*b*x + 8*a) + 3 *sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(10*b*x + 10* a) - 9*sin(10*b*x + 10*a)^2 - 6*(3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 + 6*(3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 9*sin(6*b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 - 6*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + si n(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 105*(2*(cos(12*b*x + 12*a) + 3*c os(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(14*b*x + 14*a) - cos(14*b*x + 14*a)^2 - 2 *(3*cos(10*b*x + 10*a) - 3*cos(8*b*x + 8*a) - 3*cos(6*b*x + 6*a) + 3*cos(4* b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^ 2 + 6*(3*cos(8*b*x + 8*a) + 3*cos(6*b*x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2 *b*x + 2*a) + 1)*cos(10*b*x + 10*a) - 9*cos(10*b*x + 10*a)^2 - 6*(3*cos(6*b *x + 6*a) - 3*cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9 *cos(8*b*x + 8*a)^2 + 6*(3*cos(4*b*x + 4*a) + cos(2*b*x + 2*a) - 1)*cos(6*b *x + 6*a) - 9*cos(6*b*x + 6*a)^2 - 6*(cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - cos(2*b*x + 2*a)^2 + 2*(sin(12*b*x + 12*a) + 3*s in(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(14*b*x + 14*a) - sin(14*b*x + 14*a)^2 - 2*(3* sin(10*b*x + 10*a) - 3*sin(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 + 6*(3 *sin(8*b*x + 8*a) + 3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2 *a))*sin(10*b*x + 10*a) - 9*sin(10*b*x + 10*a)^2 - 6*(3*sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 + 6*(3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 9*sin(6* b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 - 6*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*c

```
os(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(105*sin(
13*b*x + 13*a) - 70*sin(11*b*x + 11*a) - 329*sin(9*b*x + 9*a) + 204*sin(7*b
*x + 7*a) - 329*sin(5*b*x + 5*a) - 70*sin(3*b*x + 3*a) + 105*sin(b*x + a))*
sin(14*b*x + 14*a) - 420*(sin(12*b*x + 12*a) + 3*sin(10*b*x + 10*a) - 3*sin
(8*b*x + 8*a) - 3*sin(6*b*x + 6*a) + 3*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))
*sin(13*b*x + 13*a) + 4*(70*sin(11*b*x + 11*a) + 329*sin(9*b*x + 9*a) - 204
*sin(7*b*x + 7*a) + 329*sin(5*b*x + 5*a) + 70*s...
```

Fricas [A]
time = 2.14, size = 148, normalized size = 1.66

$$\frac{210 \cos(bx+a)^6 - 350 \cos(bx+a)^4 + 112 \cos(bx+a)^2 - 105 \left(\cos(bx+a)^7 - 2 \cos(bx+a)^5 + \cos(bx+a)^3\right) \log\left(\frac{1}{2} \cos(bx+a) + \frac{1}{2}\right) + 105 \left(\cos(bx+a)^7 - 2 \cos(bx+a)^5 + \cos(bx+a)^3\right) \log\left(-\frac{1}{2} \cos(bx+a) + \frac{1}{2}\right) + 16}{1536 \left(b \cos(bx+a)^7 - 2 b \cos(bx+a)^5 + b \cos(bx+a)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 1/1536*(210*cos(b*x + a)^6 - 350*cos(b*x + a)^4 + 112*cos(b*x + a)^2 - 105*
(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(1/2*cos(b*x + a) +
 1/2) + 105*(cos(b*x + a)^7 - 2*cos(b*x + a)^5 + cos(b*x + a)^3)*log(-1/2*c
os(b*x + a) + 1/2) + 16)/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b*x
 + a)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 85, normalized size = 0.96

$$\frac{\frac{6 \left(11 \cos(bx+a)^3 - 13 \cos(bx+a)\right)}{\left(\cos(bx+a)^2 - 1\right)^2} + \frac{16 \left(9 \cos(bx+a)^2 + 1\right)}{\cos(bx+a)^3} - 105 \log\left(\cos(bx+a) + 1\right) + 105 \log\left(-\cos(bx+a) + 1\right)}{1536 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/1536*(6*(11*cos(b*x + a)^3 - 13*cos(b*x + a))/(cos(b*x + a)^2 - 1)^2 + 16
*(9*cos(b*x + a)^2 + 1)/cos(b*x + a)^3 - 105*log(cos(b*x + a) + 1) + 105*lo
g(-cos(b*x + a) + 1))/b
```

Mupad [B]
time = 0.12, size = 78, normalized size = 0.88

$$\frac{\frac{35\cos(a+b\,x)^6}{256}-\frac{175\cos(a+b\,x)^4}{768}+\frac{7\cos(a+b\,x)^2}{96}+\frac{1}{96}}{b\,\left(\cos\left(a+b\,x\right)^7-2\cos\left(a+b\,x\right)^5+\cos\left(a+b\,x\right)^3\right)}-\frac{35\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^5,x)
```

```
[Out] ((7*cos(a + b*x)^2)/96 - (175*cos(a + b*x)^4)/768 + (35*cos(a + b*x)^6)/256
 + 1/96)/(b*(cos(a + b*x)^3 - 2*cos(a + b*x)^5 + cos(a + b*x)^7)) - (35*ata
nh(cos(a + b*x)))/(256*b)
```

3.141 $\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=44

$$-\frac{4\cos^8(a+bx)}{b}+\frac{32\cos^{10}(a+bx)}{5b}-\frac{8\cos^{12}(a+bx)}{3b}$$

```
[Out] -4*cos(b*x+a)^8/b+32/5*cos(b*x+a)^10/b-8/3*cos(b*x+a)^12/b
```

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4372, 2645, 272, 45}

$$-\frac{8\cos^{12}(a+bx)}{3b}+\frac{32\cos^{10}(a+bx)}{5b}-\frac{4\cos^8(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-4*Cos[a + b*x]^8)/b + (32*Cos[a + b*x]^10)/(5*b) - (8*Cos[a + b*x]^12)/(3
*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
```

```
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^5(2a+2bx)\,dx &= 32\int \cos^7(a+bx)\sin^5(a+bx)\,dx\\ &= -\frac{32\text{Subst}\left(\int x^7\left(1-x^2\right)^2\,dx,x,\cos(a+bx)\right)}{b}\\ &= -\frac{16\text{Subst}\left(\int (1-x)^2x^3\,dx,x,\cos^2(a+bx)\right)}{b}\\ &= -\frac{16\text{Subst}\left(\int \left(x^3-2x^4+x^5\right)\,dx,x,\cos^2(a+bx)\right)}{b}\\ &= -\frac{4\cos^8(a+bx)}{b}+\frac{32\cos^{10}(a+bx)}{5b}-\frac{8\cos^{12}(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.44, size = 68, normalized size = 1.55

$$-\frac{600\cos(2(a+bx))+75\cos(4(a+bx))-100\cos(6(a+bx))-30\cos(8(a+bx))+12\cos(10(a+bx))+5\cos(12(a+bx))}{3840b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] -1/3840*(600*Cos[2*(a + b*x)] + 75*Cos[4*(a + b*x)] - 100*Cos[6*(a + b*x)]
- 30*Cos[8*(a + b*x)] + 12*Cos[10*(a + b*x)] + 5*Cos[12*(a + b*x)])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(40) = 80$.
time = 0.24, size = 86, normalized size = 1.95

method	result	size
default	$-\frac{5\cos(2xb+2a)}{32b}-\frac{5\cos(4xb+4a)}{256b}+\frac{5\cos(6xb+6a)}{192b}+\frac{\cos(8xb+8a)}{128b}-\frac{\cos(10xb+10a)}{320b}-\frac{\cos(12xb+12a)}{768b}$	86
risch	$-\frac{5\cos(2xb+2a)}{32b}-\frac{5\cos(4xb+4a)}{256b}+\frac{5\cos(6xb+6a)}{192b}+\frac{\cos(8xb+8a)}{128b}-\frac{\cos(10xb+10a)}{320b}-\frac{\cos(12xb+12a)}{768b}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -5/32*cos(2*b*x+2*a)/b-5/256*cos(4*b*x+4*a)/b+5/192*cos(6*b*x+6*a)/b+1/128*
cos(8*b*x+8*a)/b-1/320*cos(10*b*x+10*a)/b-1/768*cos(12*b*x+12*a)/b
```

Maxima [A]
time = 0.26, size = 72, normalized size = 1.64

$$-\frac{5\cos(12bx+12a)+12\cos(10bx+10a)-30\cos(8bx+8a)-100\cos(6bx+6a)+75\cos(4bx+4a)+600\cos(2bx+2a)}{3840b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/3840*(5*cos(12*b*x + 12*a) + 12*cos(10*b*x + 10*a) - 30*cos(8*b*x + 8*a)
 - 100*cos(6*b*x + 6*a) + 75*cos(4*b*x + 4*a) + 600*cos(2*b*x + 2*a))/b
```

Fricas [A]
time = 2.68, size = 36, normalized size = 0.82

$$-\frac{4\left(10\cos(bx+a)^{12}-24\cos(bx+a)^{10}+15\cos(bx+a)^{8}\right)}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -4/15*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(37) = 74.
time = 15.36, size = 597, normalized size = 13.57

$$\begin{cases} -\frac{5x\sin^2(a+bx)\sin^5(2a+2bx)}{32}-\frac{5x\sin^2(a+bx)\sin^3(2a+2bx)\cos^2(2a+2bx)}{16}-\frac{5x\sin^2(a+bx)\sin(2a+2bx)\cos^4(2a+2bx)}{32}-\frac{5x\sin(a+bx)\sin^4(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{16}-\frac{5x\sin(a+bx)\sin^2(2a+2bx)\cos(a+bx)\cos^3(2a+2bx)}{8}-\frac{5x\sin(a+bx)\cos(a+bx)\cos^5(2a+2bx)}{16}+\frac{5x\sin^5(2a+2bx)\cos^2(a+bx)}{32}+\frac{5x\sin^3(2a+2bx)\cos^2(a+bx)\cos^2(2a+2bx)}{16}+\frac{5x\sin(2a+2bx)\cos^2(a+bx)\cos^4(2a+2bx)}{32}-\frac{125\sin^2(a+bx)\sin^4(2a+2bx)\cos(2a+2bx)}{384b}-\frac{2\sin^2(a+bx)\sin^2(2a+2bx)\cos^3(2a+2bx)}{3b}-\frac{217\sin^2(a+bx)\cos^5(2a+2bx)}{640b}+\frac{95\sin(a+bx)\sin^5(2a+2bx)\cos(a+bx)}{192b}+\frac{13\sin(a+bx)\sin^3(2a+2bx)\cos(a+bx)\cos^2(2a+2bx)}{12b}+\frac{109\sin(a+bx)\sin(2a+2bx)\cos(a+bx)\cos^4(2a+2bx)}{192b}-\frac{67\sin^4(2a+2bx)\cos^2(a+bx)\cos(2a+2bx)}{384b}+\frac{139\cos^2(a+bx)\cos^5(2a+2bx)}{1920b} & \text{for}\: b \neq 0 \\ x\sin^5(2a)\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**5,x)
```

```
[Out] Piecewise((-5*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**5/32 - 5*x*sin(a + b*x)**
2*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**2/16 - 5*x*sin(a + b*x)**2*sin(2*a
+ 2*b*x)*cos(2*a + 2*b*x)**4/32 - 5*x*sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(
a + b*x)*cos(2*a + 2*b*x)/16 - 5*x*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a +
 b*x)*cos(2*a + 2*b*x)**3/8 - 5*x*sin(a + b*x)*cos(a + b*x)*cos(2*a + 2*b*x
)**5/16 + 5*x*sin(2*a + 2*b*x)**5*cos(a + b*x)**2/32 + 5*x*sin(2*a + 2*b*x)
**3*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 + 5*x*sin(2*a + 2*b*x)*cos(a + b
*x)**2*cos(2*a + 2*b*x)**4/32 - 125*sin(a + b*x)**2*sin(2*a + 2*b*x)**4*cos
(2*a + 2*b*x)/(384*b) - 2*sin(a + b*x)**2*sin(2*a + 2*b*x)**2*cos(2*a + 2*b
*x)**3/(3*b) - 217*sin(a + b*x)**2*cos(2*a + 2*b*x)**5/(640*b) + 95*sin(a +
 b*x)*sin(2*a + 2*b*x)**5*cos(a + b*x)/(192*b) + 13*sin(a + b*x)*sin(2*a +
2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)**2/(12*b) + 109*sin(a + b*x)*sin(2*
a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**4/(192*b) - 67*sin(2*a + 2*b*x)**
4*cos(a + b*x)**2*cos(2*a + 2*b*x)/(384*b) + 139*cos(a + b*x)**2*cos(2*a +
2*b*x)**5/(1920*b), Ne(b, 0)), (x*sin(2*a)**5*cos(a)**2, True))
```

Giac [**A**]
time = 0.41, size = 36, normalized size = 0.82

$$-\frac{4\left(10\cos\left(bx+a\right)^{12}-24\cos\left(bx+a\right)^{10}+15\cos\left(bx+a\right)^{8}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^5,x, algorithm="giac")
[Out] -4/15*(10*cos(b*x + a)^12 - 24*cos(b*x + a)^10 + 15*cos(b*x + a)^8)/b
```

Mupad [**B**]
time = 0.18, size = 35, normalized size = 0.80

$$-\frac{4\cos\left(a+b\,x\right)^8\,\left(10\cos\left(a+b\,x\right)^4-24\cos\left(a+b\,x\right)^2+15\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^5,x)
[Out] -(4*cos(a + b*x)^8*(10*cos(a + b*x)^4 - 24*cos(a + b*x)^2 + 15))/(15*b)
```

3.142 $\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=76

$$\frac{3x}{16}-\frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b}-\frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b}+\frac{\sin^5(2a+2bx)}{20b}$$

```
[Out] 3/16*x-3/32*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b-1/16*cos(2*b*x+2*a)*sin(2*b*x+2
*a)^3/b+1/20*sin(2*b*x+2*a)^5/b
```

Rubi [A]
time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4370, 2715, 8, 2644, 30}

$$\frac{\sin^5(2a+2bx)}{20b}-\frac{\sin^3(2a+2bx)\cos(2a+2bx)}{16b}-\frac{3\sin(2a+2bx)\cos(2a+2bx)}{32b}+\frac{3x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (3*x)/16 - (3*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(32*b) - (Cos[2*a + 2*b*x]
*Sin[2*a + 2*b*x]^3)/(16*b) + Sin[2*a + 2*b*x]^5/(20*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4370

```
Int[cos[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Dist[1/2, Int[(g*Sin[c + d*x])^p, x], x] + Dist[1/2, Int[Cos[c + d*x]
*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d,
0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^4(2a+2bx)\,dx = \frac{1}{2}\int \sin^4(2a+2bx)\,dx + \frac{1}{2}\int \cos(2a+2bx)\sin^4(2a+2bx)\,dx$$
$$= -\frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} + \frac{3}{8}\int \sin^2(2a+2bx)\,dx + \frac{\mathrm{Subst}\left(\int x^4\,d\right.}{}$$
$$= -\frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b} - \frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} + \frac{\sin^5(2}{}$$
$$= \frac{3x}{16} - \frac{3\cos(2a+2bx)\sin(2a+2bx)}{32b} - \frac{\cos(2a+2bx)\sin^3(2a+2bx)}{16b} + \frac{\mathrm{si}}{}$$

Mathematica [A]
time = 0.22, size = 62, normalized size = 0.82

$$\frac{120bx + 20\sin(2(a+bx)) - 40\sin(4(a+bx)) - 10\sin(6(a+bx)) + 5\sin(8(a+bx)) + 2\sin(10(a+bx))}{640b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (120*b*x + 20*Sin[2*(a + b*x)] - 40*Sin[4*(a + b*x)] - 10*Sin[6*(a + b*x)]
+ 5*Sin[8*(a + b*x)] + 2*Sin[10*(a + b*x)])/(640*b)
```

Maple [A]
time = 0.27, size = 75, normalized size = 0.99

method	result	size
default	$\frac{3x}{16}+\frac{\sin(2xb+2a)}{32b}-\frac{\sin(4xb+4a)}{16b}-\frac{\sin(6xb+6a)}{64b}+\frac{\sin(8xb+8a)}{128b}+\frac{\sin(10xb+10a)}{320b}$	75
risch	$\frac{3x}{16}+\frac{\sin(2xb+2a)}{32b}-\frac{\sin(4xb+4a)}{16b}-\frac{\sin(6xb+6a)}{64b}+\frac{\sin(8xb+8a)}{128b}+\frac{\sin(10xb+10a)}{320b}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 3/16*x+1/32*sin(2*b*x+2*a)/b-1/16/b*sin(4*b*x+4*a)-1/64/b*sin(6*b*x+6*a)+1/
128/b*sin(8*b*x+8*a)+1/320/b*sin(10*b*x+10*a)
```

Maxima [A]
time = 0.28, size = 65, normalized size = 0.86

$$\frac{120\,bx+2\,\sin\left(10\,bx+10\,a\right)+5\,\sin\left(8\,bx+8\,a\right)-10\,\sin\left(6\,bx+6\,a\right)-40\,\sin\left(4\,bx+4\,a\right)+20\,\sin\left(2\,bx+2\,a\right)}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/640*(120*b*x + 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) - 10*sin(6*b*x +
 6*a) - 40*sin(4*b*x + 4*a) + 20*sin(2*b*x + 2*a))/b
```

Fricas [A]
time = 4.24, size = 66, normalized size = 0.87

$$\frac{15\,bx+\left(128\,\cos\left(bx+a\right)^{9}-176\,\cos\left(bx+a\right)^{7}+8\,\cos\left(bx+a\right)^{5}+10\,\cos\left(bx+a\right)^{3}+15\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{80\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/80*(15*b*x + (128*cos(b*x + a)^9 - 176*cos(b*x + a)^7 + 8*cos(b*x + a)^5
+ 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(70) = 140.
time = 6.47, size = 434, normalized size = 5.71

$$\begin{cases} \frac{3x\sin^{2}{\left(a+bx\right)}\sin^{4}{\left(2a+2bx\right)}}{16}+\frac{3x\sin^{2}{\left(a+bx\right)}\sin^{2}{\left(2a+2bx\right)}\cos^{2}{\left(2a+2bx\right)}}{8}+\frac{3x\sin^{2}{\left(a+bx\right)}\cos^{4}{\left(2a+2bx\right)}}{16}+\frac{3x\sin^{4}{\left(2a+2bx\right)}\cos^{2}{\left(a+bx\right)}}{16}+\frac{3x\sin^{2}{\left(2a+2bx\right)}\cos^{2}{\left(a+bx\right)}\cos^{2}{\left(2a+2bx\right)}}{8}+\frac{3x\cos^{2}{\left(a+bx\right)}\cos^{4}{\left(2a+2bx\right)}}{16}+\frac{7\sin^{2}{\left(a+bx\right)}\sin^{3}{\left(2a+2bx\right)}\cos{\left(2a+2bx\right)}}{160b}+\frac{19\sin^{2}{\left(a+bx\right)}\sin{\left(2a+2bx\right)}\cos^{3}{\left(2a+2bx\right)}}{480b}+\frac{\sin{\left(a+bx\right)}\sin^{4}{\left(2a+2bx\right)}\cos{\left(a+bx\right)}}{10b}+\frac{2\sin{\left(a+bx\right)}\sin^{2}{\left(2a+2bx\right)}\cos{\left(a+bx\right)}\cos^{2}{\left(2a+2bx\right)}}{5b}+\frac{4\sin{\left(a+bx\right)}\cos{\left(a+bx\right)}\cos^{4}{\left(2a+2bx\right)}}{15b}-\frac{57\sin^{3}{\left(2a+2bx\right)}\cos^{2}{\left(a+bx\right)}\cos{\left(2a+2bx\right)}}{160b}-\frac{109\sin{\left(2a+2bx\right)}\cos^{2}{\left(a+bx\right)}\cos^{3}{\left(2a+2bx\right)}}{480b} & \text{for}\: b\neq 0 \\ x\sin^{4}{\left(2a\right)}\cos^{2}{\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**4/16 + 3*x*sin(a + b*x)**2
*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*sin(a + b*x)**2*cos(2*a +
2*b*x)**4/16 + 3*x*sin(2*a + 2*b*x)**4*cos(a + b*x)**2/16 + 3*x*sin(2*a + 2
*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/8 + 3*x*cos(a + b*x)**2*cos(2*
a + 2*b*x)**4/16 + 7*sin(a + b*x)**2*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)/(
160*b) + 19*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**3/(480*b) +
sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(a + b*x)/(10*b) + 2*sin(a + b*x)*sin(2
*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**2/(5*b) + 4*sin(a + b*x)*cos(
a + b*x)*cos(2*a + 2*b*x)**4/(15*b) - 57*sin(2*a + 2*b*x)**3*cos(a + b*x)**
2*cos(2*a + 2*b*x)/(160*b) - 109*sin(2*a + 2*b*x)*cos(a + b*x)**2*cos(2*a +
 2*b*x)**3/(480*b), Ne(b, 0)), (x*sin(2*a)**4*cos(a)**2, True))
```

Giac [A]
time = 0.41, size = 68, normalized size = 0.89

$$\frac{120\,bx+120\,a+2\,\sin\left(10\,bx+10\,a\right)+5\,\sin\left(8\,bx+8\,a\right)-10\,\sin\left(6\,bx+6\,a\right)-40\,\sin\left(4\,bx+4\,a\right)+20\,\sin\left(2\,bx+2\,a\right)}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] 1/640*(120*b*x + 120*a + 2*sin(10*b*x + 10*a) + 5*sin(8*b*x + 8*a) - 10*sin
(6*b*x + 6*a) - 40*sin(4*b*x + 4*a) + 20*sin(2*b*x + 2*a))/b
```

Mupad [B]
time = 1.73, size = 109, normalized size = 1.43

$$\frac{3\,x}{16}+\frac{\frac{3\,\tan\left(a+b\,x\right)^9}{16}+\frac{7\,\tan\left(a+b\,x\right)^7}{8}+\frac{8\,\tan\left(a+b\,x\right)^5}{5}-\frac{7\,\tan\left(a+b\,x\right)^3}{8}-\frac{3\,\tan\left(a+b\,x\right)}{16}}{b\,\left({\tan\left(a+b\,x\right)}^{10}+5\,{\tan\left(a+b\,x\right)}^8+10\,{\tan\left(a+b\,x\right)}^6+10\,{\tan\left(a+b\,x\right)}^4+5\,{\tan\left(a+b\,x\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^4,x)
```

```
[Out] (3*x)/16 + ((8*tan(a + b*x)^5)/5 - (7*tan(a + b*x)^3)/8 - (3*tan(a + b*x))/
16 + (7*tan(a + b*x)^7)/8 + (3*tan(a + b*x)^9)/16)/(b*(5*tan(a + b*x)^2 + 1
0*tan(a + b*x)^4 + 10*tan(a + b*x)^6 + 5*tan(a + b*x)^8 + tan(a + b*x)^10 +
 1))
```

3.143 $\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=28

$$-\frac{4\cos^6(a+bx)}{3b}+\frac{\cos^8(a+bx)}{b}$$

```
[Out] -4/3*cos(b*x+a)^6/b+cos(b*x+a)^8/b
```

Rubi [A]
time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {4372, 2645, 14}

$$\frac{\cos^8(a+bx)}{b}-\frac{4\cos^6(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-4*Cos[a + b*x]^6)/(3*b) + Cos[a + b*x]^8/b
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^3(2a+2bx)\,dx = 8\int \cos^5(a+bx)\sin^3(a+bx)\,dx$$
$$= -\frac{8\text{Subst}\left(\int x^5(1-x^2)\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{8\text{Subst}\left(\int (x^5-x^7)\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{4\cos^6(a+bx)}{3b} + \frac{\cos^8(a+bx)}{b}$$

Mathematica [A]
time = 0.14, size = 48, normalized size = 1.71

$$\frac{-72\cos(2(a+bx)) - 12\cos(4(a+bx)) + 8\cos(6(a+bx)) + 3\cos(8(a+bx))}{384b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-72*Cos[2*(a + b*x)] - 12*Cos[4*(a + b*x)] + 8*Cos[6*(a + b*x)] + 3*Cos[8*
(a + b*x)])/(384*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.
time = 0.14, size = 58, normalized size = 2.07

method	result	size
default	$-\frac{3\cos(2xb+2a)}{16b} - \frac{\cos(4xb+4a)}{32b} + \frac{\cos(6xb+6a)}{48b} + \frac{\cos(8xb+8a)}{128b}$	58
risch	$-\frac{3\cos(2xb+2a)}{16b} - \frac{\cos(4xb+4a)}{32b} + \frac{\cos(6xb+6a)}{48b} + \frac{\cos(8xb+8a)}{128b}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/16*cos(2*b*x+2*a)/b-1/32*cos(4*b*x+4*a)/b+1/48*cos(6*b*x+6*a)/b+1/128*co
s(8*b*x+8*a)/b
```

Maxima [A]
time = 0.26, size = 50, normalized size = 1.79

$$\frac{3\cos(8bx+8a) + 8\cos(6bx+6a) - 12\cos(4bx+4a) - 72\cos(2bx+2a)}{384b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/384*(3*cos(8*b*x + 8*a) + 8*cos(6*b*x + 6*a) - 12*cos(4*b*x + 4*a) - 72*c
os(2*b*x + 2*a))/b
```

Fricas [A]
time = 3.65, size = 26, normalized size = 0.93

$$\frac{3\cos{(bx+a)}^8-4\cos{(bx+a)}^6}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(22) = 44$.
time = 2.72, size = 362, normalized size = 12.93

$$\begin{cases} -\frac{3x\sin^2(a+bx)\sin^3(2a+2bx)}{16}-\frac{3x\sin^2(a+bx)\sin(2a+2bx)\cos^2(2a+2bx)}{16}-\frac{3x\sin(a+bx)\sin^2(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{8}-\frac{3x\sin(a+bx)\cos(a+bx)\cos^3(2a+2bx)}{8}+\frac{3x\sin^3(2a+2bx)\cos^2(a+bx)}{16}+\frac{3x\sin(2a+2bx)\cos^2(a+bx)\cos^2(2a+2bx)}{16}-\frac{\sin^2(a+bx)\sin^2(2a+2bx)\cos(2a+2bx)}{2b}-\frac{49\sin^2(a+bx)\cos^3(2a+2bx)}{96b}+\frac{13\sin(a+bx)\sin^3(2a+2bx)\cos(a+bx)}{16b}+\frac{7\sin(a+bx)\sin(2a+2bx)\cos(a+bx)\cos^2(2a+2bx)}{8b}+\frac{17\cos^2(a+bx)\cos^3(2a+2bx)}{96b} & \text{for}\ b\neq 0 \\ x\sin^3(2a)\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((-3*x*sin(a + b*x)**2*sin(2*a + 2*b*x)**3/16 - 3*x*sin(a + b*x)**
2*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/16 - 3*x*sin(a + b*x)*sin(2*a + 2*b*
x)**2*cos(a + b*x)*cos(2*a + 2*b*x)/8 - 3*x*sin(a + b*x)*cos(a + b*x)*cos(2
*a + 2*b*x)**3/8 + 3*x*sin(2*a + 2*b*x)**3*cos(a + b*x)**2/16 + 3*x*sin(2*a
 + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/16 - sin(a + b*x)**2*sin(2*a
+ 2*b*x)**2*cos(2*a + 2*b*x)/(2*b) - 49*sin(a + b*x)**2*cos(2*a + 2*b*x)**3
/(96*b) + 13*sin(a + b*x)*sin(2*a + 2*b*x)**3*cos(a + b*x)/(16*b) + 7*sin(a
 + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(8*b) + 17*cos(a
+ b*x)**2*cos(2*a + 2*b*x)**3/(96*b), Ne(b, 0)), (x*sin(2*a)**3*cos(a)**2,
True))
```

Giac [A]
time = 0.53, size = 26, normalized size = 0.93

$$\frac{3\cos{(bx+a)}^8-4\cos{(bx+a)}^6}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*cos(b*x + a)^8 - 4*cos(b*x + a)^6)/b
```

Mupad [B]
time = 0.14, size = 26, normalized size = 0.93

$$-\frac{\frac{4\cos(a+b\,x)^6}{3}-\cos\left(a+b\,x\right)^8}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^3,x)
[Out] -((4*cos(a + b*x)^6)/3 - cos(a + b*x)^8)/b
```

3.144 $\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=49

$$\frac{x}{4}-\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b}+\frac{\sin^3(2a+2bx)}{12b}$$

```
[Out] 1/4*x-1/8*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b+1/12*sin(2*b*x+2*a)^3/b
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4370, 2715, 8, 2644, 30}

$$\frac{\sin^3(2a+2bx)}{12b}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{8b}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] x/4 - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(8*b) + Sin[2*a + 2*b*x]^3/(12*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4370

```
Int[cos[(a_.) + (b_.)*(x_)]^2*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbo
l] :> Dist[1/2, Int[(g*Sin[c + d*x])^p, x], x] + Dist[1/2, Int[Cos[c + d*x]
*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d,
0] && EqQ[d/b, 2] && IGtQ[p/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^2(2a+2bx)\,dx &= \frac{1}{2}\int \sin^2(2a+2bx)\,dx+\frac{1}{2}\int \cos(2a+2bx)\sin^2(2a+2bx)\,dx\\ &= -\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b}+\frac{\int 1\,dx}{4}+\frac{\text{Subst}\left(\int x^2\,dx,x,\sin(2a+2bx)\right)}{4b}\\ &= \frac{x}{4}-\frac{\cos(2a+2bx)\sin(2a+2bx)}{8b}+\frac{\sin^3(2a+2bx)}{12b}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 40, normalized size = 0.82

$$-\frac{-12bx-3\sin(2(a+bx))+3\sin(4(a+bx))+\sin(6(a+bx))}{48b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] -1/48*(-12*b*x - 3*Sin[2*(a + b*x)] + 3*Sin[4*(a + b*x)] + Sin[6*(a + b*x)]
)/b
```

Maple [A]
time = 0.14, size = 47, normalized size = 0.96

method	result	size
default	$\frac{x}{4}+\frac{\sin(2xb+2a)}{16b}-\frac{\sin(4xb+4a)}{16b}-\frac{\sin(6xb+6a)}{48b}$	47
risch	$\frac{x}{4}+\frac{\sin(2xb+2a)}{16b}-\frac{\sin(4xb+4a)}{16b}-\frac{\sin(6xb+6a)}{48b}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x+1/16*sin(2*b*x+2*a)/b-1/16/b*sin(4*b*x+4*a)-1/48/b*sin(6*b*x+6*a)
```

Maxima [A]
time = 0.27, size = 43, normalized size = 0.88

$$\frac{12\,bx-\sin\left(6\,bx+6\,a\right)-3\,\sin\left(4\,bx+4\,a\right)+3\,\sin\left(2\,bx+2\,a\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(12*b*x - sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))/
b
```

Fricas [A]
time = 2.02, size = 47, normalized size = 0.96

$$\frac{3\,bx-\left(8\,\cos\left(bx+a\right)^{5}-2\,\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*x - (8*cos(b*x + a)^5 - 2*cos(b*x + a)^3 - 3*cos(b*x + a))*sin(b*
x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(41) = 82$.
time = 1.13, size = 231, normalized size = 4.71

$$\begin{cases} \frac{x\sin^{2}\left(a+bx\right)\sin^{2}\left(2a+2bx\right)}{4}+\frac{x\sin^{2}\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{4}+\frac{x\sin^{2}\left(2a+2bx\right)\cos^{2}\left(a+bx\right)}{4}+\frac{x\cos^{2}\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{4}+\frac{\sin^{2}\left(a+bx\right)\sin\left(2a+2bx\right)\cos\left(2a+2bx\right)}{24b}+\frac{\sin\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos\left(a+bx\right)}{6b}+\frac{\sin\left(a+bx\right)\cos\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{3b}-\frac{7\sin\left(2a+2bx\right)\cos^{2}\left(a+bx\right)\cos\left(2a+2bx\right)}{24b} & \text{for}\: b\neq 0\\ x\sin^{2}\left(2a\right)\cos^{2}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2*sin(2*a + 2*b*x)**2/4 + x*sin(a + b*x)**2*cos(
2*a + 2*b*x)**2/4 + x*sin(2*a + 2*b*x)**2*cos(a + b*x)**2/4 + x*cos(a + b*x
)**2*cos(2*a + 2*b*x)**2/4 + sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(2*a + 2*b
*x)/(24*b) + sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)/(6*b) + sin(a +
b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**2/(3*b) - 7*sin(2*a + 2*b*x)*cos(a + b*
x)**2*cos(2*a + 2*b*x)/(24*b), Ne(b, 0)), (x*sin(2*a)**2*cos(a)**2, True))
```

Giac [A]
time = 0.45, size = 46, normalized size = 0.94

$$\frac{12\,bx+12\,a-\sin\left(6\,bx+6\,a\right)-3\,\sin\left(4\,bx+4\,a\right)+3\,\sin\left(2\,bx+2\,a\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] 1/48*(12*b*x + 12*a - sin(6*b*x + 6*a) - 3*sin(4*b*x + 4*a) + 3*sin(2*b*x +
 2*a))/b
```

Mupad [B]
time = 0.31, size = 43, normalized size = 0.88

$$\frac{x}{4}-\frac{\frac{\sin(4\,a+4\,b\,x)}{16}-\frac{\sin(2\,a+2\,b\,x)}{16}+\frac{\sin(6\,a+6\,b\,x)}{48}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^2,x)
[Out] x/4 - (sin(4*a + 4*b*x)/16 - sin(2*a + 2*b*x)/16 + sin(6*a + 6*b*x)/48)/b
```

3.145 $\int \cos^2(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=15

$$-\frac{\cos^4(a+bx)}{2b}$$

[Out] -1/2*cos(b*x+a)^4/b

Rubi [A]
time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4372, 2645, 30}

$$-\frac{\cos^4(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x],x]

[Out] -1/2*Cos[a + b*x]^4/b

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^2(a+bx)\sin(2a+2bx)\,dx = 2\int \cos^3(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{2\text{Subst}\left(\int x^3\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{\cos^4(a+bx)}{2b}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\frac{\cos^4(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x],x]
```

```
[Out] -1/2*Cos[a + b*x]^4/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(13) = 26$.
time = 0.14, size = 30, normalized size = 2.00

method	result
default	$-\frac{\cos(2xb+2a)}{4b} - \frac{\cos(4xb+4a)}{16b}$
risch	$-\frac{\cos(2xb+2a)}{4b} - \frac{\cos(4xb+4a)}{16b}$
norman	$\frac{x\left(\tan^3\left(\frac{a}{2}+\frac{xb}{2}\right)\right)+x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan^2(xb+a)\right)+\frac{3\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(xb+a)}{b}-x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+\frac{x\tan(xb+a)}{2}-3x\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)\right)\tan(xb+a)}{\left(1+\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*cos(2*b*x+2*a)/b-1/16*cos(4*b*x+4*a)/b
```

Maxima [A]
time = 0.29, size = 26, normalized size = 1.73

$$-\frac{\cos(4\,bx+4\,a)+4\,\cos(2\,bx+2\,a)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/16*(cos(4*b*x + 4*a) + 4*cos(2*b*x + 2*a))/b
```

Fricas [A]
time = 2.71, size = 13, normalized size = 0.87

$$-\frac{\cos\left(bx+a\right)^4}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -1/2*cos(b*x + a)^4/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(12) = 24$.
time = 0.43, size = 133, normalized size = 8.87

$$\begin{cases} -\frac{x\sin^2\left(a+bx\right)\sin\left(2a+2bx\right)}{4}-\frac{x\sin\left(a+bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{2}+\frac{x\sin\left(2a+2bx\right)\cos^2\left(a+bx\right)}{4}-\frac{\sin^2\left(a+bx\right)\cos\left(2a+2bx\right)}{2b}+\frac{3\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos\left(a+bx\right)}{4b} & \text{for}\ b\neq 0 \\ x\sin\left(2a\right)\cos^2\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-x*sin(a + b*x)**2*sin(2*a + 2*b*x)/4 - x*sin(a + b*x)*cos(a + b
*x)*cos(2*a + 2*b*x)/2 + x*sin(2*a + 2*b*x)*cos(a + b*x)**2/4 - sin(a + b*x
)**2*cos(2*a + 2*b*x)/(2*b) + 3*sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)/
(4*b), Ne(b, 0)), (x*sin(2*a)*cos(a)**2, True))
```

Giac [A]
time = 0.41, size = 13, normalized size = 0.87

$$-\frac{\cos\left(bx+a\right)^4}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -1/2*cos(b*x + a)^4/b
```

Mupad [B]
time = 0.15, size = 13, normalized size = 0.87

$$-\frac{\cos\left(a+b\,x\right)^4}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x),x)
```

```
[Out] -cos(a + b*x)^4/(2*b)
```

3.146 $\int \cos^2(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=14

$$\frac{\log(\sin(a+bx))}{2b}$$

```
[Out] 1/2*ln(sin(b*x+a))/b
```

Rubi [A]
time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4372, 3556}

$$\frac{\log(\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/(2*b)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\csc(2a+2bx)\,dx &= \frac{1}{2}\int \cot(a+bx)\,dx\\ &= \frac{\log(\sin(a+bx))}{2b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.57

$$\frac{\log(\cos(a+bx))+\log(\tan(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x],x]
```

```
[Out] (Log[Cos[a + b*x]] + Log[Tan[a + b*x]])/(2*b)
```

Maple [A]
time = 0.12, size = 13, normalized size = 0.93

method	result	size
default	$\frac{\ln(\sin(xb+a))}{2b}$	13
risch	$-\frac{ix}{2}-\frac{ia}{b}+\frac{\ln\left(e^{2i(xb+a)}-1\right)}{2b}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(sin(b*x+a))/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(12) = 24.
time = 0.27, size = 82, normalized size = 5.86

$$\frac{\log\left(\cos(bx)^2+2\cos(bx)\cos(a)+\cos(a)^2+\sin(bx)^2-2\sin(bx)\sin(a)+\sin(a)^2\right)+\log\left(\cos(bx)^2-2\cos(bx)\cos(a)+\cos(a)^2+\sin(bx)^2+2\sin(bx)\sin(a)+\sin(a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/4*(log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x
)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(
b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Fricas [A]
time = 2.82, size = 14, normalized size = 1.00

$$\frac{\log\left(\frac{1}{2}\sin(bx+a)\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/2*log(1/2*sin(b*x + a))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.41, size = 13, normalized size = 0.93

$$\frac{\log\left(\left|\sin\left(bx+a\right)\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(sin(b*x + a)))/b
```

Mupad [B]
time = 0.16, size = 14, normalized size = 1.00

$$\frac{\ln\left(\sin\left(a+b\,x\right)^2\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x),x)
```

```
[Out] log(sin(a + b*x)^2)/(4*b)
```

3.147 $\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=13

$$-\frac{\cot(a+bx)}{4b}$$

```
[Out] -1/4*cot(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4372, 3852, 8}

$$-\frac{\cot(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -1/4*Cot[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\csc^2(2a+2bx)\,dx &= \frac{1}{4}\int \csc^2(a+bx)\,dx\\ &= -\frac{\text{Subst}\left(\int 1\,dx, x, \cot(a+bx)\right)}{4b}\\ &= -\frac{\cot(a+bx)}{4b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 13, normalized size = 1.00

$$-\frac{\cot(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -1/4*Cot[a + b*x]/b
```

Maple [A]
time = 0.15, size = 12, normalized size = 0.92

method	result	size
default	$-\frac{\cot(xb+a)}{4b}$	12
risch	$-\frac{i}{2b\left(e^{2i(xb+a)}-1\right)}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*cot(b*x+a)/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(11) = 22.
time = 0.27, size = 53, normalized size = 4.08

$$-\frac{\sin(2\,bx+2\,a)}{2\,\left(b\cos(2\,bx+2\,a)^2+b\sin(2\,bx+2\,a)^2-2\,b\cos(2\,bx+2\,a)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sin(2*b*x + 2*a)/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*co
s(2*b*x + 2*a) + b)
```

Fricas [A]
time = 2.44, size = 19, normalized size = 1.46

$$-\frac{\cos(bx+a)}{4\,b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*cos(b*x + a)/(b*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.46, size = 13, normalized size = 1.00

$$-\frac{1}{4\,b\tan\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/4/(b*tan(b*x + a))
```

Mupad [**B**]
time = 0.14, size = 11, normalized size = 0.85

$$-\frac{\cot(a+b\,x)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^2,x)
```

```
[Out] -cot(a + b*x)/(4*b)
```

3.148 $\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=30

$$-\frac{\cot^2(a+bx)}{16b}+\frac{\log(\tan(a+bx))}{8b}$$

```
[Out] -1/16*cot(b*x+a)^2/b+1/8*ln(tan(b*x+a))/b
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4372, 2700, 14}

$$\frac{\log(\tan(a+bx))}{8b}-\frac{\cot^2(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] -1/16*Cot[a + b*x]^2/b + Log[Tan[a + b*x]]/(8*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^2(a+bx)\csc^3(2a+2bx)\,dx = \frac{1}{8}\int \csc^3(a+bx)\sec(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^3}\,dx, x, \tan(a+bx)\right)}{8b}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^3}+\frac{1}{x}\right)\,dx, x, \tan(a+bx)\right)}{8b}$$
$$= -\frac{\cot^2(a+bx)}{16b} + \frac{\log(\tan(a+bx))}{8b}$$

Mathematica [A]
time = 0.06, size = 34, normalized size = 1.13

$$-\frac{\csc^2(a+bx)+2\log(\cos(a+bx))-2\log(\sin(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^3,x]
[Out] -1/16*(Csc[a + b*x]^2 + 2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]])/b
```

Maple [A]
time = 0.16, size = 24, normalized size = 0.80

method	result	size
default	$\frac{-\frac{1}{2\sin(xb+a)^2}+\ln(\tan(xb+a))}{8b}$	24
risch	$\frac{e^{2i(xb+a)}}{4b\left(e^{2i(xb+a)}-1\right)^2}+\frac{\ln\left(e^{2i(xb+a)}-1\right)}{8b}-\frac{\ln\left(e^{2i(xb+a)}+1\right)}{8b}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
[Out] 1/8/b*(-1/2/sin(b*x+a)^2+ln(tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(26) = 52.
time = 0.29, size = 656, normalized size = 21.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(4*cos(4*b*x + 4*a)*cos(2*b*x + 2*a) - 8*cos(2*b*x + 2*a)^2 + (2*(2*co
s(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2
*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*
b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(
2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) - (2
*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b
*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*
sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*co
s(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*(2*cos(2*
b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^
2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x
+ 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos
(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) - 8*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a))/(b*cos(4*b*x + 4
*a)^2 + 4*b*cos(2*b*x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*sin(4*b*x + 4*a
)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*b*x + 2*a) - b)*
cos(4*b*x + 4*a) - 4*b*cos(2*b*x + 2*a) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(26) = 52$.
time = 2.75, size = 65, normalized size = 2.17

$$-\frac{(\cos(bx+a)^2-1)\log(\cos(bx+a)^2)-(\cos(bx+a)^2-1)\log\left(-\frac{1}{4}\cos(bx+a)^2+\frac{1}{4}\right)-1}{16\left(b\cos(bx+a)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*((cos(b*x + a)^2 - 1)*log(cos(b*x + a)^2) - (cos(b*x + a)^2 - 1)*log(
-1/4*cos(b*x + a)^2 + 1/4) - 1)/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 41, normalized size = 1.37

$$\frac{\frac{1}{\cos(bx+a)^2-1}+\log\left(-\cos(bx+a)^2+1\right)-2\log\left(|\cos(bx+a)|\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/16*(1/(cos(b*x + a)^2 - 1) + log(-cos(b*x + a)^2 + 1) - 2*log(abs(cos(b*x
 + a))))/b
```

Mupad [B]
time = 0.18, size = 36, normalized size = 1.20

$$-\frac{\frac{\ln(\cos(a+b\,x))}{8}-\frac{\ln\left(\sin(a+b\,x)^2\right)}{16}+\frac{1}{16\sin(a+b\,x)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^3,x)
```

```
[Out] -(log(cos(a + b*x))/8 - log(sin(a + b*x)^2)/16 + 1/(16*sin(a + b*x)^2))/b
```

3.149 $\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=42

$$-\frac{\cot(a+bx)}{8b}-\frac{\cot^3(a+bx)}{48b}+\frac{\tan(a+bx)}{16b}$$

```
[Out] -1/8*cot(b*x+a)/b-1/48*cot(b*x+a)^3/b+1/16*tan(b*x+a)/b
```

Rubi [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4372, 2700, 276}

$$\frac{\tan(a+bx)}{16b}-\frac{\cot^3(a+bx)}{48b}-\frac{\cot(a+bx)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] -1/8*Cot[a + b*x]/b - Cot[a + b*x]^3/(48*b) + Tan[a + b*x]/(16*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(a+bx)\csc^4(2a+2bx)\,dx &= \frac{1}{16}\int \csc^4(a+bx)\sec^2(a+bx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4}\,dx, x, \tan(a+bx)\right)}{16b} \\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)\,dx, x, \tan(a+bx)\right)}{16b} \\
&= -\frac{\cot(a+bx)}{8b}-\frac{\cot^3(a+bx)}{48b}+\frac{\tan(a+bx)}{16b}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 1.14

$$
-\frac{5\cot(a+bx)}{48b}-\frac{\cot(a+bx)\csc^2(a+bx)}{48b}+\frac{\tan(a+bx)}{16b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] (-5*Cot[a + b*x])/(48*b) - (Cot[a + b*x]*Csc[a + b*x]^2)/(48*b) + Tan[a + b
*x]/(16*b)
```

Maple [A]
time = 0.21, size = 51, normalized size = 1.21

method	result	size
risch	$\frac{i\left(2\,\mathrm{e}^{2i(xb+a)}-1\right)}{3b\left(\mathrm{e}^{2i(xb+a)}-1\right)^3\left(\mathrm{e}^{2i(xb+a)}+1\right)}$	46
default	$\frac{-\frac{1}{3\cos(xb+a)\sin(xb+a)^3}+\frac{4}{3\sin(xb+a)\cos(xb+a)}-\frac{8\cot(xb+a)}{3}}{16b}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/b*(-1/3/cos(b*x+a)/sin(b*x+a)^3+4/3/sin(b*x+a)/cos(b*x+a)-8/3*cot(b*x+
a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(36) = 72.
time = 0.28, size = 308, normalized size = 7.33

$$
\frac{(2\cos(2bx+2a)-1)\sin(8bx+8a)-2(2\cos(2bx+2a)-1)\sin(6bx+6a)-2\cos(8bx+8a)\sin(2bx+2a)+4\cos(6bx+6a)\sin(2bx+2a)}{3\left(b\cos(8bx+8a)^2+4b\cos(6bx+6a)^2+4b\cos(2bx+2a)^2+b\sin(8bx+8a)^2+4b\sin(6bx+6a)^2-8b\sin(6bx+6a)\sin(2bx+2a)+4b\sin(2bx+2a)^2-2(2b\cos(6bx+6a)-2b\cos(2bx+2a)+b)\cos(8bx+8a)-4(2b\cos(2bx+2a)-b)\cos(6bx+6a)-4b\cos(2bx+2a)-4(b\sin(6bx+6a)-b\sin(2bx+2a))\sin(8bx+8a)+b\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/3*((2*cos(2*b*x + 2*a) - 1)*sin(8*b*x + 8*a) - 2*(2*cos(2*b*x + 2*a) - 1)
*sin(6*b*x + 6*a) - 2*cos(8*b*x + 8*a)*sin(2*b*x + 2*a) + 4*cos(6*b*x + 6*a
)*sin(2*b*x + 2*a))/(b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2 + 4*b*co
s(2*b*x + 2*a)^2 + b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 - 8*b*sin(
6*b*x + 6*a)*sin(2*b*x + 2*a) + 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(6*b*x +
 6*a) - 2*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) - 4*(2*b*cos(2*b*x + 2*a
) - b)*cos(6*b*x + 6*a) - 4*b*cos(2*b*x + 2*a) - 4*(b*sin(6*b*x + 6*a) - b*
sin(2*b*x + 2*a))*sin(8*b*x + 8*a) + b)
```

Fricas [A]
time = 2.84, size = 54, normalized size = 1.29

$$-\frac{8 \cos\left(bx+a\right)^{4}-12 \cos\left(bx+a\right)^{2}+3}{48 \left(b\cos\left(bx+a\right)^{3}-b\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(b*x + a)^4 - 12*cos(b*x + a)^2 + 3)/((b*cos(b*x + a)^3 - b*cos
(b*x + a))*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 35, normalized size = 0.83

$$-\frac{\frac{6 \tan\left(bx+a\right)^{2}+1}{\tan\left(bx+a\right)^{3}}-3 \tan\left(bx+a\right)}{48 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^4,x, algorithm="giac")
```

```
[Out] -1/48*((6*tan(b*x + a)^2 + 1)/tan(b*x + a)^3 - 3*tan(b*x + a))/b
```

Mupad [B]
time = 0.18, size = 37, normalized size = 0.88

$$\frac{\tan(a+b\,x)}{16\,b}-\frac{\frac{\tan(a+b\,x)^2}{8}+\frac{1}{48}}{b\tan\left(a+b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^4,x)
```

```
[Out] tan(a + b*x)/(16*b) - (tan(a + b*x)^2/8 + 1/48)/(b*tan(a + b*x)^3)
```

3.150 $\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx$

Optimal. Leaf size=60

$$-\frac{3\cot^2(a+bx)}{64b}-\frac{\cot^4(a+bx)}{128b}+\frac{3\log(\tan(a+bx))}{32b}+\frac{\tan^2(a+bx)}{64b}$$

```
[Out] -3/64*cot(b*x+a)^2/b-1/128*cot(b*x+a)^4/b+3/32*ln(tan(b*x+a))/b+1/64*tan(b*
x+a)^2/b
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4372, 2700, 272, 45}

$$\frac{\tan^2(a+bx)}{64b}-\frac{\cot^4(a+bx)}{128b}-\frac{3\cot^2(a+bx)}{64b}+\frac{3\log(\tan(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] (-3*Cot[a + b*x]^2)/(64*b) - Cot[a + b*x]^4/(128*b) + (3*Log[Tan[a + b*x]])
/(32*b) + Tan[a + b*x]^2/(64*b)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2700

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
```

```
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^2(a+bx)\csc^5(2a+2bx)\,dx = \frac{1}{32}\int \csc^5(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^5}\,dx, x, \tan(a+bx)\right)}{32b}$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x^3}\,dx, x, \tan^2(a+bx)\right)}{64b}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^3}+\frac{3}{x^2}+\frac{3}{x}\right)\,dx, x, \tan^2(a+bx)\right)}{64b}$$
$$= -\frac{3\cot^2(a+bx)}{64b} - \frac{\cot^4(a+bx)}{128b} + \frac{3\log(\tan(a+bx))}{32b} + \frac{\tan^2(a+bx)}{64b}$$

Mathematica [A]
time = 0.39, size = 54, normalized size = 0.90

$$-\frac{4\csc^2(a+bx)+\csc^4(a+bx)+12\log(\cos(a+bx))-12\log(\sin(a+bx))-2\sec^2(a+bx)}{128b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Csc[2*a + 2*b*x]^5,x]
[Out] -1/128*(4*Csc[a + b*x]^2 + Csc[a + b*x]^4 + 12*Log[Cos[a + b*x]] - 12*Log[S
in[a + b*x]] - 2*Sec[a + b*x]^2)/b
```

Maple [A]
time = 0.22, size = 62, normalized size = 1.03

method	result	size
default	$\frac{-\frac{1}{4\sin(xb+a)^4\cos(xb+a)^2}+\frac{3}{4\sin(xb+a)^2\cos(xb+a)^2}-\frac{3}{2\sin(xb+a)^2}+3\ln(\tan(xb+a))}{32b}$	62
risch	$\frac{3\,e^{10i(xb+a)}-6\,e^{8i(xb+a)}-2\,e^{6i(xb+a)}-6\,e^{4i(xb+a)}+3\,e^{2i(xb+a)}}{16b\left(e^{2i(xb+a)}-1\right)^4\left(e^{2i(xb+a)}+1\right)^2}-\frac{3\ln\left(e^{2i(xb+a)}+1\right)}{32b}+\frac{3\ln\left(e^{2i(xb+a)}-1\right)}{32b}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
[Out] 1/32/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)^2+3/4/sin(b*x+a)^2/cos(b*x+a)^2-3/2/si
n(b*x+a)^2+3*ln(tan(b*x+a)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3188 vs. $2(52) = 104$.
time = 0.36, size = 3188, normalized size = 53.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/64*(4*(3*cos(10*b*x + 10*a) - 6*cos(8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 6
*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a))*cos(12*b*x + 12*a) + 4*(9*cos(8*b*x
 + 8*a) + 16*cos(6*b*x + 6*a) + 9*cos(4*b*x + 4*a) - 12*cos(2*b*x + 2*a) +
3)*cos(10*b*x + 10*a) - 24*cos(10*b*x + 10*a)^2 - 4*(22*cos(6*b*x + 6*a) -
12*cos(4*b*x + 4*a) - 9*cos(2*b*x + 2*a) + 6)*cos(8*b*x + 8*a) + 24*cos(8*b
*x + 8*a)^2 - 8*(11*cos(4*b*x + 4*a) - 8*cos(2*b*x + 2*a) + 1)*cos(6*b*x +
6*a) - 32*cos(6*b*x + 6*a)^2 + 12*(3*cos(2*b*x + 2*a) - 2)*cos(4*b*x + 4*a)
 + 24*cos(4*b*x + 4*a)^2 - 24*cos(2*b*x + 2*a)^2 + 3*(2*(2*cos(10*b*x + 10*
a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x
 + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a
) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(10*
b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) - cos(4*b*x +
4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 8*(c
os(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 16*cos(6*b*x +
 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2
- 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*x + 8*a) - 4*sin
(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) -
 sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*
x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*sin(10*b*x + 10*a)^2
+ 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x
+ 8*a) - sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(6*b*x + 6*a) - 16*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 4*sin(4*b*x +
4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(
cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*
b*x)*sin(2*a) + sin(2*a)^2) - 3*(2*(2*cos(10*b*x + 10*a) + cos(8*b*x + 8*a)
 - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b
*x + 12*a) - cos(12*b*x + 12*a)^2 - 4*(cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a
) + cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(10*b*x + 10*a) - 4*cos(1
0*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*a) - cos(4*b*x + 4*a) - 2*cos(2*b*x +
2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*x + 8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*c
os(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a) - 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*
b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^
2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4
*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(12*b*x + 12*a) - sin(12*b*x + 12*a)^2
 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*
x + 2*a))*sin(10*b*x + 10*a) - 4*sin(10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*
```

```
a) - sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a))*sin(8*b*x + 8*a) - sin(8*b*x +
8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 16*si
n(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
 - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 3*(2*(2
*cos(10*b*x + 10*a) + cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4
*a) + 2*cos(2*b*x + 2*a) - 1)*cos(12*b*x + 12*a) - cos(12*b*x + 12*a)^2 - 4
*(cos(8*b*x + 8*a) - 4*cos(6*b*x + 6*a) + cos(4*b*x + 4*a) + 2*cos(2*b*x +
2*a) - 1)*cos(10*b*x + 10*a) - 4*cos(10*b*x + 10*a)^2 + 2*(4*cos(6*b*x + 6*
a) - cos(4*b*x + 4*a) - 2*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - cos(8*b*
x + 8*a)^2 + 8*(cos(4*b*x + 4*a) + 2*cos(2*b*x + 2*a) - 1)*cos(6*b*x + 6*a)
 - 16*cos(6*b*x + 6*a)^2 - 2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - co
s(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 + 2*(2*sin(10*b*x + 10*a) + sin(8*b
*x + 8*a) - 4*sin(6*b*x + 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin
(12*b*x + 12*a) - sin(12*b*x + 12*a)^2 - 4*(sin(8*b*x + 8*a) - 4*sin(6*b*x
+ 6*a) + sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 4*sin(
10*b*x + 10*a)^2 + 2*(4*sin(6*b*x + 6*a) - sin(4*b*x + 4*a) - 2*sin(2*b*x +
 2*a))*sin(8*b*x + 8*a) - sin(8*b*x + 8*a)^2 + 8*(sin(4*b*x + 4*a) + 2*sin(
2*b*x + 2*a))*sin(6*b*x + 6*a) - 16*sin(6*b*x + 6*a)^2 - sin(4*b*x + 4*a)^2
 - 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x
 + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(a) + sin(a)^2) + 4*(3*sin(10*b*x + 10*a) - 6*sin(8*b*x + 8*a)
 - 2*sin(6*b*x + 6*a) - 6*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(12*b*x
 + 12*a) + 4*(9*sin(8*b*x + 8*a) + 16*sin(6*b*x + 6*a) + 9*sin(4*b*x + 4*a)
 - 12*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - 24*sin(10*b*x + 10*a)^2 - 4*(2
2*sin(6*b*x + 6*a) - 12*sin(4*b*x + 4*a) - 9*sin(2*b*x + 2*a))*sin(8*b*x +
8*a) + 24*sin(8*b*x + 8*a)^2 - 8*(11*sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))
*sin(6*b*x + 6*a) - 32*sin(6*b*x + 6*a)^2 + 24*sin(4*b*x + 4*a)^2 + 36*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) - 24*sin(2*b*x + 2*a)^2 + 12*cos(2*b*x + 2*a)
)/(b*cos(12*b*x + 12*a)^2 + 4*b*cos(10*b*x + 10*a)^2 + b*cos(8*b*x + 8*a)^2
 + 16*b*cos(6*b*x + 6*a)^2 + b*cos(4*b*x + 4*a)...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(52) = 104$.
time = 2.47, size = 138, normalized size = 2.30

$$\frac{6 \cos\left(bx+a\right)^{4} - 9 \cos\left(bx+a\right)^{2} - 6 \left(\cos\left(bx+a\right)^{6} - 2 \cos\left(bx+a\right)^{4} + \cos\left(bx+a\right)^{2}\right) \log\left(\cos\left(bx+a\right)^{2}\right) + 6 \left(\cos\left(bx+a\right)^{6} - 2 \cos\left(bx+a\right)^{4} + \cos\left(bx+a\right)^{2}\right) \log\left(-\frac{1}{4} \cos\left(bx+a\right)^{2} + \frac{1}{4}\right) + 2}{128 \left(b \cos\left(bx+a\right)^{6} - 2 b \cos\left(bx+a\right)^{4} + b \cos\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 1/128*(6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 - 6*(cos(b*x + a)^6 - 2*cos(b*x
+ a)^4 + cos(b*x + a)^2)*log(cos(b*x + a)^2) + 6*(cos(b*x + a)^6 - 2*cos(b*
x + a)^4 + cos(b*x + a)^2)*log(-1/4*cos(b*x + a)^2 + 1/4) + 2)/(b*cos(b*x +
 a)^6 - 2*b*cos(b*x + a)^4 + b*cos(b*x + a)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 74, normalized size = 1.23

$$\frac{\frac{6\,\cos(bx+a)^4-9\,\cos(bx+a)^2+2}{\left(\cos(bx+a)^2-1\right)^2\cos(bx+a)^2}+6\,\log\left(-\cos\left(bx+a\right)^2+1\right)-12\,\log\left(\left|\cos\left(bx+a\right)\right|\right)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/128*((6*cos(b*x + a)^4 - 9*cos(b*x + a)^2 + 2)/((cos(b*x + a)^2 - 1)^2*co
s(b*x + a)^2) + 6*log(-cos(b*x + a)^2 + 1) - 12*log(abs(cos(b*x + a))))/b
```

Mupad [B]
time = 0.14, size = 82, normalized size = 1.37

$$\frac{3\,\ln\left(\sin\left(a+b\,x\right)^2\right)}{64\,b}-\frac{3\,\ln\left(\cos\left(a+b\,x\right)\right)}{32\,b}+\frac{\frac{3\cos\left(a+b\,x\right)^4}{64}-\frac{9\cos\left(a+b\,x\right)^2}{128}+\frac{1}{64}}{b\,\left(\cos\left(a+b\,x\right)^6-2\cos\left(a+b\,x\right)^4+\cos\left(a+b\,x\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^5,x)
```

```
[Out] (3*log(sin(a + b*x)^2))/(64*b) - (3*log(cos(a + b*x)))/(32*b) + ((3*cos(a +
 b*x)^4)/64 - (9*cos(a + b*x)^2)/128 + 1/64)/(b*(cos(a + b*x)^2 - 2*cos(a +
 b*x)^4 + cos(a + b*x)^6))
```

3.151 $\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{32\cos^9(a+bx)}{9b}+\frac{64\cos^{11}(a+bx)}{11b}-\frac{32\cos^{13}(a+bx)}{13b}$$

```
[Out] -32/9*cos(b*x+a)^9/b+64/11*cos(b*x+a)^11/b-32/13*cos(b*x+a)^13/b
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4372, 2645, 276}

$$-\frac{32\cos^{13}(a+bx)}{13b}+\frac{64\cos^{11}(a+bx)}{11b}-\frac{32\cos^9(a+bx)}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (-32*Cos[a + b*x]^9)/(9*b) + (64*Cos[a + b*x]^11)/(11*b) - (32*Cos[a + b*x]
^13)/(13*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^3(a+bx)\sin^5(2a+2bx)\,dx = 32\int \cos^8(a+bx)\sin^5(a+bx)\,dx$$
$$= -\frac{32\mathrm{Subst}\left(\int x^8\left(1-x^2\right)^2\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{32\mathrm{Subst}\left(\int \left(x^8-2x^{10}+x^{12}\right)\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{32\cos^9(a+bx)}{9b}+\frac{64\cos^{11}(a+bx)}{11b}-\frac{32\cos^{13}(a+bx)}{13b}$$

Mathematica [A]
time = 0.44, size = 37, normalized size = 0.80

$$\frac{4\cos^9(a+bx)(-505+540\cos(2(a+bx))-99\cos(4(a+bx)))}{1287b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^5,x]
```

```
[Out] (4*Cos[a + b*x]^9*(-505 + 540*Cos[2*(a + b*x)] - 99*Cos[4*(a + b*x)]))/(128
7*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(40) = 80$.
time = 0.33, size = 97, normalized size = 2.11

method	result	si
default	$-\frac{5\cos(xb+a)}{32b}-\frac{25\cos(3xb+3a)}{384b}+\frac{\cos(5xb+5a)}{128b}+\frac{\cos(7xb+7a)}{64b}+\frac{\cos(9xb+9a)}{576b}-\frac{3\cos(11xb+11a)}{1408b}-\frac{\cos(13xb+13a)}{1664b}$	9
risch	$-\frac{5\cos(xb+a)}{32b}-\frac{25\cos(3xb+3a)}{384b}+\frac{\cos(5xb+5a)}{128b}+\frac{\cos(7xb+7a)}{64b}+\frac{\cos(9xb+9a)}{576b}-\frac{3\cos(11xb+11a)}{1408b}-\frac{\cos(13xb+13a)}{1664b}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -5/32*cos(b*x+a)/b-25/384*cos(3*b*x+3*a)/b+1/128*cos(5*b*x+5*a)/b+1/64*cos(
7*b*x+7*a)/b+1/576*cos(9*b*x+9*a)/b-3/1408*cos(11*b*x+11*a)/b-1/1664*cos(13
*b*x+13*a)/b
```

Maxima [A]
time = 0.28, size = 80, normalized size = 1.74

$$-\frac{99\cos\left(13\,bx+13\,a\right)+351\cos\left(11\,bx+11\,a\right)-286\cos\left(9\,bx+9\,a\right)-2574\cos\left(7\,bx+7\,a\right)-1287\cos\left(5\,bx+5\,a\right)+10725\cos\left(3\,bx+3\,a\right)+25740\cos\left(bx+a\right)}{164736\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] -1/164736*(99*cos(13*b*x + 13*a) + 351*cos(11*b*x + 11*a) - 286*cos(9*b*x +
 9*a) - 2574*cos(7*b*x + 7*a) - 1287*cos(5*b*x + 5*a) + 10725*cos(3*b*x + 3
*a) + 25740*cos(b*x + a))/b
```

Fricas [A]
time = 1.89, size = 36, normalized size = 0.78

$$-\frac{32\left(99\cos\left(bx+a\right)^{13}-234\cos\left(bx+a\right)^{11}+143\cos\left(bx+a\right)^{9}\right)}{1287\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] -32/1287*(99*cos(b*x + a)^13 - 234*cos(b*x + a)^11 + 143*cos(b*x + a)^9)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(39) = 78$.
time = 33.48, size = 447, normalized size = 9.72

$$\begin{cases} -\frac{2234\sin^{3}(a+bx)\sin^{5}(2a+2bx)}{9009b}-\frac{4544\sin^{3}(a+bx)\sin^{3}(2a+2bx)\cos^{2}(2a+2bx)}{9009b}-\frac{256\sin^{3}(a+bx)\sin(2a+2bx)\cos^{4}(2a+2bx)}{1001b}-\frac{1388\sin^{2}(a+bx)\sin^{4}(2a+2bx)\cos(a+bx)\cos(2a+2bx)}{3003b}-\frac{2944\sin^{2}(a+bx)\sin^{2}(2a+2bx)\cos(a+bx)\cos^{3}(2a+2bx)}{3003b}-\frac{512\sin^{2}(a+bx)\cos(a+bx)\cos^{5}(2a+2bx)}{1001b}+\frac{271\sin(a+bx)\sin^{5}(2a+2bx)\cos^{2}(a+bx)}{3003b}+\frac{48\sin(a+bx)\sin^{3}(2a+2bx)\cos^{2}(a+bx)\cos^{2}(2a+2bx)}{143b}+\frac{640\sin(a+bx)\sin(2a+2bx)\cos^{2}(a+bx)\cos^{4}(2a+2bx)}{3003b}-\frac{1366\sin^{4}(2a+2bx)\cos^{3}(a+bx)\cos(2a+2bx)}{3003b}-\frac{4960\sin^{2}(2a+2bx)\cos^{3}(a+bx)\cos^{3}(2a+2bx)}{9009b}-\frac{256\cos^{3}(a+bx)\cos^{5}(2a+2bx)}{1287b} & \text{for}\: b\neq 0 \\ x\sin^{5}(2a)\cos^{3}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**5,x)
```

```
[Out] Piecewise((-2234*sin(a + b*x)**3*sin(2*a + 2*b*x)**5/(9009*b) - 4544*sin(a
+ b*x)**3*sin(2*a + 2*b*x)**3*cos(2*a + 2*b*x)**2/(9009*b) - 256*sin(a + b*
x)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**4/(1001*b) - 1388*sin(a + b*x)**2*
sin(2*a + 2*b*x)**4*cos(a + b*x)*cos(2*a + 2*b*x)/(3003*b) - 2944*sin(a + b
*x)**2*sin(2*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**3/(3003*b) - 512*
sin(a + b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)**5/(1001*b) + 271*sin(a + b*x
)*sin(2*a + 2*b*x)**5*cos(a + b*x)**2/(3003*b) + 48*sin(a + b*x)*sin(2*a +
2*b*x)**3*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/(143*b) + 640*sin(a + b*x)*si
n(2*a + 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**4/(3003*b) - 1366*sin(2*a
+ 2*b*x)**4*cos(a + b*x)**3*cos(2*a + 2*b*x)/(3003*b) - 4960*sin(2*a + 2*b*
x)**2*cos(a + b*x)**3*cos(2*a + 2*b*x)**3/(9009*b) - 256*cos(a + b*x)**3*co
s(2*a + 2*b*x)**5/(1287*b), Ne(b, 0)), (x*sin(2*a)**5*cos(a)**3, True))
```

Giac [A]
time = 0.42, size = 36, normalized size = 0.78

$$-\frac{32\left(99\cos\left(bx+a\right)^{13}-234\cos\left(bx+a\right)^{11}+143\cos\left(bx+a\right)^{9}\right)}{1287\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] -32/1287*(99*cos(b*x + a)^13 - 234*cos(b*x + a)^11 + 143*cos(b*x + a)^9)/b
```

Mupad [B]
time = 0.07, size = 36, normalized size = 0.78

$$-\frac{32\,\left(99\,\cos\left(a+b\,x\right)^{13}-234\,\cos\left(a+b\,x\right)^{11}+143\,\cos\left(a+b\,x\right)^9\right)}{1287\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^5,x)
```

```
[Out] -(32*(143*cos(a + b*x)^9 - 234*cos(a + b*x)^11 + 99*cos(a + b*x)^13))/(1287
*b)
```

3.152 $\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx$

Optimal. Leaf size=61

$$\frac{16\sin^5(a+bx)}{5b}-\frac{48\sin^7(a+bx)}{7b}+\frac{16\sin^9(a+bx)}{3b}-\frac{16\sin^{11}(a+bx)}{11b}$$

```
[Out] 16/5*sin(b*x+a)^5/b-48/7*sin(b*x+a)^7/b+16/3*sin(b*x+a)^9/b-16/11*sin(b*x+a
)^11/b
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4372, 2644, 276}

$$-\frac{16\sin^{11}(a+bx)}{11b}+\frac{16\sin^9(a+bx)}{3b}-\frac{48\sin^7(a+bx)}{7b}+\frac{16\sin^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] (16*Sin[a + b*x]^5)/(5*b) - (48*Sin[a + b*x]^7)/(7*b) + (16*Sin[a + b*x]^9)
/(3*b) - (16*Sin[a + b*x]^11)/(11*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^3(a+bx)\sin^4(2a+2bx)\,dx = 16\int \cos^7(a+bx)\sin^4(a+bx)\,dx$$
$$= \frac{16\text{Subst}\left(\int x^4(1-x^2)^3\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{16\text{Subst}\left(\int (x^4-3x^6+3x^8-x^{10})\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{16\sin^5(a+bx)}{5b} - \frac{48\sin^7(a+bx)}{7b} + \frac{16\sin^9(a+bx)}{3b} - \frac{16\sin^{11}(a+bx)}{11b}$$

Mathematica [A]
time = 0.25, size = 47, normalized size = 0.77

$$\frac{(3042+3335\cos(2(a+bx))+910\cos(4(a+bx))+105\cos(6(a+bx)))\sin^5(a+bx)}{2310b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^4,x]
```

```
[Out] ((3042 + 3335*Cos[2*(a + b*x)] + 910*Cos[4*(a + b*x)] + 105*Cos[6*(a + b*x)
])*Sin[a + b*x]^5)/(2310*b)
```

Maple [A]
time = 0.29, size = 83, normalized size = 1.36

method	result	size
default	$\frac{7\sin(xb+a)}{32b}-\frac{\sin(3xb+3a)}{32b}-\frac{11\sin(5xb+5a)}{320b}-\frac{\sin(7xb+7a)}{448b}+\frac{\sin(9xb+9a)}{192b}+\frac{\sin(11xb+11a)}{704b}$	83
risch	$\frac{7\sin(xb+a)}{32b}-\frac{\sin(3xb+3a)}{32b}-\frac{11\sin(5xb+5a)}{320b}-\frac{\sin(7xb+7a)}{448b}+\frac{\sin(9xb+9a)}{192b}+\frac{\sin(11xb+11a)}{704b}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 7/32*sin(b*x+a)/b-1/32*sin(3*b*x+3*a)/b-11/320/b*sin(5*b*x+5*a)-1/448/b*sin
(7*b*x+7*a)+1/192/b*sin(9*b*x+9*a)+1/704/b*sin(11*b*x+11*a)
```

Maxima [A]
time = 0.28, size = 69, normalized size = 1.13

$$\frac{105\sin(11\,bx+11\,a)+385\sin(9\,bx+9\,a)-165\sin(7\,bx+7\,a)-2541\sin(5\,bx+5\,a)-2310\sin(3\,bx+3\,a)+16170\sin(bx+a)}{73920\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

[Out] 1/73920*(105*sin(11*b*x + 11*a) + 385*sin(9*b*x + 9*a) - 165*sin(7*b*x + 7*a) - 2541*sin(5*b*x + 5*a) - 2310*sin(3*b*x + 3*a) + 16170*sin(b*x + a))/b

Fricas [**A**]
time = 1.58, size = 63, normalized size = 1.03

$$\frac{16\left(105\cos\left(bx+a\right)^{10}-140\cos\left(bx+a\right)^{8}+5\cos\left(bx+a\right)^{6}+6\cos\left(bx+a\right)^{4}+8\cos\left(bx+a\right)^{2}+16\right)\sin\left(bx+a\right)}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="fricas")

[Out] 16/1155*(105*cos(b*x + a)^10 - 140*cos(b*x + a)^8 + 5*cos(b*x + a)^6 + 6*cos(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x + a)/b

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(53) = 106$.
time = 14.96, size = 366, normalized size = 6.00

$$\begin{cases} \frac{46\sin^{3}\left(a+bx\right)\sin^{4}\left(2a+2bx\right)}{165b}+\frac{192\sin^{3}\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos^{2}\left(2a+2bx\right)}{385b}+\frac{256\sin^{3}\left(a+bx\right)\cos^{4}\left(2a+2bx\right)}{1155b}+\frac{272\sin^{2}\left(a+bx\right)\sin^{3}\left(2a+2bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{1155b}+\frac{256\sin^{2}\left(a+bx\right)\sin\left(2a+2bx\right)\cos\left(a+bx\right)\cos^{3}\left(2a+2bx\right)}{1155b}+\frac{211\sin\left(a+bx\right)\sin^{4}\left(2a+2bx\right)\cos^{2}\left(a+bx\right)}{1155b}+\frac{304\sin\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos^{2}\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{385b}+\frac{128\sin\left(a+bx\right)\cos^{2}\left(a+bx\right)\cos^{4}\left(2a+2bx\right)}{231b}-\frac{472\sin^{3}\left(2a+2bx\right)\cos^{3}\left(a+bx\right)\cos\left(2a+2bx\right)}{1155b}-\frac{64\sin\left(2a+2bx\right)\cos^{3}\left(a+bx\right)\cos^{3}\left(2a+2bx\right)}{231b} & \text{for}\: b \neq 0 \\ x\sin^{4}\left(2a\right)\cos^{3}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**4,x)

[Out] Piecewise((46*sin(a + b*x)**3*sin(2*a + 2*b*x)**4/(165*b) + 192*sin(a + b*x)**3*sin(2*a + 2*b*x)**2*cos(2*a + 2*b*x)**2/(385*b) + 256*sin(a + b*x)**3*cos(2*a + 2*b*x)**4/(1155*b) + 272*sin(a + b*x)**2*sin(2*a + 2*b*x)**3*cos(a + b*x)*cos(2*a + 2*b*x)/(1155*b) + 256*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(a + b*x)*cos(2*a + 2*b*x)**3/(1155*b) + 211*sin(a + b*x)*sin(2*a + 2*b*x)**4*cos(a + b*x)**2/(1155*b) + 304*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/(385*b) + 128*sin(a + b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**4/(231*b) - 472*sin(2*a + 2*b*x)**3*cos(a + b*x)**3*cos(2*a + 2*b*x)/(1155*b) - 64*sin(2*a + 2*b*x)*cos(a + b*x)**3*cos(2*a + 2*b*x)**3/(231*b), Ne(b, 0)), (x*sin(2*a)**4*cos(a)**3, True))

Giac [**A**]
time = 0.41, size = 69, normalized size = 1.13

$$\frac{105\sin\left(11\,bx+11\,a\right)+385\sin\left(9\,bx+9\,a\right)-165\sin\left(7\,bx+7\,a\right)-2541\sin\left(5\,bx+5\,a\right)-2310\sin\left(3\,bx+3\,a\right)+16170\sin\left(bx+a\right)}{73920\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^4,x, algorithm="giac")

[Out] 1/73920*(105*sin(11*b*x + 11*a) + 385*sin(9*b*x + 9*a) - 165*sin(7*b*x + 7*a) - 2541*sin(5*b*x + 5*a) - 2310*sin(3*b*x + 3*a) + 16170*sin(b*x + a))/b

Mupad [B]
time = 0.06, size = 45, normalized size = 0.74

$$\frac{-\frac{16\sin(a+b\,x)^{11}}{11}+\frac{16\sin(a+b\,x)^9}{3}-\frac{48\sin(a+b\,x)^7}{7}+\frac{16\sin(a+b\,x)^5}{5}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^4,x)
[Out] ((16*sin(a + b*x)^5)/5 - (48*sin(a + b*x)^7)/7 + (16*sin(a + b*x)^9)/3 - (1
6*sin(a + b*x)^11)/11)/b
```

3.153 $\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx$

Optimal. Leaf size=31

$$-\frac{8\cos^7(a+bx)}{7b}+\frac{8\cos^9(a+bx)}{9b}$$

```
[Out] -8/7*cos(b*x+a)^7/b+8/9*cos(b*x+a)^9/b
```

Rubi [A]
time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4372, 2645, 14}

$$\frac{8\cos^9(a+bx)}{9b}-\frac{8\cos^7(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
```

```
[Out] (-8*Cos[a + b*x]^7)/(7*b) + (8*Cos[a + b*x]^9)/(9*b)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^3(a+bx)\sin^3(2a+2bx)\,dx = 8\int \cos^6(a+bx)\sin^3(a+bx)\,dx$$
$$= -\frac{8\text{Subst}\left(\int x^6(1-x^2)\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{8\text{Subst}\left(\int (x^6-x^8)\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{8\cos^7(a+bx)}{7b} + \frac{8\cos^9(a+bx)}{9b}$$

Mathematica [A]
time = 0.16, size = 27, normalized size = 0.87

$$\frac{4\cos^7(a+bx)(-11+7\cos(2(a+bx)))}{63b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^3,x]
[Out] (4*Cos[a + b*x]^7*(-11 + 7*Cos[2*(a + b*x)]))/(63*b)
```

Maple [A]
time = 0.18, size = 55, normalized size = 1.77

method	result	size
default	$-\frac{3\cos(xb+a)}{16b}-\frac{\cos(3xb+3a)}{12b}+\frac{3\cos(7xb+7a)}{224b}+\frac{\cos(9xb+9a)}{288b}$	55
risch	$-\frac{3\cos(xb+a)}{16b}-\frac{\cos(3xb+3a)}{12b}+\frac{3\cos(7xb+7a)}{224b}+\frac{\cos(9xb+9a)}{288b}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
[Out] -3/16*cos(b*x+a)/b-1/12*cos(3*b*x+3*a)/b+3/224*cos(7*b*x+7*a)/b+1/288*cos(9
*b*x+9*a)/b
```

Maxima [A]
time = 0.28, size = 47, normalized size = 1.52

$$\frac{7\cos(9\,bx+9\,a)+27\cos(7\,bx+7\,a)-168\cos(3\,bx+3\,a)-378\cos(bx+a)}{2016\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/2016*(7*cos(9*b*x + 9*a) + 27*cos(7*b*x + 7*a) - 168*cos(3*b*x + 3*a) - 3
78*cos(b*x + a))/b
```

Fricas [A]
time = 1.91, size = 26, normalized size = 0.84

$$\frac{8\left(7\cos\left(bx+a\right)^{9}-9\cos\left(bx+a\right)^{7}\right)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] 8/63*(7*cos(b*x + a)^9 - 9*cos(b*x + a)^7)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(26) = 52$.
time = 6.37, size = 284, normalized size = 9.16

$$\begin{cases} -\frac{94\sin^{3}\left(a+bx\right)\sin^{3}\left(2a+2bx\right)}{315b}-\frac{32\sin^{3}\left(a+bx\right)\sin\left(2a+2bx\right)\cos^{2}\left(2a+2bx\right)}{105b}-\frac{4\sin^{2}\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{7b}-\frac{64\sin^{2}\left(a+bx\right)\cos\left(a+bx\right)\cos^{3}\left(2a+2bx\right)}{105b}+\frac{13\sin\left(a+bx\right)\sin^{3}\left(2a+2bx\right)\cos^{2}\left(a+bx\right)}{105b}+\frac{8\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos^{2}\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{35b}-\frac{46\sin^{2}\left(2a+2bx\right)\cos^{3}\left(a+bx\right)\cos\left(2a+2bx\right)}{105b}-\frac{16\cos^{3}\left(a+bx\right)\cos^{3}\left(2a+2bx\right)}{63b} & \text{for}\: b\neq 0 \\ x\sin^{3}\left(2a\right)\cos^{3}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**3,x)
```

```
[Out] Piecewise((-94*sin(a + b*x)**3*sin(2*a + 2*b*x)**3/(315*b) - 32*sin(a + b*x
)**3*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)**2/(105*b) - 4*sin(a + b*x)**2*sin(2
*a + 2*b*x)**2*cos(a + b*x)*cos(2*a + 2*b*x)/(7*b) - 64*sin(a + b*x)**2*cos
(a + b*x)*cos(2*a + 2*b*x)**3/(105*b) + 13*sin(a + b*x)*sin(2*a + 2*b*x)**3
*cos(a + b*x)**2/(105*b) + 8*sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*x)**2*
cos(2*a + 2*b*x)**2/(35*b) - 46*sin(2*a + 2*b*x)**2*cos(a + b*x)**3*cos(2*a
 + 2*b*x)/(105*b) - 16*cos(a + b*x)**3*cos(2*a + 2*b*x)**3/(63*b), Ne(b, 0)
), (x*sin(2*a)**3*cos(a)**3, True))
```

Giac [A]
time = 0.55, size = 26, normalized size = 0.84

$$\frac{8\left(7\cos\left(bx+a\right)^{9}-9\cos\left(bx+a\right)^{7}\right)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 8/63*(7*cos(b*x + a)^9 - 9*cos(b*x + a)^7)/b
```

Mupad [B]
time = 0.15, size = 26, normalized size = 0.84

$$-\frac{8\left(9\cos\left(a+b\,x\right)^{7}-7\cos\left(a+b\,x\right)^{9}\right)}{63\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^3,x)
[Out] -(8*(9*cos(a + b*x)^7 - 7*cos(a + b*x)^9))/(63*b)
```

3.154 $\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=46

$$\frac{4\sin^3(a+bx)}{3b}-\frac{8\sin^5(a+bx)}{5b}+\frac{4\sin^7(a+bx)}{7b}$$

```
[Out] 4/3*sin(b*x+a)^3/b-8/5*sin(b*x+a)^5/b+4/7*sin(b*x+a)^7/b
```

Rubi [A]
time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {4372, 2644, 276}

$$\frac{4\sin^7(a+bx)}{7b}-\frac{8\sin^5(a+bx)}{5b}+\frac{4\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (4*Sin[a + b*x]^3)/(3*b) - (8*Sin[a + b*x]^5)/(5*b) + (4*Sin[a + b*x]^7)/(7
*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^3(a+bx)\sin^2(2a+2bx)\,dx = 4\int \cos^5(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{4\text{Subst}\left(\int x^2(1-x^2)^2\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{4\text{Subst}\left(\int (x^2-2x^4+x^6)\,dx, x, \sin(a+bx)\right)}{b}$$
$$= \frac{4\sin^3(a+bx)}{3b} - \frac{8\sin^5(a+bx)}{5b} + \frac{4\sin^7(a+bx)}{7b}$$

Mathematica [A]
time = 0.11, size = 37, normalized size = 0.80

$$\frac{(157+108\cos(2(a+bx))+15\cos(4(a+bx)))\sin^3(a+bx)}{210b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] ((157 + 108*Cos[2*(a + b*x)] + 15*Cos[4*(a + b*x)])*Sin[a + b*x]^3)/(210*b)
```

Maple [A]
time = 0.16, size = 55, normalized size = 1.20

method	result	size
default	$\frac{5\sin(xb+a)}{16b}-\frac{\sin(3xb+3a)}{48b}-\frac{3\sin(5xb+5a)}{80b}-\frac{\sin(7xb+7a)}{112b}$	55
risch	$\frac{5\sin(xb+a)}{16b}-\frac{\sin(3xb+3a)}{48b}-\frac{3\sin(5xb+5a)}{80b}-\frac{\sin(7xb+7a)}{112b}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 5/16*sin(b*x+a)/b-1/48*sin(3*b*x+3*a)/b-3/80/b*sin(5*b*x+5*a)-1/112/b*sin(7
*b*x+7*a)
```

Maxima [A]
time = 0.29, size = 47, normalized size = 1.02

$$-\frac{15\sin(7\,bx+7\,a)+63\sin(5\,bx+5\,a)+35\sin(3\,bx+3\,a)-525\sin(bx+a)}{1680\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/1680*(15*sin(7*b*x + 7*a) + 63*sin(5*b*x + 5*a) + 35*sin(3*b*x + 3*a) -
525*sin(b*x + a))/b
```

Fricas [**A**]
time = 2.36, size = 43, normalized size = 0.93

$$-\frac{4\left(15\cos\left(bx+a\right)^{6}-3\cos\left(bx+a\right)^{4}-4\cos\left(bx+a\right)^{2}-8\right)\sin\left(bx+a\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -4/105*(15*cos(b*x + a)^6 - 3*cos(b*x + a)^4 - 4*cos(b*x + a)^2 - 8)*sin(b*
x + a)/b
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(39) = 78$.
time = 2.71, size = 202, normalized size = 4.39

$$\begin{cases} \frac{38\sin^{3}\left(a+bx\right)\sin^{2}\left(2a+2bx\right)}{105b}+\frac{32\sin^{3}\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{105b}+\frac{8\sin^{2}\left(a+bx\right)\sin\left(2a+2bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{35b}+\frac{11\sin\left(a+bx\right)\sin^{2}\left(2a+2bx\right)\cos^{2}\left(a+bx\right)}{35b}+\frac{24\sin\left(a+bx\right)\cos^{2}\left(a+bx\right)\cos^{2}\left(2a+2bx\right)}{35b}-\frac{12\sin\left(2a+2bx\right)\cos^{3}\left(a+bx\right)\cos\left(2a+2bx\right)}{35b} & \text{for}\: b\neq 0 \\ x\sin^{2}\left(2a\right)\cos^{3}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((38*sin(a + b*x)**3*sin(2*a + 2*b*x)**2/(105*b) + 32*sin(a + b*x)
**3*cos(2*a + 2*b*x)**2/(105*b) + 8*sin(a + b*x)**2*sin(2*a + 2*b*x)*cos(a
+ b*x)*cos(2*a + 2*b*x)/(35*b) + 11*sin(a + b*x)*sin(2*a + 2*b*x)**2*cos(a
+ b*x)**2/(35*b) + 24*sin(a + b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)**2/(35*
b) - 12*sin(2*a + 2*b*x)*cos(a + b*x)**3*cos(2*a + 2*b*x)/(35*b), Ne(b, 0))
, (x*sin(2*a)**2*cos(a)**3, True))
```

Giac [**A**]
time = 0.46, size = 47, normalized size = 1.02

$$-\frac{15\sin\left(7\,bx+7\,a\right)+63\sin\left(5\,bx+5\,a\right)+35\sin\left(3\,bx+3\,a\right)-525\sin\left(bx+a\right)}{1680\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/1680*(15*sin(7*b*x + 7*a) + 63*sin(5*b*x + 5*a) + 35*sin(3*b*x + 3*a) -
525*sin(b*x + a))/b
```

Mupad [**B**]
time = 0.16, size = 36, normalized size = 0.78

$$\frac{4\left(15\sin\left(a+b\,x\right)^{7}-42\sin\left(a+b\,x\right)^{5}+35\sin\left(a+b\,x\right)^{3}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^2,x)
[Out] (4*(35*sin(a + b*x)^3 - 42*sin(a + b*x)^5 + 15*sin(a + b*x)^7))/(105*b)
```

3.155 $\int \cos^3(a+bx)\sin(2a+2bx)\,dx$

Optimal. Leaf size=15

$$-\frac{2\cos^5(a+bx)}{5b}$$

```
[Out] -2/5*cos(b*x+a)^5/b
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4372, 2645, 30}

$$-\frac{2\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*Cos[a + b*x]^5)/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^3(a+bx)\sin(2a+2bx)\,dx = 2\int \cos^4(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{2\text{Subst}\left(\int x^4\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{2\cos^5(a+bx)}{5b}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-\frac{2\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x],x]
```

```
[Out] (-2*Cos[a + b*x]^5)/(5*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(13) = 26$.
time = 0.10, size = 41, normalized size = 2.73

method	result	size
default	$-\frac{\cos(xb+a)}{4b}-\frac{\cos(3xb+3a)}{8b}-\frac{\cos(5xb+5a)}{40b}$	41
risch	$-\frac{\cos(xb+a)}{4b}-\frac{\cos(3xb+3a)}{8b}-\frac{\cos(5xb+5a)}{40b}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*cos(b*x+a)/b-1/8*cos(3*b*x+3*a)/b-1/40*cos(5*b*x+5*a)/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(13) = 26$.
time = 0.27, size = 34, normalized size = 2.27

$$-\frac{\cos(5\,bx+5\,a)+5\,\cos(3\,bx+3\,a)+10\,\cos(bx+a)}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] -1/40*(cos(5*b*x + 5*a) + 5*cos(3*b*x + 3*a) + 10*cos(b*x + a))/b
```

Fricas [A]
time = 1.74, size = 13, normalized size = 0.87

$$-\frac{2 \cos\left(bx+a\right)^{5}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] -2/5*cos(b*x + a)^5/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(14) = 28$.
time = 1.04, size = 117, normalized size = 7.80

$$\begin{cases} -\frac{2\sin^{3}\left(a+bx\right)\sin\left(2a+2bx\right)}{5b} - \frac{4\sin^{2}\left(a+bx\right)\cos\left(a+bx\right)\cos\left(2a+2bx\right)}{5b} + \frac{\sin\left(a+bx\right)\sin\left(2a+2bx\right)\cos^{2}\left(a+bx\right)}{5b} - \frac{2\cos^{3}\left(a+bx\right)\cos\left(2a+2bx\right)}{5b} & \text{for}\: b \neq 0 \\ x\sin\left(2a\right)\cos^{3}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a),x)
```

```
[Out] Piecewise((-2*sin(a + b*x)**3*sin(2*a + 2*b*x)/(5*b) - 4*sin(a + b*x)**2*co
s(a + b*x)*cos(2*a + 2*b*x)/(5*b) + sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b
*x)**2/(5*b) - 2*cos(a + b*x)**3*cos(2*a + 2*b*x)/(5*b), Ne(b, 0)), (x*sin(
2*a)*cos(a)**3, True))
```

Giac [A]
time = 0.40, size = 13, normalized size = 0.87

$$-\frac{2 \cos\left(bx+a\right)^{5}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] -2/5*cos(b*x + a)^5/b
```

Mupad [B]
time = 0.14, size = 13, normalized size = 0.87

$$-\frac{2\cos\left(a+b\,x\right)^5}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x),x)
```

```
[Out] -(2*cos(a + b*x)^5)/(5*b)
```

3.156 $\int \cos^3(a+bx)\csc(2a+2bx)\,dx$

Optimal. Leaf size=28

$$-\frac{\tanh^{-1}(\cos(a+bx))}{2b}+\frac{\cos(a+bx)}{2b}$$

```
[Out] -1/2*arctanh(cos(b*x+a))/b+1/2*cos(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {4372, 2672, 327, 212}

$$\frac{\cos(a+bx)}{2b}-\frac{\tanh^{-1}(\cos(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x],x]
```

```
[Out] -1/2*ArcTanh[Cos[a + b*x]]/b + Cos[a + b*x]/(2*b)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
```

```
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\csc(2a+2bx)\,dx &= \frac{1}{2}\int \cos(a+bx)\cot(a+bx)\,dx\\ &= -\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}\\ &= \frac{\cos(a+bx)}{2b} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(a+bx)\right)}{2b}\\ &= -\frac{\tanh^{-1}(\cos(a+bx))}{2b} + \frac{\cos(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 1.64

$$\frac{1}{2}\left(\frac{\cos(a+bx)}{b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x],x]
[Out] (Cos[a + b*x]/b - Log[Cos[(a + b*x)/2]]/b + Log[Sin[(a + b*x)/2]]/b)/2
```

Maple [A]
time = 0.15, size = 29, normalized size = 1.04

method	result	size
default	$\frac{\cos(xb+a)+\ln(\csc(xb+a)-\cot(xb+a))}{2b}$	29
risch	$\frac{e^{i(xb+a)}}{4b} + \frac{e^{-i(xb+a)}}{4b} + \frac{\ln(e^{i(xb+a)}-1)}{2b} - \frac{\ln(e^{i(xb+a)}+1)}{2b}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
[Out] 1/2/b*(cos(b*x+a)+ln(csc(b*x+a)-cot(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(24) = 48$.
time = 0.27, size = 92, normalized size = 3.29

$$\frac{2\cos(bx+a) - \log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2\right) + \log\left(\cos(bx)^2 - 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 + 2\sin(bx)\sin(a) + \sin(a)^2\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a),x, algorithm="maxima")
```

```
[Out] 1/4*(2*cos(b*x + a) - log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b
*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a)
+ cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b
```

Fricas [A]
time = 2.89, size = 38, normalized size = 1.36

$$\frac{2 \cos (bx+a)-\log \left(\frac{1}{2} \cos (bx+a)+\frac{1}{2}\right)+\log \left(-\frac{1}{2} \cos (bx+a)+\frac{1}{2}\right)}{4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*cos(b*x + a) - log(1/2*cos(b*x + a) + 1/2) + log(-1/2*cos(b*x + a) +
 1/2))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [A]
time = 0.42, size = 36, normalized size = 1.29

$$\frac{2 \cos (bx+a)-\log (\cos (bx+a)+1)+\log (-\cos (bx+a)+1)}{4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a),x, algorithm="giac")
```

```
[Out] 1/4*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(-cos(b*x + a) + 1))/b
```

Mupad [B]
time = 0.05, size = 22, normalized size = 0.79

$$\frac{\frac{\cos(a+b\,x)}{2}-\frac{\mathrm{atanh}(\cos(a+b\,x))}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x),x)
```

```
[Out] (cos(a + b*x)/2 - atanh(cos(a + b*x))/2)/b
```

3.157 $\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx$

Optimal. Leaf size=13

$$-\frac{\csc(a+bx)}{4b}$$

```
[Out] -1/4*csc(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4372, 2686, 8}

$$-\frac{\csc(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -1/4*Csc[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \cos^3(a+bx)\csc^2(2a+2bx)\,dx &= \frac{1}{4}\int \cot(a+bx)\csc(a+bx)\,dx\\
&= -\frac{\text{Subst}\left(\int 1\,dx, x, \csc(a+bx)\right)}{4b}\\
&= -\frac{\csc(a+bx)}{4b}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$-\frac{\csc(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^2,x]
```

```
[Out] -1/4*Csc[a + b*x]/b
```

Maple [A]
time = 0.09, size = 14, normalized size = 1.08

method	result	size
default	$-\frac{1}{4\sin(xb+a)b}$	14
risch	$-\frac{i e^{i(xb+a)}}{2b\left(e^{2i(xb+a)}-1\right)}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/sin(b*x+a)/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(11) = 22.
time = 0.27, size = 84, normalized size = 6.46

$$-\frac{\cos(bx+a)\sin(2bx+2a)-\cos(2bx+2a)\sin(bx+a)+\sin(bx+a)}{2\left(b\cos(2bx+2a)^2+b\sin(2bx+2a)^2-2b\cos(2bx+2a)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(cos(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)*sin(b*x + a) + sin(b
*x + a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 - 2*b*cos(2*b*x + 2*a
) + b)
```

Fricas [A]
time = 3.10, size = 13, normalized size = 1.00

$$-\frac{1}{4b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -1/4/(b*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.46, size = 13, normalized size = 1.00

$$-\frac{1}{4\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -1/4/(b*sin(b*x + a))
```

Mupad [**B**]
time = 0.02, size = 13, normalized size = 1.00

$$-\frac{1}{4\,b\,\sin\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^2,x)
```

```
[Out] -1/(4*b*sin(a + b*x))
```

3.158 $\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx$

Optimal. Leaf size=34

$$-\frac{\tanh^{-1}(\cos(a+bx))}{16b}-\frac{\cot(a+bx)\csc(a+bx)}{16b}$$

```
[Out] -1/16*arctanh(cos(b*x+a))/b-1/16*cot(b*x+a)*csc(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4372, 3853, 3855}

$$-\frac{\tanh^{-1}(\cos(a+bx))}{16b}-\frac{\cot(a+bx)\csc(a+bx)}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] -1/16*ArcTanh[Cos[a + b*x]]/b - (Cot[a + b*x]*Csc[a + b*x])/(16*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(a+bx)\csc^3(2a+2bx)\,dx &= \frac{1}{8}\int \csc^3(a+bx)\,dx \\
&= -\frac{\cot(a+bx)\csc(a+bx)}{16b} + \frac{1}{16}\int \csc(a+bx)\,dx \\
&= -\frac{\tanh^{-1}(\cos(a+bx))}{16b} - \frac{\cot(a+bx)\csc(a+bx)}{16b}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 79 vs. $2(34) = 68$.
time = 0.02, size = 79, normalized size = 2.32

$$
\frac{1}{8}\left(-\frac{\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^3,x]
```

```
[Out] (-1/8*Csc[(a + b*x)/2]^2/b - Log[Cos[(a + b*x)/2]]/(2*b) + Log[Sin[(a + b*x
)/2]]/(2*b) + Sec[(a + b*x)/2]^2/(8*b))/8
```

Maple [**A**]
time = 0.22, size = 39, normalized size = 1.15

method	result	size
default	$\frac{-\frac{\csc(xb+a)\cot(xb+a)}{2}+\frac{\ln(\csc(xb+a)-\cot(xb+a))}{2}}{8b}$	39
risch	$\frac{e^{3i(xb+a)}+e^{i(xb+a)}}{8b\left(e^{2i(xb+a)}-1\right)^2} - \frac{\ln\left(e^{i(xb+a)}+1\right)}{16b} + \frac{\ln\left(e^{i(xb+a)}-1\right)}{16b}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/b*(-1/2*csc(b*x+a)*cot(b*x+a)+1/2*ln(csc(b*x+a)-cot(b*x+a)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(30) = 60$.
time = 0.29, size = 558, normalized size = 16.41

$$
\frac{4(\cos(3bx+3a)+\cos(bx+a))\cos(4bx+4a) - 4(2\cos(2bx+2a)-1)\cos(3bx+3a) - 8\cos(2bx+2a)\cos(bx+a) + (2(2\cos(2bx+2a)-1)\cos(4bx+4a) - \cos(4bx+4a)^2 - 4\cos(2bx+2a)^2 - \sin(4bx+4a)^2 + 4\sin(4bx+4a)\sin(2bx+2a) - 4\sin(2bx+2a)^2 + 4\cos(2bx+2a) - 1)\log(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)^2) - (2(2\cos(2bx+2a)-1)\cos(4bx+4a) - \cos(4bx+4a)^2 - 4\cos(2bx+2a)^2 - \sin(4bx+4a)^2 + 4\sin(4bx+4a)\sin(2bx+2a) - 4\sin(2bx+2a)^2 + 4\cos(2bx+2a) - 1)\log(\cos(bx)^2 - 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 + 2\sin(bx)\sin(a) + \sin(a)^2) + 4(\sin(3bx+3a)+\sin(bx+a))\sin(4bx+4a) - 8\sin(3bx+3a)\sin(2bx+2a) - 8\sin(2bx+2a)\sin(bx+a) + 4\cos(bx+a)}{32\left(b\cos(4bx+4a)^2 + 4b\cos(2bx+2a)^2 + b\sin(4bx+4a)^2 - 4b\sin(4bx+4a)\sin(2bx+2a) + 4b\sin(2bx+2a)^2 - 2(2b\cos(2bx+2a)-b)\cos(4bx+4a) - 4b\cos(2bx+2a) + b\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(4*(cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) - 4*(2*cos(2*b*x
 + 2*a) - 1)*cos(3*b*x + 3*a) - 8*cos(2*b*x + 2*a)*cos(b*x + a) + (2*(2*cos
(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*
a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b
*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) +
cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (2*(2*cos(2*b*x + 2
*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin
(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^
2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 +
 sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + 4*(sin(3*b*x + 3*a) + sin(b*x
 + a))*sin(4*b*x + 4*a) - 8*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) - 8*sin(2*b*x
 + 2*a)*sin(b*x + a) + 4*cos(b*x + a))/(b*cos(4*b*x + 4*a)^2 + 4*b*cos(2*b*
x + 2*a)^2 + b*sin(4*b*x + 4*a)^2 - 4*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) +
 4*b*sin(2*b*x + 2*a)^2 - 2*(2*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a) - 4
*b*cos(2*b*x + 2*a) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(30) = 60$.
time = 2.23, size = 72, normalized size = 2.12

$$-\frac{\left(\cos\left(bx+a\right)^2-1\right)\log\left(\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-\left(\cos\left(bx+a\right)^2-1\right)\log\left(-\frac{1}{2}\cos\left(bx+a\right)+\frac{1}{2}\right)-2\cos\left(bx+a\right)}{32\left(b\cos\left(bx+a\right)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x, algorithm="fricas")
```

```
[Out] -1/32*((cos(b*x + a)^2 - 1)*log(1/2*cos(b*x + a) + 1/2) - (cos(b*x + a)^2 -
 1)*log(-1/2*cos(b*x + a) + 1/2) - 2*cos(b*x + a))/(b*cos(b*x + a)^2 - b)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 48, normalized size = 1.41

$$\frac{\frac{2\cos\left(bx+a\right)}{\cos\left(bx+a\right)^2-1}-\log\left(\cos\left(bx+a\right)+1\right)+\log\left(-\cos\left(bx+a\right)+1\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) - log(cos(b*x + a) + 1) + log(-co
s(b*x + a) + 1))/b
```

Mupad [B]
time = 0.06, size = 36, normalized size = 1.06

$$\frac{\cos\left(a+b\,x\right)}{16\,b\,\left({\cos\left(a+b\,x\right)}^2-1\right)}-\frac{\mathrm{atanh}\left(\cos\left(a+b\,x\right)\right)}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^3,x)
```

```
[Out] cos(a + b*x)/(16*b*(cos(a + b*x)^2 - 1)) - atanh(cos(a + b*x))/(16*b)
```

3.159 $\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}(\sin(a+bx))}{16b}-\frac{\csc(a+bx)}{16b}-\frac{\csc^3(a+bx)}{48b}$$

```
[Out] 1/16*arctanh(sin(b*x+a))/b-1/16*csc(b*x+a)/b-1/48*csc(b*x+a)^3/b
```

Rubi [A]
time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {4372, 2701, 308, 213}

$$-\frac{\csc^3(a+bx)}{48b}-\frac{\csc(a+bx)}{16b}+\frac{\tanh^{-1}(\sin(a+bx))}{16b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/(16*b) - Csc[a + b*x]/(16*b) - Csc[a + b*x]^3/(48*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(a+bx)\csc^4(2a+2bx)\,dx &= \frac{1}{16}\int \csc^4(a+bx)\sec(a+bx)\,dx \\
&= -\frac{\operatorname{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b} \\
&= -\frac{\operatorname{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \csc(a+bx)\right)}{16b} \\
&= -\frac{\csc(a+bx)}{16b}-\frac{\csc^3(a+bx)}{48b}-\frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{16b} \\
&= \frac{\tanh^{-1}(\sin(a+bx))}{16b}-\frac{\csc(a+bx)}{16b}-\frac{\csc^3(a+bx)}{48b}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 31, normalized size = 0.72

$$
-\frac{\csc^3(a+bx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\sin^2(a+bx)\right)}{48b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^4,x]
[Out] -1/48*(Csc[a + b*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[a + b*x]^2])/b
```

Maple [A]
time = 0.19, size = 41, normalized size = 0.95

method	result	size
default	$\frac{-\frac{1}{3\sin(xb+a)^3}-\frac{1}{\sin(xb+a)}+\ln(\sec(xb+a)+\tan(xb+a))}{16b}$	41
risch	$-\frac{i\left(3\,e^{5i(xb+a)}-10\,e^{3i(xb+a)}+3\,e^{i(xb+a)}\right)}{24b\left(e^{2i(xb+a)}-1\right)^3}-\frac{\ln\left(e^{i(xb+a)}-i\right)}{16b}+\frac{\ln\left(i+e^{i(xb+a)}\right)}{16b}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x,method=_RETURNVERBOSE)
[Out] 1/16/b*(-1/3/sin(b*x+a)^3-1/sin(b*x+a)+ln(sec(b*x+a)+tan(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 834 vs. 2(37) = 74.
time = 0.53, size = 834, normalized size = 19.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x, algorithm="maxima")
```

```
[Out] 1/96*(4*(3*sin(5*b*x + 5*a) - 10*sin(3*b*x + 3*a) + 3*sin(b*x + a))*cos(6*b
*x + 6*a) + 36*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + 12*
(10*sin(3*b*x + 3*a) - 3*sin(b*x + a))*cos(4*b*x + 4*a) + 3*(2*(3*cos(4*b*x
 + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - cos(6*b*x + 6*a)^2 + 6
*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 9*cos(4*b*x + 4*a)^2 - 9*cos(2
*b*x + 2*a)^2 + 6*(sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*sin(6*b*x + 6*a) -
sin(6*b*x + 6*a)^2 - 9*sin(4*b*x + 4*a)^2 + 18*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a) - 1)*log((cos(b*x + 2*a)^
2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a
)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a)
 + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) - 4*(3*cos(5*b*x
 + 5*a) - 10*cos(3*b*x + 3*a) + 3*cos(b*x + a))*sin(6*b*x + 6*a) - 12*(3*co
s(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*sin(5*b*x + 5*a) - 12*(10*cos(3*b*
x + 3*a) - 3*cos(b*x + a))*sin(4*b*x + 4*a) - 40*(3*cos(2*b*x + 2*a) - 1)*s
in(3*b*x + 3*a) + 120*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 36*cos(b*x + a)*s
in(2*b*x + 2*a) + 36*cos(2*b*x + 2*a)*sin(b*x + a) - 12*sin(b*x + a))/(b*co
s(6*b*x + 6*a)^2 + 9*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(
6*b*x + 6*a)^2 + 9*b*sin(4*b*x + 4*a)^2 - 18*b*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*cos(4*b*x + 4*a) - 3*b*cos(2*b*x +
2*a) + b)*cos(6*b*x + 6*a) - 6*(3*b*cos(2*b*x + 2*a) - b)*cos(4*b*x + 4*a)
- 6*b*cos(2*b*x + 2*a) - 6*(b*sin(4*b*x + 4*a) - b*sin(2*b*x + 2*a))*sin(6*
b*x + 6*a) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(37) = 74$.
time = 2.16, size = 94, normalized size = 2.19

$$\frac{3\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-3\left(\cos\left(bx+a\right)^{2}-1\right)\log\left(-\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right)-6\cos\left(bx+a\right)^{2}+8}{96\left(b\cos\left(bx+a\right)^{2}-b\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x, algorithm="fricas")
```

```
[Out] 1/96*(3*(cos(b*x + a)^2 - 1)*log(sin(b*x + a) + 1)*sin(b*x + a) - 3*(cos(b*
x + a)^2 - 1)*log(-sin(b*x + a) + 1)*sin(b*x + a) - 6*cos(b*x + a)^2 + 8)/(
(b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**4,x)
[Out] Timed out
```

Giac [A]
time = 0.47, size = 52, normalized size = 1.21

$$-\frac{\frac{2\left(3\,\sin(bx+a)^2+1\right)}{\sin(bx+a)^3} - 3\,\log\left(\sin\left(bx+a\right)+1\right) + 3\,\log\left(-\sin\left(bx+a\right)+1\right)}{96\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^4,x, algorithm="giac")
[Out] -1/96*(2*(3*sin(b*x + a)^2 + 1)/sin(b*x + a)^3 - 3*log(sin(b*x + a) + 1) +
3*log(-sin(b*x + a) + 1))/b
```

Mupad [B]
time = 0.06, size = 38, normalized size = 0.88

$$\frac{\mathrm{atanh}(\sin\left(a+b\,x\right))}{16\,b}-\frac{\frac{\sin(a+b\,x)^2}{16}+\frac{1}{48}}{b\sin\left(a+b\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^4,x)
[Out] atanh(sin(a + b*x))/(16*b) - (sin(a + b*x)^2/16 + 1/48)/(b*sin(a + b*x)^3)
```

3.160 $\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx$

Optimal. Leaf size=70

$$-\frac{15\tanh^{-1}(\cos(a+bx))}{256b}+\frac{15\sec(a+bx)}{256b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{256b}-\frac{\csc^4(a+bx)\sec(a+bx)}{128b}$$

```
[Out] -15/256*arctanh(cos(b*x+a))/b+15/256*sec(b*x+a)/b-5/256*csc(b*x+a)^2*sec(b*
x+a)/b-1/128*csc(b*x+a)^4*sec(b*x+a)/b
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4372, 2702, 294, 327, 213}

$$\frac{15\sec(a+bx)}{256b}-\frac{15\tanh^{-1}(\cos(a+bx))}{256b}-\frac{\csc^4(a+bx)\sec(a+bx)}{128b}-\frac{5\csc^2(a+bx)\sec(a+bx)}{256b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] (-15*ArcTanh[Cos[a + b*x]])/(256*b) + (15*Sec[a + b*x])/(256*b) - (5*Csc[a
+ b*x]^2*Sec[a + b*x])/(256*b) - (Csc[a + b*x]^4*Sec[a + b*x])/(128*b)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2702

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4372

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\int \cos^3(a+bx)\csc^5(2a+2bx)\,dx = \frac{1}{32}\int \csc^5(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^6}{(-1+x^2)^3}\,dx, x, \sec(a+bx)\right)}{32b}$$
$$= -\frac{\csc^4(a+bx)\sec(a+bx)}{128b} + \frac{5\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx, x, \sec(a+bx)\right)}{128b}$$
$$= -\frac{5\csc^2(a+bx)\sec(a+bx)}{256b} - \frac{\csc^4(a+bx)\sec(a+bx)}{128b} + \frac{15\text{Subst}\left(\int \right.}{}$$
$$= \frac{15\sec(a+bx)}{256b} - \frac{5\csc^2(a+bx)\sec(a+bx)}{256b} - \frac{\csc^4(a+bx)\sec(a+bx)}{128b}$$
$$= -\frac{15\tanh^{-1}(\cos(a+bx))}{256b} + \frac{15\sec(a+bx)}{256b} - \frac{5\csc^2(a+bx)\sec(a+bx}{256b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 195 vs. 2(70) = 140.
time = 0.37, size = 195, normalized size = 2.79

$$-\frac{7\csc^2\left(\frac{1}{2}(a+bx)\right)}{1024b} - \frac{\csc^4\left(\frac{1}{2}(a+bx)\right)}{2048b} - \frac{15\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{256b} + \frac{15\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{256b} + \frac{7\sec^2\left(\frac{1}{2}(a+bx)\right)}{1024b} + \frac{\sec^4\left(\frac{1}{2}(a+bx)\right)}{2048b} + \frac{\sin\left(\frac{1}{2}(a+bx)\right)}{32b\left(\cos\left(\frac{1}{2}(a+bx)\right) - \sin\left(\frac{1}{2}(a+bx)\right)\right)} - \frac{\sin\left(\frac{1}{2}(a+bx)\right)}{32b\left(\cos\left(\frac{1}{2}(a+bx)\right) + \sin\left(\frac{1}{2}(a+bx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Csc[2*a + 2*b*x]^5,x]
```

```
[Out] (-7*Csc[(a + b*x)/2]^2)/(1024*b) - Csc[(a + b*x)/2]^4/(2048*b) - (15*Log[Co
s[(a + b*x)/2]])/(256*b) + (15*Log[Sin[(a + b*x)/2]])/(256*b) + (7*Sec[(a +
 b*x)/2]^2)/(1024*b) + Sec[(a + b*x)/2]^4/(2048*b) + Sin[(a + b*x)/2]/(32*b
```

```
*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - Sin[(a + b*x)/2]/(32*b*(Cos[(a +
b*x)/2] + Sin[(a + b*x)/2]))
```

Maple [A]
time = 0.23, size = 71, normalized size = 1.01

method	result	size
default	$\frac{-\frac{1}{4\sin(xb+a)^4\cos(xb+a)}-\frac{5}{8\sin(xb+a)^2\cos(xb+a)}+\frac{15}{8\cos(xb+a)}+\frac{15\ln(\csc(xb+a)-\cot(xb+a))}{8}}{32b}$	71
risch	$\frac{15\,e^{9i(xb+a)}-40\,e^{7i(xb+a)}+18\,e^{5i(xb+a)}-40\,e^{3i(xb+a)}+15\,e^{i(xb+a)}}{128b\left(e^{2i(xb+a)}-1\right)^4\left(e^{2i(xb+a)}+1\right)}+\frac{15\ln\left(e^{i(xb+a)}-1\right)}{256b}-\frac{15\ln\left(e^{i(xb+a)}+1\right)}{256b}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/b*(-1/4/sin(b*x+a)^4/cos(b*x+a)-5/8/sin(b*x+a)^2/cos(b*x+a)+15/8/cos(b
*x+a)+15/8*ln(csc(b*x+a)-cot(b*x+a)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2237 vs. $2(62) = 124$.
time = 0.33, size = 2237, normalized size = 31.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x, algorithm="maxima")
```

```
[Out] 1/512*(4*(15*cos(9*b*x + 9*a) - 40*cos(7*b*x + 7*a) + 18*cos(5*b*x + 5*a) -
 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(10*b*x + 10*a) - 60*(3*cos(8*b*
x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1
)*cos(9*b*x + 9*a) + 12*(40*cos(7*b*x + 7*a) - 18*cos(5*b*x + 5*a) + 40*cos
(3*b*x + 3*a) - 15*cos(b*x + a))*cos(8*b*x + 8*a) - 160*(2*cos(6*b*x + 6*a)
 + 2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(7*b*x + 7*a) + 8*(18*co
s(5*b*x + 5*a) - 40*cos(3*b*x + 3*a) + 15*cos(b*x + a))*cos(6*b*x + 6*a) +
72*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(5*b*x + 5*a) - 40*(8*c
os(3*b*x + 3*a) - 3*cos(b*x + a))*cos(4*b*x + 4*a) + 160*(3*cos(2*b*x + 2*a
) - 1)*cos(3*b*x + 3*a) - 180*cos(2*b*x + 2*a)*cos(b*x + a) + 15*(2*(3*cos(
8*b*x + 8*a) - 2*cos(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a)
 - 1)*cos(10*b*x + 10*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) + 2
*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x
+ 8*a)^2 - 4*(2*cos(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a)
 - 4*cos(6*b*x + 6*a)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*c
os(4*b*x + 4*a)^2 - 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) - 2*sin(6*
b*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) -
sin(10*b*x + 10*a)^2 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2
*b*x + 2*a))*sin(8*b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a
```

```
) - 3*sin(2*b*x + 2*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b
*x + 4*a)^2 + 12*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 +
 6*cos(2*b*x + 2*a) - 1)*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + si
n(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - 15*(2*(3*cos(8*b*x + 8*a) - 2*co
s(6*b*x + 6*a) - 2*cos(4*b*x + 4*a) + 3*cos(2*b*x + 2*a) - 1)*cos(10*b*x +
10*a) - cos(10*b*x + 10*a)^2 + 6*(2*cos(6*b*x + 6*a) + 2*cos(4*b*x + 4*a) -
 3*cos(2*b*x + 2*a) + 1)*cos(8*b*x + 8*a) - 9*cos(8*b*x + 8*a)^2 - 4*(2*cos
(4*b*x + 4*a) - 3*cos(2*b*x + 2*a) + 1)*cos(6*b*x + 6*a) - 4*cos(6*b*x + 6*
a)^2 + 4*(3*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - 4*cos(4*b*x + 4*a)^2 -
 9*cos(2*b*x + 2*a)^2 + 2*(3*sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(
4*b*x + 4*a) + 3*sin(2*b*x + 2*a))*sin(10*b*x + 10*a) - sin(10*b*x + 10*a)^
2 + 6*(2*sin(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(8*
b*x + 8*a) - 9*sin(8*b*x + 8*a)^2 - 4*(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2
*a))*sin(6*b*x + 6*a) - 4*sin(6*b*x + 6*a)^2 - 4*sin(4*b*x + 4*a)^2 + 12*si
n(4*b*x + 4*a)*sin(2*b*x + 2*a) - 9*sin(2*b*x + 2*a)^2 + 6*cos(2*b*x + 2*a)
 - 1)*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*
x)*sin(a) + sin(a)^2) + 4*(15*sin(9*b*x + 9*a) - 40*sin(7*b*x + 7*a) + 18*s
in(5*b*x + 5*a) - 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(10*b*x + 10*a)
 - 60*(3*sin(8*b*x + 8*a) - 2*sin(6*b*x + 6*a) - 2*sin(4*b*x + 4*a) + 3*sin
(2*b*x + 2*a))*sin(9*b*x + 9*a) + 12*(40*sin(7*b*x + 7*a) - 18*sin(5*b*x +
5*a) + 40*sin(3*b*x + 3*a) - 15*sin(b*x + a))*sin(8*b*x + 8*a) - 160*(2*sin
(6*b*x + 6*a) + 2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(7*b*x + 7*a) +
 8*(18*sin(5*b*x + 5*a) - 40*sin(3*b*x + 3*a) + 15*sin(b*x + a))*sin(6*b*x
+ 6*a) + 72*(2*sin(4*b*x + 4*a) - 3*sin(2*b*x + 2*a))*sin(5*b*x + 5*a) - 40
*(8*sin(3*b*x + 3*a) - 3*sin(b*x + a))*sin(4*b*x + 4*a) + 480*sin(3*b*x + 3
*a)*sin(2*b*x + 2*a) - 180*sin(2*b*x + 2*a)*sin(b*x + a) + 60*cos(b*x + a))
/(b*cos(10*b*x + 10*a)^2 + 9*b*cos(8*b*x + 8*a)^2 + 4*b*cos(6*b*x + 6*a)^2
+ 4*b*cos(4*b*x + 4*a)^2 + 9*b*cos(2*b*x + 2*a)^2 + b*sin(10*b*x + 10*a)^2
+ 9*b*sin(8*b*x + 8*a)^2 + 4*b*sin(6*b*x + 6*a)^2 + 4*b*sin(4*b*x + 4*a)^2
- 12*b*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 9*b*sin(2*b*x + 2*a)^2 - 2*(3*b*
cos(8*b*x + 8*a) - 2*b*cos(6*b*x + 6*a) - 2*b*cos(4*b*x + 4*a) + 3*b*cos(2*
b*x + 2*a) - b)*cos(10*b*x + 10*a) - 6*(2*b*cos(6*b*x + 6*a) + 2*b*cos(4*b*
x + 4*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(8*b*x + 8*a) + 4*(2*b*cos(4*b*x +
4*a) - 3*b*cos(2*b*x + 2*a) + b)*cos(6*b*x + 6*a) - 4*(3*b*cos(2*b*x + 2*a)
 - b)*cos(4*b*x + 4*a) - 6*b*cos(2*b*x + 2*a) - 2*(3*b*sin(8*b*x + 8*a) - 2
*b*sin(6*b*x + 6*a) - 2*b*sin(4*b*x + 4*a) + 3*b*sin(2*b*x + 2*a))*sin(10*b
*x + 10*a) - 6*(2*b*sin(6*b*x + 6*a) + 2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x
 + 2*a))*sin(8*b*x + 8*a) + 4*(2*b*sin(4*b*x + 4*a) - 3*b*sin(2*b*x + 2*a))
*sin(6*b*x + 6*a) + b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(62) = 124$.
time = 2.46, size = 132, normalized size = 1.89

$$\frac{30 \cos\left(bx + a\right)^{4} - 50 \cos\left(bx + a\right)^{2} - 15 \left(\cos\left(bx + a\right)^{5} - 2 \cos\left(bx + a\right)^{3} + \cos\left(bx + a\right)\right) \log\left(\frac{1}{2} \cos\left(bx + a\right) + \frac{1}{2}\right) + 15 \left(\cos\left(bx + a\right)^{5} - 2 \cos\left(bx + a\right)^{3} + \cos\left(bx + a\right)\right) \log\left(-\frac{1}{2} \cos\left(bx + a\right) + \frac{1}{2}\right) + 16}{512 \left(b \cos\left(bx + a\right)^{5} - 2 b \cos\left(bx + a\right)^{3} + b \cos\left(bx + a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x, algorithm="fricas")
```

```
[Out] 1/512*(30*cos(b*x + a)^4 - 50*cos(b*x + a)^2 - 15*(cos(b*x + a)^5 - 2*cos(b
*x + a)^3 + cos(b*x + a))*log(1/2*cos(b*x + a) + 1/2) + 15*(cos(b*x + a)^5
- 2*cos(b*x + a)^3 + cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2) + 16)/(b*co
s(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**5,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 73, normalized size = 1.04

$$\frac{\frac{2\left(7\,\cos(bx+a)^3-9\,\cos(bx+a)\right)}{\left(\cos(bx+a)^2-1\right)^2}+\frac{16}{\cos(bx+a)}-15\,\log\left(\cos\left(bx+a\right)+1\right)+15\,\log\left(-\cos\left(bx+a\right)+1\right)}{512\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^5,x, algorithm="giac")
```

```
[Out] 1/512*(2*(7*cos(b*x + a)^3 - 9*cos(b*x + a))/(cos(b*x + a)^2 - 1)^2 + 16/co
s(b*x + a) - 15*log(cos(b*x + a) + 1) + 15*log(-cos(b*x + a) + 1))/b
```

Mupad [**B**]
time = 0.19, size = 66, normalized size = 0.94

$$\frac{\frac{15\cos(a+b\,x)^4}{256}-\frac{25\cos(a+b\,x)^2}{256}+\frac{1}{32}}{b\,\left(\cos\left(a+b\,x\right)^5-2\cos\left(a+b\,x\right)^3+\cos\left(a+b\,x\right)\right)}-\frac{15\,\mathrm{atanh}(\cos\left(a+b\,x\right))}{256\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^5,x)
```

```
[Out] ((15*cos(a + b*x)^4)/256 - (25*cos(a + b*x)^2)/256 + 1/32)/(b*(cos(a + b*x)
 - 2*cos(a + b*x)^3 + cos(a + b*x)^5)) - (15*atanh(cos(a + b*x)))/(256*b)
```

3.161 $\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$-\frac{5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{32b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b}-\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b}+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}$$

```
[Out] -5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b-5/24*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+1/6*sin(b*x+a)*sin(
2*b*x+2*a)^(5/2)/b+5/16*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4386, 4387, 4391}

$$-\frac{5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{32b}+\frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b}-\frac{5\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{24b}-\frac{5\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) - (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) - (5*Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(24*b) + (Sin[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(6*b)
```

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
```

```
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b} + \frac{5}{6}\int \sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= -\frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b} + \frac{\sin(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{6b} + \frac{5}{8}\int \cos(a$$
$$= \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b} - \frac{5\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{24b} + \frac{\sin(a+}{}$$
$$= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b} - \frac{5\log\left(\cos(a+bx)+\sin(a+bx)\right.}{32b}$$

Mathematica [A]
time = 0.38, size = 98, normalized size = 0.72

$$\frac{-5\left(\mathrm{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+\frac{2}{3}\sqrt{\sin(2(a+bx))}\,(14\sin(a+bx)-3\sin(3(a+bx))-2\sin(5(a+bx)))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]]) + (2*Sqrt[Sin[2*(a + b*x)]]*(14*Sin[a + b*x] - 3
*Sin[3*(a + b*x)] - 2*Sin[5*(a + b*x)]))/3)/(32*b)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 99.63, size = 221660564, normalized size = 1629857.09

method	result	size
default	Expression too large to display	221660564

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(118) = 236$.
time = 2.43, size = 290, normalized size = 2.13

$$-\frac{8\sqrt{2}\left(32\cos\left(bx+a\right)^{4}-12\cos\left(bx+a\right)^{2}-15\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\,\sin\left(bx+a\right)-30\arctan\left(-\frac{\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\,\left(\cos(bx+a)-\sin(bx+a)\right)+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^{2}+2\cos(bx+a)\sin(bx+a)-1}\right)+30\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)-15\log\left(-32\cos\left(bx+a\right)^{4}+4\sqrt{2}\left(4\cos\left(bx+a\right)^{3}-\left(4\cos\left(bx+a\right)^{2}+1\right)\sin\left(bx+a\right)-5\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\cos\left(bx+a\right)^{2}+16\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/384*(8*sqrt(2)*(32*cos(b*x + a)^4 - 12*cos(b*x + a)^2 - 15)*sqrt(cos(b*x
 + a)*sin(b*x + a))*sin(b*x + a) - 30*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos
(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 30*arctan(-(2*sqrt(2)*sqr
t(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) -
 sin(b*x + a))) - 15*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 -
 (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*si
n(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,{\sin\left(2\,a+2\,b\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^(5/2),x)
[Out] int(cos(a + b*x)*sin(2*a + 2*b*x)^(5/2), x)
```

3.162 $\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=110

$$-\frac{3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{16b}+\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{16b}-\frac{3\cos(a+bx)}{}$$

[Out] -3/16*arcsin(cos(b*x+a)-sin(b*x+a))/b+3/16*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+2*a)^(1/2))/b+1/4*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b-3/8*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4386, 4387, 4390}

$$-\frac{3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{16b}+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}-\frac{3\sqrt{\sin(2a+2bx)}\cos(a+bx)}{8b}+\frac{3\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{16b}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]

[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(16*b) + (3*Log[Cos[a + b*x] + Sin[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(16*b) - (3*Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(8*b) + (Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(4*b)

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
```

```
a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx &= \frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}+\frac{3}{4}\int \sin(a+bx)\sqrt{\sin(2a+2bx)}\,dx\\ &= -\frac{3\cos(a+bx)\sqrt{\sin(2a+2bx)}}{8b}+\frac{\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{4b}+\frac{3}{8}\int \frac{\quad}{\sqrt{\mathrm{s}}}\\ &= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{16b}+\frac{3\log\left(\cos(a+bx)+\sin(a+bx)-\right.}{16b}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 86, normalized size = 0.78

$$\frac{3\left(-\mathrm{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)-2(2\cos(a+bx)+\cos(3(a+bx)))\sqrt{\sin(2(a+bx))}}{16b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (3*(-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) - 2*(2*Cos[a + b*x] + Cos[3*(a + b*x)])*Sqrt[Sin
[2*(a + b*x)]])/(16*b)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 24.43, size = 85899870, normalized size = 780907.91

method	result	size
default	Expression too large to display	85899870

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")

[Out] integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(96) = 192$.
time = 3.70, size = 281, normalized size = 2.55

$$-\frac{8\sqrt{2}\left(4\cos\left(bx+a\right)^{3}-\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\left(\cos(bx+a)-\sin(bx+a)\right)+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^{2}+2\cos(bx+a)\sin(bx+a)-1}\right)+6\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)+3\log\left(-32\cos\left(bx+a\right)^{4}+4\sqrt{2}\left(4\cos\left(bx+a\right)^{3}-\left(4\cos\left(bx+a\right)^{2}+1\right)\sin\left(bx+a\right)-5\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\cos\left(bx+a\right)^{2}+16\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")

[Out] -1/64*(8*sqrt(2)*(4*cos(b*x + a)^3 - cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) - 6*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 3*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**(3/2),x)

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")

[Out] integrate(cos(b*x + a)*sin(2*b*x + 2*a)^(3/2), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,{\sin\left(2\,a+2\,b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^(3/2),x)

[Out] int(cos(a + b*x)*sin(2*a + 2*b*x)^(3/2), x)

3.163 $\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=84

$$-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b}+\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}$$

```
[Out] -1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b-1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b+1/2*sin(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]

time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4386, 4391}

$$-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{4b}+\frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}-\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/4*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b - Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) + (Sin[a + b*x]*Sqrt[Sin[2*a + 2*b*x]])/(
2*b)
```

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= \frac{\sin(a+bx)\sqrt{\sin(2a+2bx)}}{2b}+\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= -\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\right.}{4b}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 70, normalized size = 0.83

$$-\frac{\operatorname{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-2\sin(a+bx)\sqrt{\sin(2(a+bx))}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/4*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]] - 2*Sin[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 2.47, size = 5537888, normalized size = 65927.24

method	result	size
default	Expression too large to display	5537888

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(74) = 148$.
time = 2.30, size = 266, normalized size = 3.17

$$\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\sin(bx+a)+2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)+\log\left(-32\cos(bx+a)^4+4\sqrt{2}\left(4\cos(bx+a)^3-\left(4\cos(bx+a)^2+1\right)\sin(bx+a)-5\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}+32\cos(bx+a)^2+16\cos(bx+a)\sin(bx+a)+1\right)}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + 2*arctan(-(s
qrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(
b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1))
- 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin
```

```
(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + log(-32*cos(b*x + a)^4 + 4*sqrt
(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a
))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*si
n(b*x + a) + 1))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(2*a + 2*b*x)^(1/2), x)
```

3.164 $\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{2b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{2b}$$

```
[Out] -1/2*arcsin(cos(b*x+a)-sin(b*x+a))/b+1/2*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{4390\}$

$$\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{2b}-\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] -1/2*ArcSin[Cos[a + b*x] - Sin[a + b*x]]/b + Log[Cos[a + b*x] + Sin[a + b*x
] + Sqrt[Sin[2*a + 2*b*x]]]/(2*b)
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx=-\frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{2b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a\right.}}{2b}$$

Mathematica [A]
time = 0.06, size = 52, normalized size = 0.90

$$\frac{-\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + S
qrt[Sin[2*(a + b*x)]]])/(2*b)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 2.80, size = 18450099, normalized size = 318105.16

method	result	size
default	Expression too large to display	18450099

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(52) = 104$.
time = 1.62, size = 242, normalized size = 4.17

$$\frac{2\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-2\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)-\log\left(-32\cos(bx+a)^4+4\sqrt{2}\left(4\cos(bx+a)^3-\left(4\cos(bx+a)^2+1\right)\sin(bx+a)-5\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}+32\cos(bx+a)^2+16\cos(bx+a)\sin(bx+a)+1\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin
(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*si
n(b*x + a) - 1)) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - c
os(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - log(-32*cos(b*
x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a
) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 1
6*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)/sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(a+b\,x\right)}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)/sin(2*a + 2*b*x)^(1/2), x)
```

3.165 $\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=24

$$-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {4376}

$$-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -(Cos[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]]))
```

Rule 4376

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.96

$$-\frac{\cos(a+bx)}{b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -(Cos[a + b*x]/(b*Sqrt[Sin[2*(a + b*x)]]))
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 10.36, size = 63401108, normalized size = 2641712.83

method	result	size
default	Expression too large to display	63401108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [**A**]
time = 1.59, size = 39, normalized size = 1.62

$$-\frac{\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+\sin\left(bx+a\right)}{2\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + sin(b*x + a))/(b*sin(b*x +
a))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4852 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2029 vs. $2(22) = 44$.
time = 12.48, size = 2029, normalized size = 84.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)^
4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2*b
*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(1/2
*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1/2*b
*x) + tan(1/2*a))*(((sqrt(2)*tan(1/2*a)^26 + 5*sqrt(2)*tan(1/2*a)^24 - 10*s
qrt(2)*tan(1/2*a)^22 - 154*sqrt(2)*tan(1/2*a)^20 - 605*sqrt(2)*tan(1/2*a)^1
8 - 1353*sqrt(2)*tan(1/2*a)^16 - 1980*sqrt(2)*tan(1/2*a)^14 - 1980*sqrt(2)*
tan(1/2*a)^12 - 1353*sqrt(2)*tan(1/2*a)^10 - 605*sqrt(2)*tan(1/2*a)^8 - 154
*sqrt(2)*tan(1/2*a)^6 - 10*sqrt(2)*tan(1/2*a)^4 + 5*sqrt(2)*tan(1/2*a)^2 +
sqrt(2))*tan(1/2*b*x)/(tan(1/2*a)^24 + 12*tan(1/2*a)^22 + 66*tan(1/2*a)^20
+ 220*tan(1/2*a)^18 + 495*tan(1/2*a)^16 + 792*tan(1/2*a)^14 + 924*tan(1/2*a
)^12 + 792*tan(1/2*a)^10 + 495*tan(1/2*a)^8 + 220*tan(1/2*a)^6 + 66*tan(1/2
*a)^4 + 12*tan(1/2*a)^2 + 1) - 8*(sqrt(2)*tan(1/2*a)^25 + 10*sqrt(2)*tan(1/
2*a)^23 + 44*sqrt(2)*tan(1/2*a)^21 + 110*sqrt(2)*tan(1/2*a)^19 + 165*sqrt(2
)*tan(1/2*a)^17 + 132*sqrt(2)*tan(1/2*a)^15 - 132*sqrt(2)*tan(1/2*a)^11 - 1
65*sqrt(2)*tan(1/2*a)^9 - 110*sqrt(2)*tan(1/2*a)^7 - 44*sqrt(2)*tan(1/2*a)^
5 - 10*sqrt(2)*tan(1/2*a)^3 - sqrt(2)*tan(1/2*a))/(tan(1/2*a)^24 + 12*tan(1
/2*a)^22 + 66*tan(1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan(1/2*a)^16 + 792*t
an(1/2*a)^14 + 924*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 495*tan(1/2*a)^8 + 2
20*tan(1/2*a)^6 + 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 + 1))*tan(1/2*b*x) - (s
qrt(2)*tan(1/2*a)^26 + 5*sqrt(2)*tan(1/2*a)^24 - 10*sqrt(2)*tan(1/2*a)^22 -
 154*sqrt(2)*tan(1/2*a)^20 - 605*sqrt(2)*tan(1/2*a)^18 - 1353*sqrt(2)*tan(1
/2*a)^16 - 1980*sqrt(2)*tan(1/2*a)^14 - 1980*sqrt(2)*tan(1/2*a)^12 - 1353*s
qrt(2)*tan(1/2*a)^10 - 605*sqrt(2)*tan(1/2*a)^8 - 154*sqrt(2)*tan(1/2*a)^6
- 10*sqrt(2)*tan(1/2*a)^4 + 5*sqrt(2)*tan(1/2*a)^2 + sqrt(2))/(tan(1/2*a)^2
4 + 12*tan(1/2*a)^22 + 66*tan(1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan(1/2*a
)^16 + 792*tan(1/2*a)^14 + 924*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 495*tan(
1/2*a)^8 + 220*tan(1/2*a)^6 + 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 + 1))*cos(a
)/((tan(1/2*b*x)^4*tan(1/2*a)^3 + tan(1/2*b*x)^3*tan(1/2*a)^4 - tan(1/2*b*x
)^4*tan(1/2*a) - 6*tan(1/2*b*x)^3*tan(1/2*a)^2 - 6*tan(1/2*b*x)^2*tan(1/2*a
)^3 - tan(1/2*b*x)*tan(1/2*a)^4 + tan(1/2*b*x)^3 + 6*tan(1/2*b*x)^2*tan(1/2
*a) + 6*tan(1/2*b*x)*tan(1/2*a)^2 + tan(1/2*a)^3 - tan(1/2*b*x) - tan(1/2*a
))*b) + 1/2*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(
1/2*a)^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*ta
n(1/2*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*
tan(1/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + ta
```

```
n(1/2*b*x) + tan(1/2*a))*((2*(sqrt(2)*tan(1/2*a)^25 + 10*sqrt(2)*tan(1/2*a)
^23 + 44*sqrt(2)*tan(1/2*a)^21 + 110*sqrt(2)*tan(1/2*a)^19 + 165*sqrt(2)*ta
n(1/2*a)^17 + 132*sqrt(2)*tan(1/2*a)^15 - 132*sqrt(2)*tan(1/2*a)^11 - 165*s
qrt(2)*tan(1/2*a)^9 - 110*sqrt(2)*tan(1/2*a)^7 - 44*sqrt(2)*tan(1/2*a)^5 -
10*sqrt(2)*tan(1/2*a)^3 - sqrt(2)*tan(1/2*a))*tan(1/2*b*x)/(tan(1/2*a)^24 +
 12*tan(1/2*a)^22 + 66*tan(1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan(1/2*a)^1
6 + 792*tan(1/2*a)^14 + 924*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 495*tan(1/2
*a)^8 + 220*tan(1/2*a)^6 + 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 + 1) + (sqrt(2
)*tan(1/2*a)^26 + 5*sqrt(2)*tan(1/2*a)^24 - 10*sqrt(2)*tan(1/2*a)^22 - 154*
sqrt(2)*tan(1/2*a)^20 - 605*sqrt(2)*tan(1/2*a)^18 - 1353*sqrt(2)*tan(1/2*a)
^16 - 1980*sqrt(2)*tan(1/2*a)^14 - 1980*sqrt(2)*tan(1/2*a)^12 - 1353*sqrt(2
)*tan(1/2*a)^10 - 605*sqrt(2)*tan(1/2*a)^8 - 154*sqrt(2)*tan(1/2*a)^6 - 10*
sqrt(2)*tan(1/2*a)^4 + 5*sqrt(2)*tan(1/2*a)^2 + sqrt(2))/(tan(1/2*a)^24 + 1
2*tan(1/2*a)^22 + 66*tan(1/2*a)^20 + 220*tan(1/2*a)^18 + 495*tan(1/2*a)^16
+ 792*tan(1/2*a)^14 + 924*tan(1/2*a)^12 + 792*tan(1/2*a)^10 + 495*tan(1/2*a
)^8 + 220*tan(1/2*a)^6 + 66*tan(1/2*a)^4 + 12*tan(1/2*a)^2 + 1))*tan(1/2*b*
x) - 2*(sqrt(2)*tan(1/2*a)^25 + 10*sqrt(2)*tan(1/2*a)^23 + 44*sqrt(2)*tan(1
/2*a)^21 + 110*sqrt(2)*tan(1/2*a)^19 + 165*sqrt(2)*tan(1/2*a)^17 + 132*sqrt
(2)*tan(1/2*a)^15 - 132*sqrt(2)*tan(1/2*a)^11 - 165*sqrt(2)*tan(1/2*a)^9 -
110*sqrt(2)*tan(1/2*a)^7 - 44*sqrt(2)*tan(1/2*a)^5 - 10*sqrt(2)*tan(1/2*a)^
3 - sqrt(2)*tan(1/2*a))/(tan(1/2*a)^24 + 12*tan(1/2*a)^22 + 66*tan(1/2*a)^2
0 + 220*tan(1/2*a)^18 + 495*tan(1/2*a)^16 + 792*tan(1/2*a)^14 + 924*tan(1/2
*a)^12 + 792*tan(1/2*a)^10 + 495*tan(1/2*a)^8 + 220*tan(1/2*a)^6 + 66*tan(1
/2*a)^4 + 12*tan(1/2*a)^2 + 1))*sin(a)/((tan(1/2*b*x)^4*tan(1/2*a)^3 + tan(
1/2*b*x)^3*tan(1/2*a)^4 - tan(1/2*b*x)^4*tan(1/2*a) - 6*tan(1/2*b*x)^3*tan(
1/2*a)^2 - 6*tan(1/2*b*x)^2*tan(1/2*a)^3 - tan(1/2*b*x)*tan(1/2*a)^4 + tan(
1/2*b*x)^3 + 6*tan(1/2*b*x)^2*tan(1/2*a) + 6*tan(1/2*b*x)*tan(1/2*a)^2 + ta
n(1/2*a)^3 - tan(1/2*b*x) - tan(1/2*a))*b)
```

Mupad [B]
time = 0.19, size = 24, normalized size = 1.00

$$-\frac{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}{2\,b\,\sin\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^(3/2),x)
[Out] -sin(2*a + 2*b*x)^(1/2)/(2*b*sin(a + b*x))
```

3.166 $\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=53

$$-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -1/3*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+2/3*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2
)
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4388, 4377}

$$\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}-\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -1/3*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(3/2)) + (2*Sin[a + b*x])/(3*b*Sqrt[S
in[2*a + 2*b*x]])
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4388

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx &= -\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{2}{3}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{2\sin(a+bx)}{3b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 43, normalized size = 0.81

$$\frac{\left(-\frac{1}{12}\cot(a+bx)\csc(a+bx)+\frac{1}{4}\sec(a+bx)\right)\sqrt{\sin(2(a+bx))}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] ((-1/12*(Cot[a + b*x]*Csc[a + b*x]) + Sec[a + b*x]/4)*Sqrt[Sin[2*(a + b*x)]
])/b
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 17.64, size = 194, normalized size = 3.66

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1}}\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)\left(2\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1}\sqrt{-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2}\sqrt{-\tan\left(\frac{a}{2}+\frac{xb}{2}\right.}\cdots\right.}{24b\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\sqrt{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan^2\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)}\sqrt{\tan^3\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24/b*(-tan(1/2*a+1/2*x*b)/(tan(1/2*a+1/2*x*b)^2-1))^(1/2)*(tan(1/2*a+1/2
*x*b)^2-1)/tan(1/2*a+1/2*x*b)*(2*(tan(1/2*a+1/2*x*b)+1)^(1/2)*(-2*tan(1/2*a
+1/2*x*b)+2)^(1/2)*(-tan(1/2*a+1/2*x*b))^(1/2)*EllipticF((tan(1/2*a+1/2*x*b
)+1)^(1/2),1/2*2^(1/2))*tan(1/2*a+1/2*x*b)-tan(1/2*a+1/2*x*b)^4+1)/(tan(1/2
*a+1/2*x*b)*(tan(1/2*a+1/2*x*b)^2-1))^(1/2)/(tan(1/2*a+1/2*x*b)^3-tan(1/2*a
+1/2*x*b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [A]
time = 1.54, size = 74, normalized size = 1.40

$$\frac{4\cos\left(bx+a\right)^{3}+\sqrt{2}\left(4\cos\left(bx+a\right)^{2}-3\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-4\cos\left(bx+a\right)}{12\left(b\cos\left(bx+a\right)^{3}-b\cos\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")

[Out] 1/12*(4*cos(b*x + a)^3 + sqrt(2)*(4*cos(b*x + a)^2 - 3)*sqrt(cos(b*x + a)*s
in(b*x + a)) - 4*cos(b*x + a))/(b*cos(b*x + a)^3 - b*cos(b*x + a))

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(5/2),x)

[Out] Timed out

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7875 vs. 2(45) = 90.
time = 45.07, size = 7875, normalized size = 148.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")

[Out] 1/48*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)^
4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2*b
*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(1/2
*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1/2*b
*x) + tan(1/2*a))*((((((sqrt(2)*tan(1/2*a)^57 + 18*sqrt(2)*tan(1/2*a)^55 +
 132*sqrt(2)*tan(1/2*a)^53 + 374*sqrt(2)*tan(1/2*a)^51 - 1375*sqrt(2)*tan(1
/2*a)^49 - 19620*sqrt(2)*tan(1/2*a)^47 - 108560*sqrt(2)*tan(1/2*a)^45 - 399
740*sqrt(2)*tan(1/2*a)^43 - 1096755*sqrt(2)*tan(1/2*a)^41 - 2340250*sqrt(2)
*tan(1/2*a)^39 - 3941740*sqrt(2)*tan(1/2*a)^37 - 5204670*sqrt(2)*tan(1/2*a)
^35 - 5163155*sqrt(2)*tan(1/2*a)^33 - 3268760*sqrt(2)*tan(1/2*a)^31 + 32687
60*sqrt(2)*tan(1/2*a)^27 + 5163155*sqrt(2)*tan(1/2*a)^25 + 5204670*sqrt(2)*
tan(1/2*a)^23 + 3941740*sqrt(2)*tan(1/2*a)^21 + 2340250*sqrt(2)*tan(1/2*a)^
19 + 1096755*sqrt(2)*tan(1/2*a)^17 + 399740*sqrt(2)*tan(1/2*a)^15 + 108560*
sqrt(2)*tan(1/2*a)^13 + 19620*sqrt(2)*tan(1/2*a)^11 + 1375*sqrt(2)*tan(1/2*
a)^9 - 374*sqrt(2)*tan(1/2*a)^7 - 132*sqrt(2)*tan(1/2*a)^5 - 18*sqrt(2)*tan
(1/2*a)^3 - sqrt(2)*tan(1/2*a))*tan(1/2*b*x)/(tan(1/2*a)^51 + 23*tan(1/2*a)
^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/2*a)^43 + 31878*t
an(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37 + 389367*tan(1/2*
a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 534888*tan(1/2*a)^29
+ 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 534888*tan(1/2*a)^23 - 6537

52*tan(1/2*a)^21 - 572033*tan(1/2*a)^19 - 389367*tan(1/2*a)^17 - 211508*tan
(1/2*a)^15 - 92092*tan(1/2*a)^13 - 31878*tan(1/2*a)^11 - 8602*tan(1/2*a)^9
- 1748*tan(1/2*a)^7 - 252*tan(1/2*a)^5 - 23*tan(1/2*a)^3 - tan(1/2*a)) - 24
*(sqrt(2)*tan(1/2*a)^56 + 23*sqrt(2)*tan(1/2*a)^54 + 251*sqrt(2)*tan(1/2*a)
^52 + 1725*sqrt(2)*tan(1/2*a)^50 + 8350*sqrt(2)*tan(1/2*a)^48 + 30130*sqrt(
2)*tan(1/2*a)^46 + 83490*sqrt(2)*tan(1/2*a)^44 + 179630*sqrt(2)*tan(1/2*a)^
42 + 297275*sqrt(2)*tan(1/2*a)^40 + 360525*sqrt(2)*tan(1/2*a)^38 + 264385*s
qrt(2)*tan(1/2*a)^36 - 37145*sqrt(2)*tan(1/2*a)^34 - 445740*sqrt(2)*tan(1/2
*a)^32 - 742900*sqrt(2)*tan(1/2*a)^30 - 742900*sqrt(2)*tan(1/2*a)^28 - 4457
40*sqrt(2)*tan(1/2*a)^26 - 37145*sqrt(2)*tan(1/2*a)^24 + 264385*sqrt(2)*tan
(1/2*a)^22 + 360525*sqrt(2)*tan(1/2*a)^20 + 297275*sqrt(2)*tan(1/2*a)^18 +
179630*sqrt(2)*tan(1/2*a)^16 + 83490*sqrt(2)*tan(1/2*a)^14 + 30130*sqrt(2)*
tan(1/2*a)^12 + 8350*sqrt(2)*tan(1/2*a)^10 + 1725*sqrt(2)*tan(1/2*a)^8 + 25
1*sqrt(2)*tan(1/2*a)^6 + 23*sqrt(2)*tan(1/2*a)^4 + sqrt(2)*tan(1/2*a)^2)/(t
an(1/2*a)^51 + 23*tan(1/2*a)^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 +
8602*tan(1/2*a)^43 + 31878*tan(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan
(1/2*a)^37 + 389367*tan(1/2*a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a
)^31 + 534888*tan(1/2*a)^29 + 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 -
534888*tan(1/2*a)^23 - 653752*tan(1/2*a)^21 - 572033*tan(1/2*a)^19 - 38936
7*tan(1/2*a)^17 - 211508*tan(1/2*a)^15 - 92092*tan(1/2*a)^13 - 31878*tan(1/
2*a)^11 - 8602*tan(1/2*a)^9 - 1748*tan(1/2*a)^7 - 252*tan(1/2*a)^5 - 23*tan
(1/2*a)^3 - tan(1/2*a)))*tan(1/2*b*x) - 15*(sqrt(2)*tan(1/2*a)^57 + 18*sqrt
(2)*tan(1/2*a)^55 + 132*sqrt(2)*tan(1/2*a)^53 + 374*sqrt(2)*tan(1/2*a)^51 -
1375*sqrt(2)*tan(1/2*a)^49 - 19620*sqrt(2)*tan(1/2*a)^47 - 108560*sqrt(2)*
tan(1/2*a)^45 - 399740*sqrt(2)*tan(1/2*a)^43 - 1096755*sqrt(2)*tan(1/2*a)^4
1 - 2340250*sqrt(2)*tan(1/2*a)^39 - 3941740*sqrt(2)*tan(1/2*a)^37 - 5204670
*sqrt(2)*tan(1/2*a)^35 - 5163155*sqrt(2)*tan(1/2*a)^33 - 3268760*sqrt(2)*ta
n(1/2*a)^31 + 3268760*sqrt(2)*tan(1/2*a)^27 + 5163155*sqrt(2)*tan(1/2*a)^25
+ 5204670*sqrt(2)*tan(1/2*a)^23 + 3941740*sqrt(2)*tan(1/2*a)^21 + 2340250*
sqrt(2)*tan(1/2*a)^19 + 1096755*sqrt(2)*tan(1/2*a)^17 + 399740*sqrt(2)*tan(
1/2*a)^15 + 108560*sqrt(2)*tan(1/2*a)^13 + 19620*sqrt(2)*tan(1/2*a)^11 + 13
75*sqrt(2)*tan(1/2*a)^9 - 374*sqrt(2)*tan(1/2*a)^7 - 132*sqrt(2)*tan(1/2*a)
^5 - 18*sqrt(2)*tan(1/2*a)^3 - sqrt(2)*tan(1/2*a))/(tan(1/2*a)^51 + 23*tan(
1/2*a)^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/2*a)^43 + 3
1878*tan(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37 + 389367*ta
n(1/2*a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 534888*tan(1/2*
a)^29 + 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 534888*tan(1/2*a)^23
- 653752*tan(1/2*a)^21 - 572033*tan(1/2*a)^19 - 389367*tan(1/2*a)^17 - 2115
08*tan(1/2*a)^15 - 92092*tan(1/2*a)^13 - 31878*tan(1/2*a)^11 - 8602*tan(1/2
*a)^9 - 1748*tan(1/2*a)^7 - 252*tan(1/2*a)^5 - 23*tan(1/2*a)^3 - tan(1/2*a)
))*tan(1/2*b*x) + 80*(sqrt(2)*tan(1/2*a)^56 + 23*sqrt(2)*tan(1/2*a)^54 + 25
1*sqrt(2)*tan(1/2*a)^52 + 1725*sqrt(2)*tan(1/2*a)^50 + 8350*sqrt(2)*tan(1/2
*a)^48 + 30130*sqrt(2)*tan(1/2*a)^46 + 83490*sqrt(2)*tan(1/2*a)^44 + 179630
*sqrt(2)*tan(1/2*a)^42 + 297275*sqrt(2)*tan(1/2*a)^40 + 360525*sqrt(2)*tan(
1/2*a)^38 + 264385*sqrt(2)*tan(1/2*a)^36 - 37145*sqrt(2)*tan(1/2*a)^34 - 44

```
5740*sqrt(2)*tan(1/2*a)^32 - 742900*sqrt(2)*tan(1/2*a)^30 - 742900*sqrt(2)*
tan(1/2*a)^28 - 445740*sqrt(2)*tan(1/2*a)^26 - ...
```

Mupad [B]
time = 3.17, size = 104, normalized size = 1.96

$$-\frac{2\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,\left(3\,\cos\left(a+b\,x\right)-6\,\cos\left(3\,a+3\,b\,x\right)+4\,\cos\left(5\,a+5\,b\,x\right)-\cos\left(7\,a+7\,b\,x\right)\right)}{3\,b\,\left(4\,\cos\left(2\,a+2\,b\,x\right)+4\,\cos\left(4\,a+4\,b\,x\right)-4\,\cos\left(6\,a+6\,b\,x\right)+\cos\left(8\,a+8\,b\,x\right)-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^(5/2),x)
```

```
[Out] -(2*sin(2*a + 2*b*x)^(1/2)*(3*cos(a + b*x) - 6*cos(3*a + 3*b*x) + 4*cos(5*a
 + 5*b*x) - cos(7*a + 7*b*x)))/(3*b*(4*cos(2*a + 2*b*x) + 4*cos(4*a + 4*b*x
) - 4*cos(6*a + 6*b*x) + cos(8*a + 8*b*x) - 5))
```

3.167 $\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=79

$$-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -1/5*cos(b*x+a)/b/sin(2*b*x+2*a)^(5/2)+4/15*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/
2)-8/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.150$, Rules used = {4388, 4389, 4376}

$$\frac{4\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] -1/5*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(5/2)) + (4*Sin[a + b*x])/(15*b*Sin[2
*a + 2*b*x]^(3/2)) - (8*Cos[a + b*x])/(15*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4376

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4388

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4389

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + D
ist[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x],
 x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !I
```

```
ntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx &= -\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{5}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8}{15}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{15b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{15b\sqrt{\sin(2a+2bx)}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 52, normalized size = 0.66

$$-\frac{\sqrt{\sin(2(a+bx))}\,\left(27\csc(a+bx)+3\csc^3(a+bx)-5\sec(a+bx)\tan(a+bx)\right)}{120b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(7/2),x]
[Out] -1/120*(Sqrt[Sin[2*(a + b*x)]]*(27*Csc[a + b*x] + 3*Csc[a + b*x]^3 - 5*Sec[
a + b*x]*Tan[a + b*x]))/b
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(xb+a\right)}{\sin\left(2xb+2a\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
[Out] int(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [A]
time = 2.25, size = 103, normalized size = 1.30

$$-\frac{\sqrt{2}\left(32\cos\left(bx+a\right)^{4}-40\cos\left(bx+a\right)^{2}+5\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\left(\cos\left(bx+a\right)^{4}-\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)}{120\left(b\cos\left(bx+a\right)^{4}-b\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/120*(sqrt(2)*(32*cos(b*x + a)^4 - 40*cos(b*x + a)^2 + 5)*sqrt(cos(b*x +
a)*sin(b*x + a)) + 32*(cos(b*x + a)^4 - cos(b*x + a)^2)*sin(b*x + a))/((b*c
os(b*x + a)^4 - b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 18022 vs. 2(67) = 134.
time = 155.50, size = 18022, normalized size = 228.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] -1/960*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a
)^4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2
*b*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(1
/2*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1/2
*b*x) + tan(1/2*a))*((((((((((3*sqrt(2)*tan(1/2*a)^88 + 221*sqrt(2)*tan(1/
2*a)^86 + 4529*sqrt(2)*tan(1/2*a)^84 + 46575*sqrt(2)*tan(1/2*a)^82 + 267014
*sqrt(2)*tan(1/2*a)^80 + 611706*sqrt(2)*tan(1/2*a)^78 - 3503654*sqrt(2)*tan
(1/2*a)^76 - 44470106*sqrt(2)*tan(1/2*a)^74 - 259557037*sqrt(2)*tan(1/2*a)^
72 - 1054367027*sqrt(2)*tan(1/2*a)^70 - 3278963927*sqrt(2)*tan(1/2*a)^68 -
8089589961*sqrt(2)*tan(1/2*a)^66 - 16006283224*sqrt(2)*tan(1/2*a)^64 - 2518
6632744*sqrt(2)*tan(1/2*a)^62 - 30337876456*sqrt(2)*tan(1/2*a)^60 - 2468571
2920*sqrt(2)*tan(1/2*a)^58 - 5629982106*sqrt(2)*tan(1/2*a)^56 + 19969391706
```

*sqrt(2)*tan(1/2*a)^54 + 37658626338*sqrt(2)*tan(1/2*a)^52 + 36190152990*sq
rt(2)*tan(1/2*a)^50 + 18717018180*sqrt(2)*tan(1/2*a)^48 + 2040819900*sqrt(2
)*tan(1/2*a)^46 + 2040819900*sqrt(2)*tan(1/2*a)^44 + 18717018180*sqrt(2)*ta
n(1/2*a)^42 + 36190152990*sqrt(2)*tan(1/2*a)^40 + 37658626338*sqrt(2)*tan(1
/2*a)^38 + 19969391706*sqrt(2)*tan(1/2*a)^36 - 5629982106*sqrt(2)*tan(1/2*a
)^34 - 24685712920*sqrt(2)*tan(1/2*a)^32 - 30337876456*sqrt(2)*tan(1/2*a)^3
0 - 25186632744*sqrt(2)*tan(1/2*a)^28 - 16006283224*sqrt(2)*tan(1/2*a)^26 -
 8089589961*sqrt(2)*tan(1/2*a)^24 - 3278963927*sqrt(2)*tan(1/2*a)^22 - 1054
367027*sqrt(2)*tan(1/2*a)^20 - 259557037*sqrt(2)*tan(1/2*a)^18 - 44470106*s
qrt(2)*tan(1/2*a)^16 - 3503654*sqrt(2)*tan(1/2*a)^14 + 611706*sqrt(2)*tan(1
/2*a)^12 + 267014*sqrt(2)*tan(1/2*a)^10 + 46575*sqrt(2)*tan(1/2*a)^8 + 4529
*sqrt(2)*tan(1/2*a)^6 + 221*sqrt(2)*tan(1/2*a)^4 + 3*sqrt(2)*tan(1/2*a)^2)*
tan(1/2*b*x)/(tan(1/2*a)^78 + 34*tan(1/2*a)^76 + 559*tan(1/2*a)^74 + 5916*t
an(1/2*a)^72 + 45255*tan(1/2*a)^70 + 266322*tan(1/2*a)^68 + 1252713*tan(1/2
*a)^66 + 4829088*tan(1/2*a)^64 + 15512772*tan(1/2*a)^62 + 41970280*tan(1/2*
a)^60 + 96160636*tan(1/2*a)^58 + 186574864*tan(1/2*a)^56 + 304253964*tan(1/
2*a)^54 + 408239496*tan(1/2*a)^52 + 426395700*tan(1/2*a)^50 + 286097760*tan
(1/2*a)^48 - 31635810*tan(1/2*a)^46 - 450345060*tan(1/2*a)^44 - 811985790*t
an(1/2*a)^42 - 955277400*tan(1/2*a)^40 - 811985790*tan(1/2*a)^38 - 45034506
0*tan(1/2*a)^36 - 31635810*tan(1/2*a)^34 + 286097760*tan(1/2*a)^32 + 426395
700*tan(1/2*a)^30 + 408239496*tan(1/2*a)^28 + 304253964*tan(1/2*a)^26 + 186
574864*tan(1/2*a)^24 + 96160636*tan(1/2*a)^22 + 41970280*tan(1/2*a)^20 + 15
512772*tan(1/2*a)^18 + 4829088*tan(1/2*a)^16 + 1252713*tan(1/2*a)^14 + 2663
22*tan(1/2*a)^12 + 45255*tan(1/2*a)^10 + 5916*tan(1/2*a)^8 + 559*tan(1/2*a)
^6 + 34*tan(1/2*a)^4 + tan(1/2*a)^2) + 40*(5*sqrt(2)*tan(1/2*a)^87 + 52*sqr
t(2)*tan(1/2*a)^85 - 705*sqrt(2)*tan(1/2*a)^83 - 19880*sqrt(2)*tan(1/2*a)^8
1 - 215390*sqrt(2)*tan(1/2*a)^79 - 1452320*sqrt(2)*tan(1/2*a)^77 - 6815938*
sqrt(2)*tan(1/2*a)^75 - 22829080*sqrt(2)*tan(1/2*a)^73 - 51571715*sqrt(2)*t
an(1/2*a)^71 - 52560260*sqrt(2)*tan(1/2*a)^69 + 140318255*sqrt(2)*tan(1/2*a
)^67 + 890085728*sqrt(2)*tan(1/2*a)^65 + 2628022520*sqrt(2)*tan(1/2*a)^63 +
 5320760640*sqrt(2)*tan(1/2*a)^61 + 7702020680*sqrt(2)*tan(1/2*a)^59 + 7118
210080*sqrt(2)*tan(1/2*a)^57 + 995931978*sqrt(2)*tan(1/2*a)^55 - 1034040588
0*sqrt(2)*tan(1/2*a)^53 - 21876041250*sqrt(2)*tan(1/2*a)^51 - 25920327600*s
qrt(2)*tan(1/2*a)^49 - 17685038100*sqrt(2)*tan(1/2*a)^47 + 17685038100*sqrt
(2)*tan(1/2*a)^43 + 25920327600*sqrt(2)*tan(1/2*a)^41 + 21876041250*sqrt(2)
*tan(1/2*a)^39 + 10340405880*sqrt(2)*tan(1/2*a)^37 - 995931978*sqrt(2)*tan(
1/2*a)^35 - 7118210080*sqrt(2)*tan(1/2*a)^33 - 7702020680*sqrt(2)*tan(1/2*a
)^31 - 5320760640*sqrt(2)*tan(1/2*a)^29 - 2628022520*sqrt(2)*tan(1/2*a)^27
 - 890085728*sqrt(2)*tan(1/2*a)^25 - 140318255*sqrt(2)*tan(1/2*a)^23 + 52560
260*sqrt(2)*tan(1/2*a)^21 + 51571715*sqrt(2)*tan(1/2*a)^19 + 22829080*sqrt(
2)*tan(1/2*a)^17 + 6815938*sqrt(2)*tan(1/2*a)^15 + 1452320*sqrt(2)*tan(1/2*
a)^13 + 215390*sqrt(2)*tan(1/2*a)^11 + 19880*sqrt(2)*tan(1/2*a)^9 + 705*sqr
t(2)*tan(1/2*a)^7 - 52*sqrt(2)*tan(1/2*a)^5 - 5*sqrt(2)*tan(1/2*a)^3)/(tan(
1/2*a)^78 + 34*tan(1/2*a)^76 + 559*tan(1/2*a)^74 + 5916*tan(1/2*a)^72 + 452
55*tan(1/2*a)^70 + 266322*tan(1/2*a)^68 + 1252713*tan(1/2*a)^66 + 4829088*t

```
an(1/2*a)^64 + 15512772*tan(1/2*a)^62 + 41970280*tan(1/2*a)^60 + 96160636*t
an(1/2*a)^58 + 186574864*tan(1/2*a)^56 + 304253964*tan(1/2*a)^54 + 40823949
6*tan(1/2*a)^52 + 426395700*tan(1/2*a)^50 + 286097760*tan(1/2*a)^48 - 31635
810*tan(1/2*a)^46 - 450345060*tan(1/2*a)^44 - 811985790*tan(1/2*a)^42 - 955
277400*tan(1/2*a)^40 - 811985790*tan(1/2*a)^38 - 450345060*tan(1/2*a)^36 -
31635810*tan(1/2*a)^34 + 286097760*tan(1/2*a)^32 + 426395700*tan(1/2*a)^30
+ 408239496*tan(1/2*a)^28 + 304253964*tan(1/2*a)^26 + 186574864*tan(1/2*a)^
24 + 96160636*tan(1/2*a)^22 + 41970280*tan(1/2*a)^20 + 15512772*tan(1/2*a)^
18 + 4829088*tan(1/2*a)^16 + 1252713*tan(1/2*a)^14 + 266322*tan(1/2*a)^12 +
 45255*tan(1/2*a)^10 + 5916*tan(1/2*a)^8 + 559*...
```

Mupad [B]
time = 3.23, size = 136, normalized size = 1.72

$$\frac{4\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,2\mathrm{i}-\mathrm{e}^{a\,8\mathrm{i}+b\,x\,8\mathrm{i}}\,2\mathrm{i}-2\mathrm{i}\right)}{15\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^(7/2),x)
```

```
[Out] (4*exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2)*(exp(a*2i + b*x*2i)*2i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*x*
6i)*2i - exp(a*8i + b*x*8i)*2i - 2i))/(15*b*(exp(a*2i + b*x*2i) - 1)^3*(exp
(a*2i + b*x*2i) + 1)^2)
```

3.168 $\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=105

$$-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{16\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -1/7*cos(b*x+a)/b/sin(2*b*x+2*a)^(7/2)+6/35*sin(b*x+a)/b/sin(2*b*x+2*a)^(5/
2)-8/35*cos(b*x+a)/b/sin(2*b*x+2*a)^(3/2)+16/35*sin(b*x+a)/b/sin(2*b*x+2*a)
^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {4388, 4389, 4377}

$$\frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{16\sin(a+bx)}{35b\sqrt{\sin(2a+2bx)}}-\frac{8\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] -1/7*Cos[a + b*x]/(b*Sin[2*a + 2*b*x]^(7/2)) + (6*Sin[a + b*x])/(35*b*Sin[2
*a + 2*b*x]^(5/2)) - (8*Cos[a + b*x])/(35*b*Sin[2*a + 2*b*x]^(3/2)) + (16*S
in[a + b*x])/(35*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4388

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4389

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + D
```

```
ist[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x],
 x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !I
ntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx &= -\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{6}{7}\int \frac{\sin(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{24}{35}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{16}{35}\int \frac{\sin(a+}{\sin^{\frac{3}{2}}(2a+}\\ &= -\frac{\cos(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{6\sin(a+bx)}{35b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{35b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{16\sin(a+bx}{35b\sqrt{\sin(2a+2}}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 67, normalized size = 0.64

$$\frac{(5-10\cos(2(a+bx))-4\cos(4(a+bx))+4\cos(6(a+bx)))\csc^4(a+bx)\sec^3(a+bx)\sqrt{\sin(2(a+bx))}}{560b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] ((5 - 10*Cos[2*(a + b*x)] - 4*Cos[4*(a + b*x)] + 4*Cos[6*(a + b*x)])*Csc[a
+ b*x]^4*Sec[a + b*x]^3*Sqrt[Sin[2*(a + b*x)]])/(560*b)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(xb+a)}{\sin(2xb+2a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

```
[Out] int(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)/sin(2*b*x + 2*a)^(9/2), x)
```

Fricas [A]
time = 1.54, size = 118, normalized size = 1.12

$$\frac{128\cos\left(bx+a\right)^{7}-256\cos\left(bx+a\right)^{5}+128\cos\left(bx+a\right)^{3}+\sqrt{2}\left(128\cos\left(bx+a\right)^{6}-224\cos\left(bx+a\right)^{4}+84\cos\left(bx+a\right)^{2}+7\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}}{560\left(b\cos\left(bx+a\right)^{7}-2\,b\cos\left(bx+a\right)^{5}+b\cos\left(bx+a\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/560*(128*cos(b*x + a)^7 - 256*cos(b*x + a)^5 + 128*cos(b*x + a)^3 + sqrt(
2)*(128*cos(b*x + a)^6 - 224*cos(b*x + a)^4 + 84*cos(b*x + a)^2 + 7)*sqrt(c
os(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^7 - 2*b*cos(b*x + a)^5 + b*cos(b
*x + a)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 3.87, size = 350, normalized size = 3.33

$$-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{7\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^4}+\frac{\mathrm{e}^{a\,3\mathrm{i}+b\,x\,3\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,16\mathrm{i}}{35\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{1}{7\,b}-\frac{8\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}}{35\,b}\right)\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{16\mathrm{i}}{35\,b}+\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,44\mathrm{i}}{35\,b}\right)\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^3\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(2*a + 2*b*x)^(9/2),x)
```

```
[Out] (exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/
2)^(1/2)*16i)/(35*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i))
- (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i
)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i - 1i)^4) - (exp(a*1i + b*x*1i)*(1/(
7*b) - (8*exp(a*2i + b*x*2i))/(35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a
*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*x*2i) + 1)^2*(exp(a*2i + b*x*2i)
*1i - 1i)^2) + (exp(a*1i + b*x*1i)*(16i/(35*b) + (exp(a*2i + b*x*2i)*44i)/(
35*b))*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((e
xp(a*2i + b*x*2i) + 1)^3*(exp(a*2i + b*x*2i)*1i - 1i)^3)
```

3.169 $\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=98

$$\frac{5F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{42b}-\frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b}-\frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b}+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}$$

```
[Out] -5/42*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+
b*x),2^(1/2))/b-1/14*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(5/2)/b+1/18*sin(2*b*x+2
*a)^(9/2)/b-5/42*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4382, 2715, 2720}

$$\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}+\frac{5F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{42b}-\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(2a+2bx)}{14b}-\frac{5\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{42b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (5*EllipticF[a - Pi/4 + b*x, 2])/(42*b) - (5*Cos[2*a + 2*b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(42*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x]^(5/2))/(14*b) + Sin[
2*a + 2*b*x]^(9/2)/(18*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4382

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sin^{\frac{7}{2}}(2a+2bx)\,dx &= \frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}+\frac{1}{2}\int \sin^{\frac{7}{2}}(2a+2bx)\,dx\\ &= -\frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b}+\frac{\sin^{\frac{9}{2}}(2a+2bx)}{18b}+\frac{5}{14}\int \sin^{\frac{3}{2}}(2a+2\\ &= -\frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b}-\frac{\cos(2a+2bx)\sin^{\frac{5}{2}}(2a+2bx)}{14b}+\\ &= \frac{5F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{42b}-\frac{5\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{42b}-\frac{\cos(2a+2bx}{}\end{aligned}$$

Mathematica [**A**]
time = 0.38, size = 96, normalized size = 0.98

$$\frac{240F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2(a+bx))}+70\sin(2(a+bx))-156\sin(4(a+bx))-35\sin(6(a+bx))+18\sin(8(a+bx))+7\sin(10(a+bx))}{2016b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(7/2),x]
[Out] (240*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 70*Sin[2*(a + b*
x)] - 156*Sin[4*(a + b*x)] - 35*Sin[6*(a + b*x)] + 18*Sin[8*(a + b*x)] + 7*
Sin[10*(a + b*x)])/(2016*b*Sqrt[Sin[2*(a + b*x)]])
```

Maple [**F(-1)**]
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
[Out] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(2*b*x + 2*a)^2*cos(b*x + a)^2 - cos(b*x + a)^2)*sin(2*b*x +
2*a)^(3/2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(7/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(7/2), x)
```

3.170 $\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=69

$$\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b}-\frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

```
[Out] -3/10*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+
b*x),2^(1/2))/b-1/10*cos(2*b*x+2*a)*sin(2*b*x+2*a)^(3/2)/b+1/14*sin(2*b*x+2
*a)^(7/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4382, 2715, 2719}

$$\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(2a+2bx)}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) - (Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x
]^(3/2))/(10*b) + Sin[2*a + 2*b*x]^(7/2)/(14*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4382

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^{\frac{5}{2}}(2a+2bx)\,dx = \frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{1}{2}\int \sin^{\frac{5}{2}}(2a+2bx)\,dx$$
$$= -\frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}+\frac{3}{10}\int \sqrt{\sin(2a+2b}$$
$$= \frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b}-\frac{\cos(2a+2bx)\sin^{\frac{3}{2}}(2a+2bx)}{10b}+\frac{\sin^{\frac{7}{2}}(2a+2bx)}{14b}$$

Mathematica [A]
time = 0.23, size = 66, normalized size = 0.96

$$\frac{84E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sqrt{\sin(2(a+bx))}\,(15\sin(2(a+bx))-14\sin(4(a+bx))-5\sin(6(a+bx)))}{280b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] (84*EllipticE[a - Pi/4 + b*x, 2] + Sqrt[Sin[2*(a + b*x)]]*(15*Sin[2*(a + b*
x)] - 14*Sin[4*(a + b*x)] - 5*Sin[6*(a + b*x)]))/(280*b)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 186.90, size = 358108730, normalized size = 5189981.59

method	result	size
default	Expression too large to display	358108730

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")
[Out] integral(-(cos(2*b*x + 2*a)^2*cos(b*x + a)^2 - cos(b*x + a)^2)*sqrt(sin(2*b
*x + 2*a)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(5/2),x)
[Out] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(5/2), x)
```

3.171 $\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=69

$$\frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b}-\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

```
[Out] -1/6*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b+1/10*sin(2*b*x+2*a)^(5/2)/b-1/6*cos(2*b*x+2*a)*sin(2*b*x+2*a
)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4382, 2715, 2720}

$$\frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}-\frac{\sqrt{\sin(2a+2bx)}\cos(2a+2bx)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) - (Cos[2*a + 2*b*x]*Sqrt[Sin[2*a + 2*b*x
]])/(6*b) + Sin[2*a + 2*b*x]^(5/2)/(10*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4382

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b} + \frac{1}{2}\int \sin^{\frac{3}{2}}(2a+2bx)\,dx$$
$$= -\frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b} + \frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b} + \frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+$$
$$= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b} - \frac{\cos(2a+2bx)\sqrt{\sin(2a+2bx)}}{6b} + \frac{\sin^{\frac{5}{2}}(2a+2bx)}{10b}$$

Mathematica [A]
time = 0.33, size = 76, normalized size = 1.10

$$\frac{20F\left(a-\frac{\pi}{4}+bx\middle|2\right)\sqrt{\sin(2(a+bx))} + 9\sin(2(a+bx)) - 10\sin(4(a+bx)) - 3\sin(6(a+bx))}{120b\sqrt{\sin(2(a+bx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^(3/2),x]
[Out] (20*EllipticF[a - Pi/4 + b*x, 2]*Sqrt[Sin[2*(a + b*x)]] + 9*Sin[2*(a + b*x)
] - 10*Sin[4*(a + b*x)] - 3*Sin[6*(a + b*x)])/(120*b*Sqrt[Sin[2*(a + b*x)]]
)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 71.64, size = 189673602, normalized size = 2748892.78

method	result	size
default	Expression too large to display	189673602

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
[Out] integral(cos(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3003 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^2*sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(3/2),x)
[Out] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(3/2), x)
```

3.172 $\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=40

$$\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}+\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}$$

```
[Out] -1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))/b+1/6*sin(2*b*x+2*a)^(3/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4382, 2719}

$$\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}+\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] EllipticE[a - Pi/4 + b*x, 2]/(2*b) + Sin[2*a + 2*b*x]^(3/2)/(6*b)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4382

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= \frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}+\frac{1}{2}\int \sqrt{\sin(2a+2bx)}\,dx\\ &= \frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}+\frac{\sin^{\frac{3}{2}}(2a+2bx)}{6b}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 34, normalized size = 0.85

$$\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\sin^{\frac{3}{2}}(2(a+bx))}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sqrt[Sin[2*a + 2*b*x]],x]
[Out] (3*EllipticE[a - Pi/4 + b*x, 2] + Sin[2*(a + b*x)]^(3/2))/(6*b)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 6.63, size = 26159849, normalized size = 653996.22

method	result	size
default	Expression too large to display	26159849

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
[Out] integral(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(a+b\,x\right)^2\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^(1/2), x)
```

3.173 $\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=40

$$\frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}+\frac{\sqrt{\sin(2a+2bx)}}{2b}$$

```
[Out] -1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b+1/2*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4382, 2720}

$$\frac{\sqrt{\sin(2a+2bx)}}{2b}+\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(2*b) + Sqrt[Sin[2*a + 2*b*x]]/(2*b)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4382

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx &= \frac{\sqrt{\sin(2a+2bx)}}{2b}+\frac{1}{2}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}+\frac{\sqrt{\sin(2a+2bx)}}{2b}\end{aligned}$$

Mathematica [A]
time = 1.00, size = 76, normalized size = 1.90

$$\frac{2\sqrt{\sin(2(a+bx))}-\frac{\sqrt{2}\,F\left(\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\middle|\frac{1}{2}\right)(\cos(a+bx)+\sin(a+bx))}{\sqrt{1+\sin(2(a+bx))}}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (2*Sqrt[Sin[2*(a + b*x)]] - (Sqrt[2]*EllipticF[ArcSin[Cos[a + b*x] - Sin[a
+ b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Sin[2*(a + b*x)]])/(4
*b)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 15.17, size = 66249372, normalized size = 1656234.30

method	result	size
default	Expression too large to display	66249372

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(a+b\,x\right)^2}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(1/2), x)
```

3.174 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=46

$$-\frac{E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{2b}-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/2*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b*
x),2^(1/2))/b-cos(b*x+a)^2/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4380, 2719}

$$-\frac{E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{2b}-\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -1/2*EllipticE[a - Pi/4 + b*x, 2]/b - Cos[a + b*x]^2/(b*Sqrt[Sin[2*a + 2*b*
x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4380

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Dist[e^2*((m + 2*p + 2)/(4*g^2*(p + 1))), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx = -\frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}} - \frac{1}{2}\int \sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)}{2b} - \frac{\cos^2(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

Mathematica [**A**]
time = 0.13, size = 39, normalized size = 0.85

$$-\frac{E\left(a-\frac{\pi}{4}+bx\middle|\,2\right)+\cot(a+bx)\sqrt{\sin(2(a+bx))}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] -1/2*(EllipticE[a - Pi/4 + b*x, 2] + Cot[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/b
```

Maple [**B**] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 17.61, size = 106313389, normalized size = 2311160.63

method	result	size
default	Expression too large to display	106313389

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(a+b\,x\right)^2}{\sin\left(2\,a+2\,b\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(3/2), x)
```

3.175 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=48

$$\frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b}-\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] -1/6*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticF(cos(a+1/4*Pi+b
*x),2^(1/2))/b-1/3*cos(b*x+a)^2/b/sin(2*b*x+2*a)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4380, 2720}

$$\frac{F\left(a+bx-\frac{\pi}{4}\middle|2\right)}{6b}-\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] EllipticF[a - Pi/4 + b*x, 2]/(6*b) - Cos[a + b*x]^2/(3*b*Sin[2*a + 2*b*x]^(
3/2))
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4380

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Dist[e^2*((m + 2*p + 2)/(4*g^2*(p + 1))), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)} + \frac{1}{6}\int \frac{1}{\sqrt{\sin(2a+2bx)}}\,dx$$
$$= \frac{F\left(a-\frac{\pi}{4}+bx\middle|2\right)}{6b} - \frac{\cos^2(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Mathematica [A]
time = 1.14, size = 82, normalized size = 1.71

$$-\frac{\csc^2(a+bx)\sqrt{\sin(2(a+bx))} + \frac{\sqrt{2}\,F\left(\text{ArcSin}(\cos(a+bx)-\sin(a+bx))\middle|\frac{1}{2}\right)(\cos(a+bx)+\sin(a+bx))}{\sqrt{1+\sin(2(a+bx))}}}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -1/12*(Csc[a + b*x]^2*Sqrt[Sin[2*(a + b*x)]] + (Sqrt[2]*EllipticF[ArcSin[Co
s[a + b*x] - Sin[a + b*x]], 1/2]*(Cos[a + b*x] + Sin[a + b*x]))/Sqrt[1 + Si
n[2*(a + b*x)]])/b
```

Maple [A]
time = 82.68, size = 123, normalized size = 2.56

method	result
default	$\frac{\sqrt{\sin(2xb+2a)+1}\,\sqrt{-2\sin(2xb+2a)+2}\,\sqrt{-\sin(2xb+2a)}\,\text{EllipticF}\left(\sqrt{\sin(2xb+2a)+1}\right.}{12\sin(2xb+2a)^{\frac{3}{2}}\cos(2xb+2a)b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/sin(2*b*x+2*a)^(3/2)/cos(2*b*x+2*a)*((sin(2*b*x+2*a)+1)^(1/2)*(-2*sin(
2*b*x+2*a)+2)^(1/2)*(-sin(2*b*x+2*a))^(1/2)*EllipticF((sin(2*b*x+2*a)+1)^(1
/2),1/2*2^(1/2))*sin(2*b*x+2*a)-2*cos(2*b*x+2*a)^2-2*cos(2*b*x+2*a))/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")
```

[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)

Fricas [C] Result contains complex when optimal does not.
time = 0.44, size = 101, normalized size = 2.10

$$-\frac{\sqrt{2i}\left(\cos\left(bx+a\right)^2-1\right)\text{ellipticF}(\cos\left(bx+a\right)+i\sin\left(bx+a\right),-1)+\sqrt{-2i}\left(\cos\left(bx+a\right)^2-1\right)\text{ellipticF}(\cos\left(bx+a\right)-i\sin\left(bx+a\right),-1)-\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}}{12\left(b\cos\left(bx+a\right)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")

[Out] -1/12*(sqrt(2*I)*(cos(b*x + a)^2 - 1)*ellipticF(cos(b*x + a) + I*sin(b*x + a), -1) + sqrt(-2*I)*(cos(b*x + a)^2 - 1)*ellipticF(cos(b*x + a) - I*sin(b*x + a), -1) - sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)))/(b*cos(b*x + a)^2 - b)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(5/2),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")

[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(5/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(a+b\,x\right)^2}{\sin\left(2\,a+2\,b\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(5/2),x)

[Out] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(5/2), x)

3.176 $\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=77

$$-\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b}-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 3/10*(sin(a+1/4*Pi+b*x)^2)^(1/2)/sin(a+1/4*Pi+b*x)*EllipticE(cos(a+1/4*Pi+b
*x),2^(1/2))/b-1/5*cos(b*x+a)^2/b/sin(2*b*x+2*a)^(5/2)-3/10*cos(2*b*x+2*a)/
b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4380, 2716, 2719}

$$-\frac{3E\left(a+bx-\frac{\pi}{4}\middle|2\right)}{10b}-\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] (-3*EllipticE[a - Pi/4 + b*x, 2])/(10*b) - Cos[a + b*x]^2/(5*b*Sin[2*a + 2*
b*x]^(5/2)) - (3*Cos[2*a + 2*b*x])/(10*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 4380

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Dist[e^2*((m + 2*p + 2)/(4*g^2*(p + 1))), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
```

```
2*p]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{3}{10}\int \frac{1}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}} - \frac{3}{10}\int \sqrt{\sin(2a+2bx)}\,dx$$
$$= -\frac{3E\left(a-\frac{\pi}{4}+bx\middle|2\right)}{10b} - \frac{\cos^2(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{3\cos(2a+2bx)}{10b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A]
time = 0.63, size = 64, normalized size = 0.83

$$\frac{-12E\left(a-\frac{\pi}{4}+bx\middle|2\right)+\frac{2(1-6\cos(2(a+bx))+3\cos(4(a+bx)))\cot(a+bx)}{\sin^{\frac{3}{2}}(2(a+bx))}}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sin[2*a + 2*b*x]^(7/2),x]
[Out] (-12*EllipticE[a - Pi/4 + b*x, 2] + (2*(1 - 6*Cos[2*(a + b*x)] + 3*Cos[4*(a
 + b*x)])*Cot[a + b*x])/Sin[2*(a + b*x)]^(3/2))/(40*b)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(xb+a)}{\sin(2xb+2a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
[Out] int(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
```

[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)**2/sin(2*b*x+2*a)**(7/2),x)

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")

[Out] integrate(cos(b*x + a)^2/sin(2*b*x + 2*a)^(7/2), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2}{\sin\left(2\,a+2\,b\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(7/2),x)

[Out] int(cos(a + b*x)^2/sin(2*a + 2*b*x)^(7/2), x)

3.177 $\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$

Optimal. Leaf size=136

$$-\frac{7\mathrm{ArcSin}(\cos(a+bx)-\sin(a+bx))}{64b}+\frac{7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{64b}-\frac{7\cos(a+bx)}{}$$

```
[Out] -7/64*arcsin(cos(b*x+a)-sin(b*x+a))/b+7/64*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b+7/48*sin(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+1/12*cos(b*x+a)*sin
(2*b*x+2*a)^(5/2)/b-7/32*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4382, 4386, 4387, 4390}

$$-\frac{7\mathrm{ArcSin}(\cos(a+bx)-\sin(a+bx))}{64b}+\frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b}+\frac{\sin^{\frac{5}{2}}(2a+2bx)\cos(a+bx)}{12b}-\frac{7\sqrt{\sin(2a+2bx)}\cos(a+bx)}{32b}+\frac{7\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{64b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(64*b) + (7*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(64*b) - (7*Cos[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(32*b) + (7*Sin[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(48*b) + (Cos[a
 + b*x]*Sin[2*a + 2*b*x]^(5/2))/(12*b)
```

Rule 4382

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4387

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[-2*Cos[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*
```

```
(g/(2*p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \cos^3(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx = \frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b} + \frac{7}{12}\int \cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)\,dx$$

$$= \frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b} + \frac{\cos(a+bx)\sin^{\frac{5}{2}}(2a+2bx)}{12b} + \frac{7}{16}\int \sin$$

$$= -\frac{7\cos(a+bx)\sqrt{\sin(2a+2bx)}}{32b} + \frac{7\sin(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{48b} + \cos($$

$$= -\frac{7\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{64b} + \frac{7\log\left(\cos(a+bx)+\sin(a+bx\right.}{64b}$$

Mathematica [**A**]
time = 0.37, size = 99, normalized size = 0.73

$$\frac{-7\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+7\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-\frac{2}{3}(10\cos(a+bx)+9\cos(3(a+bx))+2\cos(5(a+bx)))\sqrt{\sin(2(a+bx))}}{64b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (-7*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 7*Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]] - (2*(10*Cos[a + b*x] + 9*Cos[3*(a + b*x)] + 2*C
os[5*(a + b*x)])*Sqrt[Sin[2*(a + b*x)]])/3)/(64*b)
```

Maple [**F(-1)**]
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
```

```
[Out] int(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(118) = 236$.
time = 1.95, size = 291, normalized size = 2.14

$$-\frac{8\sqrt{2}\left(32\cos\left(bx+a\right)^{5}-4\cos\left(bx+a\right)^{3}-7\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-42\arctan\left(-\frac{\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\left(\cos(bx+a)-\sin(bx+a)\right)+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^{2}+2\cos(bx+a)\sin(bx+a)-1}\right)+42\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)+21\log\left(-32\cos\left(bx+a\right)^{4}+4\sqrt{2}\left(4\cos\left(bx+a\right)^{3}-\left(4\cos\left(bx+a\right)^{2}+1\right)\sin\left(bx+a\right)-5\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\cos\left(bx+a\right)^{2}+16\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{768\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/768*(8*sqrt(2)*(32*cos(b*x + a)^5 - 4*cos(b*x + a)^3 - 7*cos(b*x + a))*s
qrt(cos(b*x + a)*sin(b*x + a)) - 42*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(
b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b
*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1)) + 42*arctan(-(2*sqrt(2)*sqrt(
cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - s
in(b*x + a))) + 21*log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (
4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(
b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3*sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+bx)^3 \sin(2a+2bx)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(3/2),x)
[Out] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(3/2), x)
```

3.178 $\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx$

Optimal. Leaf size=110

$$-\frac{5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{5\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{32b}+\frac{5\sin(a+bx)}{}$$

```
[Out] -5/32*arcsin(cos(b*x+a)-sin(b*x+a))/b-5/32*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b
*x+2*a)^(1/2))/b+1/8*cos(b*x+a)*sin(2*b*x+2*a)^(3/2)/b+5/16*sin(b*x+a)*sin(
2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4382, 4386, 4391}

$$-\frac{5\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{32b}+\frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b}+\frac{\sin^{\frac{3}{2}}(2a+2bx)\cos(a+bx)}{8b}-\frac{5\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-5*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(32*b) - (5*Log[Cos[a + b*x] + Sin
[a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(32*b) + (5*Sin[a + b*x]*Sqrt[Sin[2*a
+ 2*b*x]])/(16*b) + (Cos[a + b*x]*Sin[2*a + 2*b*x]^(3/2))/(8*b)
```

Rule 4382

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4386

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[2*Sin[a + b*x]*((g*Sin[c + d*x])^p/(d*(2*p + 1))), x] + Dist[2*p*(
g/(2*p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
```

```
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sqrt{\sin(2a+2bx)}\,dx &= \frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}+\frac{5}{8}\int \cos(a+bx)\sqrt{\sin(2a+2bx)}\,dx\\ &= \frac{5\sin(a+bx)\sqrt{\sin(2a+2bx)}}{16b}+\frac{\cos(a+bx)\sin^{\frac{3}{2}}(2a+2bx)}{8b}+\frac{5}{16}\int\\ &= -\frac{5\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{32b}-\frac{5\log\left(\cos(a+bx)+\sin(a+\right.}{32b}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 84, normalized size = 0.76

$$\frac{-5\left(\operatorname{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)\right)+2\sqrt{\sin(2(a+bx))}\,(6\sin(a+bx)+\sin(3(a+bx)))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sqrt[Sin[2*a + 2*b*x]],x]
[Out] (-5*(ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x]
+ Sqrt[Sin[2*(a + b*x)]]]) + 2*Sqrt[Sin[2*(a + b*x)]]*(6*Sin[a + b*x] + Sin
[3*(a + b*x)]))/(32*b)
```

Maple [F(-1)]
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x)
[Out] int(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^3*sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(96) = 192$.
time = 2.85, size = 280, normalized size = 2.55

$$\frac{8\sqrt{2}\left(4\cos\left(bx+a\right)^{2}+5\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\sin\left(bx+a\right)+10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\left(\cos\left(bx+a\right)-\sin\left(bx+a\right)\right)+\cos\left(bx+a\right)\sin\left(bx+a\right)}{\cos\left(bx+a\right)^{2}+2\cos\left(bx+a\right)\sin\left(bx+a\right)-1}\right)-10\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-\cos\left(bx+a\right)-\sin\left(bx+a\right)}{\cos\left(bx+a\right)-\sin\left(bx+a\right)}\right)+5\log\left(-32\cos\left(bx+a\right)^{4}+4\sqrt{2}\left(4\cos\left(bx+a\right)^{3}-\left(4\cos\left(bx+a\right)^{2}+1\right)\sin\left(bx+a\right)-5\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+32\cos\left(bx+a\right)^{2}+16\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/128*(8*sqrt(2)*(4*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a)*sin(b*x + a))*sin
(b*x + a) + 10*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x +
a) - sin(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x
 + a)*sin(b*x + a) - 1)) - 10*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x
+ a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a))) + 5*log
(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*
sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x
 + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:ext_reduce Error: Bad Argument TypeDo
ne
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^3\,\sqrt{\sin\left(2\,a+2\,b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^(1/2), x)
```

3.179 $\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$

Optimal. Leaf size=84

$$-\frac{3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{8b}+\frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{8b}+\frac{\cos(a+bx)}{}$$

```
[Out] -3/8*arcsin(cos(b*x+a)-sin(b*x+a))/b+3/8*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x
+2*a)^(1/2))/b+1/4*cos(b*x+a)*sin(2*b*x+2*a)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4382, 4390}

$$-\frac{3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{8b}+\frac{\sqrt{\sin(2a+2bx)}\cos(a+bx)}{4b}+\frac{3\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]])/(8*b) + (3*Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[2*a + 2*b*x]]])/(8*b) + (Cos[a + b*x]*Sqrt[Sin[2*a + 2*
b*x]])/(4*b)
```

Rule 4382

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(m + 2*p))), x] + Dist[e^2*((m + p - 1)/(m + 2*p)), Int[(e*Cos[a +
 b*x])^(m - 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x
] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && NeQ[
m + 2*p, 0] && IntegersQ[2*m, 2*p]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx = \frac{\cos(a+bx)\sqrt{\sin(2a+2bx)}}{4b} + \frac{3}{4}\int \frac{\cos(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx$$

$$= -\frac{3\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{8b} + \frac{3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2\ldots}\right.}{8b}$$

Mathematica [**A**]
time = 0.13, size = 73, normalized size = 0.87

$$\frac{-3\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+3\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)+\csc(a+bx)\sin^{\frac{3}{2}}(2(a+bx))}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sqrt[Sin[2*a + 2*b*x]],x]
```

```
[Out] (-3*ArcSin[Cos[a + b*x] - Sin[a + b*x]] + 3*Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*(a + b*x)]]] + Csc[a + b*x]*Sin[2*(a + b*x)]^(3/2))/(8*b)
```

Maple [**F(-1)**] grade_annotation
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

```
[Out] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(74) = 148.
time = 3.26, size = 268, normalized size = 3.19

$$\frac{8\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}\cos(bx+a)+6\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}(\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right)-6\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(bx+a)\sin(bx+a)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right)-3\log\left(-32\cos(bx+a)^4+4\sqrt{2}\left(4\cos(bx+a)^3-\left(4\cos(bx+a)^2+1\right)\sin(bx+a)-5\cos(bx+a)\right)\sqrt{\cos(bx+a)\sin(bx+a)}+32\cos(bx+a)^2+16\cos(bx+a)\sin(bx+a)+1\right)}{32b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/32*(8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + 6*arctan(-(s
qrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - sin(b*x + a)) + cos(
b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*sin(b*x + a) - 1))
- 6*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) - cos(b*x + a) - sin
(b*x + a))/(cos(b*x + a) - sin(b*x + a))) - 3*log(-32*cos(b*x + a)^4 + 4*sq
rt(2)*(4*cos(b*x + a)^3 - (4*cos(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x +
 a))*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*
sin(b*x + a) + 1))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/sqrt(sin(2*b*x + 2*a)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^3}{\sqrt{\sin\left(2\,a+2\,b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(1/2), x)
```

3.180 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=82

$$\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{4b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2bx)}\right)}{4b}-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}$$

```
[Out] 1/4*arcsin(cos(b*x+a)-sin(b*x+a))/b+1/4*ln(cos(b*x+a)+sin(b*x+a)+sin(2*b*x+
2*a)^(1/2))/b-cos(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4378, 4392, 4391}

$$\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))}{4b}-\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}+\frac{\log\left(\sin(a+bx)+\sqrt{\sin(2a+2bx)}+\cos(a+bx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] ArcSin[Cos[a + b*x] - Sin[a + b*x]]/(4*b) + Log[Cos[a + b*x] + Sin[a + b*x]
 + Sqrt[Sin[2*a + 2*b*x]]]/(4*b) - Cos[a + b*x]/(b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4378

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[e^2*(e*Cos[a + b*x])^(m - 2)*((g*Sin[c + d*x])^(p + 1
)/(2*b*g*(p + 1))), x] + Dist[e^4*((m + p - 1)/(4*g^2*(p + 1))), Int[(e*Cos
[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e
, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 2]
&& LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

Rule 4391

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4392

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/cos[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
```

```
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx &= -\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{4}\int \sec(a+bx)\sqrt{\sin(2a+2bx)}\,dx\\ &= -\frac{\cos(a+bx)}{b\sqrt{\sin(2a+2bx)}}-\frac{1}{2}\int \frac{\sin(a+bx)}{\sqrt{\sin(2a+2bx)}}\,dx\\ &= \frac{\sin^{-1}(\cos(a+bx)-\sin(a+bx))}{4b}+\frac{\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2a+2b}\right.}{4b}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 70, normalized size = 0.85

$$\frac{\text{ArcSin}(\cos(a+bx)-\sin(a+bx))+\log\left(\cos(a+bx)+\sin(a+bx)+\sqrt{\sin(2(a+bx))}\right)-2\csc(a+bx)\sqrt{\sin(2(a+bx))}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(3/2),x]
```

```
[Out] (ArcSin[Cos[a + b*x] - Sin[a + b*x]] + Log[Cos[a + b*x] + Sin[a + b*x] + Sq
rt[Sin[2*(a + b*x)]]] - 2*Csc[a + b*x]*Sqrt[Sin[2*(a + b*x)]])/(4*b)
```

Maple [B] result has leaf size over 500,000. Avoiding possible recursion issues.
time = 23.23, size = 179323150, normalized size = 2186867.68

method	result	size
default	Expression too large to display	179323150

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(74) = 148$.
time = 3.66, size = 295, normalized size = 3.60

$$-\frac{2 \arctan\left(-\frac{\sqrt{2} \sqrt{\cos\left(bx+a\right) \sin\left(bx+a\right)} (\cos(bx+a)-\sin(bx+a))+\cos(bx+a)\sin(bx+a)}{\cos(bx+a)^2+2\cos(bx+a)\sin(bx+a)-1}\right) \sin\left(bx+a\right) - 2 \arctan\left(-\frac{2\sqrt{2} \sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}-\cos(bx+a)-\sin(bx+a)}{\cos(bx+a)-\sin(bx+a)}\right) \sin\left(bx+a\right) + \log\left(-32 \cos\left(bx+a\right)^4 + 4\sqrt{2} \left(4 \cos\left(bx+a\right)^3 - \left(4\cos\left(bx+a\right)^2+1\right)\sin\left(bx+a\right) - 5\cos\left(bx+a\right)\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)} + 32\cos\left(bx+a\right)^2 + 16\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)\sin\left(bx+a\right) + 8\sqrt{2}\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)} + 8\sin\left(bx+a\right)}{16\,b\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/16*(2*arctan(-(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*(cos(b*x + a) - s
in(b*x + a)) + cos(b*x + a)*sin(b*x + a))/(cos(b*x + a)^2 + 2*cos(b*x + a)*
sin(b*x + a) - 1))*sin(b*x + a) - 2*arctan(-(2*sqrt(2)*sqrt(cos(b*x + a)*si
n(b*x + a)) - cos(b*x + a) - sin(b*x + a))/(cos(b*x + a) - sin(b*x + a)))*s
in(b*x + a) + log(-32*cos(b*x + a)^4 + 4*sqrt(2)*(4*cos(b*x + a)^3 - (4*cos
(b*x + a)^2 + 1)*sin(b*x + a) - 5*cos(b*x + a))*sqrt(cos(b*x + a)*sin(b*x +
 a)) + 32*cos(b*x + a)^2 + 16*cos(b*x + a)*sin(b*x + a) + 1)*sin(b*x + a) +
 8*sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a)) + 8*sin(b*x + a))/(b*sin(b*x + a
))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(a+b\,x\right)}^3}{{\sin\left(2\,a+2\,b\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(3/2),x)
[Out] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(3/2), x)
```

3.181 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=28

$$-\frac{\cos^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

```
[Out] -1/3*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{4376\}$

$$-\frac{\cos^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

```
[Out] -1/3*Cos[a + b*x]^3/(b*Sin[2*a + 2*b*x]^(3/2))
```

Rule 4376

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx = -\frac{\cos^3(a+bx)}{3b\sin^{\frac{3}{2}}(2a+2bx)}$$

Mathematica [A]
time = 0.06, size = 27, normalized size = 0.96

$$-\frac{\csc^3(a+bx)\sin^{\frac{3}{2}}(2(a+bx))}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(5/2),x]
```

[Out] -1/24*(Csc[a + b*x]^3*Sin[2*(a + b*x)]^(3/2))/b

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 175.78, size = 192, normalized size = 6.86

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1}}\,\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\left(4\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+2}\,\sqrt{-\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}}{24\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)}\,\sqrt{\tan^{3}\left(\frac{bx}{2}\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/24*(-tan(1/2*b*x+1/2*a)/(tan(1/2*b*x+1/2*a)^2-1))^(1/2)*(tan(1/2*b*x+1/2*a)^2-1)/tan(1/2*b*x+1/2*a)*(4*(tan(1/2*b*x+1/2*a)+1)^(1/2)*(-2*tan(1/2*b*x+1/2*a)+2)^(1/2)*(-tan(1/2*b*x+1/2*a))^(1/2)*EllipticF((tan(1/2*b*x+1/2*a)+1)^(1/2),1/2*2^(1/2))*tan(1/2*b*x+1/2*a)+tan(1/2*b*x+1/2*a)^4-1)/(tan(1/2*b*x+1/2*a)*(tan(1/2*b*x+1/2*a)^2-1))^(1/2)/(tan(1/2*b*x+1/2*a)^3-tan(1/2*b*x+1/2*a))^(1/2)/b

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="maxima")

[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(5/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(24) = 48.
time = 2.70, size = 53, normalized size = 1.89

$$\frac{\sqrt{2}\,\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}\,\cos\left(bx+a\right)+\cos\left(bx+a\right)^{2}-1}{12\left(b\cos\left(bx+a\right)^{2}-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="fricas")

[Out] 1/12*(sqrt(2)*sqrt(cos(b*x + a)*sin(b*x + a))*cos(b*x + a) + cos(b*x + a)^2 - 1)/(b*cos(b*x + a)^2 - b)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 15292 vs. $2(24) = 48$.
time = 92.58, size = 15292, normalized size = 546.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(2)*sqrt(-tan(1/2*b*x)^4*tan(1/2*a)^3 - tan(1/2*b*x)^3*tan(1/2*a)^
4 + tan(1/2*b*x)^4*tan(1/2*a) + 6*tan(1/2*b*x)^3*tan(1/2*a)^2 + 6*tan(1/2*b
*x)^2*tan(1/2*a)^3 + tan(1/2*b*x)*tan(1/2*a)^4 - tan(1/2*b*x)^3 - 6*tan(1/2
*b*x)^2*tan(1/2*a) - 6*tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*a)^3 + tan(1/2*b
*x) + tan(1/2*a))*(((((((sqrt(2)*tan(1/2*a)^57 + 50*sqrt(2)*tan(1/2*a)^55 +
 516*sqrt(2)*tan(1/2*a)^53 + 1750*sqrt(2)*tan(1/2*a)^51 - 5215*sqrt(2)*tan(
1/2*a)^49 - 77796*sqrt(2)*tan(1/2*a)^47 - 386576*sqrt(2)*tan(1/2*a)^45 - 11
86876*sqrt(2)*tan(1/2*a)^43 - 2515251*sqrt(2)*tan(1/2*a)^41 - 3759930*sqrt(
2)*tan(1/2*a)^39 - 3812844*sqrt(2)*tan(1/2*a)^37 - 2196894*sqrt(2)*tan(1/2*
a)^35 - 36499*sqrt(2)*tan(1/2*a)^33 + 824296*sqrt(2)*tan(1/2*a)^31 - 824296
*sqrt(2)*tan(1/2*a)^27 + 36499*sqrt(2)*tan(1/2*a)^25 + 2196894*sqrt(2)*tan(
1/2*a)^23 + 3812844*sqrt(2)*tan(1/2*a)^21 + 3759930*sqrt(2)*tan(1/2*a)^19 +
 2515251*sqrt(2)*tan(1/2*a)^17 + 1186876*sqrt(2)*tan(1/2*a)^15 + 386576*sqr
t(2)*tan(1/2*a)^13 + 77796*sqrt(2)*tan(1/2*a)^11 + 5215*sqrt(2)*tan(1/2*a)^
9 - 1750*sqrt(2)*tan(1/2*a)^7 - 516*sqrt(2)*tan(1/2*a)^5 - 50*sqrt(2)*tan(1
/2*a)^3 - sqrt(2)*tan(1/2*a))*tan(1/2*b*x)/(tan(1/2*a)^51 + 23*tan(1/2*a)^4
9 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*tan(1/2*a)^43 + 31878*tan
(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*a)^37 + 389367*tan(1/2*a)
^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31 + 534888*tan(1/2*a)^29 +
208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 534888*tan(1/2*a)^23 - 653752
*tan(1/2*a)^21 - 572033*tan(1/2*a)^19 - 389367*tan(1/2*a)^17 - 211508*tan(1
/2*a)^15 - 92092*tan(1/2*a)^13 - 31878*tan(1/2*a)^11 - 8602*tan(1/2*a)^9 -
1748*tan(1/2*a)^7 - 252*tan(1/2*a)^5 - 23*tan(1/2*a)^3 - tan(1/2*a)) + 24*(
sqrt(2)*tan(1/2*a)^56 - 9*sqrt(2)*tan(1/2*a)^54 - 293*sqrt(2)*tan(1/2*a)^52
 - 2499*sqrt(2)*tan(1/2*a)^50 - 11234*sqrt(2)*tan(1/2*a)^48 - 28814*sqrt(2)
*tan(1/2*a)^46 - 30750*sqrt(2)*tan(1/2*a)^44 + 62254*sqrt(2)*tan(1/2*a)^42
```

```
+ 338619*sqrt(2)*tan(1/2*a)^40 + 727149*sqrt(2)*tan(1/2*a)^38 + 874209*sqrt
(2)*tan(1/2*a)^36 + 376295*sqrt(2)*tan(1/2*a)^34 - 693804*sqrt(2)*tan(1/2*a
)^32 - 1611124*sqrt(2)*tan(1/2*a)^30 - 1611124*sqrt(2)*tan(1/2*a)^28 - 6938
04*sqrt(2)*tan(1/2*a)^26 + 376295*sqrt(2)*tan(1/2*a)^24 + 874209*sqrt(2)*ta
n(1/2*a)^22 + 727149*sqrt(2)*tan(1/2*a)^20 + 338619*sqrt(2)*tan(1/2*a)^18 +
 62254*sqrt(2)*tan(1/2*a)^16 - 30750*sqrt(2)*tan(1/2*a)^14 - 28814*sqrt(2)*
tan(1/2*a)^12 - 11234*sqrt(2)*tan(1/2*a)^10 - 2499*sqrt(2)*tan(1/2*a)^8 - 2
93*sqrt(2)*tan(1/2*a)^6 - 9*sqrt(2)*tan(1/2*a)^4 + sqrt(2)*tan(1/2*a)^2)/(t
an(1/2*a)^51 + 23*tan(1/2*a)^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 +
8602*tan(1/2*a)^43 + 31878*tan(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan
(1/2*a)^37 + 389367*tan(1/2*a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a
)^31 + 534888*tan(1/2*a)^29 + 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 -
 534888*tan(1/2*a)^23 - 653752*tan(1/2*a)^21 - 572033*tan(1/2*a)^19 - 38936
7*tan(1/2*a)^17 - 211508*tan(1/2*a)^15 - 92092*tan(1/2*a)^13 - 31878*tan(1/
2*a)^11 - 8602*tan(1/2*a)^9 - 1748*tan(1/2*a)^7 - 252*tan(1/2*a)^5 - 23*tan
(1/2*a)^3 - tan(1/2*a)))*tan(1/2*b*x) - 3*(sqrt(2)*tan(1/2*a)^57 + 178*sqrt
(2)*tan(1/2*a)^55 + 2052*sqrt(2)*tan(1/2*a)^53 + 7254*sqrt(2)*tan(1/2*a)^51
 - 20575*sqrt(2)*tan(1/2*a)^49 - 310500*sqrt(2)*tan(1/2*a)^47 - 1498640*sqr
t(2)*tan(1/2*a)^45 - 4335420*sqrt(2)*tan(1/2*a)^43 - 8189235*sqrt(2)*tan(1/
2*a)^41 - 9438650*sqrt(2)*tan(1/2*a)^39 - 3297260*sqrt(2)*tan(1/2*a)^37 + 9
834210*sqrt(2)*tan(1/2*a)^35 + 20470125*sqrt(2)*tan(1/2*a)^33 + 17196520*sq
rt(2)*tan(1/2*a)^31 - 17196520*sqrt(2)*tan(1/2*a)^27 - 20470125*sqrt(2)*tan
(1/2*a)^25 - 9834210*sqrt(2)*tan(1/2*a)^23 + 3297260*sqrt(2)*tan(1/2*a)^21
+ 9438650*sqrt(2)*tan(1/2*a)^19 + 8189235*sqrt(2)*tan(1/2*a)^17 + 4335420*s
qrt(2)*tan(1/2*a)^15 + 1498640*sqrt(2)*tan(1/2*a)^13 + 310500*sqrt(2)*tan(1
/2*a)^11 + 20575*sqrt(2)*tan(1/2*a)^9 - 7254*sqrt(2)*tan(1/2*a)^7 - 2052*sq
rt(2)*tan(1/2*a)^5 - 178*sqrt(2)*tan(1/2*a)^3 - sqrt(2)*tan(1/2*a))/(tan(1/
2*a)^51 + 23*tan(1/2*a)^49 + 252*tan(1/2*a)^47 + 1748*tan(1/2*a)^45 + 8602*
tan(1/2*a)^43 + 31878*tan(1/2*a)^41 + 92092*tan(1/2*a)^39 + 211508*tan(1/2*
a)^37 + 389367*tan(1/2*a)^35 + 572033*tan(1/2*a)^33 + 653752*tan(1/2*a)^31
+ 534888*tan(1/2*a)^29 + 208012*tan(1/2*a)^27 - 208012*tan(1/2*a)^25 - 5348
88*tan(1/2*a)^23 - 653752*tan(1/2*a)^21 - 572033*tan(1/2*a)^19 - 389367*tan
(1/2*a)^17 - 211508*tan(1/2*a)^15 - 92092*tan(1/2*a)^13 - 31878*tan(1/2*a)^
11 - 8602*tan(1/2*a)^9 - 1748*tan(1/2*a)^7 - 252*tan(1/2*a)^5 - 23*tan(1/2*
a)^3 - tan(1/2*a)))*tan(1/2*b*x) - 16*(3*sqrt(2)*tan(1/2*a)^56 - 91*sqrt(2)
*tan(1/2*a)^54 - 1967*sqrt(2)*tan(1/2*a)^52 - 15945*sqrt(2)*tan(1/2*a)^50 -
 72870*sqrt(2)*tan(1/2*a)^48 - 204330*sqrt(2)*tan(1/2*a)^46 - 320730*sqrt(2
)*tan(1/2*a)^44 - 47990*sqrt(2)*tan(1/2*a)^42 + 1098545*sqrt(2)*tan(1/2*a)^
40 + 2914695*sqrt(2)*tan(1/2*a)^38 + 3842275*sqrt(2)*tan(1/2*a)^36 + 195576
5*sqrt(2)*tan(1/2*a)^34 - 2577540*sqrt(2)*tan(1/2*a)^32 - 6569820*sqrt(2)*t
an(1/2*a)^30 - 6569820*sqrt(2)*tan(1/2*a)^28 - ...
```

Mupad [B]

time = 1.17, size = 94, normalized size = 3.36

$$\frac{\sqrt{\sin \left(2\,a+2\,b\,x\right)}\,\left(\frac{2\,{\sin \left(\frac{a}{2}+\frac{b\,x}{2}\right)}^2}{3}-{\sin \left(\frac{3\,a}{2}+\frac{3\,b\,x}{2}\right)}^2+\frac{{\sin \left(\frac{5\,a}{2}+\frac{5\,b\,x}{2}\right)}^2}{3}\right)}{b\,\left(30\,{\sin \left(a+b\,x\right)}^2-12\,{\sin \left(2\,a+2\,b\,x\right)}^2+2\,{\sin \left(3\,a+3\,b\,x\right)}^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(5/2),x)
```

```
[Out] (sin(2*a + 2*b*x)^(1/2)*((2*sin(a/2 + (b*x)/2)^2)/3 - sin((3*a)/2 + (3*b*x)
/2)^2 + sin((5*a)/2 + (5*b*x)/2)^2/3))/(b*(2*sin(3*a + 3*b*x)^2 - 12*sin(2*
a + 2*b*x)^2 + 30*sin(a + b*x)^2))
```

3.182 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=55

$$-\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -1/5*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(5/2)-1/5*cos(b*x+a)/b/sin(2*b*x+2*a)^(1
/2)
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4380, 4376}

$$-\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
```

```
[Out] -1/5*Cos[a + b*x]^3/(b*Sin[2*a + 2*b*x]^(5/2)) - Cos[a + b*x]/(5*b*Sqrt[Sin
[2*a + 2*b*x]])
```

Rule 4376

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4380

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Dist[e^2*((m + 2*p + 2)/(4*g^2*(p + 1))), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx = -\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} + \frac{1}{5}\int \frac{\cos(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx$$
$$= -\frac{\cos^3(a+bx)}{5b\sin^{\frac{5}{2}}(2a+2bx)} - \frac{\cos(a+bx)}{5b\sqrt{\sin(2a+2bx)}}$$

Mathematica [A]
time = 0.11, size = 35, normalized size = 0.64

$$-\frac{\csc(a+bx)\left(4+\csc^2(a+bx)\right)\sqrt{\sin(2(a+bx))}}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(7/2),x]
[Out] -1/40*(Csc[a + b*x]*(4 + Csc[a + b*x]^2)*Sqrt[Sin[2*(a + b*x)]])/b
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(bx+a)}{\sin(2bx+2a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x)
[Out] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(7/2), x)
```

Fricas [A]
time = 3.16, size = 76, normalized size = 1.38

$$-\frac{\sqrt{2}\left(4\cos(bx+a)^2-5\right)\sqrt{\cos(bx+a)\sin(bx+a)}+4\left(\cos(bx+a)^2-1\right)\sin(bx+a)}{40\left(b\cos(bx+a)^2-b\right)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/40*(sqrt(2)*(4*cos(b*x + a)^2 - 5)*sqrt(cos(b*x + a)*sin(b*x + a)) + 4*(
cos(b*x + a)^2 - 1)*sin(b*x + a))/((b*cos(b*x + a)^2 - b)*sin(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(7/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.07, size = 93, normalized size = 1.69

$$-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,\left(-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,3\mathrm{i}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{5\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(7/2),x)
```

```
[Out] -(exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)
/2)^(1/2)*(exp(a*4i + b*x*4i)*1i - exp(a*2i + b*x*2i)*3i + 1i))/(5*b*(exp(a
*2i + b*x*2i) - 1)^3)
```

3.183 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=81

$$-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -1/7*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(7/2)-2/21*cos(b*x+a)/b/sin(2*b*x+2*a)^(
3/2)+4/21*sin(b*x+a)/b/sin(2*b*x+2*a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {4380, 4388, 4377}

$$\frac{4\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}-\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
```

```
[Out] -1/7*Cos[a + b*x]^3/(b*Sin[2*a + 2*b*x]^(7/2)) - (2*Cos[a + b*x])/(21*b*Sin
[2*a + 2*b*x]^(3/2)) + (4*Sin[a + b*x])/(21*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4377

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(e*Sin[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(b*g*m)
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4380

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Dist[e^2*((m + 2*p + 2)/(4*g^2*(p + 1))), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rule 4388

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist
```

```
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)}{\sin^{\frac{9}{2}}(2a+2bx)}\,dx &= -\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}+\frac{2}{7}\int \frac{\cos(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx \\
&= -\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{4}{21}\int \frac{\sin(a+bx)}{\sin^{\frac{3}{2}}(2a+2bx)}\,dx \\
&= -\frac{\cos^3(a+bx)}{7b\sin^{\frac{7}{2}}(2a+2bx)}-\frac{2\cos(a+bx)}{21b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{21b\sqrt{\sin(2a+2bx)}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 55, normalized size = 0.68

$$\frac{(5-12\cos(2(a+bx))+4\cos(4(a+bx)))\csc^4(a+bx)\sec(a+bx)\sqrt{\sin(2(a+bx))}}{336b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(9/2),x]
[Out] ((5 - 12*Cos[2*(a + b*x)] + 4*Cos[4*(a + b*x)])*Csc[a + b*x]^4*Sec[a + b*x]
*Sqrt[Sin[2*(a + b*x)]])/(336*b)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^3(bx+a)}{\sin(2bx+2a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
[Out] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(9/2), x)
```

Fricas [**A**]
time = 2.24, size = 104, normalized size = 1.28

$$\frac{32 \cos (bx+a)^5-64 \cos (bx+a)^3+\sqrt{2}\,(32 \cos (bx+a)^4-56 \cos (bx+a)^2+21)\sqrt{\cos (bx+a) \sin (bx+a)}+32 \cos (bx+a)}{336\,(b\cos (bx+a)^5-2\,b\cos (bx+a)^3+b\cos (bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/336*(32*cos(b*x + a)^5 - 64*cos(b*x + a)^3 + sqrt(2)*(32*cos(b*x + a)^4 -
 56*cos(b*x + a)^2 + 21)*sqrt(cos(b*x + a)*sin(b*x + a)) + 32*cos(b*x + a))
/(b*cos(b*x + a)^5 - 2*b*cos(b*x + a)^3 + b*cos(b*x + a))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(9/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.66, size = 302, normalized size = 3.73

$$-\frac{5\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{84\,b\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})^2}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,3\mathrm{i}}{14\,b\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})^3}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{7\,b\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})^4}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{5\mathrm{i}}{84\,b}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,4\mathrm{i}}{21\,b}\right)\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1)\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(9/2),x)
```

```
[Out] (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/
2)^(1/2)*3i)/(14*b*(exp(a*2i + b*x*2i)*1i - 1i)^3) - (5*exp(a*1i + b*x*1i)*
```

```
((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(84*b*(exp
(a*2i + b*x*2i)*1i - 1i)^2) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i
)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(7*b*(exp(a*2i + b*x*2i)*1i - 1i)^4
) - (exp(a*1i + b*x*1i)*(5i/(84*b) - (exp(a*2i + b*x*2i)*4i)/(21*b))*((exp(
- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*x
*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i))
```

3.184 $\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx$

Optimal. Leaf size=107

$$-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

```
[Out] -1/9*cos(b*x+a)^3/b/sin(2*b*x+2*a)^(9/2)-1/15*cos(b*x+a)/b/sin(2*b*x+2*a)^(
5/2)+4/45*sin(b*x+a)/b/sin(2*b*x+2*a)^(3/2)-8/45*cos(b*x+a)/b/sin(2*b*x+2*a
)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4380, 4388, 4389, 4376}

$$\frac{4\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}-\frac{8\cos(a+bx)}{45b\sqrt{\sin(2a+2bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

```
[Out] -1/9*Cos[a + b*x]^3/(b*Sin[2*a + 2*b*x]^(9/2)) - Cos[a + b*x]/(15*b*Sin[2*a
 + 2*b*x]^(5/2)) + (4*Sin[a + b*x])/(45*b*Sin[2*a + 2*b*x]^(3/2)) - (8*Cos[
a + b*x])/(45*b*Sqrt[Sin[2*a + 2*b*x]])
```

Rule 4376

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[(-(e*Cos[a + b*x])^m)*((g*Sin[c + d*x])^(p + 1)/(b*g
*m)), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[
d/b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rule 4380

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e*Cos[a + b*x])^m*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(
p + 1))), x] + Dist[e^2*((m + 2*p + 2)/(4*g^2*(p + 1))), Int[(e*Cos[a + b*x
])^(m - 2)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e, g}, x]
 && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 1] && LtQ[p
, -1] && NeQ[m + 2*p + 2, 0] && (LtQ[p, -2] || EqQ[m, 2]) && IntegersQ[2*m,
 2*p]
```

Rule 4388

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[Cos[a + b*x]*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4389

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-Sin[a + b*x])*((g*Sin[c + d*x])^(p + 1)/(2*b*g*(p + 1))), x] + D
ist[(2*p + 3)/(2*g*(p + 1)), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1), x],
 x] /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !I
ntegerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{\sin^{\frac{11}{2}}(2a+2bx)}\,dx &= -\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}+\frac{1}{3}\int \frac{\cos(a+bx)}{\sin^{\frac{7}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4}{15}\int \frac{\sin(a+bx)}{\sin^{\frac{5}{2}}(2a+2bx)}\,dx\\ &= -\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}+\frac{8}{45}\int \frac{\cos(a+}{\sin^{\frac{3}{2}}(2a+}\\ &= -\frac{\cos^3(a+bx)}{9b\sin^{\frac{9}{2}}(2a+2bx)}-\frac{\cos(a+bx)}{15b\sin^{\frac{5}{2}}(2a+2bx)}+\frac{4\sin(a+bx)}{45b\sin^{\frac{3}{2}}(2a+2bx)}-\frac{8\cos(a+bx}{45b\sqrt{\sin(2a+}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 62, normalized size = 0.58

$$-\frac{\sqrt{\sin(2(a+bx))}\,(113\csc(a+bx)+17\csc^3(a+bx)+5\csc^5(a+bx)-15\sec(a+bx)\tan(a+bx))}{1440b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sin[2*a + 2*b*x]^(11/2),x]
```

```
[Out] -1/1440*(Sqrt[Sin[2*(a + b*x)]]*(113*Csc[a + b*x] + 17*Csc[a + b*x]^3 + 5*C
sc[a + b*x]^5 - 15*Sec[a + b*x]*Tan[a + b*x]))/b
```

Maple [F(-1)]
time = 180.00, size = 0, normalized size = 0.00 hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)
```

[Out] int(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^3/sin(2*b*x + 2*a)^(11/2), x)
```

Fricas [A]
time = 2.19, size = 131, normalized size = 1.22

$$-\frac{\sqrt{2}\,\left(128\,\cos\left(bx+a\right)^{6}-288\,\cos\left(bx+a\right)^{4}+180\,\cos\left(bx+a\right)^{2}-15\right)\sqrt{\cos\left(bx+a\right)\sin\left(bx+a\right)}+128\,\left(\cos\left(bx+a\right)^{6}-2\,\cos\left(bx+a\right)^{4}+\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)}{1440\,\left(b\cos\left(bx+a\right)^{6}-2\,b\cos\left(bx+a\right)^{4}+b\cos\left(bx+a\right)^{2}\right)\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="fricas")
```

```
[Out] -1/1440*(sqrt(2)*(128*cos(b*x + a)^6 - 288*cos(b*x + a)^4 + 180*cos(b*x + a
)^2 - 15)*sqrt(cos(b*x + a)*sin(b*x + a)) + 128*(cos(b*x + a)^6 - 2*cos(b*x
 + a)^4 + cos(b*x + a)^2)*sin(b*x + a))/((b*cos(b*x + a)^6 - 2*b*cos(b*x +
a)^4 + b*cos(b*x + a)^2)*sin(b*x + a))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/sin(2*b*x+2*a)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/sin(2*b*x+2*a)^(11/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 4.88, size = 383, normalized size = 3.58

$$-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{60\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^3}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}\,2\mathrm{i}}{9\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^4}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{9\,b\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^5}+\frac{8\,\mathrm{e}^{a\,3\mathrm{i}+b\,x\,3\mathrm{i}}\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{45\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\frac{49\mathrm{i}}{180\,b}+\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,19\mathrm{i}}{180\,b}\right)\,\sqrt{\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}}{2}}}{{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/sin(2*a + 2*b*x)^(11/2),x)
```

```
[Out] (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/
2)^(1/2))/(9*b*(exp(a*2i + b*x*2i)*1i - 1i)^5) - (exp(a*1i + b*x*1i)*((exp(
- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2)*2i)/(9*b*(exp(a*2
i + b*x*2i)*1i - 1i)^4) - (exp(a*1i + b*x*1i)*((exp(- a*2i - b*x*2i)*1i)/2
- (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/(60*b*(exp(a*2i + b*x*2i)*1i - 1i)^3) +
 (8*exp(a*3i + b*x*3i)*((exp(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1
i)/2)^(1/2))/(45*b*(exp(a*2i + b*x*2i) + 1)*(exp(a*2i + b*x*2i)*1i - 1i)) -
 (exp(a*1i + b*x*1i)*(49i/(180*b) + (exp(a*2i + b*x*2i)*19i)/(180*b))*((exp
(- a*2i - b*x*2i)*1i)/2 - (exp(a*2i + b*x*2i)*1i)/2)^(1/2))/((exp(a*2i + b*
x*2i) + 1)^2*(exp(a*2i + b*x*2i)*1i - 1i)^2)
```

3.185 $\int \frac{\cos(x)}{\sqrt{\sin(2x)}} \, dx$

Optimal. Leaf size=31

$$-\frac{1}{2}\text{ArcSin}(\cos(x)-\sin(x))+\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

```
[Out] -1/2*arcsin(cos(x)-sin(x))+1/2*ln(cos(x)+sin(x)+sin(2*x)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{4390\}$

$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\text{ArcSin}(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[Sin[2*x]],x]
```

```
[Out] -1/2*ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}} \, dx = -\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))+\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 0.94

$$\frac{1}{2}\left(-\text{ArcSin}(\cos(x)-\sin(x))+\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[Sin[2*x]],x]
```

[Out] (-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.18, size = 98, normalized size = 3.16

method	result
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\right.}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="maxima")

[Out] integrate(cos(x)/sqrt(sin(2*x)), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(25) = 50.
time = 2.69, size = 137, normalized size = 4.42

$$\frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)-\frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right)-\frac{1}{8}\log\left(-32\cos(x)^4+4\sqrt{2}\left(4\cos(x)^3-\left(4\cos(x)^2+1\right)\sin(x)-5\cos(x)\right)\sqrt{\cos(x)\sin(x)}+32\cos(x)^2+16\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="fricas")

[Out] 1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/8*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(x)/sqrt(sin(2*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos (x)}{\sqrt{\sin (2\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(2*x)^(1/2),x)
```

```
[Out] int(cos(x)/sin(2*x)^(1/2), x)
```

3.186 $\int \csc(x)\sqrt{\sin(2x)}\,dx$

Optimal. Leaf size=25

$$-\text{ArcSin}(\cos(x)-\sin(x))+\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

```
[Out] -arcsin(cos(x)-sin(x))+ln(cos(x)+sin(x)+sin(2*x)^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4393, 4390}

$$\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\text{ArcSin}(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Sqrt[Sin[2*x]],x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]
```

Rule 4390

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Sim
p[-ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4393

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \csc(x)\sqrt{\sin(2x)}\,dx &= 2\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx\\ &= -\sin^{-1}(\cos(x)-\sin(x))+\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$-\text{ArcSin}(\cos(x)-\sin(x))+\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Sqrt[Sin[2*x]],x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.14, size = 99, normalized size = 3.96

method	result
default	$\frac{2\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}}\,\left(\tan^2\left(\frac{x}{2}\right)-1\right)\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\mathrm{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1}\,,\ldots\right.}{\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)-1\right)}\,\sqrt{\tan^3\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1/
2*x)^2-1))^(1/2)*(tan(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2),1/2*2
^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(x)*sqrt(sin(2*x)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(23) = 46.
time = 2.91, size = 137, normalized size = 5.48

$$\frac{1}{2}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}\,(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)-\frac{1}{2}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right)-\frac{1}{4}\log\left(-32\cos(x)^4+4\sqrt{2}\left(4\cos(x)^3-\left(4\cos(x)^2+1\right)\sin(x)-5\cos(x)\right)\sqrt{\cos(x)\sin(x)}+32\cos(x)^2+16\cos(x)\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))
/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/2*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin
(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/4*log(-32*cos(x)^4 + 4*sqrt(
2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) +
32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(2*x)**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(2*x)^(1/2),x, algorithm="giac")
[Out] integrate(csc(x)*sqrt(sin(2*x)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\sin\left(2\,x\right)}}{\sin\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)^(1/2)/sin(x),x)
[Out] int(sin(2*x)^(1/2)/sin(x), x)
```

3.187 $\int \cos^3(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=85

$$-\frac{\cos^3(a+bx)\cot(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{4+m}{2};\frac{6+m}{2};\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)}{b(4+m)}$$

```
[Out] -cos(b*x+a)^3*cot(b*x+a)*hypergeom([2+1/2*m, 1/2-1/2*m],[3+1/2*m],cos(b*x+a
)^2)*(sin(b*x+a)^2)^(1/2-1/2*m)*sin(2*b*x+2*a)^m/b/(4+m)
```

Rubi [A]
time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4394, 2656}

$$-\frac{\cos^3(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{m+4}{2};\frac{m+6}{2};\cos^2(a+bx)\right)}{b(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] -((Cos[a + b*x]^3*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (4 + m)/2, (6 +
 m)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(b*
(4 + m)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 4394

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p), Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^3(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^{3+m}(a+bx)\sin^m(a$$

$$= -\frac{\cos^3(a+bx)\cot(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{4+m}{2};\frac{6+m}{2};\cos^2(a+bx)\right)\sin^2(a+bx}{b(4+m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 13.38, size = 2472, normalized size = 29.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]^3*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (2^(1 + m)*(6*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m)/2, -m, 2*(2 + m), (3 + m)/2,
 Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1 + m)/2, -m, 1 + 2*m
, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12*AppellF1[(1 + m)
/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[a
 + b*x]^3*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] +
 Sin[(3*(a + b*x))/2]))^m*Sin[2*(a + b*x)]^m*Tan[(a + b*x)/2])/(b*(1 + m)*(
Cos[a + b*x]*Sec[(a + b*x)/2]^2)^m*((2^m*(6*AppellF1[(1 + m)/2, -m, 2*(1 +
m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m
)/2, -m, 2*(2 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - A
ppellF1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*
x)/2]^2] - 12*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2])*(Sec[(a + b*x)/2]^2)^(1 + 2*m)*(Cos[(a + b*x)/2]*(
-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m)/((1 + m)*(Cos[a + b*x]*Sec[(a
 + b*x)/2]^2)^m) + (2^(1 + m)*m*(6*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 +
m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m)/2, -m,
2*(2 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - AppellF1[(
1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
- 12*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2])*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x
)/2] + Sin[(3*(a + b*x))/2]))^(-1 + m)*(Cos[(a + b*x)/2]*(-1/2*Cos[(a + b*x
)/2] + (3*Cos[(3*(a + b*x))/2])/2) - (Sin[(a + b*x)/2]*(-Sin[(a + b*x)/2] +
 Sin[(3*(a + b*x))/2]))/2)*Tan[(a + b*x)/2])/((1 + m)*(Cos[a + b*x]*Sec[(a
+ b*x)/2]^2)^m) + (2^(2 + m)*m*(6*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m
)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m)/2, -m, 2
*(2 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1
 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] -
 12*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a
 + b*x)/2]^2])*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)
/2] + Sin[(3*(a + b*x))/2]))^m*Tan[(a + b*x)/2]^2)/((1 + m)*(Cos[a + b*x]*S
ec[(a + b*x)/2]^2)^m) - (2^(1 + m)*m*(6*AppellF1[(1 + m)/2, -m, 2*(1 + m),
(3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] + 8*AppellF1[(1 + m)/2,
 -m, 2*(2 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - Appel
lF1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2
]^2] - 12*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -
Tan[(a + b*x)/2]^2])*(Sec[(a + b*x)/2]^2)^(2*m)*(Cos[a + b*x]*Sec[(a + b*x)
```

```
/2]^2)^(-1 - m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]
))^m*Tan[(a + b*x)/2]*(-(Sec[(a + b*x)/2]^2*Sin[a + b*x]) + Cos[a + b*x]*Se
c[(a + b*x)/2]^2*Tan[(a + b*x)/2]))/(1 + m) + (2^(1 + m)*(Sec[(a + b*x)/2]^
2)^(2*m)*(Cos[(a + b*x)/2]*(-Sin[(a + b*x)/2] + Sin[(3*(a + b*x))/2]))^m*Ta
n[(a + b*x)/2]*((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 - m, 1 + 2*m, 1 + (3 +
 m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a +
 b*x)/2])/(3 + m) + ((1 + m)*(1 + 2*m)*AppellF1[1 + (1 + m)/2, -m, 2 + 2*m,
 1 + (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2
*Tan[(a + b*x)/2])/(3 + m) - 12*(-((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 - m
, 3 + 2*m, 1 + (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a +
 b*x)/2]^2*Tan[(a + b*x)/2])/(3 + m)) - ((1 + m)*(3 + 2*m)*AppellF1[1 + (1
+ m)/2, -m, 4 + 2*m, 1 + (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2
]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(3 + m)) + 6*(-((m*(1 + m)*AppellF1[
1 + (1 + m)/2, 1 - m, 2*(1 + m), 1 + (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a
 + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(3 + m)) - (2*(1 + m)^2*
AppellF1[1 + (1 + m)/2, -m, 1 + 2*(1 + m), 1 + (3 + m)/2, Tan[(a + b*x)/2]^
2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(3 + m)) + 8*(
-((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 - m, 2*(2 + m), 1 + (3 + m)/2, Tan[(
a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[(a + b*x)/2])/(3
 + m)) - (2*(1 + m)*(2 + m)*AppellF1[1 + (1 + m)/2, -m, 1 + 2*(2 + m), 1 +
(3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Sec[(a + b*x)/2]^2*Tan[
(a + b*x)/2])/(3 + m))))/((1 + m)*(Cos[a + b*x]*Sec[(a + b*x)/2]^2)^m)))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \left(\cos^3\left(bx+a\right)\right)\left(\sin^m\left(2bx+2a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x)
[Out] int(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="maxima")
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*cos(b*x + a)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^m(2a+2bx)\cos^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(2*b*x+2*a)**m,x)
```

```
[Out] Integral(sin(2*a + 2*b*x)**m*cos(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\cos\left(a+b\,x\right)}^3\,{\sin\left(2\,a+2\,b\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^m,x)
```

```
[Out] int(cos(a + b*x)^3*sin(2*a + 2*b*x)^m, x)
```

3.188 $\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=85

$$-\frac{\cos^2(a+bx)\cot(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{3+m}{2};\frac{5+m}{2};\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)}{b(3+m)}$$

```
[Out] -cos(b*x+a)^2*cot(b*x+a)*hypergeom([1/2-1/2*m, 3/2+1/2*m],[5/2+1/2*m],cos(b
*x+a)^2)*(sin(b*x+a)^2)^(1/2-1/2*m)*sin(2*b*x+2*a)^m/b/(3+m)
```

Rubi [A]

time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4394, 2656}

$$-\frac{\cos^2(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{m+3}{2};\frac{m+5}{2};\cos^2(a+bx)\right)}{b(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] -((Cos[a + b*x]^2*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (3 + m)/2, (5 +
 m)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(b*
(3 + m)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 4394

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p), Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^{2+m}(a+bx)\sin^m(a$$

$$= -\frac{\cos^2(a+bx)\cot(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{3+m}{2};\frac{5+m}{2};\cos^2(a+bx)\right)\sin^2(a+bx}{b(3+m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 8.23, size = 890, normalized size = 10.47

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[a + b*x]^2*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] (4*(3 + m)*(4*AppellF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2
]^2, -Tan[(a + b*x)/2]^2] - AppellF1[(1 + m)/2, -m, 1 + 2*m, (3 + m)/2, Tan
[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 4*AppellF1[(1 + m)/2, -m, 3 + 2*m,
(3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*Cos[(a + b*x)/2]^3*Cos
[a + b*x]^2*Sin[(a + b*x)/2]*Sin[2*(a + b*x)]^m)/(b*(1 + m)*(8*(3 + m)*Appe
llF1[(1 + m)/2, -m, 2*(1 + m), (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x
)/2]^2]*Cos[(a + b*x)/2]^2 - 2*(3 + m)*AppellF1[(1 + m)/2, -m, 1 + 2*m, (3
+ m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 - 8*(3
+ m)*AppellF1[(1 + m)/2, -m, 3 + 2*m, (3 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(
a + b*x)/2]^2]*Cos[(a + b*x)/2]^2 + 2*(4*m*AppellF1[(3 + m)/2, 1 - m, 2*(1
+ m), (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - m*AppellF1[(3 +
 m)/2, 1 - m, 1 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2]
- 4*m*AppellF1[(3 + m)/2, 1 - m, 3 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]^2, -T
an[(a + b*x)/2]^2] - AppellF1[(3 + m)/2, -m, 2*(1 + m), (5 + m)/2, Tan[(a +
 b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 2*m*AppellF1[(3 + m)/2, -m, 2*(1 + m), (
5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 12*AppellF1[(3 + m)/2,
 -m, 2*(2 + m), (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2] - 8*m*A
ppellF1[(3 + m)/2, -m, 2*(2 + m), (5 + m)/2, Tan[(a + b*x)/2]^2, -Tan[(a +
b*x)/2]^2] + 8*AppellF1[(3 + m)/2, -m, 3 + 2*m, (5 + m)/2, Tan[(a + b*x)/2]
^2, -Tan[(a + b*x)/2]^2] + 8*m*AppellF1[(3 + m)/2, -m, 3 + 2*m, (5 + m)/2,
Tan[(a + b*x)/2]^2, -Tan[(a + b*x)/2]^2])*(-1 + Cos[a + b*x])))
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(\cos^{2}\left(b x +a \right)\right) \left(\sin^{m}\left(2 b x +2 a \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

```
[Out] int(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="fricas")
```

```
[Out] integral(sin(2*b*x + 2*a)^m*cos(b*x + a)^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{m}(2a+2bx)\cos^{2}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(2*b*x+2*a)**m,x)
```

```
[Out] Integral(sin(2*a + 2*b*x)**m*cos(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(2\,a+2\,b\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^m,x)
```

```
[Out] int(cos(a + b*x)^2*sin(2*a + 2*b*x)^m, x)
```

3.189 $\int \cos(a+bx)\sin^m(2a+2bx)\,dx$

Optimal. Leaf size=83

$$-\frac{\cos(a+bx)\cot(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{2+m}{2};\frac{4+m}{2};\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)}{b(2+m)}$$

```
[Out] -cos(b*x+a)*cot(b*x+a)*hypergeom([1+1/2*m, 1/2-1/2*m],[2+1/2*m],cos(b*x+a)^
2)*(sin(b*x+a)^2)^(1/2-1/2*m)*sin(2*b*x+2*a)^m/b/(2+m)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4394, 2656}

$$-\frac{\cos(a+bx)\cot(a+bx)\sin^2(a+bx)^{\frac{1-m}{2}}\sin^m(2a+2bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{m+2}{2};\frac{m+4}{2};\cos^2(a+bx)\right)}{b(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sin[2*a + 2*b*x]^m,x]
```

```
[Out] -((Cos[a + b*x]*Cot[a + b*x]*Hypergeometric2F1[(1 - m)/2, (2 + m)/2, (4 + m
)/2, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 - m)/2)*Sin[2*a + 2*b*x]^m)/(b*(2
 + m)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 4394

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p), Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos(a+bx)\sin^m(2a+2bx)\,dx = \left(\cos^{-m}(a+bx)\sin^{-m}(a+bx)\sin^m(2a+2bx)\right)\int \cos^{1+m}(a+bx)\sin^m($$

$$= -\frac{\cos(a+bx)\cot(a+bx)\,{}_2F_1\left(\frac{1-m}{2},\frac{2+m}{2};\frac{4+m}{2};\cos^2(a+bx)\right)\sin^2(a+bx}{b(2+m)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.30, size = 149, normalized size = 1.80

$$\frac{2^{-1-m}e^{i(a+bx)}\left(-ie^{-2i(a+bx)}\left(-1+e^{4i(a+bx)}\right)\right)^{1+m}\left((-1+2m)\,{}_2F_1\left(1,\tfrac{1}{4}(3+2m);\tfrac{1}{4}(3-2m);e^{4i(a+bx)}\right)+e^{2i(a+bx)}(1+2m)\,{}_2F_1\left(1,\tfrac{1}{4}(5+2m);\tfrac{1}{4}(5-2m);e^{4i(a+bx)}\right)\right)}{b\left(-1+4m^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sin[2*a + 2*b*x]^m,x]
[Out] (2^(-1 - m)*E^(I*(a + b*x))*(((-I)*(-1 + E^((4*I)*(a + b*x))))/E^((2*I)*(a
+ b*x)))^(1 + m)*((-1 + 2*m)*Hypergeometric2F1[1, (3 + 2*m)/4, (3 - 2*m)/4,
 E^((4*I)*(a + b*x))] + E^((2*I)*(a + b*x))*(1 + 2*m)*Hypergeometric2F1[1,
(5 + 2*m)/4, (5 - 2*m)/4, E^((4*I)*(a + b*x))]))/(b*(-1 + 4*m^2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \cos\left(bx+a\right)\left(\sin^{m}\left(2bx+2a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(2*b*x+2*a)^m,x)
[Out] int(cos(b*x+a)*sin(2*b*x+2*a)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="maxima")
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="fricas")
[Out] integral(sin(2*b*x + 2*a)^m*cos(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin^{m}\left(2a+2bx\right)\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)**m,x)
```

```
[Out] Integral(sin(2*a + 2*b*x)**m*cos(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(2*b*x+2*a)^m,x, algorithm="giac")
```

```
[Out] integrate(sin(2*b*x + 2*a)^m*cos(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,{\sin\left(2\,a+2\,b\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(2*a + 2*b*x)^m,x)
```

```
[Out] int(cos(a + b*x)*sin(2*a + 2*b*x)^m, x)
```

3.190 $\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx$

Optimal. Leaf size=46

$$-\frac{4\cos^5(a+bx)}{5b}+\frac{8\cos^7(a+bx)}{7b}-\frac{4\cos^9(a+bx)}{9b}$$

```
[Out] -4/5*cos(b*x+a)^5/b+8/7*cos(b*x+a)^7/b-4/9*cos(b*x+a)^9/b
```

Rubi [A]
time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {4397, 2645, 276}

$$-\frac{4\cos^9(a+bx)}{9b}+\frac{8\cos^7(a+bx)}{7b}-\frac{4\cos^5(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (-4*Cos[a + b*x]^5)/(5*b) + (8*Cos[a + b*x]^7)/(7*b) - (4*Cos[a + b*x]^9)/(
9*b)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4397

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((f_.)*sin[(a_.) + (b_.)*(x_)])^(
n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_Symbol] :> Dist[2^p/(e^p*f^p), Int[(e
*Cos[a + b*x])^(m + p)*(f*Sin[a + b*x])^(n + p), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^2(a+bx)\sin^3(a+bx)\sin^2(2a+2bx)\,dx = 4\int \cos^4(a+bx)\sin^5(a+bx)\,dx$$
$$= -\frac{4\text{Subst}\left(\int x^4(1-x^2)^2\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{4\text{Subst}\left(\int (x^4-2x^6+x^8)\,dx, x, \cos(a+bx)\right)}{b}$$
$$= -\frac{4\cos^5(a+bx)}{5b}+\frac{8\cos^7(a+bx)}{7b}-\frac{4\cos^9(a+bx)}{9b}$$

Mathematica [A]
time = 0.18, size = 37, normalized size = 0.80

$$\frac{\cos^5(a+bx)(-249+220\cos(2(a+bx))-35\cos(4(a+bx)))}{630b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sin[a + b*x]^3*Sin[2*a + 2*b*x]^2,x]
```

```
[Out] (Cos[a + b*x]^5*(-249 + 220*Cos[2*(a + b*x)] - 35*Cos[4*(a + b*x)]))/(630*b
)
```

Maple [A]
time = 0.22, size = 69, normalized size = 1.50

method	result	size
default	$-\frac{3\cos(bx+a)}{32b}-\frac{\cos(3bx+3a)}{48b}+\frac{\cos(5bx+5a)}{80b}+\frac{\cos(7bx+7a)}{448b}-\frac{\cos(9bx+9a)}{576b}$	69
risch	$-\frac{3\cos(bx+a)}{32b}-\frac{\cos(3bx+3a)}{48b}+\frac{\cos(5bx+5a)}{80b}+\frac{\cos(7bx+7a)}{448b}-\frac{\cos(9bx+9a)}{576b}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/32*cos(b*x+a)/b-1/48*cos(3*b*x+3*a)/b+1/80/b*cos(5*b*x+5*a)+1/448/b*cos(
7*b*x+7*a)-1/576/b*cos(9*b*x+9*a)
```

Maxima [A]
time = 0.27, size = 58, normalized size = 1.26

$$-\frac{35\cos(9bx+9a)-45\cos(7bx+7a)-252\cos(5bx+5a)+420\cos(3bx+3a)+1890\cos(bx+a)}{20160b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="maxima")
```

```
[Out] -1/20160*(35*cos(9*b*x + 9*a) - 45*cos(7*b*x + 7*a) - 252*cos(5*b*x + 5*a)
+ 420*cos(3*b*x + 3*a) + 1890*cos(b*x + a))/b
```

Fricas [A]
time = 2.70, size = 36, normalized size = 0.78

$$-\frac{4\left(35\cos\left(bx+a\right)^{9}-90\cos\left(bx+a\right)^{7}+63\cos\left(bx+a\right)^{5}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="fricas")
```

```
[Out] -4/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(39) = 78$.
time = 14.95, size = 318, normalized size = 6.91

$$\begin{cases} -\frac{8\sin^{5}(a+bx)\sin(2a+2bx)\cos(2a+2bx)}{315b}+\frac{16\sin^{4}(a+bx)\sin^{2}(2a+2bx)\cos(a+bx)}{315b}-\frac{16\sin^{4}(a+bx)\cos(a+bx)\cos^{2}(2a+2bx)}{315b}+\frac{44\sin^{3}(a+bx)\sin(2a+2bx)\cos^{2}(a+bx)\cos(2a+2bx)}{315b}-\frac{113\sin^{2}(a+bx)\sin^{2}(2a+2bx)\cos^{3}(a+bx)}{315b}+\frac{8\sin^{2}(a+bx)\cos^{3}(a+bx)\cos^{2}(2a+2bx)}{315b}-\frac{88\sin(a+bx)\sin(2a+2bx)\cos^{4}(a+bx)\cos(2a+2bx)}{315b}-\frac{2\sin^{2}(2a+2bx)\cos^{5}(a+bx)}{63b}-\frac{32\cos^{5}(a+bx)\cos^{2}(2a+2bx)}{315b} & \text{for}\ b\neq 0 \\ x\sin^{3}(a)\sin^{2}(2a)\cos^{2}(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3*sin(2*b*x+2*a)**2,x)
```

```
[Out] Piecewise((-8*sin(a + b*x)**5*sin(2*a + 2*b*x)*cos(2*a + 2*b*x)/(315*b) + 1
6*sin(a + b*x)**4*sin(2*a + 2*b*x)**2*cos(a + b*x)/(315*b) - 16*sin(a + b*x
)**4*cos(a + b*x)*cos(2*a + 2*b*x)**2/(315*b) + 44*sin(a + b*x)**3*sin(2*a
+ 2*b*x)*cos(a + b*x)**2*cos(2*a + 2*b*x)/(315*b) - 113*sin(a + b*x)**2*sin
(2*a + 2*b*x)**2*cos(a + b*x)**3/(315*b) + 8*sin(a + b*x)**2*cos(a + b*x)**
3*cos(2*a + 2*b*x)**2/(315*b) - 88*sin(a + b*x)*sin(2*a + 2*b*x)*cos(a + b*
x)**4*cos(2*a + 2*b*x)/(315*b) - 2*sin(2*a + 2*b*x)**2*cos(a + b*x)**5/(63*
b) - 32*cos(a + b*x)**5*cos(2*a + 2*b*x)**2/(315*b), Ne(b, 0)), (x*sin(a)**
3*sin(2*a)**2*cos(a)**2, True))
```

Giac [A]
time = 0.46, size = 36, normalized size = 0.78

$$-\frac{4\left(35\cos\left(bx+a\right)^{9}-90\cos\left(bx+a\right)^{7}+63\cos\left(bx+a\right)^{5}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3*sin(2*b*x+2*a)^2,x, algorithm="giac")
```

```
[Out] -4/315*(35*cos(b*x + a)^9 - 90*cos(b*x + a)^7 + 63*cos(b*x + a)^5)/b
```

Mupad [B]
time = 0.20, size = 36, normalized size = 0.78

$$-\frac{4\left(35\cos\left(a+b\,x\right)^9-90\cos\left(a+b\,x\right)^7+63\cos\left(a+b\,x\right)^5\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*sin(2*a + 2*b*x)^2,x)
[Out] -(4*(63*cos(a + b*x)^5 - 90*cos(a + b*x)^7 + 35*cos(a + b*x)^9))/(315*b)
```

3.191 $\int \sin(a+bx)\sin^n(c+dx)\,dx$

Optimal. Leaf size=293

$$-\frac{2^{-1-n}e^{i(a-cn)+i(b-dn)x+in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n {}_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(2+\frac{b}{d}-n\right);e^{2i(c+dx}\right.}{b-dn}$$

```
[Out] -2^(-1-n)*exp(I*(-c*n+a)+I*(-d*n+b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(
I*(d*x+c)))^n*hypergeom([-n, 1/2*(-d*n+b)/d],[1+1/2*b/d-1/2*n],exp(2*I*(d*x
+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(-d*n+b)-2^(-1-n)*exp(-I*(c*n+a)-I*(d*n+b)
*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-n, 1/2*(-
d*n-b)/d],[1+1/2*(-d*n-b)/d],exp(2*I*(d*x+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(
d*n+b)
```

Rubi [A]
time = 0.59, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4649, 2323, 2285, 2284, 2283}

$$-\frac{2^{-n-1}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}{}_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(\frac{b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(i(a-cn)+ix(b-dn)+in(c+dx))}{b-dn}-\frac{2^{-n-1}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}{}_2F_1\left(-n,-\frac{b+dn}{2d};1-\frac{b+dn}{2d};e^{2i(c+dx)}\right)\exp(-i(a+cn)-ix(b+dn)+in(c+dx))}{b+dn}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + d*x]^n,x]
```

```
[Out] -((2^(-1 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(I/E^(I*(c +
d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d), (2 + b/
d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*(b - d*n))
) - (2^(-1 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*x))*(I/E^(I*
(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -1/2*(b + d*n)/d, 1
 - (b + d*n)/(2*d), E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*
(b + d*n))
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2284

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Dist[(a + b*F^(e*(c + d*x)))^p/(1 + (b/a)*F^
(e*(c + d*x)))^p, Int[G^(h*(f + g*x))*(1 + (b/a)*F^(e*(c + d*x)))^p, x], x]
 /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] &&  !(ILtQ[p, 0] || GtQ[a,
```

```
0])
```

Rule 2285

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2323

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 4649

```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \sin(a+bx)\sin^n(c+dx)\,dx = 2^{-1-n}\int \left(ie^{-ia-ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n - ie^{ia+ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)\right.$$

$$= \left(i2^{-1-n}\right)\int e^{-ia-ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\,dx - \left(i2^{-1-n}\right)\int e^{ia+ibx}\left(ie^{-i(c-\right.$$

$$= \left(i2^{-1-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{-ia-ibx-in(c+}$$

$$= -\left(\left(i2^{-1-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(a-cn)+i(}\right.$$

$$= -\left(\left(i2^{-1-n}e^{in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(a-cn)+i(}\right.$$

$$= -\frac{2^{-1-n}\exp(i(a-cn)+i(b-dn)x+in(c+dx))\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c-\right.}{b-dn}$$

Mathematica [A]
time = 0.94, size = 199, normalized size = 0.68

$$-\frac{e^{-ibx}\left(2-2e^{2i(c+dx)}\right)^{-n}\left(-ie^{-i(c+dx)}\left(-1+e^{2i(c+dx)}\right)\right)^n\left((b-dn)\,{}_2F_1\left(-n,-\frac{b+dn}{2d};-\frac{b+d(-2+n)}{2d};e^{2i(c+dx)}\right)(\cos(a)-i\sin(a))+e^{2ibx}(b+dn)\,{}_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(2+\frac{b}{d}-n\right);e^{2i(c+dx)}\right)(\cos(a)+i\sin(a))\right)}{2(b-dn)(b+dn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[c + d*x]^n,x]
```

```
[Out] -1/2*((((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x)))^n*((b - d*n)*Hype
rgeometric2F1[-n, -1/2*(b + d*n)/d, -1/2*(b + d*(-2 + n))/d, E^((2*I)*(c +
d*x))]*(Cos[a] - I*Sin[a]) + E^((2*I)*b*x)*(b + d*n)*Hypergeometric2F1[-n,
(b - d*n)/(2*d), (2 + b/d - n)/2, E^((2*I)*(c + d*x))]*(Cos[a] + I*Sin[a]))
)/(E^(I*b*x)*(2 - 2*E^((2*I)*(c + d*x)))^n*(b - d*n)*(b + d*n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \sin\left(bx+a\right)\left(\sin^{n}\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(d*x+c)^n,x)
```

```
[Out] int(sin(b*x+a)*sin(d*x+c)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral(sin(d*x + c)^n*sin(b*x + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,x\right)\,{\sin\left(c+d\,x\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(c + d*x)^n,x)
```

```
[Out] int(sin(a + b*x)*sin(c + d*x)^n, x)
```

3.192 $\int \sin(a+bx)\sin^3(c+dx)\,dx$

Optimal. Leaf size=91

$$-\frac{\sin(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)}-\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(a+3c+(b+3d)x)}{8(b+3d)}$$

```
[Out] -1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sin(a-c+(b-d)*x)/(b-d)-3/8*sin(a+c+(b
+d)*x)/(b+d)+1/8*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4665, 2717}

$$-\frac{\sin(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sin(a+x(b-d)-c)}{8(b-d)}-\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + d*x]^3,x]
```

```
[Out] -1/8*Sin[a - 3*c + (b - 3*d)*x]/(b - 3*d) + (3*Sin[a - c + (b - d)*x])/(8*(
b - d)) - (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[a + 3*c + (b + 3*d)*
x]/(8*(b + 3*d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4665

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^3(c+dx)\,dx &= \int \left(-\frac{1}{8}\cos(a-3c+(b-3d)x)+\frac{3}{8}\cos(a-c+(b-d)x)-\frac{3}{8}\cos(a+c+\right.\\ &= -\left(\frac{1}{8}\int \cos(a-3c+(b-3d)x)\,dx\right)+\frac{1}{8}\int \cos(a+3c+(b+3d)x)\,dx+\frac{3}{8}\\ &= -\frac{\sin(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)}-\frac{3\sin(a+c+(b+d)}{8(b+d)}\end{aligned}$$

Mathematica [**A**]
time = 0.58, size = 86, normalized size = 0.95

$$\frac{1}{8}\left(-\frac{\sin(a-3c+bx-3dx)}{b-3d}+\frac{3\sin(a-c+bx-dx)}{b-d}+\frac{\sin(a+3c+bx+3dx)}{b+3d}-\frac{3\sin(a+c+(b+d)x)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[c + d*x]^3,x]
```

```
[Out] (-(Sin[a - 3*c + b*x - 3*d*x]/(b - 3*d)) + (3*Sin[a - c + b*x - d*x])/(b -
d) + Sin[a + 3*c + b*x + 3*d*x]/(b + 3*d) - (3*Sin[a + c + (b + d)*x])/(b +
 d))/8
```

Maple [**A**]
time = 0.21, size = 84, normalized size = 0.92

method	result	size
default	$-\frac{\sin(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)}-\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(a+3c+(b+3d)x)}{8b+24d}$	84
risch	$-\frac{\sin(bx-3dx+a-3c)}{8(b-3d)}+\frac{3\sin(bx-dx+a-c)}{8(b-d)}-\frac{3\sin(bx+dx+a+c)}{8(b+d)}+\frac{\sin(bx+3dx+a+3c)}{8b+24d}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sin(a-c+(b-d)*x)/(b-d)-3/8*sin(a+c+(b
+d)*x)/(b+d)+1/8*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(83) = 166$.
time = 0.32, size = 916, normalized size = 10.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))*
cos((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3
*c) + 3*d^3*sin(3*c))*cos((b + 3*d)*x + a) - 3*(b^3*sin(3*c) - b^2*d*sin(3*
c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x + a + 4*c) + 3*(b^3*s
in(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x
 + a - 2*c) - 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9*d^3*s
in(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b
*d^2*sin(3*c) - 9*d^3*sin(3*c))*cos(-(b - d)*x - a - 2*c) + (b^3*sin(3*c) +
 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))*cos(-(b - 3*d)*x - a +
```

```
6*c) - (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))
*cos(-(b - 3*d)*x - a) - (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos(3*c)
+ 3*d^3*cos(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*cos(3*c) - 3*b^2*d*cos(
3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))*sin((b + 3*d)*x + a) + 3*(b^3*cos(3
*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*sin((b + d)*x + a
 + 4*c) + 3*(b^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3
*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*
cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 3*(b^3*cos(3*c) + b^
2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a - 2*c)
 - (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3*cos(3*c))*sin(
-(b - 3*d)*x - a + 6*c) - (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c)
 - 3*d^3*cos(3*c))*sin(-(b - 3*d)*x - a))/(b^4*cos(3*c)^2 + b^4*sin(3*c)^2
+ 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 - 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^
2)
```

Fricas [A]
time = 2.85, size = 122, normalized size = 1.34

$$-\frac{3\left(\left(b^{2}d-d^{3}\right)\cos\left(dx+c\right)^{3}-\left(b^{2}d-3\,d^{3}\right)\cos\left(dx+c\right)\right)\sin\left(bx+a\right)-\left(\left(b^{3}-bd^{2}\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^{2}-\left(b^{3}-7\,bd^{2}\right)\cos\left(bx+a\right)\right)\sin\left(dx+c\right)}{b^{4}-10\,b^{2}d^{2}+9\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -(3*((b^2*d - d^3)*cos(d*x + c)^3 - (b^2*d - 3*d^3)*cos(d*x + c))*sin(b*x +
 a) - ((b^3 - b*d^2)*cos(b*x + a)*cos(d*x + c)^2 - (b^3 - 7*b*d^2)*cos(b*x
+ a))*sin(d*x + c))/(b^4 - 10*b^2*d^2 + 9*d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 918 vs. 2(76) = 152.
time = 2.36, size = 918, normalized size = 10.09

$$\begin{cases} x\sin\left(a\right)\sin^{3}\left(c\right) & \text{for}\: b=0\wedge d=0 \\ \frac{x\sin\left(a-3dx\right)\sin^{3}\left(c+dx\right)}{8}-\frac{3x\sin\left(a-3dx\right)\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}-\frac{3x\sin^{2}\left(c+dx\right)\cos\left(a-3dx\right)\cos\left(c+dx\right)}{8}+\frac{x\cos\left(a-3dx\right)\cos^{3}\left(c+dx\right)}{8}+\frac{\sin\left(a-3dx\right)\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{4d}+\frac{\sin\left(a-3dx\right)\cos^{3}\left(c+dx\right)}{24d}+\frac{3\sin^{3}\left(c+dx\right)\cos\left(a-3dx\right)}{8d} & \text{for}\: b=-3d \\ \frac{3x\sin\left(a-dx\right)\sin^{3}\left(c+dx\right)}{8}+\frac{3x\sin\left(a-dx\right)\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}-\frac{3x\sin^{2}\left(c+dx\right)\cos\left(a-dx\right)\cos\left(c+dx\right)}{8}-\frac{3x\cos\left(a-dx\right)\cos^{3}\left(c+dx\right)}{8}-\frac{3\sin\left(a-dx\right)\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{4d}-\frac{3\sin\left(a-dx\right)\cos^{3}\left(c+dx\right)}{8d}-\frac{\sin^{3}\left(c+dx\right)\cos\left(a-dx\right)}{8d} & \text{for}\: b=-d \\ \frac{3x\sin\left(a+dx\right)\sin^{3}\left(c+dx\right)}{8}+\frac{3x\sin\left(a+dx\right)\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}+\frac{3x\sin^{2}\left(c+dx\right)\cos\left(a+dx\right)\cos\left(c+dx\right)}{8}+\frac{3x\cos\left(a+dx\right)\cos^{3}\left(c+dx\right)}{8}-\frac{3\sin\left(a+dx\right)\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{4d}-\frac{3\sin\left(a+dx\right)\cos^{3}\left(c+dx\right)}{8d}+\frac{\sin^{3}\left(c+dx\right)\cos\left(a+dx\right)}{8d} & \text{for}\: b=d \\ \frac{x\sin\left(a+3dx\right)\sin^{3}\left(c+dx\right)}{8}-\frac{3x\sin\left(a+3dx\right)\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}+\frac{3x\sin^{2}\left(c+dx\right)\cos\left(a+3dx\right)\cos\left(c+dx\right)}{8}-\frac{x\cos\left(a+3dx\right)\cos^{3}\left(c+dx\right)}{8}+\frac{\sin\left(a+3dx\right)\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{4d}+\frac{\sin\left(a+3dx\right)\cos^{3}\left(c+dx\right)}{24d}-\frac{3\sin^{3}\left(c+dx\right)\cos\left(a+3dx\right)}{8d} & \text{for}\: b=3d \\ -\frac{b^{3}\sin^{3}\left(c+dx\right)\cos\left(a+bx\right)}{b^{4}-10b^{2}d^{2}+9d^{4}}+\frac{3b^{2}d\sin\left(a+bx\right)\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{b^{4}-10b^{2}d^{2}+9d^{4}}+\frac{7bd^{2}\sin^{3}\left(c+dx\right)\cos\left(a+bx\right)}{b^{4}-10b^{2}d^{2}+9d^{4}}+\frac{6bd^{2}\sin\left(c+dx\right)\cos\left(a+bx\right)\cos^{2}\left(c+dx\right)}{b^{4}-10b^{2}d^{2}+9d^{4}}-\frac{9d^{3}\sin\left(a+bx\right)\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{b^{4}-10b^{2}d^{2}+9d^{4}}-\frac{6d^{3}\sin\left(a+bx\right)\cos^{3}\left(c+dx\right)}{b^{4}-10b^{2}d^{2}+9d^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)**3,x)
```

```
[Out] Piecewise((x*sin(a)*sin(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sin(a - 3*d*x)*sin(
c + d*x)**3/8 - 3*x*sin(a - 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*sin
(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/8 + x*cos(a - 3*d*x)*cos(c + d*x)*
*3/8 + sin(a - 3*d*x)*sin(c + d*x)**2*cos(c + d*x)/(4*d) + sin(a - 3*d*x)*c
os(c + d*x)**3/(24*d) + 3*sin(c + d*x)**3*cos(a - 3*d*x)/(8*d), Eq(b, -3*d)
), (3*x*sin(a - d*x)*sin(c + d*x)**3/8 + 3*x*sin(a - d*x)*sin(c + d*x)*cos(
```

```
c + d*x)**2/8 - 3*x*sin(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/8 - 3*x*cos(a
 - d*x)*cos(c + d*x)**3/8 - 3*sin(a - d*x)*sin(c + d*x)**2*cos(c + d*x)/(4*
d) - 3*sin(a - d*x)*cos(c + d*x)**3/(8*d) - sin(c + d*x)**3*cos(a - d*x)/(8
*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**3/8 + 3*x*sin(a + d*x)*sin
(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a + d*x)*cos(c + d*x)
/8 + 3*x*cos(a + d*x)*cos(c + d*x)**3/8 - 3*sin(a + d*x)*sin(c + d*x)**2*co
s(c + d*x)/(4*d) - 3*sin(a + d*x)*cos(c + d*x)**3/(8*d) + sin(c + d*x)**3*c
os(a + d*x)/(8*d), Eq(b, d)), (x*sin(a + 3*d*x)*sin(c + d*x)**3/8 - 3*x*sin
(a + 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a + 3*
d*x)*cos(c + d*x)/8 - x*cos(a + 3*d*x)*cos(c + d*x)**3/8 + sin(a + 3*d*x)*s
in(c + d*x)**2*cos(c + d*x)/(4*d) + sin(a + 3*d*x)*cos(c + d*x)**3/(24*d) -
 3*sin(c + d*x)**3*cos(a + 3*d*x)/(8*d), Eq(b, 3*d)), (-b**3*sin(c + d*x)**
3*cos(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 3*b**2*d*sin(a + b*x)*sin(c
 + d*x)**2*cos(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 7*b*d**2*sin(c + d
*x)**3*cos(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*sin(c + d*x)*
cos(a + b*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) - 9*d**3*sin(a
+ b*x)*sin(c + d*x)**2*cos(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 6*d**3
*sin(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4), True))
```

Giac [A]
time = 0.39, size = 84, normalized size = 0.92

$$\frac{\sin\left(bx+3\,dx+a+3\,c\right)}{8\,(b+3\,d)}-\frac{3\,\sin\left(bx+dx+a+c\right)}{8\,(b+d)}+\frac{3\,\sin\left(bx-dx+a-c\right)}{8\,(b-d)}-\frac{\sin\left(bx-3\,dx+a-3\,c\right)}{8\,(b-3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/8*sin(b*x + d*x + a + c)/(b +
d) + 3/8*sin(b*x - d*x + a - c)/(b - d) - 1/8*sin(b*x - 3*d*x + a - 3*c)/(b
 - 3*d)
```

Mupad [B]
time = 1.57, size = 311, normalized size = 3.42

$$-\mathrm{e}^{a\,1\mathrm{i}-c\,3\mathrm{i}+b\,x\,1\mathrm{i}-d\,x\,3\mathrm{i}}\,\left(\frac{b+3\,d}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(b-3\,d\right)}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}\right)+\mathrm{e}^{a\,1\mathrm{i}+c\,3\mathrm{i}+b\,x\,1\mathrm{i}+d\,x\,3\mathrm{i}}\,\left(\frac{b-3\,d}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(b+3\,d\right)}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}\right)+\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}+b\,x\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\frac{3\,b+3\,d}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(3\,b-3\,d\right)}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}\right)-\mathrm{e}^{a\,1\mathrm{i}+c\,1\mathrm{i}+b\,x\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,\left(\frac{3\,b-3\,d}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(3\,b+3\,d\right)}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(c + d*x)^3,x)
```

```
[Out] exp(a*1i + c*3i + b*x*1i + d*x*3i)*((b - 3*d)/(b^2*16i - d^2*144i) + (exp(-
 a*2i - b*x*2i)*(b + 3*d))/(b^2*16i - d^2*144i)) - exp(a*1i - c*3i + b*x*1i
 - d*x*3i)*((b + 3*d)/(b^2*16i - d^2*144i) + (exp(- a*2i - b*x*2i)*(b - 3*d
))/(b^2*16i - d^2*144i)) + exp(a*1i - c*1i + b*x*1i - d*x*1i)*((3*b + 3*d)/
(b^2*16i - d^2*16i) + (exp(- a*2i - b*x*2i)*(3*b - 3*d))/(b^2*16i - d^2*16i
)) - exp(a*1i + c*1i + b*x*1i + d*x*1i)*((3*b - 3*d)/(b^2*16i - d^2*16i) +
(exp(- a*2i - b*x*2i)*(3*b + 3*d))/(b^2*16i - d^2*16i))
```

3.193 $\int \sin(a+bx)\sin^2(c+dx)\,dx$

Optimal. Leaf size=62

$$-\frac{\cos(a+bx)}{2b}+\frac{\cos(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}$$

```
[Out] -1/2*cos(b*x+a)/b+1/4*cos(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cos(a+2*c+(b+2*d)*x)
/(b+2*d)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4665, 2718}

$$\frac{\cos(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\cos(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cos(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + d*x]^2,x]
```

```
[Out] -1/2*Cos[a + b*x]/b + Cos[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Cos[a + 2*
c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4665

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\sin^2(c+dx)\,dx &= \int \left(\frac{1}{2}\sin(a+bx)-\frac{1}{4}\sin(a-2c+(b-2d)x)-\frac{1}{4}\sin(a+2c+(b+2d)x)\right)\\ &= -\left(\frac{1}{4}\int \sin(a-2c+(b-2d)x)\,dx\right)-\frac{1}{4}\int \sin(a+2c+(b+2d)x)\,dx+\frac{1}{2}\\ &= -\frac{\cos(a+bx)}{2b}+\frac{\cos(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}\end{aligned}$$

Mathematica [**A**]
time = 0.75, size = 69, normalized size = 1.11

$$\frac{1}{4}\left(-\frac{2\cos(a)\cos(bx)}{b}+\frac{\cos(a-2c+bx-2dx)}{b-2d}+\frac{\cos(a+2c+bx+2dx)}{b+2d}+\frac{2\sin(a)\sin(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[c + d*x]^2,x]
```

```
[Out] ((-2*Cos[a]*Cos[b*x])/b + Cos[a - 2*c + b*x - 2*d*x]/(b - 2*d) + Cos[a + 2*
c + b*x + 2*d*x]/(b + 2*d) + (2*Sin[a]*Sin[b*x])/b)/4
```

Maple [**A**]
time = 0.22, size = 57, normalized size = 0.92

method	result
default	$-\frac{\cos(bx+a)}{2b}+\frac{\cos(a-2c+(b-2d)x)}{4b-8d}+\frac{\cos(a+2c+(b+2d)x)}{4b+8d}$
risch	$-\frac{\cos(bx+a)}{2b}+\frac{\cos(bx-2dx+a-2c)}{4b-8d}+\frac{\cos(bx+2dx+a+2c)}{4b+8d}$
norman	$\frac{\frac{4d^2}{b\left(b^2-4d^2\right)}+\frac{4d^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b\left(b^2-4d^2\right)}+\frac{8d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-4d^2}-\frac{8d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2-4d^2}+\frac{4b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2-4d^2}+\frac{2(-2\cdots}{}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cos(b*x+a)/b+1/4*cos(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*cos(a+2*c+(b+2*d)*x)
/(b+2*d)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(56) = 112$.
time = 0.30, size = 414, normalized size = 6.68

$$\frac{\left(b^2\cos(2c)-2bd\cos(2c)\right)\cos\left((b+2d)x+a+4c\right)+\left(b^2\cos(2c)-2bd\cos(2c)\right)\cos\left((b+2d)x+a\right)+\left(b^2\cos(2c)+2bd\cos(2c)\right)\cos\left(-(b-2d)x-a+4c\right)+\left(b^2\cos(2c)+2bd\cos(2c)\right)\cos\left(-(b-2d)x-a\right)-2\left(b^2\cos(2c)-4d^2\cos(2c)\right)\cos(bx+a+2c)-2\left(b^2\cos(2c)-4d^2\cos(2c)\right)\cos(bx+a-2c)+\left(b^2\sin(2c)-2bd\sin(2c)\right)\sin\left((b+2d)x+a+4c\right)-\left(b^2\sin(2c)-2bd\sin(2c)\right)\sin\left((b+2d)x+a\right)+\left(b^2\sin(2c)+2bd\sin(2c)\right)\sin\left(-(b-2d)x-a+4c\right)-\left(b^2\sin(2c)+2bd\sin(2c)\right)\sin\left(-(b-2d)x-a\right)-2\left(b^2\sin(2c)-4d^2\sin(2c)\right)\sin(bx+a+2c)+2\left(b^2\sin(2c)-4d^2\sin(2c)\right)\sin(bx+a-2c)}{8\left(b^3\cos(2c)^2+b^3\sin(2c)^2-4\left(b\cos(2c)^2+b\sin(2c)^2\right)d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*((b^2*cos(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a + 4*c) + (b^2*cos(
2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(2*c
))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*cos(-(b -
2*d)*x - a) - 2*(b^2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a + 2*c) - 2*(b^2
*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a - 2*c) + (b^2*sin(2*c) - 2*b*d*sin(
2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*sin(2*c) - 2*b*d*sin(2*c))*sin((b +
 2*d)*x + a) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a + 4*c)
```

```
- (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a) - 2*(b^2*sin(2*c) -
 4*d^2*sin(2*c))*sin(b*x + a + 2*c) + 2*(b^2*sin(2*c) - 4*d^2*sin(2*c))*sin
(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c)^2 + b*sin
(2*c)^2)*d^2)
```

Fricas [A]
time = 2.16, size = 71, normalized size = 1.15

$$\frac{b^{2} \cos\left(bx+a\right) \cos\left(dx+c\right)^{2} + 2\, bd \cos\left(dx+c\right) \sin\left(bx+a\right) \sin\left(dx+c\right) - \left(b^{2} - 2\, d^{2}\right) \cos\left(bx+a\right)}{b^{3} - 4\, bd^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*cos(b*x + a)*cos(d*x + c)^2 + 2*b*d*cos(d*x + c)*sin(b*x + a)*sin(d*x
+ c) - (b^2 - 2*d^2)*cos(b*x + a))/(b^3 - 4*b*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(49) = 98$.
time = 0.80, size = 405, normalized size = 6.53

$$\begin{cases} x \sin\left(a\right) \sin^{2}\left(c\right) & \text{for}\: b = 0 \wedge d = 0 \\ \left(\frac{x \sin^{2}\left(c+dx\right)}{2} + \frac{x \cos^{2}\left(c+dx\right)}{2} - \frac{\sin\left(c+dx\right) \cos\left(c+dx\right)}{2d}\right) \sin\left(a\right) & \text{for}\: b = 0 \\ \frac{x \sin\left(a-2dx\right) \sin^{2}\left(c+dx\right)}{4} - \frac{x \sin\left(a-2dx\right) \cos^{2}\left(c+dx\right)}{4} - \frac{x \sin\left(c+dx\right) \cos\left(a-2dx\right) \cos\left(c+dx\right)}{2} + \frac{\sin\left(a-2dx\right) \sin\left(c+dx\right) \cos\left(c+dx\right)}{4d} + \frac{\sin^{2}\left(c+dx\right) \cos\left(a-2dx\right)}{2d} & \text{for}\: b = -2d \\ \frac{x \sin\left(a+2dx\right) \sin^{2}\left(c+dx\right)}{4} - \frac{x \sin\left(a+2dx\right) \cos^{2}\left(c+dx\right)}{4} + \frac{x \sin\left(c+dx\right) \cos\left(a+2dx\right) \cos\left(c+dx\right)}{2} + \frac{\sin\left(a+2dx\right) \sin\left(c+dx\right) \cos\left(c+dx\right)}{4d} - \frac{\sin^{2}\left(c+dx\right) \cos\left(a+2dx\right)}{2d} & \text{for}\: b = 2d \\ - \frac{b^{2} \sin^{2}\left(c+dx\right) \cos\left(a+bx\right)}{b^{3}-4bd^{2}} + \frac{2bd \sin\left(a+bx\right) \sin\left(c+dx\right) \cos\left(c+dx\right)}{b^{3}-4bd^{2}} + \frac{2d^{2} \sin^{2}\left(c+dx\right) \cos\left(a+bx\right)}{b^{3}-4bd^{2}} + \frac{2d^{2} \cos\left(a+bx\right) \cos^{2}\left(c+dx\right)}{b^{3}-4bd^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)**2,x)
```

```
[Out] Piecewise((x*sin(a)*sin(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
+ x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a), Eq(b, 0)),
 (x*sin(a - 2*d*x)*sin(c + d*x)**2/4 - x*sin(a - 2*d*x)*cos(c + d*x)**2/4 -
 x*sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/2 + sin(a - 2*d*x)*sin(c + d*x)
*cos(c + d*x)/(4*d) + sin(c + d*x)**2*cos(a - 2*d*x)/(2*d), Eq(b, -2*d)), (
x*sin(a + 2*d*x)*sin(c + d*x)**2/4 - x*sin(a + 2*d*x)*cos(c + d*x)**2/4 + x
*sin(c + d*x)*cos(a + 2*d*x)*cos(c + d*x)/2 + sin(a + 2*d*x)*sin(c + d*x)*c
os(c + d*x)/(4*d) - sin(c + d*x)**2*cos(a + 2*d*x)/(2*d), Eq(b, 2*d)), (-b*
*2*sin(c + d*x)**2*cos(a + b*x)/(b**3 - 4*b*d**2) + 2*b*d*sin(a + b*x)*sin(
c + d*x)*cos(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*sin(c + d*x)**2*cos(a + b*
x)/(b**3 - 4*b*d**2) + 2*d**2*cos(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2
), True))
```

Giac [A]
time = 0.40, size = 56, normalized size = 0.90

$$\frac{\cos\left(bx + 2\, dx + a + 2\, c\right)}{4\, \left(b + 2\, d\right)} + \frac{\cos\left(bx - 2\, dx + a - 2\, c\right)}{4\, \left(b - 2\, d\right)} - \frac{\cos\left(bx + a\right)}{2\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*cos(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*cos(b*x - 2*d*x + a - 2*c)/(
b - 2*d) - 1/2*cos(b*x + a)/b
```

Mupad [B]
time = 0.74, size = 98, normalized size = 1.58

$$-\frac{d\,\left(2\,b\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)-2\,b\,\cos\left(a+2\,c+b\,x+2\,d\,x\right)\right)+b^2\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)+b^2\,\cos\left(a+2\,c+b\,x+2\,d\,x\right)}{16\,b\,d^2-4\,b^3}-\frac{\cos\left(a+b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(c + d*x)^2,x)
```

```
[Out] - (d*(2*b*cos(a - 2*c + b*x - 2*d*x) - 2*b*cos(a + 2*c + b*x + 2*d*x)) + b^
2*cos(a - 2*c + b*x - 2*d*x) + b^2*cos(a + 2*c + b*x + 2*d*x))/(16*b*d^2 -
4*b^3) - cos(a + b*x)/(2*b)
```

3.194 $\int \sin(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=43

$$\frac{\sin(a-c+(b-d)x)}{2(b-d)}-\frac{\sin(a+c+(b+d)x)}{2(b+d)}$$

```
[Out] 1/2*sin(a-c+(b-d)*x)/(b-d)-1/2*sin(a+c+(b+d)*x)/(b+d)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4665, 2717}

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}-\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Sin[c + d*x],x]
```

```
[Out] Sin[a - c + (b - d)*x]/(2*(b - d)) - Sin[a + c + (b + d)*x]/(2*(b + d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4665

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin(a+bx)\sin(c+dx)\,dx &= \int \left(\frac{1}{2}\cos(a-c+(b-d)x)-\frac{1}{2}\cos(a+c+(b+d)x)\right)dx\\
&= \frac{1}{2}\int \cos(a-c+(b-d)x)\,dx-\frac{1}{2}\int \cos(a+c+(b+d)x)\,dx\\
&= \frac{\sin(a-c+(b-d)x)}{2(b-d)}-\frac{\sin(a+c+(b+d)x)}{2(b+d)}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 43, normalized size = 1.00

$$\frac{\sin(a-c+(b-d)x)}{2(b-d)}-\frac{\sin(a+c+(b+d)x)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Sin[c + d*x],x]
```

```
[Out] Sin[a - c + (b - d)*x]/(2*(b - d)) - Sin[a + c + (b + d)*x]/(2*(b + d))
```

Maple [A]
time = 0.12, size = 40, normalized size = 0.93

method	result	size
default	$\frac{\sin(a-c+(b-d)x)}{2b-2d}-\frac{\sin(a+c+(b+d)x)}{2(b+d)}$	40
risch	$\frac{\sin(bx-dx+a-c)}{2b-2d}-\frac{\sin(bx+dx+a+c)}{2(b+d)}$	41
norman	$\frac{-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-d^2}+\frac{2d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^2-d^2}+\frac{2b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-d^2}-\frac{2d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2-d^2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	147

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(a-c+(b-d)*x)/(b-d)-1/2*sin(a+c+(b+d)*x)/(b+d)
```

Maxima [A]
time = 0.27, size = 40, normalized size = 0.93

$$-\frac{\sin\left(bx+dx+a+c\right)}{2\left(b+d\right)}-\frac{\sin\left(-bx+dx-a+c\right)}{2\left(b-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*sin(b*x + d*x + a + c)/(b + d) - 1/2*sin(-b*x + d*x - a + c)/(b - d)
```

Fricas [A]
time = 3.00, size = 42, normalized size = 0.98

$$\frac{d\cos\left(dx+c\right)\sin\left(bx+a\right)-b\cos\left(bx+a\right)\sin\left(dx+c\right)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] (d*cos(d*x + c)*sin(b*x + a) - b*cos(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(32) = 64$.

time = 0.31, size = 153, normalized size = 3.56

$$\begin{cases} x\sin(a)\sin(c) & \text{for } b=0 \wedge d=0 \\ \frac{x\sin(a-dx)\sin(c+dx)}{2} - \frac{x\cos(a-dx)\cos(c+dx)}{2} + \frac{\sin(c+dx)\cos(a-dx)}{2d} & \text{for } b=-d \\ \frac{x\sin(a+dx)\sin(c+dx)}{2} + \frac{x\cos(a+dx)\cos(c+dx)}{2} - \frac{\sin(c+dx)\cos(a+dx)}{2d} & \text{for } b=d \\ -\frac{b\sin(c+dx)\cos(a+bx)}{b^2-d^2} + \frac{d\sin(a+bx)\cos(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c),x)
```

```
[Out] Piecewise((x*sin(a)*sin(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x)*sin(c + d
*x)/2 - x*cos(a - d*x)*cos(c + d*x)/2 + sin(c + d*x)*cos(a - d*x)/(2*d), Eq
(b, -d)), (x*sin(a + d*x)*sin(c + d*x)/2 + x*cos(a + d*x)*cos(c + d*x)/2 -
sin(c + d*x)*cos(a + d*x)/(2*d), Eq(b, d)), (-b*sin(c + d*x)*cos(a + b*x)/(
b**2 - d**2) + d*sin(a + b*x)*cos(c + d*x)/(b**2 - d**2), True))
```

Giac [A]
time = 0.40, size = 40, normalized size = 0.93

$$-\frac{\sin(bx+dx+a+c)}{2(b+d)} + \frac{\sin(bx-dx+a-c)}{2(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -1/2*sin(b*x + d*x + a + c)/(b + d) + 1/2*sin(b*x - d*x + a - c)/(b - d)
```

Mupad [B]
time = 1.06, size = 84, normalized size = 1.95

$$\frac{d\left(\frac{\sin(a+c+b\,x+d\,x)}{2} + \frac{\sin(a-c+b\,x-d\,x)}{2}\right)}{b^2-d^2} - \frac{b\left(\frac{\sin(a+c+b\,x+d\,x)}{2} - \frac{\sin(a-c+b\,x-d\,x)}{2}\right)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*sin(c + d*x),x)
```

```
[Out] (d*(sin(a + c + b*x + d*x)/2 + sin(a - c + b*x - d*x)/2))/(b^2 - d^2) - (b*
(sin(a + c + b*x + d*x)/2 - sin(a - c + b*x - d*x)/2))/(b^2 - d^2)
```

3.195 $\int \csc(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=26

$$x\cos(a-c)+\frac{\log(\sin(c+bx))\sin(a-c)}{b}$$

```
[Out] x*cos(a-c)+ln(sin(b*x+c))*sin(a-c)/b
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4678, 3556, 8}

$$\frac{\sin(a-c)\log(\sin(bx+c))}{b}+x\cos(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]*Sin[a + b*x],x]
```

```
[Out] x*Cos[a - c] + (Log[Sin[c + b*x]]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4678

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \csc(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int 1\,dx+\sin(a-c)\int \cot(c+bx)\,dx\\ &= x\cos(a-c)+\frac{\log(\sin(c+bx))\sin(a-c)}{b}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 26, normalized size = 1.00

$$x\cos(a-c)+\frac{\log(\sin(c+bx))\sin(a-c)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]*Sin[a + b*x],x]
```

```
[Out] x*Cos[a - c] + (Log[Sin[c + b*x]]*Sin[a - c])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(26) = 52.
time = 0.38, size = 161, normalized size = 6.19

method	result
risch	$x\,\mathrm{e}^{i(a-c)}-2i\sin\left(a-c\right)x-\frac{2i\sin(a-c)a}{b}+\frac{\ln\left(\mathrm{e}^{2i(bx+a)}-\mathrm{e}^{2i(a-c)}\right)\sin(a-c)}{b}$
default	$\frac{\frac{\frac{(\cos(a)\sin(c)-\sin(a)\cos(c))\ln\left(1+\tan^2(bx+a)\right)}{2}+(\cos(a)\cos(c)+\sin(a)\sin(c))\arctan(\tan(bx+a))}{\left(\cos^2(c)+\sin^2(c)\right)\left(\cos^2(a)+\sin^2(a)\right)}+\frac{(\sin(a)\cos(c)-\cos(a)\sin(c))\ln(\tan(bx+a)\cos(a)\cos}{\left(\cos^2(a)\right)\left(\cos^2(c)\right)+\left(\cos^2(c)\right)\left(\sin^2(\right.}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(a)^2)*(1/2*(cos(a)*sin(c)-sin(a)*c
os(c))*ln(1+tan(b*x+a)^2)+(cos(a)*cos(c)+sin(a)*sin(c))*arctan(tan(b*x+a)))
+(sin(a)*cos(c)-cos(a)*sin(c))/(cos(a)^2*cos(c)^2+cos(c)^2*sin(a)^2+cos(a)^
2*sin(c)^2+sin(a)^2*sin(c)^2)*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)
*sin(c)+cos(a)*sin(c)-sin(a)*cos(c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(26) = 52.
time = 0.30, size = 108, normalized size = 4.15

$$\frac{2\,bx\cos\left(-a+c\right)-\log\left(\cos\left(bx\right)^2+2\,\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^2+\sin\left(bx\right)^2-2\,\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^2\right)\sin\left(-a+c\right)-\log\left(\cos\left(bx\right)^2-2\,\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^2+\sin\left(bx\right)^2+2\,\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^2\right)\sin\left(-a+c\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*cos(-a + c) - log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + si
n(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2)*sin(-a + c) - log(cos(b*x)^2 - 2*c
os(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*sin(
-a + c))/b
```

Fricas [A]
time = 2.85, size = 31, normalized size = 1.19

$$\frac{bx\cos\left(-a+c\right)-\log\left(\frac{1}{2}\sin\left(bx+c\right)\right)\sin\left(-a+c\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x*cos(-a + c) - log(1/2*sin(b*x + c))*sin(-a + c))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(20) = 40$.
time = 4.10, size = 335, normalized size = 12.88

$$\left(\begin{cases} 0 & \text{for}\: b=0 \wedge \left(b=0 \vee c=0\right) \\ x & \text{for}\: c=0 \\ -\frac{bx\tan^{2}\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}+\frac{bx}{b\tan^{2}\left(\frac{c}{2}\right)+b}-\frac{2\log\left(\tan\left(\frac{c}{2}\right)+\tan\left(\frac{bx}{2}\right)\right)\tan\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}-\frac{2\log\left(\tan\left(\frac{bx}{2}\right)-\frac{1}{\tan\left(\frac{c}{2}\right)}\right)\tan\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}+\frac{2\log\left(\tan^{2}\left(\frac{bx}{2}\right)+1\right)\tan\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b} & \text{otherwise} \end{cases}\right)\cos\left(a\right)+\left(\begin{cases} \tilde{\infty}x & \text{for}\: b=0 \wedge c=0 \\ \frac{x}{\sin\left(c\right)} & \text{for}\: b=0 \\ \frac{\log\left(\sin\left(bx\right)\right)}{b} & \text{for}\: c=0 \\ \frac{2bx\tan\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}-\frac{\log\left(\tan\left(\frac{c}{2}\right)+\tan\left(\frac{bx}{2}\right)\right)\tan^{2}\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}+\frac{\log\left(\tan\left(\frac{c}{2}\right)+\tan\left(\frac{bx}{2}\right)\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}-\frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{1}{\tan\left(\frac{c}{2}\right)}\right)\tan^{2}\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}+\frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{1}{\tan\left(\frac{c}{2}\right)}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}+\frac{\log\left(\tan^{2}\left(\frac{bx}{2}\right)+1\right)\tan^{2}\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}-\frac{\log\left(\tan^{2}\left(\frac{bx}{2}\right)+1\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b} & \text{otherwise} \end{cases}\right)\sin\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)*sin(b*x+a),x)
```

```
[Out] Piecewise((0, Eq(b, 0) & (Eq(b, 0) | Eq(c, 0))), (x, Eq(c, 0)), (-b*x*tan(c
/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) - 2*log(tan(c/2) + tan
(b*x/2))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2))*tan(
c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2
 + b), True))*cos(a) + Piecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (x/sin(c), E
q(b, 0)), (log(sin(b*x))/b, Eq(c, 0)), (2*b*x*tan(c/2)/(b*tan(c/2)**2 + b)
- log(tan(c/2) + tan(b*x/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) + log(tan(c/2)
 + tan(b*x/2))/(b*tan(c/2)**2 + b) - log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)*
*2/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - 1/tan(c/2))/(b*tan(c/2)**2 + b) +
 log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2
 + 1)/(b*tan(c/2)**2 + b), True))*sin(a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(26) = 52$.
time = 0.41, size = 236, normalized size = 9.08

$$\frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}a\right)^{2}+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^{2}+1\right)\left(bx+c\right)}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1}-\frac{2\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)\log\left(\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)^{2}+1\right)}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1}+\frac{2\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)\right|\right)}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] ((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(
1/2*c)^2 + 1)*(b*x + c)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2
*c)^2 + 1) - 2*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2
```

```
*a) - tan(1/2*c))*log(tan(1/2*b*x + 1/2*c)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^
2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2
*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c)))/
(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1))/b
```

Mupad [B]
time = 0.87, size = 111, normalized size = 4.27

$$x\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}}{2}-\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}}{2}\right)+x\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}}{2}\right)+\frac{\ln\left(-\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(c + b*x),x)
```

```
[Out] x*(exp(c*1i - a*1i)/2 - exp(a*1i - c*1i)/2) + x*(exp(c*1i - a*1i)/2 + exp(a
*1i - c*1i)/2) + (log(exp(a*2i + b*x*2i) - exp(a*2i - c*2i))*((exp(c*1i - a
*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2))/b
```

3.196 $\int \csc^2(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b}-\frac{\csc(c+bx)\sin(a-c)}{b}$$

```
[Out] -arctanh(cos(b*x+c))*cos(a-c)/b-csc(b*x+c)*sin(a-c)/b
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4678, 2686, 8, 3855}

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a-c)\csc(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) - (Csc[c + b*x]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4678

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \csc^2(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \csc(c+bx)\,dx + \sin(a-c)\int \cot(c+bx)\csc(c+bx)\,dx$$
$$= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b} - \frac{\sin(a-c)\text{Subst}(\int 1\,dx, x, \csc(c+bx))}{b}$$
$$= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b} - \frac{\csc(c+bx)\sin(a-c)}{b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.12, size = 90, normalized size = 2.50

$$-\frac{2i\text{ArcTan}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b} - \frac{\csc(c+bx)\sin(a-c)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)
/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Csc[c
 + b*x]*Sin[a - c])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(36) = 72.
time = 0.62, size = 347, normalized size = 9.64

method	result
risch	$\frac{e^{i(bx+3a)}-e^{i(bx+a+2c)}}{b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)} - \frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos(a-c)}{b} + \frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\cos(a-c)}{b}$
default	$\frac{\frac{4(-2\cos(a)\cos(c)-2\sin(a)\sin(c))\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+8\sin(a)\cos(c)-8\cos}{\left(-4\left(\cos^2(c)\right)\left(\sin^2(a)\right)-4\left(\cos^2(a)\right)\left(\cos^2(c)\right)-4\left(\sin^2(a)\right)\left(\sin^2(c)\right)-4\left(\cos^2(a)\right)\left(\sin^2(c)\right)\right)\left(\cos(c)\sin(a)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\sin(c)\cos(a)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right.\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(4*((-2*cos(a)*cos(c)-2*sin(a)*sin(c))*tan(1/2*b*x+1/2*a)+2*sin(a)*cos(
c)-2*cos(a)*sin(c))/(-4*cos(c)^2*sin(a)^2-4*cos(a)^2*cos(c)^2-4*sin(a)^2*si
n(c)^2-4*cos(a)^2*sin(c)^2)/(cos(c)*sin(a)*tan(1/2*b*x+1/2*a)^2-sin(c)*cos(
a)*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+2*tan(1/2*b*x+1/
2*a)*sin(a)*sin(c)-sin(a)*cos(c)+cos(a)*sin(c))+4*(-2*cos(a)*cos(c)-2*sin(a
)*sin(c))/(-4*cos(c)^2*sin(a)^2-4*cos(a)^2*cos(c)^2-4*sin(a)^2*sin(c)^2-4*c
```

```
os(a)^2*sin(c)^2)/(-cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin(c)^2-c
os(a)^2*sin(c)^2)^(1/2)*arctan(1/2*(2*(sin(a)*cos(c)-cos(a)*sin(c))*tan(1/2
*b*x+1/2*a)+2*cos(a)*cos(c)+2*sin(a)*sin(c))/(-cos(c)^2*sin(a)^2-cos(a)^2*c
os(c)^2-sin(a)^2*sin(c)^2-cos(a)^2*sin(c)^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(36) = 72$.
time = 0.31, size = 454, normalized size = 12.61

$$-\frac{2\left(\cos\left(bx+2a\right)-\cos\left(bx+2c\right)\right)\cos\left(2bx+a+2c\right)-2\cos\left(bx+2a\right)\cos\left(a\right)+2\cos\left(bx+2c\right)\cos\left(a\right)+\left(\cos\left(2bx+a+2c\right)^{2}\cos\left(-a+c\right)-2\cos\left(2bx+a+2c\right)\cos\left(a\right)\cos\left(-a+c\right)+\cos\left(-a+c\right)\sin\left(2bx+a+2c\right)^{2}-2\cos\left(-a+c\right)\sin\left(2bx+a+2c\right)\sin\left(a\right)+\left(\cos\left(a\right)^{2}+\sin\left(a\right)^{2}\right)\cos\left(-a+c\right)\right)\log\left(\cos\left(bx\right)^{2}+2\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^{2}+\sin\left(bx\right)^{2}-2\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^{2}\right)-\left(\cos\left(2bx+a+2c\right)^{2}\cos\left(-a+c\right)-2\cos\left(2bx+a+2c\right)\cos\left(a\right)\cos\left(-a+c\right)+\cos\left(-a+c\right)\sin\left(2bx+a+2c\right)^{2}-2\cos\left(-a+c\right)\sin\left(2bx+a+2c\right)\sin\left(a\right)+\left(\cos\left(a\right)^{2}+\sin\left(a\right)^{2}\right)\cos\left(-a+c\right)\right)\log\left(\cos\left(bx\right)^{2}-2\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^{2}+\sin\left(bx\right)^{2}+2\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^{2}\right)+2\left(\sin\left(bx+2a\right)-\sin\left(bx+2c\right)\right)\sin\left(2bx+a+2c\right)-2\sin\left(bx+2a\right)\sin\left(a\right)+2\sin\left(bx+2c\right)\sin\left(a\right)}{2\left(b\cos\left(2bx+a+2c\right)^{2}-2b\cos\left(2bx+a+2c\right)\cos\left(a\right)+b\sin\left(2bx+a+2c\right)^{2}-2b\sin\left(2bx+a+2c\right)\sin\left(a\right)+\left(\cos\left(a\right)^{2}+\sin\left(a\right)^{2}\right)b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*(cos(b*x + 2*a) - cos(b*x + 2*c))*cos(2*b*x + a + 2*c) - 2*cos(b*x
+ 2*a)*cos(a) + 2*cos(b*x + 2*c)*cos(a) + (cos(2*b*x + a + 2*c)^2*cos(-a +
c) - 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(2*b*x + a
+ 2*c)^2 - 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2
)*cos(-a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 -
 2*sin(b*x)*sin(c) + sin(c)^2) - (cos(2*b*x + a + 2*c)^2*cos(-a + c) - 2*co
s(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(2*b*x + a + 2*c)^2
- 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*cos(-a
+ c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*
x)*sin(c) + sin(c)^2) + 2*(sin(b*x + 2*a) - sin(b*x + 2*c))*sin(2*b*x + a +
 2*c) - 2*sin(b*x + 2*a)*sin(a) + 2*sin(b*x + 2*c)*sin(a))/(b*cos(2*b*x + a
 + 2*c)^2 - 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^2 - 2*
b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Fricas [A]
time = 2.73, size = 71, normalized size = 1.97

$$-\frac{\cos\left(-a+c\right)\log\left(\frac{1}{2}\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)-\cos\left(-a+c\right)\log\left(-\frac{1}{2}\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)-2\sin\left(-a+c\right)}{2\,b\sin\left(bx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(-a + c)*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - cos(-a + c)*lo
g(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - 2*sin(-a + c))/(b*sin(b*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(29) = 58$.
time = 60.43, size = 3264, normalized size = 90.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**2*sin(b*x+a),x)

[Out] Piecewise((0, Eq(b, 0) & Eq(c, 0)), (log(tan(b*x/2))/b, Eq(c, 0)), (0, Eq(b
, 0)), (-log(tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*t
an(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*
x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(c/2)*
*3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 -
b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log
(tan(c/2) + tan(b*x/2))*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)*
*3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) -
b*tan(b*x/2)) + 2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**2*tan(b*x/2)/(b*tan(
c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/
2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(c/2) + tan(b*x/2))*
tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)
**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2))
 - log(tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/
2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2)
 - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(b*x/2)/(b*tan(c/2)**4*tan
(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/
2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)*
*4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*t
an(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(ta
n(b*x/2) - 1/tan(c/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)
+ b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 -
b*tan(c/2) - b*tan(b*x/2)) - log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3/(b*ta
n(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(
c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - 2*log(tan(b*x/2) - 1/tan(
c/2))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(
b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b
*x/2)) - log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4
*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(
b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1/tan(c/2))*tan(c
/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3
 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) -
 1/tan(c/2))*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2
)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)
) + tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*
x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x
/2)) - 2*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**
2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) -
 2*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan
(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - tan(b*x/
2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3
+ b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)), True))*cos(a) + Pi
ecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (x/sin(c)**2, Eq(b, 0)), (-1/(b*sin(b
```

```
*x)), Eq(c, 0)), (4*log(tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(2*b*
tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 +
2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2))
+ 4*log(tan(c/2) + tan(b*x/2))*tan(c/2)**3*tan(b*x/2)**2/(2*b*tan(c/2)**5*t
an(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)*
*2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) - 4*log(tan(c
/2) + tan(b*x/2))*tan(c/2)**3/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4
*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(
c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) - 4*log(tan(c/2) + tan(b*x/2))*tan(c/2)*
*2*tan(b*x/2)/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 -
 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*ta
n(c/2)*tan(b*x/2)) - 4*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**4*tan(b*x/2)/
(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)*
*4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x
/2)) - 4*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3*tan(b*x/2)**2/(2*b*tan(c/
2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*ta
n(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) + 4*lo
g(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*ta
n(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 -
 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) + 4*log(tan(b*x/2) - 1/tan(c/2)
)*tan(c/2)**2*tan(b*x/2)/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(
b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)*
*2 - 2*b*tan(c/2)*tan(b*x/2)) + tan(c/2)**6*tan(b*x/2)/(2*b*tan(c/2)**5*tan
(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*t...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(36) = 72$.

time = 0.40, size = 349, normalized size = 9.69

$$\frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}a\right)^{2}+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^{2}+1\right)\log\left(\left|\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)\right|\right)}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1}-\frac{\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)\tan\left(\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1}-\frac{\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)\tan\left(\frac{1}{2}a\right)^{2}+4\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1\right)\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] ((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(
1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + t
an(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/
2*c) - tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*b*x + 1/2*c)*
tan(1/2*a) - tan(1/2*b*x + 1/2*c)*tan(1/2*c))/(tan(1/2*a)^2*tan(1/2*c)^2 +
tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1
/2*c)^2 - tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2 + 4*tan(1/2*b*x + 1/2*c)*tan(1/
2*a)*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c) - tan(1/2*b*x + 1/2*c)*tan(1/2*c)
^2 - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*b*x + 1/2*c) + tan(1/2*a) - tan(1/2*
c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*tan(1/2*
b*x + 1/2*c)))/b
```

Mupad [B]
time = 5.21, size = 252, normalized size = 7.00

$$-\frac{\ln\left(-\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)-\frac{\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}+1\right)\,1\mathrm{i}}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}}}\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+1\right)}{2\,b\,\sqrt{\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}}}+\frac{\ln\left(-\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)+\frac{\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}+1\right)\,1\mathrm{i}}{\sqrt{\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}}}\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+1\right)}{2\,b\,\sqrt{\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}}}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-1\right)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(c + b*x)^2,x)
```

```
[Out] (log((exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c
*2i))^(1/2) - exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i))*(exp(a*
2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log(- exp(a*1i)*exp(b*x*1i
)*(exp(a*2i)*exp(-c*2i)*1i + 1i) - (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*
2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) + 1))/(2*b*exp
(a*2i - c*2i)^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) - 1))/(b*(exp(
a*2i - c*2i) - exp(a*2i + b*x*2i)))
```

3.197 $\int \csc^3(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=39

$$-\frac{\cos(a-c)\cot(c+bx)}{b}-\frac{\csc^2(c+bx)\sin(a-c)}{2b}$$

```
[Out] -cos(a-c)*cot(b*x+c)/b-1/2*csc(b*x+c)^2*sin(a-c)/b
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4678, 2686, 30, 3852, 8}

$$-\frac{\cos(a-c)\cot(bx+c)}{b}-\frac{\sin(a-c)\csc^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((Cos[a - c]*Cot[c + b*x])/b) - (Csc[c + b*x]^2*Sin[a - c])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4678

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
```

```
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$
\begin{aligned}
\int \csc^3(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int \csc^2(c+bx)\,dx+\sin(a-c)\int \cot(c+bx)\csc^2(c+bx)\,dx\\
&= -\frac{\cos(a-c)\text{Subst}(\int 1\,dx,x,\cot(c+bx))}{b}-\frac{\sin(a-c)\text{Subst}(\int x\,dx,x,\csc(c}{b}\\
&= -\frac{\cos(a-c)\cot(c+bx)}{b}-\frac{\csc^2(c+bx)\sin(a-c)}{2b}
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 35, normalized size = 0.90

$$\frac{(\cos(a)-\cos(a-c)\cos(c+2bx))\csc(c)\csc^2(c+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] ((Cos[a] - Cos[a - c]*Cos[c + 2*b*x])*Csc[c]*Csc[c + b*x]^2)/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(37) = 74.
time = 0.95, size = 120, normalized size = 3.08

method	result
risch	$\frac{i\left(-2\,e^{i(2bx+5a+c)}+e^{i(5a-c)}+e^{i(3a+c)}\right)}{\left(-e^{2i(bx+a+c)}+e^{2ia}\right)^2 b}$
default	$\frac{-\frac{1}{(\cos(a)\cos(c)+\sin(a)\sin(c))^2(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)+\cos(a)\sin(c)-\sin(a)\cos(c))}-\frac{}{2(\cos(a)\cos(c)+\sin(a)\sin(c))^2(\tan(bx-}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/(cos(a)*cos(c)+sin(a)*sin(c))^2/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a
)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))-1/2*(sin(a)*cos(c)-cos(a)*sin(
c))/(cos(a)*cos(c)+sin(a)*sin(c))^2/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*si
n(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. 2(37) = 74.
time = 0.28, size = 399, normalized size = 10.23

$$\frac{(2\sin(2bx+2a+2c)-\sin(2a)-\sin(2c))\cos(4bx+a+5c)-2(2\sin(2bx+2a+2c)-\sin(2a)-\sin(2c))\cos(2bx+a+3c)-(\sin(2a)+\sin(2c))\cos(a+c)-(2\cos(2bx+2a+2c)-\cos(2a)-\cos(2c))\sin(4bx+a+5c)+2\cos(a+c)\sin(2bx+2a+2c)+2(2\cos(2bx+2a+2c)-\cos(2a)-\cos(2c))\sin(2bx+a+3c)+(\cos(2a)+\cos(2c))\sin(a+c)-2\cos(2bx+2a+2c)\sin(a+c)}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2-4b\cos(2bx+a+3c)\cos(a+c)+b\cos(a+c)^2+b\sin(4bx+a+5c)^2+4b\sin(2bx+a+3c)^2-4b\sin(2bx+a+3c)\sin(a+c)+b\sin(a+c)^2-2(2b\cos(2bx+a+3c)-b\cos(a+c))\cos(4bx+a+5c)-2(2b\sin(2bx+a+3c)-b\sin(a+c))\sin(4bx+a+5c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] ((2*sin(2*b*x + 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(4*b*x + a + 5*c) - 2*
(2*sin(2*b*x + 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(2*b*x + a + 3*c) - (si
n(2*a) + sin(2*c))*cos(a + c) - (2*cos(2*b*x + 2*a + 2*c) - cos(2*a) - cos(
2*c))*sin(4*b*x + a + 5*c) + 2*cos(a + c)*sin(2*b*x + 2*a + 2*c) + 2*(2*cos
(2*b*x + 2*a + 2*c) - cos(2*a) - cos(2*c))*sin(2*b*x + a + 3*c) + (cos(2*a)
 + cos(2*c))*sin(a + c) - 2*cos(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos(4*b*x
 + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 - 4*b*cos(2*b*x + a + 3*c)*cos(a
 + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x + a + 3*c
)^2 - 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(2*b*cos(2*b
*x + a + 3*c) - b*cos(a + c))*cos(4*b*x + a + 5*c) - 2*(2*b*sin(2*b*x + a +
 3*c) - b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Fricas [**A**]
time = 2.03, size = 47, normalized size = 1.21

$$\frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)-\sin(-a+c)}{2\left(b\cos(bx+c)^2-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c) - sin(-a + c))/(b*cos(b*x + c)
^2 - b)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(37) = 74.
time = 0.43, size = 145, normalized size = 3.72

$$-\frac{\tan(bx+c)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan(bx+c)\tan\left(\frac{1}{2}a\right)^2+4\tan(bx+c)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan(bx+c)\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan(bx+c)+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1\right)b\tan(bx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -(tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + c)*tan(1/2*a)^2 + 4*ta
n(b*x + c)*tan(1/2*a)*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c) - tan(b*x + c)*t
an(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^2 + tan(b*x + c) + tan(1/2*a) - tan(1/2
*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b*tan(b
*x + c)^2)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(c + b*x)^3,x)
```

```
[Out] \text{Hanged}
```

3.198 $\int \csc^4(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=67

$$-\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{2b}-\frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b}-\frac{\csc^3(c+bx)\sin(a-c)}{3b}$$

```
[Out] -1/2*arctanh(cos(b*x+c))*cos(a-c)/b-1/2*cos(a-c)*cot(b*x+c)*csc(b*x+c)/b-1/
3*csc(b*x+c)^3*sin(a-c)/b
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4678, 2686, 30, 3853, 3855}

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{2b}-\frac{\sin(a-c)\csc^3(bx+c)}{3b}-\frac{\cos(a-c)\cot(bx+c)\csc(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^4*Sin[a + b*x],x]
```

```
[Out] -1/2*(ArcTanh[Cos[c + b*x]]*Cos[a - c])/b - (Cos[a - c]*Cot[c + b*x]*Csc[c
+ b*x])/(2*b) - (Csc[c + b*x]^3*Sin[a - c])/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4678

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \csc^4(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \csc^3(c+bx)\,dx + \sin(a-c)\int \cot(c+bx)\csc^3(c+bx)\,dx$$
$$= -\frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} + \frac{1}{2}\cos(a-c)\int \csc(c+bx)\,dx - \frac{\sin(a}{}$$
$$= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{2b} - \frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} - \frac{\csc}{}$$

Mathematica [**A**]
time = 0.61, size = 67, normalized size = 1.00

$$-\frac{6\tanh^{-1}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+3\cos(a-c)\cot(c+bx)\csc(c+bx)+2\csc^3(c+bx)\sin(a-c)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^4*Sin[a + b*x],x]
```

```
[Out] -1/6*(6*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + 3*Cos[a - c]*Cot
[c + b*x]*Csc[c + b*x] + 2*Csc[c + b*x]^3*Sin[a - c])/b
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1747 vs. $2(61) = 122$.
time = 1.88, size = 1748, normalized size = 26.09

method	result
risch	$\frac{-3\,e^{i(5bx+7a+4c)}-3\,e^{i(5bx+5a+6c)}-8\,e^{i(3bx+7a+2c)}+8\,e^{i(3bx+5a+4c)}+3\,e^{i(bx+7a)}+3\,e^{i(bx+5a+2c)}}{6b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)^3}+\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\cos(a-c)}{2b}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(32*(1/32*(cos(a)*cos(c)+sin(a)*sin(c))*(cos(c)^2*sin(a)^2-2*cos(a)*cos
(c)*sin(a)*sin(c)+cos(a)^2*sin(c)^2)/(cos(a)^4*cos(c)^4+2*cos(a)^2*cos(c)^4
*sin(a)^2+cos(c)^4*sin(a)^4+2*cos(a)^4*cos(c)^2*sin(c)^2+4*cos(a)^2*cos(c)^
2*sin(a)^2*sin(c)^2+2*sin(a)^4*sin(c)^2*cos(c)^2+sin(c)^4*cos(a)^4+2*sin(a)
```

```
^2*cos(a)^2*sin(c)^4+sin(c)^4*sin(a)^4)*tan(1/2*b*x+1/2*a)^5-1/32*(2*cos(a)
^4*cos(c)^4-cos(a)^2*cos(c)^4*sin(a)^2+2*cos(c)^4*sin(a)^4+10*cos(a)^3*cos(
c)^3*sin(a)*sin(c)-10*cos(c)^3*sin(c)*cos(a)*sin(a)^3-cos(a)^4*cos(c)^2*sin
(c)^2+28*cos(a)^2*cos(c)^2*sin(a)^2*sin(c)^2-sin(a)^4*sin(c)^2*cos(c)^2-10*
cos(c)*sin(c)^3*cos(a)^3*sin(a)+10*sin(a)^3*cos(a)*sin(c)^3*cos(c)+2*sin(c)
^4*cos(a)^4-sin(a)^2*cos(a)^2*sin(c)^4+2*sin(c)^4*sin(a)^4)/(sin(a)*cos(c)-
cos(a)*sin(c))/(cos(a)^4*cos(c)^4+2*cos(a)^2*cos(c)^4*sin(a)^2+cos(c)^4*sin
(a)^4+2*cos(a)^4*cos(c)^2*sin(c)^2+4*cos(a)^2*cos(c)^2*sin(a)^2*sin(c)^2+2*
sin(a)^4*sin(c)^2*cos(c)^2+sin(c)^4*cos(a)^4+2*sin(a)^2*cos(a)^2*sin(c)^4+s
in(c)^4*sin(a)^4)*tan(1/2*b*x+1/2*a)^4-1/48*(cos(a)*cos(c)+sin(a)*sin(c))*(
2*cos(a)^4*cos(c)^4-7*cos(a)^2*cos(c)^4*sin(a)^2+6*cos(c)^4*sin(a)^4+22*cos
(a)^3*cos(c)^3*sin(a)*sin(c)-38*cos(c)^3*sin(c)*cos(a)*sin(a)^3-7*cos(a)^4*
cos(c)^2*sin(c)^2+76*cos(a)^2*cos(c)^2*sin(a)^2*sin(c)^2-7*sin(a)^4*sin(c)^
2*cos(c)^2-38*cos(c)*sin(c)^3*cos(a)^3*sin(a)+22*sin(a)^3*cos(a)*sin(c)^3*c
os(c)+6*sin(c)^4*cos(a)^4-7*sin(a)^2*cos(a)^2*sin(c)^4+2*sin(c)^4*sin(a)^4)
/(cos(a)^4*cos(c)^4+2*cos(a)^2*cos(c)^4*sin(a)^2+cos(c)^4*sin(a)^4+2*cos(a)
^4*cos(c)^2*sin(c)^2+4*cos(a)^2*cos(c)^2*sin(a)^2*sin(c)^2+2*sin(a)^4*sin(c
)^2*cos(c)^2+sin(c)^4*cos(a)^4+2*sin(a)^2*cos(a)^2*sin(c)^4+sin(c)^4*sin(a)
^4)/(sin(a)*cos(c)-cos(a)*sin(c))^2*tan(1/2*b*x+1/2*a)^3+1/16*(cos(a)^3*cos
(c)^3-4*cos(c)^3*cos(a)*sin(a)^2+11*cos(a)^2*cos(c)^2*sin(a)*sin(c)-4*cos(c
)^2*sin(c)*sin(a)^3-4*cos(c)*sin(c)^2*cos(a)^3+11*cos(c)*sin(c)^2*cos(a)*si
n(a)^2-4*sin(c)^3*cos(a)^2*sin(a)+sin(c)^3*sin(a)^3)/(sin(a)*cos(c)-cos(a)*
sin(c))*(cos(a)*cos(c)+sin(a)*sin(c))/(cos(a)^4*cos(c)^4+2*cos(a)^2*cos(c)^
4*sin(a)^2+cos(c)^4*sin(a)^4+2*cos(a)^4*cos(c)^2*sin(c)^2+4*cos(a)^2*cos(c)
^2*sin(a)^2*sin(c)^2+2*sin(a)^4*sin(c)^2*cos(c)^2+sin(c)^4*cos(a)^4+2*sin(a
)^2*cos(a)^2*sin(c)^4+sin(c)^4*sin(a)^4)*tan(1/2*b*x+1/2*a)^2-1/32*(cos(a)*
cos(c)+sin(a)*sin(c))*(2*cos(a)^2*cos(c)^2-3*cos(c)^2*sin(a)^2+10*cos(a)*co
s(c)*sin(a)*sin(c)-3*cos(a)^2*sin(c)^2+2*sin(a)^2*sin(c)^2)/(cos(a)^4*cos(c
)^4+2*cos(a)^2*cos(c)^4*sin(a)^2+cos(c)^4*sin(a)^4+2*cos(a)^4*cos(c)^2*sin(
c)^2+4*cos(a)^2*cos(c)^2*sin(a)^2*sin(c)^2+2*sin(a)^4*sin(c)^2*cos(c)^2+sin
(c)^4*cos(a)^4+2*sin(a)^2*cos(a)^2*sin(c)^4+sin(c)^4*sin(a)^4)*tan(1/2*b*x+
1/2*a)+1/96*(cos(a)^2*cos(c)^2-2*cos(c)^2*sin(a)^2+6*cos(a)*cos(c)*sin(a)*s
in(c)-2*cos(a)^2*sin(c)^2+sin(a)^2*sin(c)^2)*(sin(a)*cos(c)-cos(a)*sin(c))/
(cos(a)^4*cos(c)^4+2*cos(a)^2*cos(c)^4*sin(a)^2+cos(c)^4*sin(a)^4+2*cos(a)^
4*cos(c)^2*sin(c)^2+4*cos(a)^2*cos(c)^2*sin(a)^2*sin(c)^2+2*sin(a)^4*sin(c)
^2*cos(c)^2+sin(c)^4*cos(a)^4+2*sin(a)^2*cos(a)^2*sin(c)^4+sin(c)^4*sin(a)^
4))/(cos(c)*sin(a)*tan(1/2*b*x+1/2*a)^2-sin(c)*cos(a)*tan(1/2*b*x+1/2*a)^2+
2*tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+2*tan(1/2*b*x+1/2*a)*sin(a)*sin(c)-sin(a
)*cos(c)+cos(a)*sin(c))^3+8*(cos(a)*cos(c)+sin(a)*sin(c))/(8*cos(a)^4*cos(c
)^4+16*cos(a)^2*cos(c)^4*sin(a)^2+8*cos(c)^4*sin(a)^4+16*cos(a)^4*cos(c)^2*
sin(c)^2+32*cos(a)^2*cos(c)^2*sin(a)^2*sin(c)^2+16*sin(a)^4*sin(c)^2*cos(c)
^2+8*sin(c)^4*cos(a)^4+16*sin(a)^2*cos(a)^2*sin(c)^4+8*sin(c)^4*sin(a)^4)/(
-cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin(c)^2-cos(a)^2*sin(c)^2)^(
1/2)*arctan(1/2*(2*(sin(a)*cos(c)-cos(a)*sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a
)*cos(c)+2*sin(a)*sin(c))/(-cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)^2*si
```

```
n(c)^2-cos(a)^2*sin(c)^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1773 vs. $2(61) = 122$.
time = 0.35, size = 1773, normalized size = 26.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/12*(2*(3*cos(5*b*x + 2*a + 4*c) + 3*cos(5*b*x + 6*c) + 8*cos(3*b*x + 2*a
+ 2*c) - 8*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(6*b*
x + a + 6*c) - 6*(3*cos(4*b*x + a + 4*c) - 3*cos(2*b*x + a + 2*c) + cos(a))
*cos(5*b*x + 2*a + 4*c) - 6*(3*cos(4*b*x + a + 4*c) - 3*cos(2*b*x + a + 2*c
) + cos(a))*cos(5*b*x + 6*c) - 6*(8*cos(3*b*x + 2*a + 2*c) - 8*cos(3*b*x +
4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) + 16*(3*co
s(2*b*x + a + 2*c) - cos(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*cos(2*b*x + a +
 2*c) - cos(a))*cos(3*b*x + 4*c) - 18*(cos(b*x + 2*a) + cos(b*x + 2*c))*cos
(2*b*x + a + 2*c) + 6*cos(b*x + 2*a)*cos(a) + 6*cos(b*x + 2*c)*cos(a) - 3*(
cos(6*b*x + a + 6*c)^2*cos(-a + c) + 9*cos(4*b*x + a + 4*c)^2*cos(-a + c) +
 9*cos(2*b*x + a + 2*c)^2*cos(-a + c) - 6*cos(2*b*x + a + 2*c)*cos(a)*cos(-
a + c) + cos(-a + c)*sin(6*b*x + a + 6*c)^2 + 9*cos(-a + c)*sin(4*b*x + a +
 4*c)^2 + 9*cos(-a + c)*sin(2*b*x + a + 2*c)^2 - 6*cos(-a + c)*sin(2*b*x +
a + 2*c)*sin(a) - 2*(3*cos(4*b*x + a + 4*c)*cos(-a + c) - 3*cos(2*b*x + a +
 2*c)*cos(-a + c) + cos(a)*cos(-a + c))*cos(6*b*x + a + 6*c) - 6*(3*cos(2*b
*x + a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(4*b*x + a + 4*c) + (cos
(a)^2 + sin(a)^2)*cos(-a + c) - 2*(3*cos(-a + c)*sin(4*b*x + a + 4*c) - 3*c
os(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)*sin(a))*sin(6*b*x + a + 6*c)
- 6*(3*cos(-a + c)*sin(2*b*x + a + 2*c) - cos(-a + c)*sin(a))*sin(4*b*x + a
 + 4*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin
(b*x)*sin(c) + sin(c)^2) + 3*(cos(6*b*x + a + 6*c)^2*cos(-a + c) + 9*cos(4*
b*x + a + 4*c)^2*cos(-a + c) + 9*cos(2*b*x + a + 2*c)^2*cos(-a + c) - 6*cos
(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(6*b*x + a + 6*c)^2 +
 9*cos(-a + c)*sin(4*b*x + a + 4*c)^2 + 9*cos(-a + c)*sin(2*b*x + a + 2*c)^
2 - 6*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a) - 2*(3*cos(4*b*x + a + 4*c)*c
os(-a + c) - 3*cos(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*cos(-a + c))*cos(6
*b*x + a + 6*c) - 6*(3*cos(2*b*x + a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c
))*cos(4*b*x + a + 4*c) + (cos(a)^2 + sin(a)^2)*cos(-a + c) - 2*(3*cos(-a +
 c)*sin(4*b*x + a + 4*c) - 3*cos(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)
*sin(a))*sin(6*b*x + a + 6*c) - 6*(3*cos(-a + c)*sin(2*b*x + a + 2*c) - cos
(-a + c)*sin(a))*sin(4*b*x + a + 4*c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) +
 cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) + 2*(3*sin(5*b*x + 2
*a + 4*c) + 3*sin(5*b*x + 6*c) + 8*sin(3*b*x + 2*a + 2*c) - 8*sin(3*b*x + 4
*c) - 3*sin(b*x + 2*a) - 3*sin(b*x + 2*c))*sin(6*b*x + a + 6*c) - 6*(3*sin(
```

```
4*b*x + a + 4*c) - 3*sin(2*b*x + a + 2*c) + sin(a))*sin(5*b*x + 2*a + 4*c)
- 6*(3*sin(4*b*x + a + 4*c) - 3*sin(2*b*x + a + 2*c) + sin(a))*sin(5*b*x +
6*c) - 6*(8*sin(3*b*x + 2*a + 2*c) - 8*sin(3*b*x + 4*c) - 3*sin(b*x + 2*a)
- 3*sin(b*x + 2*c))*sin(4*b*x + a + 4*c) + 16*(3*sin(2*b*x + a + 2*c) - sin
(a))*sin(3*b*x + 2*a + 2*c) - 16*(3*sin(2*b*x + a + 2*c) - sin(a))*sin(3*b*
x + 4*c) - 18*(sin(b*x + 2*a) + sin(b*x + 2*c))*sin(2*b*x + a + 2*c) + 6*si
n(b*x + 2*a)*sin(a) + 6*sin(b*x + 2*c)*sin(a))/(b*cos(6*b*x + a + 6*c)^2 +
9*b*cos(4*b*x + a + 4*c)^2 + 9*b*cos(2*b*x + a + 2*c)^2 - 6*b*cos(2*b*x + a
 + 2*c)*cos(a) + b*sin(6*b*x + a + 6*c)^2 + 9*b*sin(4*b*x + a + 4*c)^2 + 9*
b*sin(2*b*x + a + 2*c)^2 - 6*b*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + si
n(a)^2)*b - 2*(3*b*cos(4*b*x + a + 4*c) - 3*b*cos(2*b*x + a + 2*c) + b*cos(
a))*cos(6*b*x + a + 6*c) - 6*(3*b*cos(2*b*x + a + 2*c) - b*cos(a))*cos(4*b*
x + a + 4*c) - 2*(3*b*sin(4*b*x + a + 4*c) - 3*b*sin(2*b*x + a + 2*c) + b*s
in(a))*sin(6*b*x + a + 6*c) - 6*(3*b*sin(2*b*x + a + 2*c) - b*sin(a))*sin(4
*b*x + a + 4*c))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(61) = 122$.
time = 3.45, size = 141, normalized size = 2.10

$$\frac{6\cos(bx+c)\cos(-a+c)\sin(bx+c) - 3\left(\cos(bx+c)^2\cos(-a+c) - \cos(-a+c)\right)\log\left(\frac{1}{2}\cos(bx+c) + \frac{1}{2}\right)\sin(bx+c) + 3\left(\cos(bx+c)^2\cos(-a+c) - \cos(-a+c)\right)\log\left(-\frac{1}{2}\cos(bx+c) + \frac{1}{2}\right)\sin(bx+c) - 4\sin(-a+c)}{12\left(b\cos(bx+c)^2 - b\right)\sin(bx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(6*cos(b*x + c)*cos(-a + c)*sin(b*x + c) - 3*(cos(b*x + c)^2*cos(-a +
c) - cos(-a + c))*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) + 3*(cos(b*x + c
)^2*cos(-a + c) - cos(-a + c))*log(-1/2*cos(b*x + c) + 1/2)*sin(b*x + c) -
4*sin(-a + c))/((b*cos(b*x + c)^2 - b)*sin(b*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**4*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2221 vs. $2(61) = 122$.
time = 0.44, size = 2221, normalized size = 33.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^4*sin(b*x+a),x, algorithm="giac")

[Out] 1/24*(12*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^2*tan(1/2*
c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (2*tan(1/2*b*x + 1/2*c)^3*tan(1/2
*a)^6*tan(1/2*c)^5 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5*tan(1/2*c)^6 - 3
*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^6 + 4*tan(1/2*b*x + 1/2*c)^
3*tan(1/2*a)^6*tan(1/2*c)^3 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5*tan(1/2
*c)^4 - 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^4 + 2*tan(1/2*b*x
+ 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^5 - 12*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)
^5*tan(1/2*c)^5 + 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^6*tan(1/2*c)^5 - 4*tan(
1/2*b*x + 1/2*c)^3*tan(1/2*a)^3*tan(1/2*c)^6 - 3*tan(1/2*b*x + 1/2*c)^2*tan
(1/2*a)^4*tan(1/2*c)^6 - 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^5*tan(1/2*c)^6 +
 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^6*tan(1/2*c) + 2*tan(1/2*b*x + 1/2*c)^
3*tan(1/2*a)^5*tan(1/2*c)^2 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2
*c)^2 + 4*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^4*tan(1/2*c)^3 - 24*tan(1/2*b*x
 + 1/2*c)^2*tan(1/2*a)^5*tan(1/2*c)^3 + 12*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^
6*tan(1/2*c)^3 - 4*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^3*tan(1/2*c)^4 - 3*tan
(1/2*b*x + 1/2*c)^2*tan(1/2*a)^4*tan(1/2*c)^4 - 6*tan(1/2*b*x + 1/2*c)*tan(
1/2*a)^5*tan(1/2*c)^4 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^2*tan(1/2*c)^5
- 24*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c)^5 + 6*tan(1/2*b*x + 1/2
*c)*tan(1/2*a)^4*tan(1/2*c)^5 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)*tan(1/2
*c)^6 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^2*tan(1/2*c)^6 - 12*tan(1/2*b*x
 + 1/2*c)*tan(1/2*a)^3*tan(1/2*c)^6 + 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5
 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^6 + 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2
*a)^4*tan(1/2*c) - 12*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^5*tan(1/2*c) + 6*ta
n(1/2*b*x + 1/2*c)*tan(1/2*a)^6*tan(1/2*c) + 4*tan(1/2*b*x + 1/2*c)^3*tan(1
/2*a)^3*tan(1/2*c)^2 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^4*tan(1/2*c)^2 +
 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^5*tan(1/2*c)^2 - 4*tan(1/2*b*x + 1/2*c)^
3*tan(1/2*a)^2*tan(1/2*c)^3 - 48*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/
2*c)^3 + 12*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(1/2*b*x
+ 1/2*c)^3*tan(1/2*a)*tan(1/2*c)^4 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^2*
tan(1/2*c)^4 - 12*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3*tan(1/2*c)^4 - 2*tan(1/
2*b*x + 1/2*c)^3*tan(1/2*c)^5 - 12*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)*tan(1/
2*c)^5 - 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c)^5 + 3*tan(1/2*b*x +
 1/2*c)^2*tan(1/2*c)^6 - 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^6 + 4
*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^3 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^
4 + 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^5 - 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*
a)^2*tan(1/2*c) - 24*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^3*tan(1/2*c) + 6*tan
(1/2*b*x + 1/2*c)*tan(1/2*a)^4*tan(1/2*c) + 2*tan(1/2*b*x + 1/2*c)^3*tan(1/
2*a)*tan(1/2*c)^2 - 3*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 12
*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3*tan(1/2*c)^2 - 4*tan(1/2*b*x + 1/2*c)^3*
tan(1/2*c)^3 - 24*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)*tan(1/2*c)^3 - 12*tan(1
/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c)^3 + 3*tan(1/2*b*x + 1/2*c)^2*tan(1/
2*c)^4 - 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^4 - 6*tan(1/2*b*x + 1
```

```
/2*c)*tan(1/2*c)^5 + 2*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a) - 3*tan(1/2*b*x +
1/2*c)^2*tan(1/2*a)^2 + 12*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^3 - 2*tan(1/2*b*
x + 1/2*c)^3*tan(1/2*c) - 12*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)*tan(1/2*c) -
 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c) - 3*tan(1/2*b*x + 1/2*c)^2*
tan(1/2*c)^2 + 6*tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c)^2 - 12*tan(1/2*
b*x + 1/2*c)*tan(1/2*c)^3 - 3*tan(1/2*b*x + 1/2*c)^2 + 6*tan(1/2*b*x + 1/2*
c)*tan(1/2*a) - 6*tan(1/2*b*x + 1/2*c)*tan(1/2*c))/(tan(1/2*a)^6*tan(1/2*c)
^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a)^4*tan(1/2*c)^6 + 3*tan(1/2*
a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^4 + 3*tan(1/2*a)^2*tan(1/2*c)
^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^2 + 9*tan(1/2*a)^2*tan(1/2*c)
^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2*a)^2*tan(1/2*c)^2 + 3*tan(1/
2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1) - (22*tan(1/2*b*x + 1/2*c)^3*
tan(1/2*a)^2*tan(1/2*c)^2 - 22*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^2 + 88*tan
(1/2*b*x + 1/2*c)^3*tan(1/2*a)*tan(1/2*c) + 6*tan(1/2*b*x + 1/2*c)^2*tan(1/
2*a)^2*tan(1/2*c) - 22*tan(1/2*b*x + 1/2*c)^3*tan(1/2*c)^2 - 6*tan(1/2*b*x
+ 1/2*c)^2*tan(1/2*a)*tan(1/2*c)^2 + 3*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*ta
n(1/2*c)^2 + 22*tan(1/2*b*x + 1/2*c)^3 + 6*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a
) - 3*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2 - 6*tan(1/2*b*x + 1/2*c)^2*tan(1/2*
c) + 12*tan(1/2*b*x + 1/2*c)*tan(1/2*a)*tan(1/2*c) + 2*tan(1/2*a)^2*tan(1/2
*c) - 3*tan(1/2*b*x + 1/2*c)*tan(1/2*c)^2 - 2*tan(1/2*a)*tan(1/2*c)^2 + 3*t
an(1/2*b*x + 1/2*c) + 2*tan(1/2*a) - 2*tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c
)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*tan(1/2*b*x + 1/2*c)^3))/b
```

Mupad [F(-1)]

time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(c + b*x)^4,x)
```

```
[Out] \text{Hanged}
```

3.199 $\int \csc^5(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=60

$$-\frac{\cos(a-c)\cot(c+bx)}{b}-\frac{\cos(a-c)\cot^3(c+bx)}{3b}-\frac{\csc^4(c+bx)\sin(a-c)}{4b}$$

```
[Out] -cos(a-c)*cot(b*x+c)/b-1/3*cos(a-c)*cot(b*x+c)^3/b-1/4*csc(b*x+c)^4*sin(a-c
)/b
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4678, 2686, 30, 3852}

$$-\frac{\cos(a-c)\cot^3(bx+c)}{3b}-\frac{\cos(a-c)\cot(bx+c)}{b}-\frac{\sin(a-c)\csc^4(bx+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^5*Sin[a + b*x],x]
```

```
[Out] -((Cos[a - c]*Cot[c + b*x])/b) - (Cos[a - c]*Cot[c + b*x]^3)/(3*b) - (Csc[c
 + b*x]^4*Sin[a - c])/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4678

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \csc^5(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \csc^4(c+bx)\,dx + \sin(a-c)\int \cot(c+bx)\csc^4(c+bx)\,dx$$
$$= -\frac{\cos(a-c)\text{Subst}\left(\int (1+x^2)\,dx, x, \cot(c+bx)\right)}{b} - \frac{\sin(a-c)\text{Subst}\left(\int x^3\,d\right.}{b}$$
$$= -\frac{\cos(a-c)\cot(c+bx)}{b} - \frac{\cos(a-c)\cot^3(c+bx)}{3b} - \frac{\csc^4(c+bx)\sin(a-c}{4b}$$

Mathematica [**A**]
time = 0.41, size = 58, normalized size = 0.97

$$\frac{(3\cos(a)+\cos(a-c)(-4\cos(c+2bx)+\cos(3c+4bx)))\csc\left(\frac{c}{2}\right)\csc^4(c+bx)\sec\left(\frac{c}{2}\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^5*Sin[a + b*x],x]
```

```
[Out] ((3*Cos[a] + Cos[a - c]*(-4*Cos[c + 2*b*x] + Cos[3*c + 4*b*x]))*Csc[c/2]*Cs
c[c + b*x]^4*Sec[c/2])/(24*b)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(56) = 112.
time = 2.85, size = 321, normalized size = 5.35

method	result
risch	$\frac{2i\left(6\,e^{i(4bx+9a+3c)}-4\,e^{i(2bx+9a+c)}-4\,e^{i(2bx+7a+3c)}+e^{i(9a-c)}+e^{i(7a+c)}\right)}{3\left(-e^{2i(bx+a+c)}+e^{2ia}\right)^4 b}$
default	$-\frac{1}{(\cos(a)\cos(c)+\sin(a)\sin(c))^4(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)+\cos(a)\sin(c)-\sin(a)\cos(c))} - \frac{}{2(\cos(a)\cos(c)+\sin(a)\sin(c))^4(\tan(b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^5*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/(cos(a)*cos(c)+sin(a)*sin(c))^4/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a
)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))-1/2*(3*sin(a)*cos(c)-3*cos(a)*
sin(c))/(cos(a)*cos(c)+sin(a)*sin(c))^4/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a
)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))^2-1/3*(cos(a)^2*cos(c)^2+3*cos
(c)^2*sin(a)^2-4*cos(a)*cos(c)*sin(a)*sin(c)+3*cos(a)^2*sin(c)^2+sin(a)^2*s
in(c)^2)/(cos(a)*cos(c)+sin(a)*sin(c))^4/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+
a)*sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))^3-1/4*(sin(a)*cos(c)-cos(a)*s
in(c))*(cos(a)^2*cos(c)^2+cos(c)^2*sin(a)^2+cos(a)^2*sin(c)^2+sin(a)^2*sin(
```

```
c)^2)/(cos(a)*cos(c)+sin(a)*sin(c))^4/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*
sin(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))^4)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(56) = 112$.
time = 0.32, size = 1076, normalized size = 17.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -2/3*((6*sin(4*b*x + 2*a + 4*c) - 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x +
4*c) + sin(2*a) + sin(2*c))*cos(8*b*x + a + 9*c) - 4*(6*sin(4*b*x + 2*a + 4
*c) - 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) + sin(2*c))*
cos(6*b*x + a + 7*c) + 6*(4*sin(2*b*x + a + 3*c) - sin(a + c))*cos(4*b*x +
2*a + 4*c) + 6*(6*sin(4*b*x + 2*a + 4*c) - 4*sin(2*b*x + 2*a + 2*c) - 4*sin
(2*b*x + 4*c) + sin(2*a) + sin(2*c))*cos(4*b*x + a + 5*c) + 4*(4*sin(2*b*x
+ 2*a + 2*c) - sin(2*a) - sin(2*c))*cos(2*b*x + a + 3*c) - 4*(4*sin(2*b*x +
 a + 3*c) - sin(a + c))*cos(2*b*x + 4*c) + (sin(2*a) + sin(2*c))*cos(a + c)
 - (6*cos(4*b*x + 2*a + 4*c) - 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4*c
) + cos(2*a) + cos(2*c))*sin(8*b*x + a + 9*c) + 4*(6*cos(4*b*x + 2*a + 4*c)
 - 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4*c) + cos(2*a) + cos(2*c))*sin
(6*b*x + a + 7*c) - 6*(4*cos(2*b*x + a + 3*c) - cos(a + c))*sin(4*b*x + 2*a
 + 4*c) - 6*(6*cos(4*b*x + 2*a + 4*c) - 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*
b*x + 4*c) + cos(2*a) + cos(2*c))*sin(4*b*x + a + 5*c) - 4*cos(a + c)*sin(2
*b*x + 2*a + 2*c) - 4*(4*cos(2*b*x + 2*a + 2*c) - cos(2*a) - cos(2*c))*sin(
2*b*x + a + 3*c) + 4*(4*cos(2*b*x + a + 3*c) - cos(a + c))*sin(2*b*x + 4*c)
 - (cos(2*a) + cos(2*c))*sin(a + c) + 4*cos(2*b*x + 2*a + 2*c)*sin(a + c))/
(b*cos(8*b*x + a + 9*c)^2 + 16*b*cos(6*b*x + a + 7*c)^2 + 36*b*cos(4*b*x +
a + 5*c)^2 + 16*b*cos(2*b*x + a + 3*c)^2 - 8*b*cos(2*b*x + a + 3*c)*cos(a +
 c) + b*cos(a + c)^2 + b*sin(8*b*x + a + 9*c)^2 + 16*b*sin(6*b*x + a + 7*c)
^2 + 36*b*sin(4*b*x + a + 5*c)^2 + 16*b*sin(2*b*x + a + 3*c)^2 - 8*b*sin(2*
b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(4*b*cos(6*b*x + a + 7*c) -
6*b*cos(4*b*x + a + 5*c) + 4*b*cos(2*b*x + a + 3*c) - b*cos(a + c))*cos(8*b
*x + a + 9*c) - 8*(6*b*cos(4*b*x + a + 5*c) - 4*b*cos(2*b*x + a + 3*c) + b*
cos(a + c))*cos(6*b*x + a + 7*c) - 12*(4*b*cos(2*b*x + a + 3*c) - b*cos(a +
 c))*cos(4*b*x + a + 5*c) - 2*(4*b*sin(6*b*x + a + 7*c) - 6*b*sin(4*b*x + a
 + 5*c) + 4*b*sin(2*b*x + a + 3*c) - b*sin(a + c))*sin(8*b*x + a + 9*c) - 8
*(6*b*sin(4*b*x + a + 5*c) - 4*b*sin(2*b*x + a + 3*c) + b*sin(a + c))*sin(6
*b*x + a + 7*c) - 12*(4*b*sin(2*b*x + a + 3*c) - b*sin(a + c))*sin(4*b*x +
a + 5*c))
```

Fricas [**A**]

time = 2.68, size = 75, normalized size = 1.25

$$\frac{4 \left(2 \cos \left(bx+c\right)^{3} \cos \left(-a+c\right)-3 \cos \left(bx+c\right) \cos \left(-a+c\right)\right) \sin \left(bx+c\right)+3 \sin \left(-a+c\right)}{12 \left(b \cos \left(bx+c\right)^{4}-2 \, b \cos \left(bx+c\right)^{2}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(4*(2*cos(b*x + c)^3*cos(-a + c) - 3*cos(b*x + c)*cos(-a + c))*sin(b*x
 + c) + 3*sin(-a + c))/(b*cos(b*x + c)^4 - 2*b*cos(b*x + c)^2 + b)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**5*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(56) = 112.
time = 0.42, size = 301, normalized size = 5.02

$$-\frac{6 \tan \left(bx+c\right)^{3} \tan \left(\frac{1}{2} a\right)^{2} \tan \left(\frac{1}{2} c\right)^{2}-6 \tan \left(bx+c\right)^{3} \tan \left(\frac{1}{2} a\right)^{2}+24 \tan \left(bx+c\right)^{3} \tan \left(\frac{1}{2} a\right) \tan \left(\frac{1}{2} c\right)+6 \tan \left(bx+c\right)^{2} \tan \left(\frac{1}{2} a\right)^{2} \tan \left(\frac{1}{2} c\right)-6 \tan \left(bx+c\right)^{3} \tan \left(\frac{1}{2} c\right)^{2}-6 \tan \left(bx+c\right)^{2} \tan \left(\frac{1}{2} a\right) \tan \left(\frac{1}{2} c\right)^{2}+2 \tan \left(bx+c\right) \tan \left(\frac{1}{2} a\right)^{2} \tan \left(\frac{1}{2} c\right)^{2}+6 \tan \left(bx+c\right)^{3}+6 \tan \left(bx+c\right)^{2} \tan \left(\frac{1}{2} a\right)-2 \tan \left(bx+c\right) \tan \left(\frac{1}{2} a\right)^{2}-6 \tan \left(bx+c\right)^{2} \tan \left(\frac{1}{2} c\right)+8 \tan \left(bx+c\right) \tan \left(\frac{1}{2} a\right) \tan \left(\frac{1}{2} c\right)+3 \tan \left(\frac{1}{2} a\right)^{2} \tan \left(\frac{1}{2} c\right)-2 \tan \left(bx+c\right) \tan \left(\frac{1}{2} c\right)^{2}-3 \tan \left(\frac{1}{2} a\right) \tan \left(\frac{1}{2} c\right)^{2}+2 \tan \left(bx+c\right)+3 \tan \left(\frac{1}{2} a\right)-3 \tan \left(\frac{1}{2} c\right)}{6 \left(\tan \left(\frac{1}{2} a\right)^{2} \tan \left(\frac{1}{2} c\right)^{2}+\tan \left(\frac{1}{2} a\right)^{2}+\tan \left(\frac{1}{2} c\right)^{2}+1\right) b \tan \left(bx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^5*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*(6*tan(b*x + c)^3*tan(1/2*a)^2*tan(1/2*c)^2 - 6*tan(b*x + c)^3*tan(1/2
*a)^2 + 24*tan(b*x + c)^3*tan(1/2*a)*tan(1/2*c) + 6*tan(b*x + c)^2*tan(1/2*
a)^2*tan(1/2*c) - 6*tan(b*x + c)^3*tan(1/2*c)^2 - 6*tan(b*x + c)^2*tan(1/2*
a)*tan(1/2*c)^2 + 2*tan(b*x + c)*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(b*x + c)
^3 + 6*tan(b*x + c)^2*tan(1/2*a) - 2*tan(b*x + c)*tan(1/2*a)^2 - 6*tan(b*x
+ c)^2*tan(1/2*c) + 8*tan(b*x + c)*tan(1/2*a)*tan(1/2*c) + 3*tan(1/2*a)^2*t
an(1/2*c) - 2*tan(b*x + c)*tan(1/2*c)^2 - 3*tan(1/2*a)*tan(1/2*c)^2 + 2*tan
(b*x + c) + 3*tan(1/2*a) - 3*tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(
1/2*a)^2 + tan(1/2*c)^2 + 1)*b*tan(b*x + c)^4)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(c + b*x)^5,x)
```

```
[Out] \text{Hanged}
```

3.200 $\int \csc^6(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=94

$$-\frac{3\tanh^{-1}(\cos(c+bx))\cos(a-c)}{8b}-\frac{3\cos(a-c)\cot(c+bx)\csc(c+bx)}{8b}-\frac{\cos(a-c)\cot(c+bx)\csc^3(c+bx)}{4b}$$

```
[Out] -3/8*arctanh(cos(b*x+c))*cos(a-c)/b-3/8*cos(a-c)*cot(b*x+c)*csc(b*x+c)/b-1/
4*cos(a-c)*cot(b*x+c)*csc(b*x+c)^3/b-1/5*csc(b*x+c)^5*sin(a-c)/b
```

Rubi [A]
time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4678, 2686, 30, 3853, 3855}

$$-\frac{3\cos(a-c)\tanh^{-1}(\cos(bx+c))}{8b}-\frac{\sin(a-c)\csc^5(bx+c)}{5b}-\frac{\cos(a-c)\cot(bx+c)\csc^3(bx+c)}{4b}-\frac{3\cos(a-c)\cot(bx+c)\csc(bx+c)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + b*x]^6*Sin[a + b*x],x]
```

```
[Out] (-3*ArcTanh[Cos[c + b*x]]*Cos[a - c])/(8*b) - (3*Cos[a - c]*Cot[c + b*x]*Cs
c[c + b*x])/(8*b) - (Cos[a - c]*Cot[c + b*x]*Csc[c + b*x]^3)/(4*b) - (Csc[c
 + b*x]^5*Sin[a - c])/(5*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4678

```
Int[Csc[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Sin[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] + Dist[Cos[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \csc^6(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \csc^5(c+bx)\,dx + \sin(a-c)\int \cot(c+bx)\csc^5(c+bx)\,dx$$
$$= -\frac{\cos(a-c)\cot(c+bx)\csc^3(c+bx)}{4b} + \frac{1}{4}(3\cos(a-c))\int \csc^3(c+bx)\,dx -$$
$$= -\frac{3\cos(a-c)\cot(c+bx)\csc(c+bx)}{8b} - \frac{\cos(a-c)\cot(c+bx)\csc^3(c+bx)}{4b}$$
$$= -\frac{3\tanh^{-1}(\cos(c+bx))\cos(a-c)}{8b} - \frac{3\cos(a-c)\cot(c+bx)\csc(c+bx)}{8b} -$$

Mathematica [A]
time = 1.23, size = 79, normalized size = 0.84

$$-\frac{480\tanh^{-1}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+2\csc^5(c+bx)(64\sin(a-c)+5\cos(a-c)(14\sin(2(c+bx))-3\sin(4(c+bx))))}{640b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + b*x]^6*Sin[a + b*x],x]
[Out] -1/640*(480*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + 2*Csc[c + b*
x]^5*(64*Sin[a - c] + 5*Cos[a - c]*(14*Sin[2*(c + b*x)] - 3*Sin[4*(c + b*x)
])))/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6745 vs. 2(86) = 172.
time = 6.25, size = 6746, normalized size = 71.77

method	result
risch	$\frac{-15\,e^{i(9bx+11a+8c)}-15\,e^{i(9bx+9a+10c)}+70\,e^{i(7bx+11a+6c)}+70\,e^{i(7bx+9a+8c)}+128\,e^{i(5bx+11a+4c)}-128\,e^{i(5bx+9a+6c)}-70\,e^{i(3bx+11a}}{40b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)^5}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+c)^6*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3879 vs. 2(86) = 172.
time = 0.54, size = 3879, normalized size = 41.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/80*(2*(15*cos(9*b*x + 2*a + 8*c) + 15*cos(9*b*x + 10*c) - 70*cos(7*b*x +
2*a + 6*c) - 70*cos(7*b*x + 8*c) - 128*cos(5*b*x + 2*a + 4*c) + 128*cos(5*b
*x + 6*c) + 70*cos(3*b*x + 2*a + 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*x +
2*a) - 15*cos(b*x + 2*c))*cos(10*b*x + a + 10*c) - 30*(5*cos(8*b*x + a + 8*
c) - 10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) - 5*cos(2*b*x + a +
2*c) + cos(a))*cos(9*b*x + 2*a + 8*c) - 30*(5*cos(8*b*x + a + 8*c) - 10*cos
(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) - 5*cos(2*b*x + a + 2*c) + cos(
a))*cos(9*b*x + 10*c) + 10*(70*cos(7*b*x + 2*a + 6*c) + 70*cos(7*b*x + 8*c)
 + 128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*x + 6*c) - 70*cos(3*b*x + 2*a +
 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x + 2*a) + 15*cos(b*x + 2*c))*cos(8*
b*x + a + 8*c) - 140*(10*cos(6*b*x + a + 6*c) - 10*cos(4*b*x + a + 4*c) + 5
*cos(2*b*x + a + 2*c) - cos(a))*cos(7*b*x + 2*a + 6*c) - 140*(10*cos(6*b*x
+ a + 6*c) - 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c) - cos(a))*cos
(7*b*x + 8*c) - 20*(128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*x + 6*c) - 70*
cos(3*b*x + 2*a + 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x + 2*a) + 15*cos(b
*x + 2*c))*cos(6*b*x + a + 6*c) + 256*(10*cos(4*b*x + a + 4*c) - 5*cos(2*b*
x + a + 2*c) + cos(a))*cos(5*b*x + 2*a + 4*c) - 256*(10*cos(4*b*x + a + 4*c
) - 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 6*c) - 100*(14*cos(3*b*x +
 2*a + 2*c) + 14*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*co
s(4*b*x + a + 4*c) + 140*(5*cos(2*b*x + a + 2*c) - cos(a))*cos(3*b*x + 2*a
+ 2*c) + 140*(5*cos(2*b*x + a + 2*c) - cos(a))*cos(3*b*x + 4*c) - 150*(cos(
b*x + 2*a) + cos(b*x + 2*c))*cos(2*b*x + a + 2*c) + 30*cos(b*x + 2*a)*cos(a
) + 30*cos(b*x + 2*c)*cos(a) - 15*(cos(10*b*x + a + 10*c)^2*cos(-a + c) + 2
5*cos(8*b*x + a + 8*c)^2*cos(-a + c) + 100*cos(6*b*x + a + 6*c)^2*cos(-a +
c) + 100*cos(4*b*x + a + 4*c)^2*cos(-a + c) + 25*cos(2*b*x + a + 2*c)^2*cos
(-a + c) - 10*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos(-a + c)*sin(10*
b*x + a + 10*c)^2 + 25*cos(-a + c)*sin(8*b*x + a + 8*c)^2 + 100*cos(-a + c)
*sin(6*b*x + a + 6*c)^2 + 100*cos(-a + c)*sin(4*b*x + a + 4*c)^2 + 25*cos(-
a + c)*sin(2*b*x + a + 2*c)^2 - 10*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a)
- 2*(5*cos(8*b*x + a + 8*c)*cos(-a + c) - 10*cos(6*b*x + a + 6*c)*cos(-a +
c) + 10*cos(4*b*x + a + 4*c)*cos(-a + c) - 5*cos(2*b*x + a + 2*c)*cos(-a +
c) + cos(a)*cos(-a + c))*cos(10*b*x + a + 10*c) - 10*(10*cos(6*b*x + a + 6*
c)*cos(-a + c) - 10*cos(4*b*x + a + 4*c)*cos(-a + c) + 5*cos(2*b*x + a + 2*
c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(8*b*x + a + 8*c) - 20*(10*cos(4*b*
x + a + 4*c)*cos(-a + c) - 5*cos(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*cos(
-a + c))*cos(6*b*x + a + 6*c) - 20*(5*cos(2*b*x + a + 2*c)*cos(-a + c) - co
```

```
s(a)*cos(-a + c))*cos(4*b*x + a + 4*c) + (cos(a)^2 + sin(a)^2)*cos(-a + c)
- 2*(5*cos(-a + c)*sin(8*b*x + a + 8*c) - 10*cos(-a + c)*sin(6*b*x + a + 6*
c) + 10*cos(-a + c)*sin(4*b*x + a + 4*c) - 5*cos(-a + c)*sin(2*b*x + a + 2*
c) + cos(-a + c)*sin(a))*sin(10*b*x + a + 10*c) - 10*(10*cos(-a + c)*sin(6*
b*x + a + 6*c) - 10*cos(-a + c)*sin(4*b*x + a + 4*c) + 5*cos(-a + c)*sin(2*
b*x + a + 2*c) - cos(-a + c)*sin(a))*sin(8*b*x + a + 8*c) - 20*(10*cos(-a +
 c)*sin(4*b*x + a + 4*c) - 5*cos(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)
*sin(a))*sin(6*b*x + a + 6*c) - 20*(5*cos(-a + c)*sin(2*b*x + a + 2*c) - co
s(-a + c)*sin(a))*sin(4*b*x + a + 4*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c)
+ cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + 15*(cos(10*b*x +
a + 10*c)^2*cos(-a + c) + 25*cos(8*b*x + a + 8*c)^2*cos(-a + c) + 100*cos(6
*b*x + a + 6*c)^2*cos(-a + c) + 100*cos(4*b*x + a + 4*c)^2*cos(-a + c) + 25
*cos(2*b*x + a + 2*c)^2*cos(-a + c) - 10*cos(2*b*x + a + 2*c)*cos(a)*cos(-a
 + c) + cos(-a + c)*sin(10*b*x + a + 10*c)^2 + 25*cos(-a + c)*sin(8*b*x + a
 + 8*c)^2 + 100*cos(-a + c)*sin(6*b*x + a + 6*c)^2 + 100*cos(-a + c)*sin(4*
b*x + a + 4*c)^2 + 25*cos(-a + c)*sin(2*b*x + a + 2*c)^2 - 10*cos(-a + c)*s
in(2*b*x + a + 2*c)*sin(a) - 2*(5*cos(8*b*x + a + 8*c)*cos(-a + c) - 10*cos
(6*b*x + a + 6*c)*cos(-a + c) + 10*cos(4*b*x + a + 4*c)*cos(-a + c) - 5*cos
(2*b*x + a + 2*c)*cos(-a + c) + cos(a)*cos(-a + c))*cos(10*b*x + a + 10*c)
- 10*(10*cos(6*b*x + a + 6*c)*cos(-a + c) - 10*cos(4*b*x + a + 4*c)*cos(-a
+ c) + 5*cos(2*b*x + a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(8*b*x +
 a + 8*c) - 20*(10*cos(4*b*x + a + 4*c)*cos(-a + c) - 5*cos(2*b*x + a + 2*c
)*cos(-a + c) + cos(a)*cos(-a + c))*cos(6*b*x + a + 6*c) - 20*(5*cos(2*b*x
+ a + 2*c)*cos(-a + c) - cos(a)*cos(-a + c))*cos(4*b*x + a + 4*c) + (cos(a)
^2 + sin(a)^2)*cos(-a + c) - 2*(5*cos(-a + c)*sin(8*b*x + a + 8*c) - 10*cos
(-a + c)*sin(6*b*x + a + 6*c) + 10*cos(-a + c)*sin(4*b*x + a + 4*c) - 5*cos
(-a + c)*sin(2*b*x + a + 2*c) + cos(-a + c)*sin(a))*sin(10*b*x + a + 10*c)
- 10*(10*cos(-a + c)*sin(6*b*x + a + 6*c) - 10*cos(-a + c)*sin(4*b*x + a +
4*c) + 5*cos(-a + c)*sin(2*b*x + a + 2*c) - cos(-a + c)*sin(a))*sin(8*b*x +
 a + 8*c) - 20*(10*cos(-a + c)*sin(4*b*x + a + 4*c) - 5*cos(-a + c)*sin(2*b
*x + a + 2*c) + cos(-a + c)*sin(a))*sin(6*b*x +...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(86) = 172$.
time = 2.91, size = 197, normalized size = 2.10

$$-\frac{15\left(\cos(bx+c)^4\cos(-a+c)-2\cos(bx+c)^2\cos(-a+c)+\cos(-a+c)\right)\log\left(\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c)-15\left(\cos(bx+c)^4\cos(-a+c)-2\cos(bx+c)^2\cos(-a+c)+\cos(-a+c)\right)\log\left(-\frac{1}{2}\cos(bx+c)+\frac{1}{2}\right)\sin(bx+c)-10\left(3\cos(bx+c)^3\cos(-a+c)-5\cos(bx+c)\cos(-a+c)\right)\sin(bx+c)-16\sin(-a+c)}{80\left(b\cos(bx+c)^4-2b\cos(bx+c)^2+b\right)\sin(bx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/80*(15*(cos(b*x + c)^4*cos(-a + c) - 2*cos(b*x + c)^2*cos(-a + c) + cos(
-a + c))*log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c) - 15*(cos(b*x + c)^4*cos(
-a + c) - 2*cos(b*x + c)^2*cos(-a + c) + cos(-a + c))*log(-1/2*cos(b*x + c)
 + 1/2)*sin(b*x + c) - 10*(3*cos(b*x + c)^3*cos(-a + c) - 5*cos(b*x + c)*co
```

```
s(-a + c))*sin(b*x + c) - 16*sin(-a + c))/((b*cos(b*x + c)^4 - 2*b*cos(b*x
+ c)^2 + b)*sin(b*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)**6*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 8035 vs. 2(86) = 172.
time = 0.48, size = 8035, normalized size = 85.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+c)^6*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/320*(120*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2
*c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x + 1/2*c)))/(tan(1/2*a)^2*tan(1/
2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (4*tan(1/2*b*x + 1/2*c)^5*tan(1
/2*a)^10*tan(1/2*c)^9 - 4*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^9*tan(1/2*c)^10
 - 5*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^10*tan(1/2*c)^10 + 16*tan(1/2*b*x +
1/2*c)^5*tan(1/2*a)^10*tan(1/2*c)^7 - 12*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^
9*tan(1/2*c)^8 - 15*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^10*tan(1/2*c)^8 + 12*
tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^8*tan(1/2*c)^9 - 20*tan(1/2*b*x + 1/2*c)^
4*tan(1/2*a)^9*tan(1/2*c)^9 + 20*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^10*tan(1
/2*c)^9 - 16*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^7*tan(1/2*c)^10 - 15*tan(1/2
*b*x + 1/2*c)^4*tan(1/2*a)^8*tan(1/2*c)^10 - 20*tan(1/2*b*x + 1/2*c)^3*tan(
1/2*a)^9*tan(1/2*c)^10 - 40*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^10*tan(1/2*c)
^10 + 24*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^10*tan(1/2*c)^5 - 8*tan(1/2*b*x
+ 1/2*c)^5*tan(1/2*a)^9*tan(1/2*c)^6 - 10*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)
^10*tan(1/2*c)^6 + 48*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^8*tan(1/2*c)^7 - 80
*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^9*tan(1/2*c)^7 + 80*tan(1/2*b*x + 1/2*c)
^3*tan(1/2*a)^10*tan(1/2*c)^7 - 48*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^7*tan(
1/2*c)^8 - 45*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^8*tan(1/2*c)^8 - 60*tan(1/2
*b*x + 1/2*c)^3*tan(1/2*a)^9*tan(1/2*c)^8 - 120*tan(1/2*b*x + 1/2*c)^2*tan(
1/2*a)^10*tan(1/2*c)^8 + 8*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^6*tan(1/2*c)^9
 - 80*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^7*tan(1/2*c)^9 + 60*tan(1/2*b*x + 1
/2*c)^3*tan(1/2*a)^8*tan(1/2*c)^9 - 160*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^9
*tan(1/2*c)^9 + 40*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^10*tan(1/2*c)^9 - 24*tan
(1/2*b*x + 1/2*c)^5*tan(1/2*a)^5*tan(1/2*c)^10 - 10*tan(1/2*b*x + 1/2*c)^4*
```

```
tan(1/2*a)^6*tan(1/2*c)^10 - 80*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^7*tan(1/2
*c)^10 - 120*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^8*tan(1/2*c)^10 - 40*tan(1/2
*b*x + 1/2*c)*tan(1/2*a)^9*tan(1/2*c)^10 + 16*tan(1/2*b*x + 1/2*c)^5*tan(1/
2*a)^10*tan(1/2*c)^3 + 8*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^9*tan(1/2*c)^4 +
 10*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^10*tan(1/2*c)^4 + 72*tan(1/2*b*x + 1/
2*c)^5*tan(1/2*a)^8*tan(1/2*c)^5 - 120*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^9*
tan(1/2*c)^5 + 120*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^10*tan(1/2*c)^5 - 32*t
an(1/2*b*x + 1/2*c)^5*tan(1/2*a)^7*tan(1/2*c)^6 - 30*tan(1/2*b*x + 1/2*c)^4
*tan(1/2*a)^8*tan(1/2*c)^6 - 40*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^9*tan(1/2
*c)^6 - 80*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^10*tan(1/2*c)^6 + 32*tan(1/2*b
*x + 1/2*c)^5*tan(1/2*a)^6*tan(1/2*c)^7 - 320*tan(1/2*b*x + 1/2*c)^4*tan(1/
2*a)^7*tan(1/2*c)^7 + 240*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^8*tan(1/2*c)^7
- 640*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^9*tan(1/2*c)^7 + 160*tan(1/2*b*x +
1/2*c)*tan(1/2*a)^10*tan(1/2*c)^7 - 72*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^5*
tan(1/2*c)^8 - 30*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^6*tan(1/2*c)^8 - 240*ta
n(1/2*b*x + 1/2*c)^3*tan(1/2*a)^7*tan(1/2*c)^8 - 360*tan(1/2*b*x + 1/2*c)^2
*tan(1/2*a)^8*tan(1/2*c)^8 - 120*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^9*tan(1/2*
c)^8 - 8*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^4*tan(1/2*c)^9 - 120*tan(1/2*b*x
 + 1/2*c)^4*tan(1/2*a)^5*tan(1/2*c)^9 + 40*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a
)^6*tan(1/2*c)^9 - 640*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c)^9 + 1
20*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^8*tan(1/2*c)^9 - 16*tan(1/2*b*x + 1/2*c)
^5*tan(1/2*a)^3*tan(1/2*c)^10 + 10*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^4*tan(
1/2*c)^10 - 120*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^5*tan(1/2*c)^10 - 80*tan(
1/2*b*x + 1/2*c)^2*tan(1/2*a)^6*tan(1/2*c)^10 - 160*tan(1/2*b*x + 1/2*c)*ta
n(1/2*a)^7*tan(1/2*c)^10 + 4*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^10*tan(1/2*c
) + 12*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^9*tan(1/2*c)^2 + 15*tan(1/2*b*x +
1/2*c)^4*tan(1/2*a)^10*tan(1/2*c)^2 + 48*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^
8*tan(1/2*c)^3 - 80*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^9*tan(1/2*c)^3 + 80*t
an(1/2*b*x + 1/2*c)^3*tan(1/2*a)^10*tan(1/2*c)^3 + 32*tan(1/2*b*x + 1/2*c)^
5*tan(1/2*a)^7*tan(1/2*c)^4 + 30*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^8*tan(1/
2*c)^4 + 40*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^9*tan(1/2*c)^4 + 80*tan(1/2*b
*x + 1/2*c)^2*tan(1/2*a)^10*tan(1/2*c)^4 + 48*tan(1/2*b*x + 1/2*c)^5*tan(1/
2*a)^6*tan(1/2*c)^5 - 480*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^7*tan(1/2*c)^5
+ 360*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^8*tan(1/2*c)^5 - 960*tan(1/2*b*x +
1/2*c)^2*tan(1/2*a)^9*tan(1/2*c)^5 + 240*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^10
*tan(1/2*c)^5 - 48*tan(1/2*b*x + 1/2*c)^5*tan(1/2*a)^5*tan(1/2*c)^6 - 20*ta
n(1/2*b*x + 1/2*c)^4*tan(1/2*a)^6*tan(1/2*c)^6 - 160*tan(1/2*b*x + 1/2*c)^3
*tan(1/2*a)^7*tan(1/2*c)^6 - 240*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^8*tan(1/
2*c)^6 - 80*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^9*tan(1/2*c)^6 - 32*tan(1/2*b*x
 + 1/2*c)^5*tan(1/2*a)^4*tan(1/2*c)^7 - 480*tan(1/2*b*x + 1/2*c)^4*tan(1/2*
a)^5*tan(1/2*c)^7 + 160*tan(1/2*b*x + 1/2*c)^3*tan(1/2*a)^6*tan(1/2*c)^7 -
2560*tan(1/2*b*x + 1/2*c)^2*tan(1/2*a)^7*tan(1/2*c)^7 + 480*tan(1/2*b*x + 1
/2*c)*tan(1/2*a)^8*tan(1/2*c)^7 - 48*tan(1/2*b*...
```

<u>Mupad</u> [F(-1)]

time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/sin(c + b*x)^6,x)
[Out] \text{Hanged}
```

3.201 $\int \sin^2(a+bx)\sin^n(c+dx)\,dx$

Optimal. Leaf size=410

$$-\frac{i2^{-2-n}e^{-i(2a+cn)-i(2b+dn)x+in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n{}_2F_1\left(\frac{1}{2}\left(-\frac{2b}{d}-n\right),-n;\frac{1}{2}\left(2-\frac{2b}{d}\right.\right.}{2b+dn}$$

[Out] -I*2^(-2-n)*exp(-I*(c*n+2*a)-I*(d*n+2*b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-n, -b/d-1/2*n],[1-b/d-1/2*n],exp(2*I*(d*x+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(d*n+2*b)+I*2^(-2-n)*exp(I*(-c*n+2*a)+I*(-d*n+2*b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-n, b/d-1/2*n],[1+b/d-1/2*n],exp(2*I*(d*x+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(-d*n+2*b)+I*2^(-1-n)*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-n, -1/2*n],[1-1/2*n],exp(2*I*(d*x+c)))/d/((1-exp(2*I*(d*x+c)))^n)/n

Rubi [A]
time = 0.69, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$,
Rules used = {4649, 2320, 2005, 2057, 372, 371, 2323, 2285, 2284, 2283}

$$-\frac{i2^{-n-2}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(-\frac{2b}{d}-n\right),-n;\frac{1}{2}\left(-\frac{2b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(-i(2a+cn)-ix(2b+dn)+in(c+dx))}{2b+dn}+\frac{i2^{-n-2}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(\frac{2b}{d}-n\right),-n;\frac{1}{2}\left(\frac{2b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(i(2a-cn)+ix(2b-dn)+in(c+dx))}{2b-dn}+\frac{i2^{-n-1}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2i(c+dx)}\right)^{-n}{}_2F_1\left(-n,-\frac{n}{2};1-\frac{n}{2};e^{2i(c+dx)}\right)}{dn}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b*x]^2*Sin[c + d*x]^n,x]

[Out] ((-I)*2^(-2 - n)*E^((-I)*(2*a + c*n) - I*(2*b + d*n)*x + I*n*(c + d*x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((-2*b)/d - n)/2, -n, (2 - (2*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*(2*b + d*n)) + (I*2^(-2 - n)*E^(I*(2*a - c*n) + I*(2*b - d*n)*x + I*n*(c + d*x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((2*b)/d - n)/2, -n, (2 + (2*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*(2*b - d*n)) + (I*2^(-1 - n)*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -1/2*n, 1 - n/2, E^((2*I)*(c + d*x))])/(d*(1 - E^((2*I)*(c + d*x)))^n*n)

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
```

```
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2005

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !Generali
zedBinomialMatchQ[u, x]
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2284

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Dist[(a + b*F^(e*(c + d*x)))^p/(1 + (b/a)*F^
(e*(c + d*x)))^p, Int[G^(h*(f + g*x))*(1 + (b/a)*F^(e*(c + d*x)))^p, x], x]
 /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] &&  !(ILtQ[p, 0] || GtQ[a,
0])
```

Rule 2285

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2323

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 4649

```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \sin^2(a+bx)\sin^n(c+dx)\,dx = 2^{-2-n}\int \left(2\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n - e^{-2ia-2ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n - \right.$$

$$= -\left(2^{-2-n}\int e^{-2ia-2ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\,dx\right) - 2^{-2-n}\int e^{2ia+2ibx}\left(ie\right.$$

$$= -\frac{\left(i2^{-1-n}\right)\text{Subst}\left(\int \frac{\left(-\frac{i\left(-1+x^2\right)}{x}\right)^n}{x}\,dx, x, e^{i(c+dx)}\right)}{d} - \left(2^{-2-n}e^{in(c+dx)}\left(i-i\right.\right.$$

$$= -\frac{\left(i2^{-1-n}\right)\text{Subst}\left(\int \frac{\left(\frac{i}{x}-ix\right)^n}{x}\,dx, x, e^{i(c+dx)}\right)}{d} - \left(2^{-2-n}e^{in(c+dx)}\left(i-ie^{2ic+2id}\right.\right.$$

$$= -\left(\left(2^{-2-n}e^{in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(2a-cn)+}\right.$$

$$= -\frac{i2^{-2-n}\exp(-i(2a+cn)-i(2b+dn)x+in(c+dx))\left(1-e^{2ic+2idx}\right)^{-n}\left(\right.}{2b+}$$

$$= -\frac{i2^{-2-n}\exp(-i(2a+cn)-i(2b+dn)x+in(c+dx))\left(1-e^{2ic+2idx}\right)^{-n}\left(\right.}{2b+}$$

Mathematica [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int \sin^2(a+bx)\sin^n(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b*x]^2*Sin[c + d*x]^n,x]
```

```
[Out] Integrate[Sin[a + b*x]^2*Sin[c + d*x]^n, x]
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\sin^2\left(bx+a\right)\right)\left(\sin^n\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(d*x+c)^n,x)
```

```
[Out] int(sin(b*x+a)^2*sin(d*x+c)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(d*x + c)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(a+b\,x)^2\,\sin(c+d\,x)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(c + d*x)^n,x)
```

```
[Out] int(sin(a + b*x)^2*sin(c + d*x)^n, x)
```

3.202 $\int \sin^2(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=68

$$-\frac{\cos(2a-c+(2b-d)x)}{4(2b-d)}-\frac{\cos(c+dx)}{2d}+\frac{\cos(2a+c+(2b+d)x)}{4(2b+d)}$$

```
[Out] -1/4*cos(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*cos(d*x+c)/d+1/4*cos(2*a+c+(2*b+d)*x)
/(2*b+d)
```

Rubi [A]

time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {4665, 2718}

$$-\frac{\cos(2a+x(2b-d)-c)}{4(2b-d)}+\frac{\cos(2a+x(2b+d)+c)}{4(2b+d)}-\frac{\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[c + d*x],x]
```

```
[Out] -1/4*Cos[2*a - c + (2*b - d)*x]/(2*b - d) - Cos[c + d*x]/(2*d) + Cos[2*a +
c + (2*b + d)*x]/(4*(2*b + d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4665

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin(c+dx)\,dx &= \int \left(\frac{1}{4}\sin(2a-c+(2b-d)x)+\frac{1}{2}\sin(c+dx)-\frac{1}{4}\sin(2a+c+(2b+d)x)\right)\\ &= \frac{1}{4}\int \sin(2a-c+(2b-d)x)\,dx-\frac{1}{4}\int \sin(2a+c+(2b+d)x)\,dx+\frac{1}{2}\int \sin\\ &= -\frac{\cos(2a-c+(2b-d)x)}{4(2b-d)}-\frac{\cos(c+dx)}{2d}+\frac{\cos(2a+c+(2b+d)x)}{4(2b+d)}\end{aligned}$$

Mathematica [A]
time = 0.37, size = 80, normalized size = 1.18

$$-\frac{\cos(2a-c+2bx-dx)}{4(2b-d)}+\frac{\cos(2a+c+(2b+d)x)}{4(2b+d)}+\frac{1}{2}\left(-\frac{\cos(c)\cos(dx)}{d}+\frac{\sin(c)\sin(dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[c + d*x],x]
```

```
[Out] -1/4*Cos[2*a - c + 2*b*x - d*x]/(2*b - d) + Cos[2*a + c + (2*b + d)*x]/(4*(
2*b + d)) + (-((Cos[c]*Cos[d*x])/d) + (Sin[c]*Sin[d*x])/d)/2
```

Maple [A]
time = 0.21, size = 63, normalized size = 0.93

method	result
default	$-\frac{\cos(2a-c+(2b-d)x)}{4(2b-d)}-\frac{\cos(dx+c)}{2d}+\frac{\cos(2a+c+(2b+d)x)}{8b+4d}$
risch	$-\frac{2\cos(dx+c)b^2}{(2b+d)(2b-d)d}+\frac{d\cos(dx+c)}{2(2b+d)(2b-d)}-\frac{\cos(2bx-dx+2a-c)b}{2(2b+d)(2b-d)}-\frac{d\cos(2bx-dx+2a-c)}{4(2b+d)(2b-d)}+\frac{\cos(2bx+dx+2a+c)b}{2(2b+d)(2b-d)}-\frac{d\cos(2bx+\cdots}{4(2b+d)\cdots}$
norman	$\frac{-\frac{4b^2}{d(4b^2-d^2)}-\frac{4b^2\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{d(4b^2-d^2)}-\frac{8b\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4b^2-d^2}+\frac{8b\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4b^2-d^2}-\frac{4d\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4b^2-d^2}+\frac{2(-\cdots}{\cdots}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*cos(2*a-c+(2*b-d)*x)/(2*b-d)-1/2*cos(d*x+c)/d+1/4*cos(2*a+c+(2*b+d)*x)
/(2*b+d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(62) = 124.
time = 0.31, size = 371, normalized size = 5.46

$$-\frac{(2bd\cos(c)-d^2\cos(c))\cos((2b+d)x+2a+2c)+(2bd\cos(c)-d^2\cos(c))\cos((2b+d)x+2a)-(2bd\cos(c)+d^2\cos(c))\cos(-(2b-d)x-2a+2c)-(2bd\cos(c)+d^2\cos(c))\cos(-(2b-d)x-2a)-2(4b^2\cos(c)-d^2\cos(c))\cos(dx+2c)-2(4b^2\cos(c)-d^2\cos(c))\cos(dx)+(2bd\sin(c)-d^2\sin(c))\sin((2b+d)x+2a+2c)-(2bd\sin(c)-d^2\sin(c))\sin((2b+d)x+2a)-(2bd\sin(c)+d^2\sin(c))\sin(-(2b-d)x-2a+2c)+(2bd\sin(c)+d^2\sin(c))\sin(-(2b-d)x-2a)-2(4b^2\sin(c)-d^2\sin(c))\sin(dx+2c)+2(4b^2\sin(c)-d^2\sin(c))\sin(dx)}{8((\cos(c)^2+\sin(c)^2)d^3-4(b^2\cos(c)^2+b^2\sin(c)^2)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*((2*b*d*cos(c) - d^2*cos(c))*cos((2*b + d)*x + 2*a + 2*c) + (2*b*d*cos
(c) - d^2*cos(c))*cos((2*b + d)*x + 2*a) - (2*b*d*cos(c) + d^2*cos(c))*cos(
-(2*b - d)*x - 2*a + 2*c) - (2*b*d*cos(c) + d^2*cos(c))*cos(-(2*b - d)*x -
2*a) - 2*(4*b^2*cos(c) - d^2*cos(c))*cos(d*x + 2*c) - 2*(4*b^2*cos(c) - d^2
*cos(c))*cos(d*x) + (2*b*d*sin(c) - d^2*sin(c))*sin((2*b + d)*x + 2*a + 2*c
) - (2*b*d*sin(c) - d^2*sin(c))*sin((2*b + d)*x + 2*a) - (2*b*d*sin(c) + d^
2*sin(c))*sin(-(2*b - d)*x - 2*a + 2*c) + (2*b*d*sin(c) + d^2*sin(c))*sin(-
```

```
(2*b - d)*x - 2*a) - 2*(4*b^2*sin(c) - d^2*sin(c))*sin(d*x + 2*c) + 2*(4*b^
2*sin(c) - d^2*sin(c))*sin(d*x))/((cos(c)^2 + sin(c)^2)*d^3 - 4*(b^2*cos(c)
^2 + b^2*sin(c)^2)*d)
```

Fricas [A]
time = 3.17, size = 69, normalized size = 1.01

$$-\frac{2\,bd\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(dx+c\right)+\left(d^2\cos\left(bx+a\right)^2+2\,b^2-d^2\right)\cos\left(dx+c\right)}{4\,b^2d-d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(2*b*d*cos(b*x + a)*sin(b*x + a)*sin(d*x + c) + (d^2*cos(b*x + a)^2 + 2*b^
2 - d^2)*cos(d*x + c))/(4*b^2*d - d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(49) = 98$.
time = 0.85, size = 410, normalized size = 6.03

$$\begin{cases} x\sin^2\left(a\right)\sin\left(c\right) & \text{for}\ b=0\wedge d=0 \\ \frac{x\sin^2\left(a-\frac{dx}{2}\right)\sin\left(c+dx\right)}{4}-\frac{x\sin\left(a-\frac{dx}{2}\right)\cos\left(a-\frac{dx}{2}\right)\cos\left(c+dx\right)}{2}-\frac{x\sin\left(c+dx\right)\cos^2\left(a-\frac{dx}{2}\right)}{4}+\frac{3\sin\left(a-\frac{dx}{2}\right)\sin\left(c+dx\right)\cos\left(a-\frac{dx}{2}\right)}{2d}-\frac{\cos^2\left(a-\frac{dx}{2}\right)\cos\left(c+dx\right)}{d} & \text{for}\ b=-\frac{d}{2} \\ \frac{x\sin^2\left(a+\frac{dx}{2}\right)\sin\left(c+dx\right)}{4}+\frac{x\sin\left(a+\frac{dx}{2}\right)\cos\left(a+\frac{dx}{2}\right)\cos\left(c+dx\right)}{2}-\frac{x\sin\left(c+dx\right)\cos^2\left(a+\frac{dx}{2}\right)}{4}-\frac{\sin^2\left(a+\frac{dx}{2}\right)\cos\left(c+dx\right)}{d}+\frac{\sin\left(a+\frac{dx}{2}\right)\sin\left(c+dx\right)\cos\left(a+\frac{dx}{2}\right)}{2d} & \text{for}\ b=\frac{d}{2} \\ \left(\frac{x\sin^2\left(a+bx\right)}{2}+\frac{x\cos^2\left(a+bx\right)}{2}-\frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b}\right)\sin\left(c\right) & \text{for}\ d=0 \\ -\frac{2b^2\sin^2\left(a+bx\right)\cos\left(c+dx\right)}{4b^2d-d^3}-\frac{2b^2\cos^2\left(a+bx\right)\cos\left(c+dx\right)}{4b^2d-d^3}-\frac{2bd\sin\left(a+bx\right)\sin\left(c+dx\right)\cos\left(a+bx\right)}{4b^2d-d^3}+\frac{d^2\sin^2\left(a+bx\right)\cos\left(c+dx\right)}{4b^2d-d^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(d*x+c),x)
```

```
[Out] Piecewise((x*sin(a)**2*sin(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x/2)**2*s
in(c + d*x)/4 - x*sin(a - d*x/2)*cos(a - d*x/2)*cos(c + d*x)/2 - x*sin(c +
d*x)*cos(a - d*x/2)**2/4 + 3*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)/(2*
d) - cos(a - d*x/2)**2*cos(c + d*x)/d, Eq(b, -d/2)), (x*sin(a + d*x/2)**2*s
in(c + d*x)/4 + x*sin(a + d*x/2)*cos(a + d*x/2)*cos(c + d*x)/2 - x*sin(c +
d*x)*cos(a + d*x/2)**2/4 - sin(a + d*x/2)**2*cos(c + d*x)/d + sin(a + d*x/2
)*sin(c + d*x)*cos(a + d*x/2)/(2*d), Eq(b, d/2)), ((x*sin(a + b*x)**2/2 + x
*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a + b*x)/(2*b))*sin(c), Eq(d, 0)), (-
2*b**2*sin(a + b*x)**2*cos(c + d*x)/(4*b**2*d - d**3) - 2*b**2*cos(a + b*x)
**2*cos(c + d*x)/(4*b**2*d - d**3) - 2*b*d*sin(a + b*x)*sin(c + d*x)*cos(a
+ b*x)/(4*b**2*d - d**3) + d**2*sin(a + b*x)**2*cos(c + d*x)/(4*b**2*d - d*
*3), True))
```

Giac [A]
time = 0.41, size = 61, normalized size = 0.90

$$\frac{\cos\left(2\,bx+dx+2\,a+c\right)}{4\,\left(2\,b+d\right)}-\frac{\cos\left(2\,bx-dx+2\,a-c\right)}{4\,\left(2\,b-d\right)}-\frac{\cos\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*cos(2*b*x + d*x + 2*a + c)/(2*b + d) - 1/4*cos(2*b*x - d*x + 2*a - c)/(
2*b - d) - 1/2*cos(d*x + c)/d
```

Mupad [B]
time = 0.75, size = 105, normalized size = 1.54

$$-\frac{d^2\cos\left(2a+c+2bx+dx\right)-b\left(2d\cos\left(2a+c+2bx+dx\right)-2d\cos\left(2a-c+2bx-dx\right)\right)+d^2\cos\left(2a-c+2bx-dx\right)}{16b^2d-4d^3}-\frac{\cos\left(c+dx\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(c + d*x),x)
```

```
[Out] - (d^2*cos(2*a + c + 2*b*x + d*x) - b*(2*d*cos(2*a + c + 2*b*x + d*x) - 2*d
*cos(2*a - c + 2*b*x - d*x)) + d^2*cos(2*a - c + 2*b*x - d*x))/(16*b^2*d -
4*d^3) - cos(c + d*x)/(2*d)
```

3.203 $\int \sin^2(a+bx)\sin^2(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{x}{4}-\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)}-\frac{\sin(2c+2dx)}{8d}+\frac{\sin(2(a+c)+2(b+d)x)}{16(b+d)}$$

```
[Out] 1/4*x-1/8*sin(2*b*x+2*a)/b+1/16*sin(2*a-2*c+2*(b-d)*x)/(b-d)-1/8*sin(2*d*x+
2*c)/d+1/16*sin(2*a+2*c+2*(b+d)*x)/(b+d)
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4665, 2717}

$$\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sin(2a+2bx)}{8b}-\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[c + d*x]^2,x]
```

```
[Out] x/4 - Sin[2*a + 2*b*x]/(8*b) + Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) -
Sin[2*c + 2*d*x]/(8*d) + Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4665

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin^2(c+dx)\,dx &= \int \left(\frac{1}{4}-\frac{1}{4}\cos(2a+2bx)+\frac{1}{8}\cos(2(a-c)+2(b-d)x)-\frac{1}{4}\cos(2c+2dx)\right.\\ &= \frac{x}{4}+\frac{1}{8}\int \cos(2(a-c)+2(b-d)x)\,dx+\frac{1}{8}\int \cos(2(a+c)+2(b+d)x)\,dx\\ &= \frac{x}{4}-\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)}-\frac{\sin(2c+2dx)}{8d}+\frac{\sin(2(a\end{aligned}$$

Mathematica [**A**]
time = 0.83, size = 106, normalized size = 1.20

$$\frac{(-2b^2d+2d^3)\sin(2(a+bx))+bd(b+d)\sin(2(a-c+(b-d)x))+b(b-d)(-2(b+d)\sin(2(c+dx))+d(4(b+d)x+\sin(2(a+c+(b+d)x))))}{16b(b-d)d(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[c + d*x]^2,x]
```

```
[Out] ((-2*b^2*d + 2*d^3)*Sin[2*(a + b*x)] + b*d*(b + d)*Sin[2*(a - c + (b - d)*x
)] + b*(b - d)*(-2*(b + d)*Sin[2*(c + d*x)] + d*(4*(b + d)*x + Sin[2*(a + c
 + (b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

Maple [**A**]
time = 0.18, size = 83, normalized size = 0.94

method	result
default	$\frac{x}{4}-\frac{\sin(2bx+2a)}{8b}-\frac{\sin(2dx+2c)}{8d}+\frac{\sin((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sin((2b+2d)x+2a+2c)}{16b+16d}$
risch	$\frac{x}{4}-\frac{\sin(2bx+2a)}{8b}-\frac{\sin(2dx+2c)b^2}{8d(b-d)(b+d)}+\frac{d\sin(2dx+2c)}{8(b-d)(b+d)}+\frac{\sin(2bx-2dx+2a-2c)b}{16(b-d)(b+d)}+\frac{d\sin(2bx-2dx+2a-2c)}{16(b-d)(b+d)}+\frac{\sin(2bx+2dx+}{16(b-d)(b}$ [...]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x-1/8*sin(2*b*x+2*a)/b-1/8*sin(2*d*x+2*c)/d+1/16/(b-d)*sin((2*b-2*d)*x+
2*a-2*c)+1/16/(b+d)*sin((2*b+2*d)*x+2*a+2*c)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(78) = 156$.
time = 0.31, size = 620, normalized size = 7.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)
^2)*d)*x + (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c) -
 (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) - (b^2*d*sin(2*c)
 + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) + (b^2*d*sin(2*c) + b*d^2*
sin(2*c))*cos(-2*(b - d)*x - 2*a) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2
*b*x + 2*a + 2*c) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a - 2*c
) + 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) - 2*(b^3*sin(2*c) - b*d^2*
sin(2*c))*cos(2*d*x + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d
)*x + 2*a + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)*x + 2*a)
 + (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c) + (b^2*d
```

*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) + 2*(b^2*d*cos(2*c) - d
^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) + 2*(b^2*d*cos(2*c) - d^3*cos(2*c))*sin
(2*b*x + 2*a - 2*c) + 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x) + 2*(b^3
*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2 + b*sin(2*c)^2
)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)

Fricas [A]
time = 3.07, size = 118, normalized size = 1.34

$$-\frac{\left(2\,bd^2\cos\left(bx+a\right)^2+b^3-2\,bd^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(b^3d-bd^3\right)x-\left(2\,b^2d\cos\left(bx+a\right)\cos\left(dx+c\right)^2-\left(2\,b^2d-d^3\right)\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{4\left(b^3d-bd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="fricas")

[Out] -1/4*((2*b*d^2*cos(b*x + a)^2 + b^3 - 2*b*d^2)*cos(d*x + c)*sin(d*x + c) -
(b^3*d - b*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - (2*b^2*d - d^3)*
cos(b*x + a))*sin(b*x + a))/(b^3*d - b*d^3)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. 2(76) = 152.
time = 1.93, size = 1027, normalized size = 11.67

$$\begin{cases} x\sin^2(a)\sin^2(c) & \text{for } b=0\wedge d=0 \\ \left(\frac{x\sin^2(c+dx)}{2}+\frac{x\cos^2(c+dx)}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\sin^2(a) & \text{for } b=0 \\ \frac{3x\sin^2(a-dx)\sin^2(c+dx)}{8}+\frac{x\sin^2(a-dx)\cos^2(c+dx)}{8}-\frac{x\sin(a-dx)\sin(c+dx)\cos(a-dx)\cos(c+dx)}{2}+\frac{x\sin^2(c+dx)\cos^2(a-dx)}{8}+\frac{3x\cos^2(a-dx)\cos^2(c+dx)}{8}-\frac{\sin^2(a-dx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{\sin(a-dx)\sin^2(c+dx)\cos(a-dx)}{8d}+\frac{3\sin(a-dx)\cos(a-dx)\cos^2(c+dx)}{8d} & \text{for } b=-d \\ \frac{3x\sin^2(a+dx)\sin^2(c+dx)}{8}+\frac{x\sin^2(a+dx)\cos^2(c+dx)}{8}+\frac{x\sin(a+dx)\sin(c+dx)\cos(a+dx)\cos(c+dx)}{2}+\frac{x\sin^2(c+dx)\cos^2(a+dx)}{8}+\frac{3x\cos^2(a+dx)\cos^2(c+dx)}{8}-\frac{5\sin(a+dx)\sin^2(c+dx)\cos(a+dx)}{8d}+\frac{\sin(a+dx)\cos(a+dx)\cos^2(c+dx)}{8d}-\frac{\sin(c+dx)\cos^2(a+dx)\cos(c+dx)}{2d} & \text{for } b=d \\ \left(\frac{x\sin^2(a+bx)}{2}+\frac{x\cos^2(a+bx)}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)\sin^2(c) & \text{for } d=0 \\ \frac{b^3dx\sin^2(a+bx)\sin^2(c+dx)}{4b^3d-4bd^3}+\frac{b^3dx\sin^2(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3}+\frac{b^3dx\sin^2(c+dx)\cos^2(a+bx)}{4b^3d-4bd^3}+\frac{b^3dx\cos^2(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3}-\frac{b^3\sin^2(a+bx)\sin(c+dx)\cos(c+dx)}{4b^3d-4bd^3}-\frac{b^3\sin(c+dx)\cos^2(a+bx)\cos(c+dx)}{4b^3d-4bd^3}-\frac{2b^2d\sin(a+bx)\sin^2(c+dx)\cos(a+bx)}{4b^3d-4bd^3}-\frac{bd^3x\sin^2(a+bx)\sin^2(c+dx)}{4b^3d-4bd^3}-\frac{bd^3x\sin^2(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3}-\frac{bd^3x\sin^2(c+dx)\cos^2(a+bx)}{4b^3d-4bd^3}-\frac{bd^3x\cos^2(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3}+\frac{2bd^2\sin^2(a+bx)\sin(c+dx)\cos(c+dx)}{4b^3d-4bd^3}+\frac{d^3\sin(a+bx)\sin^2(c+dx)\cos(a+bx)}{4b^3d-4bd^3}+\frac{d^3\sin(a+bx)\cos(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)**2*sin(d*x+c)**2,x)

[Out] Piecewise((x*sin(a)**2*sin(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2
/2 + x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**2, Eq(b
, 0)), (3*x*sin(a - d*x)**2*sin(c + d*x)**2/8 + x*sin(a - d*x)**2*cos(c + d
*x)**2/8 - x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + x*sin(
c + d*x)**2*cos(a - d*x)**2/8 + 3*x*cos(a - d*x)**2*cos(c + d*x)**2/8 - sin
(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(2*d) + sin(a - d*x)*sin(c + d*x)**2
*cos(a - d*x)/(8*d) + 3*sin(a - d*x)*cos(a - d*x)*cos(c + d*x)**2/(8*d), Eq
(b, -d)), (3*x*sin(a + d*x)**2*sin(c + d*x)**2/8 + x*sin(a + d*x)**2*cos(c
+ d*x)**2/8 + x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x)/2 + x*s
in(c + d*x)**2*cos(a + d*x)**2/8 + 3*x*cos(a + d*x)**2*cos(c + d*x)**2/8 -
5*sin(a + d*x)*sin(c + d*x)**2*cos(a + d*x)/(8*d) + sin(a + d*x)*cos(a + d*
x)*cos(c + d*x)**2/(8*d) - sin(c + d*x)*cos(a + d*x)**2*cos(c + d*x)/(2*d),
 Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(
a + b*x)/(2*b))*sin(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)**2*sin(c + d*x
)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c + d*x)**2/(4*b*
*3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2/(4*b**3*d - 4*b
*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b

```
**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d - 4*b*d**3) - b**3*
sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*b*d**3) - 2*b**2*d*
sin(a + b*x)*sin(c + d*x)**2*cos(a + b*x)/(4*b**3*d - 4*b*d**3) - b*d**3*x*
sin(a + b*x)**2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(a + b*
x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(c + d*x)**2*cos(
a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cos(a + b*x)**2*cos(c + d*x)**
2/(4*b**3*d - 4*b*d**3) + 2*b*d**2*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x
)/(4*b**3*d - 4*b*d**3) + d**3*sin(a + b*x)*sin(c + d*x)**2*cos(a + b*x)/(4
*b**3*d - 4*b*d**3) + d**3*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**
3*d - 4*b*d**3), True))
```

Giac [A]
time = 0.41, size = 80, normalized size = 0.91

$$\frac{1}{4}\,x + \frac{\sin\left(2\,bx + 2\,dx + 2\,a + 2\,c\right)}{16\,(b + d)} + \frac{\sin\left(2\,bx - 2\,dx + 2\,a - 2\,c\right)}{16\,(b - d)} - \frac{\sin\left(2\,bx + 2\,a\right)}{8\,b} - \frac{\sin\left(2\,dx + 2\,c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*x + 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/16*sin(2*b*x - 2*d*
x + 2*a - 2*c)/(b - d) - 1/8*sin(2*b*x + 2*a)/b - 1/8*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 0.89, size = 177, normalized size = 2.01

$$\frac{2\,d^3\,\sin\left(2\,a+2\,b\,x\right)-2\,b^3\,\sin\left(2\,c+2\,d\,x\right)+b\,d^2\,\sin\left(2\,a-2\,c+2\,b\,x-2\,d\,x\right)-b\,d^2\,\sin\left(2\,a+2\,c+2\,b\,x+2\,d\,x\right)+b^2\,d\,\sin\left(2\,a-2\,c+2\,b\,x-2\,d\,x\right)+b^2\,d\,\sin\left(2\,a+2\,c+2\,b\,x+2\,d\,x\right)-2\,b^2\,d\,\sin\left(2\,a+2\,b\,x\right)+2\,b\,d^2\,\sin\left(2\,c+2\,d\,x\right)-4\,b\,d^3\,x+4\,b^3\,d\,x}{16\,b\,d\,\left(b^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(c + d*x)^2,x)
```

```
[Out] (2*d^3*sin(2*a + 2*b*x) - 2*b^3*sin(2*c + 2*d*x) + b*d^2*sin(2*a - 2*c + 2*
b*x - 2*d*x) - b*d^2*sin(2*a + 2*c + 2*b*x + 2*d*x) + b^2*d*sin(2*a - 2*c +
 2*b*x - 2*d*x) + b^2*d*sin(2*a + 2*c + 2*b*x + 2*d*x) - 2*b^2*d*sin(2*a +
2*b*x) + 2*b*d^2*sin(2*c + 2*d*x) - 4*b*d^3*x + 4*b^3*d*x)/(16*b*d*(b^2 - d
^2))
```

3.204 $\int \sin^2(a+bx)\sin^3(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{\cos(2a-3c+(2b-3d)x)}{16(2b-3d)}-\frac{3\cos(2a-c+(2b-d)x)}{16(2b-d)}-\frac{3\cos(c+dx)}{8d}+\frac{\cos(3c+3dx)}{24d}+\frac{3\cos(2a+c+(2b+$$

```
[Out] 1/16*cos(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3/16*cos(2*a-c+(2*b-d)*x)/(2*b-d)-3
/8*cos(d*x+c)/d+1/24*cos(3*d*x+3*c)/d+3/16*cos(2*a+c+(2*b+d)*x)/(2*b+d)-1/1
6*cos(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Rubi [A]
time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$,
Rules used = {4665, 2718}

$$\frac{\cos(2a+x(2b-3d)-3c)}{16(2b-3d)}-\frac{3\cos(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\cos(2a+x(2b+d)+c)}{16(2b+d)}-\frac{\cos(2a+x(2b+3d)+3c)}{16(2b+3d)}-\frac{3\cos(c+dx)}{8d}+\frac{\cos(3c+3dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2*Sin[c + d*x]^3,x]
```

```
[Out] Cos[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) - (3*Cos[2*a - c + (2*b - d
)*x])/(16*(2*b - d)) - (3*Cos[c + d*x])/(8*d) + Cos[3*c + 3*d*x]/(24*d) + (
3*Cos[2*a + c + (2*b + d)*x])/(16*(2*b + d)) - Cos[2*a + 3*c + (2*b + 3*d)*
x]/(16*(2*b + 3*d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4665

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\sin^3(c+dx)\,dx &= \int \left(-\frac{1}{16}\sin(2a-3c+(2b-3d)x)+\frac{3}{16}\sin(2a-c+(2b-d)x)+\frac{3}{8}\sin(c\right.\\ &= -\left(\frac{1}{16}\int \sin(2a-3c+(2b-3d)x)\,dx\right)+\frac{1}{16}\int \sin(2a+3c+(2b+3d)x)\\ &= \frac{\cos(2a-3c+(2b-3d)x)}{16(2b-3d)}-\frac{3\cos(2a-c+(2b-d)x)}{16(2b-d)}-\frac{3\cos(c+dx)}{8d}+\end{aligned}$$

Mathematica [A]
time = 1.61, size = 158, normalized size = 1.10

$$\frac{1}{48}\left(-\frac{18\cos(c)\cos(dx)}{d}+\frac{2\cos(3c)\cos(3dx)}{d}+\frac{3\cos(2a-3c+2bx-3dx)}{2b-3d}-\frac{9\cos(2a-c+2bx-dx)}{2b-d}+\frac{9\cos(2a+c+2bx+dx)}{2b+d}-\frac{3\cos(2a+3c+2bx+3dx)}{2b+3d}+\frac{18\sin(c)\sin(dx)}{d}-\frac{2\sin(3c)\sin(3dx)}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2*Sin[c + d*x]^3,x]
```

```
[Out] ((-18*Cos[c]*Cos[d*x])/d + (2*Cos[3*c]*Cos[3*d*x])/d + (3*Cos[2*a - 3*c + 2
*b*x - 3*d*x])/(2*b - 3*d) - (9*Cos[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*
Cos[2*a + c + 2*b*x + d*x])/(2*b + d) - (3*Cos[2*a + 3*c + 2*b*x + 3*d*x])/
(2*b + 3*d) + (18*Sin[c]*Sin[d*x])/d - (2*Sin[3*c]*Sin[3*d*x])/d)/48
```

Maple [A]
time = 0.23, size = 133, normalized size = 0.92

method	result
default	$\frac{\cos(2a-3c+(2b-3d)x)}{32b-48d}-\frac{3\cos(2a-c+(2b-d)x)}{16(2b-d)}-\frac{3\cos(dx+c)}{8d}+\frac{\cos(3dx+3c)}{24d}+\frac{3\cos(2a+c+(2b+d)x)}{16(2b+d)}-\frac{\cos(2a+3c+(2b+}{16(2b+3d)}$
risch	$-\frac{3\cos(dx+c)b^2}{2(2b+d)(2b-d)d}+\frac{3d\cos(dx+c)}{8(2b+d)(2b-d)}+\frac{\cos(2bx-3dx+2a-3c)b}{8(2b+3d)(2b-3d)}+\frac{3d\cos(2bx-3dx+2a-3c)}{16(2b+3d)(2b-3d)}-\frac{3\cos(2bx-dx+2a-c)b}{8(2b+d)(2b-d)}-3d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*cos(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3/16*cos(2*a-c+(2*b-d)*x)/(2*b-d)-3
/8*cos(d*x+c)/d+1/24*cos(3*d*x+3*c)/d+3/16*cos(2*a+c+(2*b+d)*x)/(2*b+d)-1/1
6*cos(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. 2(132) = 264.
time = 0.37, size = 1362, normalized size = 9.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/96*(3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4
*cos(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) + 3*(8*b^3*d*cos(3*c) - 12*b^2*d^
2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*cos((2*b + 3*d)*x + 2*a) -
9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*cos(3*
c))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c)
 - 18*b*d^3*cos(3*c) + 9*d^4*cos(3*c))*cos((2*b + d)*x + 2*a - 2*c) + 9*(8*
b^3*d*cos(3*c) + 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) - 9*d^4*cos(3*c))*c
os(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*cos(3*c) + 4*b^2*d^2*cos(3*c) - 1
```

```
8*b*d^3*cos(3*c) - 9*d^4*cos(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) - 3*(8*b^3
*d*cos(3*c) + 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) - 3*d^4*cos(3*c))*cos(
-(2*b - 3*d)*x - 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) + 12*b^2*d^2*cos(3*c) - 2
*b*d^3*cos(3*c) - 3*d^4*cos(3*c))*cos(-(2*b - 3*d)*x - 2*a) - 2*(16*b^4*cos
(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(3*d*x) - 2*(16*b^4*cos(3*
c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(3*d*x + 6*c) + 18*(16*b^4*co
s(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(d*x + 4*c) + 18*(16*b^4*
cos(3*c) - 40*b^2*d^2*cos(3*c) + 9*d^4*cos(3*c))*cos(d*x - 2*c) + 3*(8*b^3*
d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*sin((
2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b
*d^3*sin(3*c) + 3*d^4*sin(3*c))*sin((2*b + 3*d)*x + 2*a) - 9*(8*b^3*d*sin(3
*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c))*sin((2*b + d
)*x + 2*a + 4*c) + 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(
3*c) + 9*d^4*sin(3*c))*sin((2*b + d)*x + 2*a - 2*c) + 9*(8*b^3*d*sin(3*c) +
 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*sin(-(2*b - d)*x
- 2*a + 4*c) - 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c)
 - 9*d^4*sin(3*c))*sin(-(2*b - d)*x - 2*a - 2*c) - 3*(8*b^3*d*sin(3*c) + 12
*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*sin(-(2*b - 3*d)*x -
 2*a + 6*c) + 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c)
- 3*d^4*sin(3*c))*sin(-(2*b - 3*d)*x - 2*a) + 2*(16*b^4*sin(3*c) - 40*b^2*d
^2*sin(3*c) + 9*d^4*sin(3*c))*sin(3*d*x) - 2*(16*b^4*sin(3*c) - 40*b^2*d^2*
sin(3*c) + 9*d^4*sin(3*c))*sin(3*d*x + 6*c) + 18*(16*b^4*sin(3*c) - 40*b^2*
d^2*sin(3*c) + 9*d^4*sin(3*c))*sin(d*x + 4*c) - 18*(16*b^4*sin(3*c) - 40*b^
2*d^2*sin(3*c) + 9*d^4*sin(3*c))*sin(d*x - 2*c))/(9*(cos(3*c)^2 + sin(3*c)^
2)*d^5 - 40*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^3 + 16*(b^4*cos(3*c)^2 + b^
4*sin(3*c)^2)*d)
```

Fricas [A]
time = 2.16, size = 192, normalized size = 1.33

$$\frac{\left(8 b^4-38 b^2 d^2+9 d^4+9 \left(4 b^2 d^2-d^4\right) \cos \left(bx+a\right)^2\right) \cos \left(dx+c\right)^3+6 \left(\left(4 b^3 d-b d^3\right) \cos \left(bx+a\right) \cos \left(dx+c\right)^2-\left(4 b^3 d-7 b d^3\right) \cos \left(bx+a\right)\right) \sin \left(bx+a\right) \sin \left(dx+c\right)-3 \left(8 b^4-26 b^2 d^2+9 d^4+3 \left(4 b^2 d^2-3 d^4\right) \cos \left(bx+a\right)^2\right) \cos \left(dx+c\right)}{3 \left(16 b^4 d-40 b^2 d^3+9 d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/3*((8*b^4 - 38*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cos(b*x + a)^2)*cos(
d*x + c)^3 + 6*((4*b^3*d - b*d^3)*cos(b*x + a)*cos(d*x + c)^2 - (4*b^3*d -
7*b*d^3)*cos(b*x + a))*sin(b*x + a)*sin(d*x + c) - 3*(8*b^4 - 26*b^2*d^2 +
9*d^4 + 3*(4*b^2*d^2 - 3*d^4)*cos(b*x + a)^2)*cos(d*x + c))/(16*b^4*d - 40*
b^2*d^3 + 9*d^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2006 vs. 2(116) = 232.
time = 6.59, size = 2006, normalized size = 13.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2*sin(d*x+c)**3,x)
```

```
[Out] Piecewise((x*sin(a)**2*sin(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sin(a - 3*d*x/2)
**2*sin(c + d*x)**3/16 - 3*x*sin(a - 3*d*x/2)**2*sin(c + d*x)*cos(c + d*x)*
*2/16 - 3*x*sin(a - 3*d*x/2)*sin(c + d*x)**2*cos(a - 3*d*x/2)*cos(c + d*x)/
8 + x*sin(a - 3*d*x/2)*cos(a - 3*d*x/2)*cos(c + d*x)**3/8 - x*sin(c + d*x)*
*3*cos(a - 3*d*x/2)**2/16 + 3*x*sin(c + d*x)*cos(a - 3*d*x/2)**2*cos(c + d*
x)**2/16 - 7*sin(a - 3*d*x/2)**2*cos(c + d*x)**3/(16*d) + 5*sin(a - 3*d*x/2
)*sin(c + d*x)**3*cos(a - 3*d*x/2)/(8*d) - 3*sin(a - 3*d*x/2)*sin(c + d*x)*
cos(a - 3*d*x/2)*cos(c + d*x)**2/(4*d) - sin(c + d*x)**2*cos(a - 3*d*x/2)**
2*cos(c + d*x)/d - 11*cos(a - 3*d*x/2)**2*cos(c + d*x)**3/(48*d), Eq(b, -3*
d/2)), (3*x*sin(a - d*x/2)**2*sin(c + d*x)**3/16 + 3*x*sin(a - d*x/2)**2*si
n(c + d*x)*cos(c + d*x)**2/16 - 3*x*sin(a - d*x/2)*sin(c + d*x)**2*cos(a -
d*x/2)*cos(c + d*x)/8 - 3*x*sin(a - d*x/2)*cos(a - d*x/2)*cos(c + d*x)**3/8
 - 3*x*sin(c + d*x)**3*cos(a - d*x/2)**2/16 - 3*x*sin(c + d*x)*cos(a - d*x/
2)**2*cos(c + d*x)**2/16 + 17*sin(a - d*x/2)**2*cos(c + d*x)**3/(48*d) + 13
*sin(a - d*x/2)*sin(c + d*x)**3*cos(a - d*x/2)/(8*d) + 7*sin(a - d*x/2)*sin
(c + d*x)*cos(a - d*x/2)*cos(c + d*x)**2/(4*d) - sin(c + d*x)**2*cos(a - d*
x/2)**2*cos(c + d*x)/d - 49*cos(a - d*x/2)**2*cos(c + d*x)**3/(48*d), Eq(b,
 -d/2)), (3*x*sin(a + d*x/2)**2*sin(c + d*x)**3/16 + 3*x*sin(a + d*x/2)**2*
sin(c + d*x)*cos(c + d*x)**2/16 + 3*x*sin(a + d*x/2)*sin(c + d*x)**2*cos(a
+ d*x/2)*cos(c + d*x)/8 + 3*x*sin(a + d*x/2)*cos(a + d*x/2)*cos(c + d*x)**3
/8 - 3*x*sin(c + d*x)**3*cos(a + d*x/2)**2/16 - 3*x*sin(c + d*x)*cos(a + d*
x/2)**2*cos(c + d*x)**2/16 - sin(a + d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)
/d - 31*sin(a + d*x/2)**2*cos(c + d*x)**3/(48*d) + 3*sin(a + d*x/2)*sin(c +
 d*x)**3*cos(a + d*x/2)/(8*d) + sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)*
cos(c + d*x)**2/(4*d) - cos(a + d*x/2)**2*cos(c + d*x)**3/(48*d), Eq(b, d/2
)), (x*sin(a + 3*d*x/2)**2*sin(c + d*x)**3/16 - 3*x*sin(a + 3*d*x/2)**2*sin
(c + d*x)*cos(c + d*x)**2/16 + 3*x*sin(a + 3*d*x/2)*sin(c + d*x)**2*cos(a +
 3*d*x/2)*cos(c + d*x)/8 - x*sin(a + 3*d*x/2)*cos(a + 3*d*x/2)*cos(c + d*x)
**3/8 - x*sin(c + d*x)**3*cos(a + 3*d*x/2)**2/16 + 3*x*sin(c + d*x)*cos(a +
 3*d*x/2)**2*cos(c + d*x)**2/16 - 7*sin(a + 3*d*x/2)**2*cos(c + d*x)**3/(16
*d) - 5*sin(a + 3*d*x/2)*sin(c + d*x)**3*cos(a + 3*d*x/2)/(8*d) + 3*sin(a +
 3*d*x/2)*sin(c + d*x)*cos(a + 3*d*x/2)*cos(c + d*x)**2/(4*d) - sin(c + d*x
)**2*cos(a + 3*d*x/2)**2*cos(c + d*x)/d - 11*cos(a + 3*d*x/2)**2*cos(c + d*
x)**3/(48*d), Eq(b, 3*d/2)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 -
sin(a + b*x)*cos(a + b*x)/(2*b))*sin(c)**3, Eq(d, 0)), (-24*b**4*sin(a + b*
x)**2*sin(c + d*x)**2*cos(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) -
16*b**4*sin(a + b*x)**2*cos(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**
5) - 24*b**4*sin(c + d*x)**2*cos(a + b*x)**2*cos(c + d*x)/(48*b**4*d - 120*
b**2*d**3 + 27*d**5) - 16*b**4*cos(a + b*x)**2*cos(c + d*x)**3/(48*b**4*d -
 120*b**2*d**3 + 27*d**5) - 24*b**3*d*sin(a + b*x)*sin(c + d*x)**3*cos(a +
b*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 78*b**2*d**2*sin(a + b*x)**2*s
```

```
in(c + d*x)**2*cos(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 40*b**2
*d**2*sin(a + b*x)**2*cos(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5)
 + 42*b**2*d**2*sin(c + d*x)**2*cos(a + b*x)**2*cos(c + d*x)/(48*b**4*d - 1
20*b**2*d**3 + 27*d**5) + 40*b**2*d**2*cos(a + b*x)**2*cos(c + d*x)**3/(48*
b**4*d - 120*b**2*d**3 + 27*d**5) + 42*b*d**3*sin(a + b*x)*sin(c + d*x)**3*
cos(a + b*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 36*b*d**3*sin(a + b*x)
*sin(c + d*x)*cos(a + b*x)*cos(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*
d**5) - 27*d**4*sin(a + b*x)**2*sin(c + d*x)**2*cos(c + d*x)/(48*b**4*d - 1
20*b**2*d**3 + 27*d**5) - 18*d**4*sin(a + b*x)**2*cos(c + d*x)**3/(48*b**4*
d - 120*b**2*d**3 + 27*d**5), True))
```

Giac [A]
time = 0.43, size = 129, normalized size = 0.90

$$-\frac{\cos\left(2\,b x+3\,d x+2\,a+3\,c\right)}{16\,\left(2\,b+3\,d\right)}+\frac{3\,\cos\left(2\,b x+d x+2\,a+c\right)}{16\,\left(2\,b+d\right)}-\frac{3\,\cos\left(2\,b x-d x+2\,a-c\right)}{16\,\left(2\,b-d\right)}+\frac{\cos\left(2\,b x-3\,d x+2\,a-3\,c\right)}{16\,\left(2\,b-3\,d\right)}+\frac{\cos\left(3\,d x+3\,c\right)}{24\,d}-\frac{3\,\cos\left(d x+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/16*cos(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/16*cos(2*b*x + d*x + 2
*a + c)/(2*b + d) - 3/16*cos(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/16*cos(2*
b*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*cos(3*d*x + 3*c)/d - 3/8*cos(d*
x + c)/d
```

Mupad [B]
time = 1.87, size = 469, normalized size = 3.26

$$\mathrm{e}^{a\,2\mathrm{i}-c\,3\mathrm{i}+b\,x\,2\mathrm{i}-d\,x\,3\mathrm{i}}\,\left(\frac{3\,d\,\left(2\,b+3\,d\right)}{384\,b^2\,d-864\,d^3}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(8\,b^2-18\,d^2\right)}{384\,b^2\,d-864\,d^3}-\frac{3\,d\,\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(2\,b-3\,d\right)}{384\,b^2\,d-864\,d^3}\right)+\mathrm{e}^{a\,2\mathrm{i}+c\,3\mathrm{i}+b\,x\,2\mathrm{i}+d\,x\,3\mathrm{i}}\,\left(-\frac{3\,d\,\left(2\,b-3\,d\right)}{384\,b^2\,d-864\,d^3}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(8\,b^2-18\,d^2\right)}{384\,b^2\,d-864\,d^3}+\frac{3\,d\,\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(2\,b+3\,d\right)}{384\,b^2\,d-864\,d^3}\right)-\mathrm{e}^{a\,2\mathrm{i}-c\,1\mathrm{i}+b\,x\,2\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\frac{3\,\left(2\,b+d\right)}{32\,\left(4\,b^2-d^2\right)}-\frac{3\,\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(2\,b-d\right)}{32\,\left(4\,b^2-d^2\right)}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(24\,b^2-6\,d^2\right)}{32\,d\,\left(4\,b^2-d^2\right)}\right)-\mathrm{e}^{a\,2\mathrm{i}+c\,1\mathrm{i}+b\,x\,2\mathrm{i}+d\,x\,1\mathrm{i}}\,\left(-\frac{3\,\left(2\,b-d\right)}{32\,\left(4\,b^2-d^2\right)}+\frac{3\,\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(2\,b+d\right)}{32\,\left(4\,b^2-d^2\right)}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(24\,b^2-6\,d^2\right)}{32\,d\,\left(4\,b^2-d^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2*sin(c + d*x)^3,x)
```

```
[Out] exp(a*2i - c*3i + b*x*2i - d*x*3i)*((3*d*(2*b + 3*d))/(384*b^2*d - 864*d^3)
 + (exp(- a*2i - b*x*2i)*(8*b^2 - 18*d^2))/(384*b^2*d - 864*d^3) - (3*d*exp
(- a*4i - b*x*4i)*(2*b - 3*d))/(384*b^2*d - 864*d^3)) + exp(a*2i + c*3i + b
*x*2i + d*x*3i)*((exp(- a*2i - b*x*2i)*(8*b^2 - 18*d^2))/(384*b^2*d - 864*d
^3) - (3*d*(2*b - 3*d))/(384*b^2*d - 864*d^3) + (3*d*exp(- a*4i - b*x*4i)*(
2*b + 3*d))/(384*b^2*d - 864*d^3)) - exp(a*2i - c*1i + b*x*2i - d*x*1i)*((3
*(2*b + d))/(32*(4*b^2 - d^2)) - (3*exp(- a*4i - b*x*4i)*(2*b - d))/(32*(4*
b^2 - d^2)) + (exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(32*d*(4*b^2 - d^2)))
 - exp(a*2i + c*1i + b*x*2i + d*x*1i)*((3*exp(- a*4i - b*x*4i)*(2*b + d))/(
32*(4*b^2 - d^2)) - (3*(2*b - d))/(32*(4*b^2 - d^2)) + (exp(- a*2i - b*x*2i
)*(24*b^2 - 6*d^2))/(32*d*(4*b^2 - d^2)))
```

3.205 $\int \sin^3(a+bx)\sin^n(c+dx)\,dx$

Optimal. Leaf size=600

$$\frac{2^{-3-n}e^{i(3a-cn)+i(3b-dn)x+in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n {}_2F_1\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-n;\frac{1}{2}\left(2+\frac{3b}{d}-n\right)\right.}{3b-dn}$$

```
[Out] 2^(-3-n)*exp(I*(-c*n+3*a)+I*(-d*n+3*b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*e
xp(I*(d*x+c)))^n*hypergeom([-n, 3/2*b/d-1/2*n],[1+3/2*b/d-1/2*n],exp(2*I*(d
*x+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(-d*n+3*b)-3*2^(-3-n)*exp(I*(-c*n+a)+I*(
-d*n+b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-n,
 1/2*(-d*n+b)/d],[1+1/2*b/d-1/2*n],exp(2*I*(d*x+c)))/((1-exp(2*I*c+2*I*d*x)
)^n)/(-d*n+b)-3*2^(-3-n)*exp(-I*(c*n+a)-I*(d*n+b)*x+I*n*(d*x+c))*(I/exp(I*(
d*x+c))-I*exp(I*(d*x+c)))^n*hypergeom([-n, 1/2*(-d*n-b)/d],[1+1/2*(-d*n-b)/
d],exp(2*I*(d*x+c)))/((1-exp(2*I*c+2*I*d*x))^n)/(d*n+b)+2^(-3-n)*exp(-I*(c*
n+3*a)-I*(d*n+3*b)*x+I*n*(d*x+c))*(I/exp(I*(d*x+c))-I*exp(I*(d*x+c)))^n*hyp
ergeom([-n, 1/2*(-d*n-3*b)/d],[1-3/2*b/d-1/2*n],exp(2*I*(d*x+c)))/((1-exp(2
*I*c+2*I*d*x))^n)/(d*n+3*b)
```

Rubi [A]

time = 1.19, antiderivative size = 600, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {4649, 2323, 2285, 2284, 2283}

$$\frac{2^{-n-3}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-n;\frac{1}{2}\left(\frac{3b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(i(3a-cn)+ix(3b-dn)+in(c+dx))}{3b-dn}-\frac{3\,2^{-n-3}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}{}_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(\frac{b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(i(a-cn)+ix(b-dn)+in(c+dx))}{b-dn}-\frac{3\,2^{-n-3}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}{}_2F_1\left(-n,-\frac{b+dn}{2d};1-\frac{b+dn}{2d};e^{2i(c+dx)}\right)\exp(-i(a+cn)-ix(b+dn)+in(c+dx))}{b+dn}+\frac{2^{-n-3}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\left(1-e^{2ic+2idx}\right)^{-n}{}_2F_1\left(-n,-\frac{3b+dn}{2d};\frac{1}{2}\left(-\frac{3b}{d}-n+2\right);e^{2i(c+dx)}\right)\exp(-i(3a+cn)-ix(3b+dn)+in(c+dx))}{3b+dn}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[c + d*x]^n,x]
```

```
[Out] (2^(-3 - n)*E^(I*(3*a - c*n) + I*(3*b - d*n)*x + I*n*(c + d*x))*(I/E^(I*(c
+ d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[((3*b)/d - n)/2, -n, (2 +
(3*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n*(3*b
 - d*n)) - (3*2^(-3 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(I
/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*
d), (2 + b/d - n)/2, E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2*I)*d*x))^n
*(b - d*n)) - (3*2^(-3 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*
x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -1/2*(b
 + d*n)/d, 1 - (b + d*n)/(2*d), E^((2*I)*(c + d*x))])/((1 - E^((2*I)*c + (2
*I)*d*x))^n*(b + d*n)) + (2^(-3 - n)*E^((-I)*(3*a + c*n) - I*(3*b + d*n)*x
+ I*n*(c + d*x))*(I/E^(I*(c + d*x)) - I*E^(I*(c + d*x)))^n*Hypergeometric2F
1[-n, -1/2*(3*b + d*n)/d, (2 - (3*b)/d - n)/2, E^((2*I)*(c + d*x))])/((1 -
E^((2*I)*c + (2*I)*d*x))^n*(3*b + d*n))
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
```

```
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2284

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Dist[(a + b*F^(e*(c + d*x)))^p/(1 + (b/a)*F^
(e*(c + d*x)))^p, Int[G^(h*(f + g*x))*(1 + (b/a)*F^(e*(c + d*x)))^p, x], x]
 /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] &&  !(ILtQ[p, 0] || GtQ[a,
0])
```

Rule 2285

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2323

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 4649

```
Int[Sin[(a_.) + (b_.)*(x_)]^(p_.)*Sin[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(I/E^(I*(c + d*x)) - I*E^(I*(c + d
*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
 b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \sin^3(a+bx)\sin^n(c+dx)\,dx = 2^{-3-n}\int \Big(3ie^{-ia-ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n - 3ie^{ia+ibx}\left(ie^{-i(c+dx)}-ie^{i(c+}\right.$$

$$= -\left(\left(i2^{-3-n}\right)\int e^{-3ia-3ibx}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n dx\right) + \left(i2^{-3-n}\right)\int e^{3ia+3}$$

$$= -\left(\left(i2^{-3-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{-3ia-3ib}\right.$$

$$= \left(i2^{-3-n}e^{in(c+dx)}\left(i-ie^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(3a-cn)+i(3b}$$

$$= \left(i2^{-3-n}e^{in(c+dx)}\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(c+dx)}-ie^{i(c+dx)}\right)^n\right)\int e^{i(3a-cn)+i(3b}$$

$$= \frac{2^{-3-n}\exp(i(3a-cn)+i(3b-dn)x+in(c+dx))\left(1-e^{2ic+2idx}\right)^{-n}\left(ie^{-i(\right.}{3b-dn}$$

Mathematica [F]
time = 0.60, size = 0, normalized size = 0.00

$$\int \sin^3(a+bx)\sin^n(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sin[a + b*x]^3*Sin[c + d*x]^n,x]
[Out] Integrate[Sin[a + b*x]^3*Sin[c + d*x]^n, x]
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\sin^3(bx+a)\right)\left(\sin^n(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(d*x+c)^n,x)
[Out] int(sin(b*x+a)^3*sin(d*x+c)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sin(d*x + c)^n*sin(b*x + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(d*x+c)**n,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^n*sin(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(a+b\,x\right)^3\,\sin\left(c+d\,x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(c + d*x)^n,x)
```

```
[Out] int(sin(a + b*x)^3*sin(c + d*x)^n, x)
```

3.206 $\int \sin^3(a+bx)\sin(c+dx)\,dx$

Optimal. Leaf size=97

$$\frac{3\sin(a-c+(b-d)x)}{8(b-d)}-\frac{\sin(3a-c+(3b-d)x)}{8(3b-d)}-\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(3a+c+(3b+d)x)}{8(3b+d)}$$

```
[Out] 3/8*sin(a-c+(b-d)*x)/(b-d)-1/8*sin(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*sin(a+c+(b+
d)*x)/(b+d)+1/8*sin(3*a+c+(3*b+d)*x)/(3*b+d)
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {4665, 2717}

$$\frac{3\sin(a+x(b-d)-c)}{8(b-d)}-\frac{\sin(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(3a+x(3b+d)+c)}{8(3b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[c + d*x],x]
```

```
[Out] (3*Sin[a - c + (b - d)*x])/(8*(b - d)) - Sin[3*a - c + (3*b - d)*x]/(8*(3*b
 - d)) - (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[3*a + c + (3*b + d)*x
]/(8*(3*b + d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4665

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin(c+dx)\,dx &= \int \left(\frac{3}{8}\cos(a-c+(b-d)x)-\frac{1}{8}\cos(3a-c+(3b-d)x)-\frac{3}{8}\cos(a+c+(\right.\\ &= -\left(\frac{1}{8}\int \cos(3a-c+(3b-d)x)\,dx\right)+\frac{1}{8}\int \cos(3a+c+(3b+d)x)\,dx+\\ &= \frac{3\sin(a-c+(b-d)x)}{8(b-d)}-\frac{\sin(3a-c+(3b-d)x)}{8(3b-d)}-\frac{3\sin(a+c+(b+d)x}{8(b+d)}\end{aligned}$$

Mathematica [A]
time = 0.58, size = 91, normalized size = 0.94

$$\frac{1}{8}\left(\frac{3\sin(a-c+bx-dx)}{b-d}-\frac{\sin(3a-c+3bx-dx)}{3b-d}+\frac{\sin(3a+c+3bx+dx)}{3b+d}-\frac{3\sin(a+c+(b+d)x)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[c + d*x],x]
```

```
[Out] ((3*Sin[a - c + b*x - d*x])/(b - d) - Sin[3*a - c + 3*b*x - d*x]/(3*b - d)
+ Sin[3*a + c + 3*b*x + d*x]/(3*b + d) - (3*Sin[a + c + (b + d)*x])/(b + d)
)/8
```

Maple [A]
time = 0.22, size = 90, normalized size = 0.93

method	result
default	$\frac{3\sin(a-c+(b-d)x)}{8(b-d)}-\frac{\sin(3a-c+(3b-d)x)}{8(3b-d)}-\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(3a+c+(3b+d)x)}{24b+8d}$
risch	$\frac{27\sin(bx-dx+a-c)b^{3}}{8(-3b+d)(-b+d)(3b+d)(b+d)}+\frac{27\sin(bx-dx+a-c)b^{2}d}{8(-3b+d)(-b+d)(3b+d)(b+d)}-\frac{3\sin(bx-dx+a-c)b\,d^{2}}{8(-3b+d)(-b+d)(3b+d)(b+d)}-\frac{3\sin(bx-dx+a-c)d^{3}}{8(-3b+d)(-b+d)(3b+d)(b-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*sin(a-c+(b-d)*x)/(b-d)-1/8*sin(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*sin(a+c+(b+
d)*x)/(b+d)+1/8*sin(3*a+c+(3*b+d)*x)/(3*b+d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 789 vs. 2(89) = 178.
time = 0.33, size = 789, normalized size = 8.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/16*((3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c) + d^3*sin(c))*cos((3*b
 + d)*x + 3*a + 2*c) - (3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c) + d^3*
sin(c))*cos((3*b + d)*x + 3*a) + (3*b^3*sin(c) + b^2*d*sin(c) - 3*b*d^2*sin
(c) - d^3*sin(c))*cos(-(3*b - d)*x - 3*a + 2*c) - (3*b^3*sin(c) + b^2*d*sin
(c) - 3*b*d^2*sin(c) - d^3*sin(c))*cos(-(3*b - d)*x - 3*a) - 3*(9*b^3*sin(c
) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*cos((b + d)*x + a + 2*c) +
3*(9*b^3*sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*cos((b + d)*x
 + a) - 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*sin(c) - d^3*sin(c))*cos(-
(b - d)*x - a + 2*c) + 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*sin(c) - d^
```

```
3*sin(c))*cos(-(b - d)*x - a) - (3*b^3*cos(c) - b^2*d*cos(c) - 3*b*d^2*cos(
c) + d^3*cos(c))*sin((3*b + d)*x + 3*a + 2*c) - (3*b^3*cos(c) - b^2*d*cos(c
) - 3*b*d^2*cos(c) + d^3*cos(c))*sin((3*b + d)*x + 3*a) - (3*b^3*cos(c) + b
^2*d*cos(c) - 3*b*d^2*cos(c) - d^3*cos(c))*sin(-(3*b - d)*x - 3*a + 2*c) -
(3*b^3*cos(c) + b^2*d*cos(c) - 3*b*d^2*cos(c) - d^3*cos(c))*sin(-(3*b - d)*
x - 3*a) + 3*(9*b^3*cos(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) + d^3*cos(c))*si
n((b + d)*x + a + 2*c) + 3*(9*b^3*cos(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) +
d^3*cos(c))*sin((b + d)*x + a) + 3*(9*b^3*cos(c) + 9*b^2*d*cos(c) - b*d^2*c
os(c) - d^3*cos(c))*sin(-(b - d)*x - a + 2*c) + 3*(9*b^3*cos(c) + 9*b^2*d*c
os(c) - b*d^2*cos(c) - d^3*cos(c))*sin(-(b - d)*x - a))/(9*b^4*cos(c)^2 + 9
*b^4*sin(c)^2 + (cos(c)^2 + sin(c)^2)*d^4 - 10*(b^2*cos(c)^2 + b^2*sin(c)^2
)*d^2)
```

Fricas [A]
time = 2.83, size = 115, normalized size = 1.19

$$\frac{\left(7\,b^2d-d^3-\left(b^2d-d^3\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)\sin\left(bx+a\right)+3\,\left(\left(b^3-bd^2\right)\cos\left(bx+a\right)^3-\left(3\,b^3-bd^2\right)\cos\left(bx+a\right)\right)\sin\left(dx+c\right)}{9\,b^4-10\,b^2d^2+d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] ((7*b^2*d - d^3 - (b^2*d - d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a) +
 3*((b^3 - b*d^2)*cos(b*x + a)^3 - (3*b^3 - b*d^2)*cos(b*x + a))*sin(d*x +
c))/(9*b^4 - 10*b^2*d^2 + d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(76) = 152$.
time = 2.38, size = 937, normalized size = 9.66

$$\begin{cases} x\sin^3(a)\sin(c) & \text{for } b=0\wedge d=0 \\ \frac{3x\sin^3(a-dx)\sin(c+dx)}{8}-\frac{3x\sin^2(a-dx)\cos(a-dx)\cos(c+dx)}{8}+\frac{3x\sin(a-dx)\sin(c+dx)\cos^2(a-dx)}{8}-\frac{3x\cos^3(a-dx)\cos(c+dx)}{8}+\frac{\sin^3(a-dx)\cos(c+dx)}{8d}+\frac{3\sin^2(a-dx)\sin(c+dx)\cos(a-dx)}{4d}+\frac{3\sin(c+dx)\cos^3(a-dx)}{8d} & \text{for } b=-d \\ \frac{x\sin^3\left(a-\frac{dx}{3}\right)\sin(c+dx)}{8}-\frac{3x\sin^2\left(a-\frac{dx}{3}\right)\cos\left(a-\frac{dx}{3}\right)\cos(c+dx)}{8}-\frac{3x\sin\left(a-\frac{dx}{3}\right)\sin(c+dx)\cos^2\left(a-\frac{dx}{3}\right)}{8}+\frac{x\cos^3\left(a-\frac{dx}{3}\right)\cos(c+dx)}{8}-\frac{7\sin^3\left(a-\frac{dx}{3}\right)\cos(c+dx)}{8d}-\frac{3\sin\left(a-\frac{dx}{3}\right)\cos^2\left(a-\frac{dx}{3}\right)\cos(c+dx)}{4d}-\frac{3\sin(c+dx)\cos^3\left(a-\frac{dx}{3}\right)}{8d} & \text{for } b=-\frac{d}{3} \\ \frac{x\sin^3\left(a+\frac{dx}{3}\right)\sin(c+dx)}{8}+\frac{3x\sin^2\left(a+\frac{dx}{3}\right)\cos\left(a+\frac{dx}{3}\right)\cos(c+dx)}{8}-\frac{3x\sin\left(a+\frac{dx}{3}\right)\sin(c+dx)\cos^2\left(a+\frac{dx}{3}\right)}{8}-\frac{x\cos^3\left(a+\frac{dx}{3}\right)\cos(c+dx)}{8}-\frac{7\sin^3\left(a+\frac{dx}{3}\right)\cos(c+dx)}{8d}-\frac{3\sin\left(a+\frac{dx}{3}\right)\cos^2\left(a+\frac{dx}{3}\right)\cos(c+dx)}{4d}+\frac{3\sin(c+dx)\cos^3\left(a+\frac{dx}{3}\right)}{8d} & \text{for } b=\frac{d}{3} \\ \frac{3x\sin^3(a+dx)\sin(c+dx)}{8}+\frac{3x\sin^2(a+dx)\cos(a+dx)\cos(c+dx)}{8}+\frac{3x\sin(a+dx)\sin(c+dx)\cos^2(a+dx)}{8}+\frac{3x\cos^3(a+dx)\cos(c+dx)}{8}-\frac{5\sin^3(a+dx)\cos(c+dx)}{8d}-\frac{3\sin(a+dx)\cos^2(a+dx)\cos(c+dx)}{4d}+\frac{3\sin(c+dx)\cos^3(a+dx)}{8d} & \text{for } b=d \\ -\frac{9b^3\sin^2(a+bx)\sin(c+dx)\cos(a+bx)}{9b^4-10b^2d^2+d^4}-\frac{6b^3\sin(c+dx)\cos^3(a+bx)}{9b^4-10b^2d^2+d^4}+\frac{7b^2d\sin^3(a+bx)\cos(c+dx)}{9b^4-10b^2d^2+d^4}+\frac{6b^2d\sin(a+bx)\cos^2(a+bx)\cos(c+dx)}{9b^4-10b^2d^2+d^4}+\frac{3bd^2\sin^2(a+bx)\sin(c+dx)\cos(a+bx)}{9b^4-10b^2d^2+d^4}-\frac{d^3\sin^3(a+bx)\cos(c+dx)}{9b^4-10b^2d^2+d^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(d*x+c),x)
```

```
[Out] Piecewise((x*sin(a)**3*sin(c), Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - d*x)**3*s
in(c + d*x)/8 - 3*x*sin(a - d*x)**2*cos(a - d*x)*cos(c + d*x)/8 + 3*x*sin(a
 - d*x)*sin(c + d*x)*cos(a - d*x)**2/8 - 3*x*cos(a - d*x)**3*cos(c + d*x)/8
 + sin(a - d*x)**3*cos(c + d*x)/(8*d) + 3*sin(a - d*x)**2*sin(c + d*x)*cos(
a - d*x)/(4*d) + 3*sin(c + d*x)*cos(a - d*x)**3/(8*d), Eq(b, -d)), (x*sin(a
 - d*x/3)**3*sin(c + d*x)/8 - 3*x*sin(a - d*x/3)**2*cos(a - d*x/3)*cos(c +
d*x)/8 - 3*x*sin(a - d*x/3)*sin(c + d*x)*cos(a - d*x/3)**2/8 + x*cos(a - d*
x/3)**3*cos(c + d*x)/8 - 7*sin(a - d*x/3)**3*cos(c + d*x)/(8*d) - 3*sin(a -
```

```
 d*x/3)*cos(a - d*x/3)**2*cos(c + d*x)/(4*d) - 3*sin(c + d*x)*cos(a - d*x/3
)**3/(8*d), Eq(b, -d/3)), (x*sin(a + d*x/3)**3*sin(c + d*x)/8 + 3*x*sin(a +
 d*x/3)**2*cos(a + d*x/3)*cos(c + d*x)/8 - 3*x*sin(a + d*x/3)*sin(c + d*x)*
cos(a + d*x/3)**2/8 - x*cos(a + d*x/3)**3*cos(c + d*x)/8 - 7*sin(a + d*x/3)
**3*cos(c + d*x)/(8*d) - 3*sin(a + d*x/3)*cos(a + d*x/3)**2*cos(c + d*x)/(4
*d) + 3*sin(c + d*x)*cos(a + d*x/3)**3/(8*d), Eq(b, d/3)), (3*x*sin(a + d*x
)**3*sin(c + d*x)/8 + 3*x*sin(a + d*x)**2*cos(a + d*x)*cos(c + d*x)/8 + 3*x
*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)**2/8 + 3*x*cos(a + d*x)**3*cos(c +
d*x)/8 - 5*sin(a + d*x)**3*cos(c + d*x)/(8*d) - 3*sin(a + d*x)*cos(a + d*x)
**2*cos(c + d*x)/(4*d) + 3*sin(c + d*x)*cos(a + d*x)**3/(8*d), Eq(b, d)), (
-9*b**3*sin(a + b*x)**2*sin(c + d*x)*cos(a + b*x)/(9*b**4 - 10*b**2*d**2 +
d**4) - 6*b**3*sin(c + d*x)*cos(a + b*x)**3/(9*b**4 - 10*b**2*d**2 + d**4)
+ 7*b**2*d*sin(a + b*x)**3*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) + 6*
b**2*d*sin(a + b*x)*cos(a + b*x)**2*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 + d
**4) + 3*b*d**2*sin(a + b*x)**2*sin(c + d*x)*cos(a + b*x)/(9*b**4 - 10*b**2
*d**2 + d**4) - d**3*sin(a + b*x)**3*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 +
d**4), True))
```

Giac [A]
time = 0.43, size = 89, normalized size = 0.92

$$\frac{\sin\left(3\,bx+dx+3\,a+c\right)}{8\,(3\,b+d)}-\frac{\sin\left(3\,bx-dx+3\,a-c\right)}{8\,(3\,b-d)}-\frac{3\,\sin\left(bx+dx+a+c\right)}{8\,(b+d)}+\frac{3\,\sin\left(bx-dx+a-c\right)}{8\,(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c),x, algorithm="giac")
```

```
[Out] 1/8*sin(3*b*x + d*x + 3*a + c)/(3*b + d) - 1/8*sin(3*b*x - d*x + 3*a - c)/(
3*b - d) - 3/8*sin(b*x + d*x + a + c)/(b + d) + 3/8*sin(b*x - d*x + a - c)/
(b - d)
```

Mupad [B]
time = 1.72, size = 494, normalized size = 5.09

$$\mathrm{e}^{a\,3\mathrm{i}-c\,1\mathrm{i}+b\,x\,3\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\frac{-3\,b^3-b^2\,d+3\,b\,d^2+d^3}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}}+\frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,\left(-3\,b^3+b^2\,d+3\,b\,d^2-d^3\right)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}}-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(-27\,b^3-27\,b^2\,d+3\,b\,d^2+3\,d^3\right)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}}-\frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(-27\,b^3+27\,b^2\,d+3\,b\,d^2-3\,d^3\right)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}}\right)-\mathrm{e}^{a\,3\mathrm{i}+c\,1\mathrm{i}+b\,x\,3\mathrm{i}+d\,x\,1\mathrm{i}}\,\left(\frac{-3\,b^3+b^2\,d+3\,b\,d^2-d^3}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}}+\frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,\left(-3\,b^3-b^2\,d+3\,b\,d^2+d^3\right)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}}-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(-27\,b^3+27\,b^2\,d+3\,b\,d^2-3\,d^3\right)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}}-\frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(-27\,b^3-27\,b^2\,d+3\,b\,d^2+3\,d^3\right)}{b^4\,144\mathrm{i}-b^2\,d^2\,160\mathrm{i}+d^4\,16\mathrm{i}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(c + d*x),x)
```

```
[Out] exp(a*3i - c*1i + b*x*3i - d*x*1i)*((3*b*d^2 - b^2*d - 3*b^3 + d^3)/(b^4*14
4i + d^4*16i - b^2*d^2*160i) + (exp(- a*6i - b*x*6i)*(3*b*d^2 + b^2*d - 3*b
^3 - d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i) - (exp(- a*2i - b*x*2i)*(3*b
*d^2 - 27*b^2*d - 27*b^3 + 3*d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i) - (e
xp(- a*4i - b*x*4i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(b^4*144i + d^4*
16i - b^2*d^2*160i)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((3*b*d^2 + b^2*d
 - 3*b^3 - d^3)/(b^4*144i + d^4*16i - b^2*d^2*160i) + (exp(- a*6i - b*x*6i)
```

```
*(3*b*d^2 - b^2*d - 3*b^3 + d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i) - (ex
p(- a*2i - b*x*2i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(b^4*144i + d^4*1
6i - b^2*d^2*160i) - (exp(- a*4i - b*x*4i)*(3*b*d^2 - 27*b^2*d - 27*b^3 + 3
*d^3))/(b^4*144i + d^4*16i - b^2*d^2*160i))
```

3.207 $\int \sin^3(a+bx)\sin^2(c+dx)\,dx$

Optimal. Leaf size=138

$$-\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}+\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}-\frac{\cos(3a-2c+(3b-2d)x)}{16(3b-2d)}+\frac{3\cos(a+2c+(b+$$

```
[Out] -3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b+3/16*cos(a-2*c+(b-2*d)*x)/(b-2*d)-1
/16*cos(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)+3/16*cos(a+2*c+(b+2*d)*x)/(b+2*d)-1/
16*cos(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

Rubi [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4665, 2718}

$$\frac{3\cos(a+x(b-2d)-2c)}{16(b-2d)}-\frac{\cos(3a+x(3b-2d)-2c)}{16(3b-2d)}+\frac{3\cos(a+x(b+2d)+2c)}{16(b+2d)}-\frac{\cos(3a+x(3b+2d)+2c)}{16(3b+2d)}-\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[c + d*x]^2,x]
```

```
[Out] (-3*Cos[a + b*x])/(8*b) + Cos[3*a + 3*b*x]/(24*b) + (3*Cos[a - 2*c + (b - 2
*d)*x])/(16*(b - 2*d)) - Cos[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d)) +
(3*Cos[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) - Cos[3*a + 2*c + (3*b + 2*d)
*x]/(16*(3*b + 2*d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4665

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin^2(c+dx)\,dx &= \int \left(\frac{3}{8}\sin(a+bx)-\frac{1}{8}\sin(3a+3bx)-\frac{3}{16}\sin(a-2c+(b-2d)x)+\frac{1}{16}\sin\right.\\ &= \frac{1}{16}\int \sin(3a-2c+(3b-2d)x)\,dx+\frac{1}{16}\int \sin(3a+2c+(3b+2d)x)\,dx-\\ &= -\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}+\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}-\frac{\cos(3a-2}{16(}\end{aligned}$$

Mathematica [A]
time = 1.65, size = 153, normalized size = 1.11

$$\frac{1}{48}\left(-\frac{18\cos(a)\cos(bx)}{b}+\frac{2\cos(3a)\cos(3bx)}{b}+\frac{9\cos(a-2c+bx-2dx)}{b-2d}-\frac{3\cos(3a-2c+3bx-2dx)}{3b-2d}+\frac{9\cos(a+2c+bx+2dx)}{b+2d}-\frac{3\cos(3a+2c+3bx+2dx)}{3b+2d}+\frac{18\sin(a)\sin(bx)}{b}-\frac{2\sin(3a)\sin(3bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[c + d*x]^2,x]
```

```
[Out] ((-18*Cos[a]*Cos[b*x])/b + (2*Cos[3*a]*Cos[3*b*x])/b + (9*Cos[a - 2*c + b*x
 - 2*d*x])/(b - 2*d) - (3*Cos[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*d) + (9*
Cos[a + 2*c + b*x + 2*d*x])/(b + 2*d) - (3*Cos[3*a + 2*c + 3*b*x + 2*d*x])/
(3*b + 2*d) + (18*Sin[a]*Sin[b*x])/b - (2*Sin[3*a]*Sin[3*b*x])/b)/48
```

Maple [A]
time = 0.22, size = 127, normalized size = 0.92

method	result
default	$-\frac{3\cos(bx+a)}{8b}+\frac{\cos(3bx+3a)}{24b}+\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}-\frac{\cos(3a-2c+(3b-2d)x)}{16(3b-2d)}+\frac{3\cos(a+2c+(b+2d)x)}{16(b+2d)}-\frac{\cos(3a+2c+(3}{16(3b+2}$
risch	$-\frac{3\cos(bx+a)}{8b}+\frac{27\cos(bx-2dx+a-2c)b^3}{16(b+2d)(3b+2d)(3b-2d)(b-2d)}+\frac{27\cos(bx-2dx+a-2c)b^2d}{8(b+2d)(3b+2d)(3b-2d)(b-2d)}-\frac{3\cos(bx-2dx+a-2c)b\,d^2}{4(b+2d)(3b+2d)(3b-2d)(b-2d)}-\frac{}{2(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b+3/16*cos(a-2*c+(b-2*d)*x)/(b-2*d)-1
/16*cos(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)+3/16*cos(a+2*c+(b+2*d)*x)/(b+2*d)-1/
16*cos(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(126) = 252$.
time = 0.35, size = 1360, normalized size = 9.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/96*(3*(3*b^4*cos(2*c) - 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3
*cos(2*c))*cos((3*b + 2*d)*x + 3*a + 4*c) + 3*(3*b^4*cos(2*c) - 2*b^3*d*cos
(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((3*b + 2*d)*x + 3*a) +
3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*
c))*cos(-(3*b - 2*d)*x - 3*a + 4*c) + 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c)
- 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(3*b - 2*d)*x - 3*a) - 9*(9*
b^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*c
os((b + 2*d)*x + a + 4*c) - 9*(9*b^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d
```

```
^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((b + 2*d)*x + a) - 9*(9*b^4*cos(2*c) +
18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(b - 2*d)*x
 - a + 4*c) - 9*(9*b^4*cos(2*c) + 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) -
8*b*d^3*cos(2*c))*cos(-(b - 2*d)*x - a) - 2*(9*b^4*cos(2*c) - 40*b^2*d^2*co
s(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a + 2*c) - 2*(9*b^4*cos(2*c) - 40*b
^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a - 2*c) + 18*(9*b^4*cos(2
*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a + 2*c) + 18*(9*b^4
*cos(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a - 2*c) + 3*(
3*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))
*sin((3*b + 2*d)*x + 3*a + 4*c) - 3*(3*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12
*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((3*b + 2*d)*x + 3*a) + 3*(3*b^4*s
in(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(
3*b - 2*d)*x - 3*a + 4*c) - 3*(3*b^4*sin(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d
^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(3*b - 2*d)*x - 3*a) - 9*(9*b^4*sin(2*
c) - 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((b + 2*
d)*x + a + 4*c) + 9*(9*b^4*sin(2*c) - 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c
) + 8*b*d^3*sin(2*c))*sin((b + 2*d)*x + a) - 9*(9*b^4*sin(2*c) + 18*b^3*d*s
in(2*c) - 4*b^2*d^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(b - 2*d)*x - a + 4*c
) + 9*(9*b^4*sin(2*c) + 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c) - 8*b*d^3*si
n(2*c))*sin(-(b - 2*d)*x - a) - 2*(9*b^4*sin(2*c) - 40*b^2*d^2*sin(2*c) + 1
6*d^4*sin(2*c))*sin(3*b*x + 3*a + 2*c) + 2*(9*b^4*sin(2*c) - 40*b^2*d^2*sin
(2*c) + 16*d^4*sin(2*c))*sin(3*b*x + 3*a - 2*c) + 18*(9*b^4*sin(2*c) - 40*b
^2*d^2*sin(2*c) + 16*d^4*sin(2*c))*sin(b*x + a + 2*c) - 18*(9*b^4*sin(2*c)
- 40*b^2*d^2*sin(2*c) + 16*d^4*sin(2*c))*sin(b*x + a - 2*c))/(9*b^5*cos(2*c
)^2 + 9*b^5*sin(2*c)^2 + 16*(b*cos(2*c)^2 + b*sin(2*c)^2)*d^4 - 40*(b^3*cos
(2*c)^2 + b^3*sin(2*c)^2)*d^2)
```

Fricas [**A**]
time = 2.23, size = 189, normalized size = 1.37

$$\frac{(9\,b^4-38\,b^2d^2+8\,d^4)\cos(bx+a)^3+6\left(7\,b^3d-4\,bd^3-(b^3d-4\,bd^3)\cos(bx+a)^2\right)\cos(dx+c)\sin(bx+a)\sin(dx+c)-9\left((b^4-4\,b^2d^2)\cos(bx+a)^3-(3\,b^4-4\,b^2d^2)\cos(bx+a)\right)\cos(dx+c)^2-3\,(9\,b^4-26\,b^2d^2+8\,d^4)\cos(bx+a)}{3\,(9\,b^5-40\,b^3d^2+16\,bd^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((9*b^4 - 38*b^2*d^2 + 8*d^4)*cos(b*x + a)^3 + 6*(7*b^3*d - 4*b*d^3 - (
b^3*d - 4*b*d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a)*sin(d*x + c) - 9
*((b^4 - 4*b^2*d^2)*cos(b*x + a)^3 - (3*b^4 - 4*b^2*d^2)*cos(b*x + a))*cos(
d*x + c)^2 - 3*(9*b^4 - 26*b^2*d^2 + 8*d^4)*cos(b*x + a))/(9*b^5 - 40*b^3*d
^2 + 16*b*d^4)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2020 vs. 2(116) = 232.
time = 6.56, size = 2020, normalized size = 14.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(d*x+c)**2,x)
```

```
[Out] Piecewise((x*sin(a)**3*sin(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2
/2 + x*cos(c + d*x)**2/2 - sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**3, Eq(b
, 0)), (3*x*sin(a - 2*d*x)**3*sin(c + d*x)**2/16 - 3*x*sin(a - 2*d*x)**3*co
s(c + d*x)**2/16 - 3*x*sin(a - 2*d*x)**2*sin(c + d*x)*cos(a - 2*d*x)*cos(c
+ d*x)/8 + 3*x*sin(a - 2*d*x)*sin(c + d*x)**2*cos(a - 2*d*x)**2/16 - 3*x*si
n(a - 2*d*x)*cos(a - 2*d*x)**2*cos(c + d*x)**2/16 - 3*x*sin(c + d*x)*cos(a
- 2*d*x)**3*cos(c + d*x)/8 + 3*sin(a - 2*d*x)**3*sin(c + d*x)*cos(c + d*x)/
(16*d) + sin(a - 2*d*x)**2*sin(c + d*x)**2*cos(a - 2*d*x)/(2*d) + sin(a - 2
*d*x)*sin(c + d*x)*cos(a - 2*d*x)**2*cos(c + d*x)/(8*d) + 31*sin(c + d*x)**
2*cos(a - 2*d*x)**3/(96*d) + cos(a - 2*d*x)**3*cos(c + d*x)**2/(96*d), Eq(b
, -2*d)), (x*sin(a - 2*d*x/3)**3*sin(c + d*x)**2/16 - x*sin(a - 2*d*x/3)**3
*cos(c + d*x)**2/16 - 3*x*sin(a - 2*d*x/3)**2*sin(c + d*x)*cos(a - 2*d*x/3)
*cos(c + d*x)/8 - 3*x*sin(a - 2*d*x/3)*sin(c + d*x)**2*cos(a - 2*d*x/3)**2/
16 + 3*x*sin(a - 2*d*x/3)*cos(a - 2*d*x/3)**2*cos(c + d*x)**2/16 + x*sin(c
+ d*x)*cos(a - 2*d*x/3)**3*cos(c + d*x)/8 + sin(a - 2*d*x/3)**3*sin(c + d*x
)*cos(c + d*x)/(16*d) + 3*sin(a - 2*d*x/3)**2*sin(c + d*x)**2*cos(a - 2*d*x
/3)/(2*d) - 15*sin(a - 2*d*x/3)*sin(c + d*x)*cos(a - 2*d*x/3)**2*cos(c + d*
x)/(8*d) + 5*sin(c + d*x)**2*cos(a - 2*d*x/3)**3/(32*d) + 27*cos(a - 2*d*x/
3)**3*cos(c + d*x)**2/(32*d), Eq(b, -2*d/3)), (x*sin(a + 2*d*x/3)**3*sin(c
+ d*x)**2/16 - x*sin(a + 2*d*x/3)**3*cos(c + d*x)**2/16 + 3*x*sin(a + 2*d*x
/3)**2*sin(c + d*x)*cos(a + 2*d*x/3)*cos(c + d*x)/8 - 3*x*sin(a + 2*d*x/3)*
sin(c + d*x)**2*cos(a + 2*d*x/3)**2/16 + 3*x*sin(a + 2*d*x/3)*cos(a + 2*d*x
/3)**2*cos(c + d*x)**2/16 - x*sin(c + d*x)*cos(a + 2*d*x/3)**3*cos(c + d*x)
/8 + sin(a + 2*d*x/3)**3*sin(c + d*x)*cos(c + d*x)/(16*d) - 3*sin(a + 2*d*x
/3)**2*sin(c + d*x)**2*cos(a + 2*d*x/3)/(2*d) - 15*sin(a + 2*d*x/3)*sin(c +
 d*x)*cos(a + 2*d*x/3)**2*cos(c + d*x)/(8*d) - 5*sin(c + d*x)**2*cos(a + 2*
d*x/3)**3/(32*d) - 27*cos(a + 2*d*x/3)**3*cos(c + d*x)**2/(32*d), Eq(b, 2*d
/3)), (3*x*sin(a + 2*d*x)**3*sin(c + d*x)**2/16 - 3*x*sin(a + 2*d*x)**3*cos
(c + d*x)**2/16 + 3*x*sin(a + 2*d*x)**2*sin(c + d*x)*cos(a + 2*d*x)*cos(c +
 d*x)/8 + 3*x*sin(a + 2*d*x)*sin(c + d*x)**2*cos(a + 2*d*x)**2/16 - 3*x*sin
(a + 2*d*x)*cos(a + 2*d*x)**2*cos(c + d*x)**2/16 + 3*x*sin(c + d*x)*cos(a +
 2*d*x)**3*cos(c + d*x)/8 + 3*sin(a + 2*d*x)**3*sin(c + d*x)*cos(c + d*x)/(
16*d) - sin(a + 2*d*x)**2*sin(c + d*x)**2*cos(a + 2*d*x)/(2*d) + sin(a + 2*
d*x)*sin(c + d*x)*cos(a + 2*d*x)**2*cos(c + d*x)/(8*d) - 31*sin(c + d*x)**2
*cos(a + 2*d*x)**3/(96*d) - cos(a + 2*d*x)**3*cos(c + d*x)**2/(96*d), Eq(b,
 2*d)), (-27*b**4*sin(a + b*x)**2*sin(c + d*x)**2*cos(a + b*x)/(27*b**5 - 1
20*b**3*d**2 + 48*b*d**4) - 18*b**4*sin(c + d*x)**2*cos(a + b*x)**3/(27*b**
5 - 120*b**3*d**2 + 48*b*d**4) + 42*b**3*d*sin(a + b*x)**3*sin(c + d*x)*cos
(c + d*x)/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) + 36*b**3*d*sin(a + b*x)*si
n(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(27*b**5 - 120*b**3*d**2 + 48*b*d**
4) + 78*b**2*d**2*sin(a + b*x)**2*sin(c + d*x)**2*cos(a + b*x)/(27*b**5 - 1
```

```
20*b**3*d**2 + 48*b*d**4) + 42*b**2*d**2*sin(a + b*x)**2*cos(a + b*x)*cos(c
 + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) + 40*b**2*d**2*sin(c + d*x
)**2*cos(a + b*x)**3/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) + 40*b**2*d**2*c
os(a + b*x)**3*cos(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 24*b
*d**3*sin(a + b*x)**3*sin(c + d*x)*cos(c + d*x)/(27*b**5 - 120*b**3*d**2 +
48*b*d**4) - 24*d**4*sin(a + b*x)**2*sin(c + d*x)**2*cos(a + b*x)/(27*b**5
- 120*b**3*d**2 + 48*b*d**4) - 24*d**4*sin(a + b*x)**2*cos(a + b*x)*cos(c +
 d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 16*d**4*sin(c + d*x)**2*co
s(a + b*x)**3/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 16*d**4*cos(a + b*x)*
*3*cos(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4), True))
```

Giac [**A**]
time = 0.41, size = 124, normalized size = 0.90

$$-\frac{\cos\left(3\,bx+2\,dx+3\,a+2\,c\right)}{16\,\left(3\,b+2\,d\right)}-\frac{\cos\left(3\,bx-2\,dx+3\,a-2\,c\right)}{16\,\left(3\,b-2\,d\right)}+\frac{\cos\left(3\,bx+3\,a\right)}{24\,b}+\frac{3\,\cos\left(bx+2\,dx+a+2\,c\right)}{16\,\left(b+2\,d\right)}+\frac{3\,\cos\left(bx-2\,dx+a-2\,c\right)}{16\,\left(b-2\,d\right)}-\frac{3\,\cos\left(bx+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/16*cos(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) - 1/16*cos(3*b*x - 2*d*x +
 3*a - 2*c)/(3*b - 2*d) + 1/24*cos(3*b*x + 3*a)/b + 3/16*cos(b*x + 2*d*x +
a + 2*c)/(b + 2*d) + 3/16*cos(b*x - 2*d*x + a - 2*c)/(b - 2*d) - 3/8*cos(b*
x + a)/b
```

Mupad [**B**]
time = 2.13, size = 437, normalized size = 3.17

$$\frac{81\,b^4\cos\left(a-2\,c+b\,x-2\,d\,x\right)+81\,b^4\cos\left(a+2\,c+b\,x+2\,d\,x\right)-162\,b^4\cos\left(a+b\,x\right)-288\,d^4\cos\left(a+b\,x\right)-9\,b^4\cos\left(3\,a-2\,c+3\,b\,x-2\,d\,x\right)-9\,b^4\cos\left(3\,a+2\,c+3\,b\,x+2\,d\,x\right)+18\,b^4\cos\left(3\,a+3\,b\,x\right)+32\,d^4\cos\left(3\,a+3\,b\,x\right)+24\,b\,d^3\cos\left(3\,a-2\,c+3\,b\,x-2\,d\,x\right)-24\,b\,d^3\cos\left(3\,a+2\,c+3\,b\,x+2\,d\,x\right)-6\,b^3\,d\cos\left(3\,a-2\,c+3\,b\,x-2\,d\,x\right)+6\,b^3\,d\cos\left(3\,a+2\,c+3\,b\,x+2\,d\,x\right)-36\,b^2\,d^2\cos\left(a-2\,c+b\,x-2\,d\,x\right)-36\,b^2\,d^2\cos\left(a+2\,c+b\,x+2\,d\,x\right)+720\,b^2\,d^2\cos\left(a+b\,x\right)+36\,b^2\,d^2\cos\left(3\,a-2\,c+3\,b\,x-2\,d\,x\right)+36\,b^2\,d^2\cos\left(3\,a+2\,c+3\,b\,x+2\,d\,x\right)-80\,b^2\,d^2\cos\left(3\,a+3\,b\,x\right)-72\,b\,d^3\cos\left(a-2\,c+b\,x-2\,d\,x\right)+72\,b\,d^3\cos\left(a+2\,c+b\,x+2\,d\,x\right)+162\,b^3\,d\cos\left(a-2\,c+b\,x-2\,d\,x\right)-162\,b^3\,d\cos\left(a+2\,c+b\,x+2\,d\,x\right)}{48\,\left(9\,b^5-40\,b^3\,d^2+16\,b\,d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(c + d*x)^2,x)
```

```
[Out] (81*b^4*cos(a - 2*c + b*x - 2*d*x) + 81*b^4*cos(a + 2*c + b*x + 2*d*x) - 16
2*b^4*cos(a + b*x) - 288*d^4*cos(a + b*x) - 9*b^4*cos(3*a - 2*c + 3*b*x - 2
*d*x) - 9*b^4*cos(3*a + 2*c + 3*b*x + 2*d*x) + 18*b^4*cos(3*a + 3*b*x) + 32
*d^4*cos(3*a + 3*b*x) + 24*b*d^3*cos(3*a - 2*c + 3*b*x - 2*d*x) - 24*b*d^3*
cos(3*a + 2*c + 3*b*x + 2*d*x) - 6*b^3*d*cos(3*a - 2*c + 3*b*x - 2*d*x) + 6
*b^3*d*cos(3*a + 2*c + 3*b*x + 2*d*x) - 36*b^2*d^2*cos(a - 2*c + b*x - 2*d*
x) - 36*b^2*d^2*cos(a + 2*c + b*x + 2*d*x) + 720*b^2*d^2*cos(a + b*x) + 36*
b^2*d^2*cos(3*a - 2*c + 3*b*x - 2*d*x) + 36*b^2*d^2*cos(3*a + 2*c + 3*b*x +
 2*d*x) - 80*b^2*d^2*cos(3*a + 3*b*x) - 72*b*d^3*cos(a - 2*c + b*x - 2*d*x)
 + 72*b*d^3*cos(a + 2*c + b*x + 2*d*x) + 162*b^3*d*cos(a - 2*c + b*x - 2*d*
x) - 162*b^3*d*cos(a + 2*c + b*x + 2*d*x))/(48*(16*b*d^4 + 9*b^5 - 40*b^3*d
^2))
```

3.208 $\int \sin^3(a+bx)\sin^3(c+dx)\,dx$

Optimal. Leaf size=195

$$-\frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}+\frac{\sin(3(a-c)+3(b-d)x)}{96(b-d)}-\frac{3\sin(3a-c+(3b-d)x)}{32(3b-d)}$$

```
[Out] -3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sin(a-c+(b-d)*x)/(b-d)+1/96*sin(3*a
-3*c+3*(b-d)*x)/(b-d)-3/32*sin(3*a-c+(3*b-d)*x)/(3*b-d)-9/32*sin(a+c+(b+d)*
x)/(b+d)-1/96*sin(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*sin(3*a+c+(3*b+d)*x)/(3*b+d
)+3/32*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A]

time = 0.10, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4665, 2717}

$$-\frac{3\sin(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sin(a+x(b-d)-c)}{32(b-d)}+\frac{\sin(3(a-c)+3x(b-d))}{96(b-d)}-\frac{3\sin(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\sin(a+x(b+d)+c)}{32(b+d)}-\frac{\sin(3(a+c)+3x(b+d))}{96(b+d)}+\frac{3\sin(3a+x(3b+d)+c)}{32(3b+d)}+\frac{3\sin(a+x(b+3d)+3c)}{32(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3*Sin[c + d*x]^3,x]
```

```
[Out] (-3*Sin[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sin[a - c + (b - d)*x])
/(32*(b - d)) + Sin[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) - (3*Sin[3*a - c
+ (3*b - d)*x])/(32*(3*b - d)) - (9*Sin[a + c + (b + d)*x])/(32*(b + d)) -
Sin[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sin[3*a + c + (3*b + d)*x])/
(32*(3*b + d)) + (3*Sin[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4665

```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\sin^3(c+dx)\,dx &= \int \left(-\frac{3}{32}\cos(a-3c+(b-3d)x)+\frac{9}{32}\cos(a-c+(b-d)x)+\frac{1}{32}\cos(3(\right.\\ &= \frac{1}{32}\int \cos(3(a-c)+3(b-d)x)\,dx-\frac{1}{32}\int \cos(3(a+c)+3(b+d)x)\,dx-\\ &= -\frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}+\frac{\sin(3(a-c)+3(}{96(b-d)}\end{aligned}$$

Mathematica [A]
time = 1.71, size = 177, normalized size = 0.91

$$\frac{1}{96}\left(-\frac{9\sin(a-3c+bx-3dx)}{b-3d}+\frac{27\sin(a-c+bx-dx)}{b-d}+\frac{\sin(3(a-c+bx-dx))}{b-d}-\frac{9\sin(3a-c+3bx-dx)}{3b-d}+\frac{9\sin(3a+c+3bx+dx)}{3b+d}+\frac{9\sin(a+3c+bx+3dx)}{b+3d}-\frac{27\sin(a+c+(b+d)x)}{b+d}-\frac{\sin(3(a+c+(b+d)x))}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3*Sin[c + d*x]^3,x]
```

```
[Out] ((-9*Sin[a - 3*c + b*x - 3*d*x])/(b - 3*d) + (27*Sin[a - c + b*x - d*x])/(b
 - d) + Sin[3*(a - c + b*x - d*x)]/(b - d) - (9*Sin[3*a - c + 3*b*x - d*x])
/(3*b - d) + (9*Sin[3*a + c + 3*b*x + d*x])/(3*b + d) + (9*Sin[a + 3*c + b*
x + 3*d*x])/(b + 3*d) - (27*Sin[a + c + (b + d)*x])/(b + d) - Sin[3*(a + c
+ (b + d)*x)]/(b + d))/96
```

Maple [A]
time = 0.38, size = 184, normalized size = 0.94

method	result
default	$-\frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}-\frac{9\sin(a+c+(b+d)x)}{32(b+d)}+\frac{3\sin(a+3c+(b+3d)x)}{32(b+3d)}+\frac{\sin((3b-3d)x+3a-3c)}{96b-96d}-\frac{3\sin}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3*sin(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sin(a-c+(b-d)*x)/(b-d)-9/32*sin(a+c
+(b+d)*x)/(b+d)+3/32*sin(a+3*c+(b+3*d)*x)/(b+3*d)+1/96/(b-d)*sin((3*b-3*d)*
x+3*a-3*c)-3/32*sin(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*sin(3*a+c+(3*b+d)*x)/(3*b
+d)-1/96/(b+d)*sin((3*b+3*d)*x+3*a+3*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2612 vs. 2(179) = 358.
time = 0.44, size = 2612, normalized size = 13.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*d
^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a + 4
*c) - 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*d^3
*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a - 2*c
) + 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*d^3*s
in(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a + 4*c)
```

- 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*d^3*si
n(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a - 2*c)
+ 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^3*
sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a + 6*c) - 9*
(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^3*sin(
3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a) - (9*b^5*sin(3
*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^
4*sin(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a + 6*c) + (9*b^5*sin(3*c)
- 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*s
in(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a) - 27*(9*b^5*sin(3*c) - 9*b
^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c
) - 9*d^5*sin(3*c))*cos((b + d)*x + a + 4*c) + 27*(9*b^5*sin(3*c) - 9*b^4*d
*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) -
9*d^5*sin(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5*sin(3*c) + 9*b^4*d*sin
(3*c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^
5*sin(3*c))*cos(-(b - d)*x - a + 4*c) + 27*(9*b^5*sin(3*c) + 9*b^4*d*sin(3*
c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*s
in(3*c))*cos(-(b - d)*x - a - 2*c) - (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c) - 8
2*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin(3*c
))*cos(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c) - 82*
b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin(3*c))
cos(-3(b - d)*x - 3*a) + 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^3*d
^2*sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*cos(-(
b - 3*d)*x - a + 6*c) - 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^3*d^2*
sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*cos(-(b -
3*d)*x - a) - 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 1
0*b^2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin((3*b + d)*x +
3*a + 4*c) - 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*
b^2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin((3*b + d)*x + 3*
a - 2*c) - 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^
2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*sin(-(3*b - d)*x - 3*a
+ 4*c) - 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2
*d^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*sin(-(3*b - d)*x - 3*a
- 2*c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b
^2*d^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*sin((b + 3*d)*x + a + 6*
c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b^2*d
^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*sin((b + 3*d)*x + a) + (9*b^
5*cos(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) +
9*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin(3*(b + d)*x + 3*a + 6*c) + (9*b^5*c
os(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*
b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin(3*(b + d)*x + 3*a) + 27*(9*b^5*cos(3*c
) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*
cos(3*c) - 9*d^5*cos(3*c))*sin((b + d)*x + a + 4*c) + 27*(9*b^5*cos(3*c) -
9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(
3*c) - 9*d^5*cos(3*c))*sin((b + d)*x + a - 2*c) + 27*(9*b^5*cos(3*c) + 9*b^

```
4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c)
 + 9*d^5*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 27*(9*b^5*cos(3*c) + 9*b^4*d
*cos(3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) +
9*d^5*cos(3*c))*sin(-(b - d)*x - a - 2*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3
*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*
cos(3*c))*sin(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3*c
) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*co
s(3*c))*sin(-3*(b - d)*x - 3*a) - 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 1
0*b^3*d^2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))
*sin(-(b - 3*d)*x - a + 6*c) - 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 10*b
^3*d^2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))*si
n(-(b - 3*d)*x - a))/(9*b^6*cos(3*c)^2 + 9*b^6*sin(3*c)^2 - 9*(cos(3*c)^2 +
 sin(3*c)^2)*d^6 + 91*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^4 - 91*(b^4*cos(3
*c)^2 + b^4*sin(3*c)^2)*d^2)
```

Fricas [**A**]
time = 1.96, size = 291, normalized size = 1.49

$$-\frac{\left(\left(63\,b^4d-88\,b^2d^3+9\,d^5-\left(9\,b^4d-82\,b^2d^3+9\,d^5\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)^3-3\left(21\,b^4d-70\,b^2d^3+9\,d^5-\left(3\,b^4d-28\,b^2d^3+9\,d^5\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)\right)\sin\left(bx+a\right)-\left(\left(9\,b^5-88\,b^3d^2+63\,bd^4\right)\cos\left(bx+a\right)^3-\left(\left(9\,b^5-82\,b^3d^2+9\,bd^4\right)\cos\left(bx+a\right)^3-3\left(9\,b^5-28\,b^3d^2+3\,bd^4\right)\cos\left(bx+a\right)\right)\cos\left(dx+c\right)^2-3\left(9\,b^5-70\,b^3d^2+21\,bd^4\right)\cos\left(bx+a\right)\right)\sin\left(dx+c\right)}{3\left(9\,b^6-91\,b^4d^2+91\,b^2d^4-9\,d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(((63*b^4*d - 88*b^2*d^3 + 9*d^5 - (9*b^4*d - 82*b^2*d^3 + 9*d^5)*cos(
b*x + a)^2)*cos(d*x + c)^3 - 3*(21*b^4*d - 70*b^2*d^3 + 9*d^5 - (3*b^4*d -
28*b^2*d^3 + 9*d^5)*cos(b*x + a)^2)*cos(d*x + c))*sin(b*x + a) - ((9*b^5 -
88*b^3*d^2 + 63*b*d^4)*cos(b*x + a)^3 - ((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cos
(b*x + a)^3 - 3*(9*b^5 - 28*b^3*d^2 + 3*b*d^4)*cos(b*x + a))*cos(d*x + c)^2
 - 3*(9*b^5 - 70*b^3*d^2 + 21*b*d^4)*cos(b*x + a))*sin(d*x + c))/(9*b^6 - 9
1*b^4*d^2 + 91*b^2*d^4 - 9*d^6)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3580 vs. $2(172) = 344$.
time = 21.22, size = 3580, normalized size = 18.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3*sin(d*x+c)**3,x)
```

```
[Out] Piecewise((x*sin(a)**3*sin(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - 3*d*x)
**3*sin(c + d*x)**3/32 - 9*x*sin(a - 3*d*x)**3*sin(c + d*x)*cos(c + d*x)**2
/32 - 9*x*sin(a - 3*d*x)**2*sin(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/32
+ 3*x*sin(a - 3*d*x)**2*cos(a - 3*d*x)*cos(c + d*x)**3/32 + 3*x*sin(a - 3*d
*x)*sin(c + d*x)**3*cos(a - 3*d*x)**2/32 - 9*x*sin(a - 3*d*x)*sin(c + d*x)*
cos(a - 3*d*x)**2*cos(c + d*x)**2/32 - 9*x*sin(c + d*x)**2*cos(a - 3*d*x)**
```

```
3*cos(c + d*x)/32 + 3*x*cos(a - 3*d*x)**3*cos(c + d*x)**3/32 + 13*sin(a - 3
*d*x)**3*sin(c + d*x)**2*cos(c + d*x)/(320*d) + sin(a - 3*d*x)**3*cos(c + d
*x)**3/(12*d) + 101*sin(a - 3*d*x)**2*sin(c + d*x)**3*cos(a - 3*d*x)/(320*d
) + 3*sin(a - 3*d*x)**2*sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/(20*d)
+ 27*sin(a - 3*d*x)*cos(a - 3*d*x)**2*cos(c + d*x)**3/(320*d) + sin(c + d*x
)**3*cos(a - 3*d*x)**3/(5*d) + 51*sin(c + d*x)*cos(a - 3*d*x)**3*cos(c + d*
x)**2/(320*d), Eq(b, -3*d)), (5*x*sin(a - d*x)**3*sin(c + d*x)**3/16 + 3*x*
sin(a - d*x)**3*sin(c + d*x)*cos(c + d*x)**2/16 - 9*x*sin(a - d*x)**2*sin(c
 + d*x)**2*cos(a - d*x)*cos(c + d*x)/16 - 3*x*sin(a - d*x)**2*cos(a - d*x)*
cos(c + d*x)**3/16 + 3*x*sin(a - d*x)*sin(c + d*x)**3*cos(a - d*x)**2/16 +
9*x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)**2/16 - 3*x*sin(
c + d*x)**2*cos(a - d*x)**3*cos(c + d*x)/16 - 5*x*cos(a - d*x)**3*cos(c + d
*x)**3/16 - 3*sin(a - d*x)**3*sin(c + d*x)**2*cos(c + d*x)/(16*d) - sin(a -
 d*x)**3*cos(c + d*x)**3/(16*d) + sin(a - d*x)**2*sin(c + d*x)**3*cos(a - d
*x)/(2*d) - 3*sin(a - d*x)*sin(c + d*x)**2*cos(a - d*x)**2*cos(c + d*x)/(4*
d) + sin(c + d*x)**3*cos(a - d*x)**3/(48*d) + 5*sin(c + d*x)*cos(a - d*x)**
3*cos(c + d*x)**2/(16*d), Eq(b, -d)), (3*x*sin(a - d*x/3)**3*sin(c + d*x)**
3/32 + 3*x*sin(a - d*x/3)**3*sin(c + d*x)*cos(c + d*x)**2/32 - 9*x*sin(a -
d*x/3)**2*sin(c + d*x)**2*cos(a - d*x/3)*cos(c + d*x)/32 - 9*x*sin(a - d*x/
3)**2*cos(a - d*x/3)*cos(c + d*x)**3/32 - 9*x*sin(a - d*x/3)*sin(c + d*x)**
3*cos(a - d*x/3)**2/32 - 9*x*sin(a - d*x/3)*sin(c + d*x)*cos(a - d*x/3)**2*
cos(c + d*x)**2/32 + 3*x*sin(c + d*x)**2*cos(a - d*x/3)**3*cos(c + d*x)/32
+ 3*x*cos(a - d*x/3)**3*cos(c + d*x)**3/32 - 303*sin(a - d*x/3)**3*sin(c +
d*x)**2*cos(c + d*x)/(320*d) - 3*sin(a - d*x/3)**3*cos(c + d*x)**3/(5*d) -
39*sin(a - d*x/3)**2*sin(c + d*x)**3*cos(a - d*x/3)/(320*d) - 9*sin(a - d*x
/3)*sin(c + d*x)**2*cos(a - d*x/3)**2*cos(c + d*x)/(20*d) - 153*sin(a - d*x
/3)*cos(a - d*x/3)**2*cos(c + d*x)**3/(320*d) - sin(c + d*x)**3*cos(a - d*x
/3)**3/(4*d) - 81*sin(c + d*x)*cos(a - d*x/3)**3*cos(c + d*x)**2/(320*d), E
q(b, -d/3)), (3*x*sin(a + d*x/3)**3*sin(c + d*x)**3/32 + 3*x*sin(a + d*x/3)
**3*sin(c + d*x)*cos(c + d*x)**2/32 + 9*x*sin(a + d*x/3)**2*sin(c + d*x)**2
*cos(a + d*x/3)*cos(c + d*x)/32 + 9*x*sin(a + d*x/3)**2*cos(a + d*x/3)*cos(
c + d*x)**3/32 - 9*x*sin(a + d*x/3)*sin(c + d*x)**3*cos(a + d*x/3)**2/32 -
9*x*sin(a + d*x/3)*sin(c + d*x)*cos(a + d*x/3)**2*cos(c + d*x)**2/32 - 3*x*
sin(c + d*x)**2*cos(a + d*x/3)**3*cos(c + d*x)/32 - 3*x*cos(a + d*x/3)**3*c
os(c + d*x)**3/32 - 303*sin(a + d*x/3)**3*sin(c + d*x)**2*cos(c + d*x)/(320
*d) - 3*sin(a + d*x/3)**3*cos(c + d*x)**3/(5*d) + 39*sin(a + d*x/3)**2*sin(
c + d*x)**3*cos(a + d*x/3)/(320*d) - 9*sin(a + d*x/3)*sin(c + d*x)**2*cos(a
 + d*x/3)**2*cos(c + d*x)/(20*d) - 153*sin(a + d*x/3)*cos(a + d*x/3)**2*cos
(c + d*x)**3/(320*d) + sin(c + d*x)**3*cos(a + d*x/3)**3/(4*d) + 81*sin(c +
 d*x)*cos(a + d*x/3)**3*cos(c + d*x)**2/(320*d), Eq(b, d/3)), (5*x*sin(a +
d*x)**3*sin(c + d*x)**3/16 + 3*x*sin(a + d*x)**3*sin(c + d*x)*cos(c + d*x)*
*2/16 + 9*x*sin(a + d*x)**2*sin(c + d*x)**2*cos(a + d*x)*cos(c + d*x)/16 +
3*x*sin(a + d*x)**2*cos(a + d*x)*cos(c + d*x)**3/16 + 3*x*sin(a + d*x)*sin(
c + d*x)**3*cos(a + d*x)**2/16 + 9*x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)
**2*cos(c + d*x)**2/16 + 3*x*sin(c + d*x)**2*cos(a + d*x)**3*cos(c + d*x)/1
```

```
6 + 5*x*cos(a + d*x)**3*cos(c + d*x)**3/16 - sin(a + d*x)**3*sin(c + d*x)**
2*cos(c + d*x)/(2*d) - 13*sin(a + d*x)**3*cos(c + d*x)**3/(48*d) - 3*sin(a
+ d*x)**2*sin(c + d*x)**3*cos(a + d*x)/(16*d) - 3*sin(a + d*x)*sin(c + d*x)
**2*cos(a + d*x)**2*cos(c + d*x)/(4*d) - 5*sin(a + d*x)*cos(a + d*x)**2*cos
(c + d*x)**3/(16*d) + 3*sin(c + d*x)**3*cos(a + d*x)**3/(16*d), Eq(b, d)),
(3*x*sin(a + 3*d*x)**3*sin(c + d*x)**3/32 - 9*x*sin(a + 3*d*x)**3*sin(c + d
*x)*cos(c + d*x)**2/32 + 9*x*sin(a + 3*d*x)**2*sin(c + d*x)**2*cos(a + 3*d*
x)*cos(c + d*x)/32 - 3*x*sin(a + 3*d*x)**2*cos(a + 3*d*x)*cos(c + d*x)**3/3
2 + 3*x*sin(a + 3*d*x)*sin(c + d*x)**3*cos(a + 3*d*x)**2/32 - 9*x*sin(a + 3
*d*x)*sin(c + d*x)*cos(a + 3*d*x)**2*cos(c + d*x)**2/32 + 9*x*sin(c + d*x)*
*2*cos(a + 3*d*x)**3*cos(c + d*x)/32 - 3*x*cos(a + 3*d*x)**3*cos(c + d*x)**
3/32 + 61*sin(a + 3*d*x)**3*sin(c + d*x)**2*cos(c + d*x)/(320*d) + sin(a +
3*d*x)**3*cos(c + d*x)**3/(30*d) - 117*sin(a + 3*d*x)**2*sin(c + d*x)**3*co
s(a + 3*d*x)/(320*d) + 3*sin(a + 3*d*x)*sin(c + d*x)**2*cos(a + 3*d*x)**2*c
os(c + d*x)/(20*d) + 11*sin(a + 3*d*x)*cos(a + 3*d*x)**2*cos(c + d*x)**3/(3
20*d) - sin(c + d*x)**3*cos(a + 3*d*x)**3/(4*d)...
```

Giac [A]
time = 0.42, size = 181, normalized size = 0.93

$$-\frac{\sin(3bx+3dx+3a+3c)}{96(b+d)}+\frac{3\sin(3bx+dx+3a+c)}{32(3b+d)}-\frac{3\sin(3bx-dx+3a-c)}{32(3b-d)}+\frac{\sin(3bx-3dx+3a-3c)}{96(b-d)}+\frac{3\sin(bx+3dx+a+3c)}{32(b+3d)}-\frac{9\sin(bx+dx+a+c)}{32(b+d)}+\frac{9\sin(bx-dx+a-c)}{32(b-d)}-\frac{3\sin(bx-3dx+a-3c)}{32(b-3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/96*sin(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*sin(3*b*x + d*x + 3*a +
 c)/(3*b + d) - 3/32*sin(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*sin(3*b*x
- 3*d*x + 3*a - 3*c)/(b - d) + 3/32*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) -
9/32*sin(b*x + d*x + a + c)/(b + d) + 9/32*sin(b*x - d*x + a - c)/(b - d) -
 3/32*sin(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

Mupad [B]
time = 4.31, size = 997, normalized size = 5.11

$$e^{a\,3i-c\,1i+b\,x\,3i-d\,x\,1i}\left(\frac{-9b^3-3b^2d+9bd^2+3d^3}{b^4\,576i-b^2d^2\,640i+d^4\,64i}+\frac{e^{-a\,6i-b\,x\,6i}\left(-9b^3+3b^2d+9bd^2-3d^3\right)}{b^4\,576i-b^2d^2\,640i+d^4\,64i}-\frac{e^{-a\,2i-b\,x\,2i}\left(-81b^3-81b^2d+9bd^2+9d^3\right)}{b^4\,576i-b^2d^2\,640i+d^4\,64i}-\frac{e^{-a\,4i-b\,x\,4i}\left(-81b^3+81b^2d+9bd^2-9d^3\right)}{b^4\,576i-b^2d^2\,640i+d^4\,64i}\right)-e^{a\,3i+c\,1i+b\,x\,3i+d\,x\,1i}\left(\frac{-9b^3+3b^2d+9bd^2-3d^3}{b^4\,576i-b^2d^2\,640i+d^4\,64i}+\frac{e^{-a\,6i-b\,x\,6i}\left(-9b^3-3b^2d+9bd^2+3d^3\right)}{b^4\,576i-b^2d^2\,640i+d^4\,64i}-\frac{e^{-a\,2i-b\,x\,2i}\left(-81b^3+81b^2d+9bd^2-9d^3\right)}{b^4\,576i-b^2d^2\,640i+d^4\,64i}-\frac{e^{-a\,4i-b\,x\,4i}\left(-81b^3-81b^2d+9bd^2+9d^3\right)}{b^4\,576i-b^2d^2\,640i+d^4\,64i}\right)-e^{a\,3i-c\,3i+b\,x\,3i-d\,x\,3i}\left(\frac{-b^3-b^2d+9bd^2+9d^3}{b^4\,192i-b^2d^2\,1920i+d^4\,1728i}+\frac{e^{-a\,6i-b\,x\,6i}\left(-b^3+b^2d+9bd^2-9d^3\right)}{b^4\,192i-b^2d^2\,1920i+d^4\,1728i}-\frac{e^{-a\,2i-b\,x\,2i}\left(-9b^3-27b^2d+9bd^2+27d^3\right)}{b^4\,192i-b^2d^2\,1920i+d^4\,1728i}-\frac{e^{-a\,4i-b\,x\,4i}\left(-9b^3+27b^2d+9bd^2-27d^3\right)}{b^4\,192i-b^2d^2\,1920i+d^4\,1728i}\right)+e^{a\,3i+c\,3i+b\,x\,3i+d\,x\,3i}\left(\frac{-b^3+b^2d+9bd^2-9d^3}{b^4\,192i-b^2d^2\,1920i+d^4\,1728i}+\frac{e^{-a\,6i-b\,x\,6i}\left(-b^3-b^2d+9bd^2+9d^3\right)}{b^4\,192i-b^2d^2\,1920i+d^4\,1728i}-\frac{e^{-a\,2i-b\,x\,2i}\left(-9b^3+27b^2d+9bd^2-27d^3\right)}{b^4\,192i-b^2d^2\,1920i+d^4\,1728i}-\frac{e^{-a\,4i-b\,x\,4i}\left(-9b^3-27b^2d+9bd^2+27d^3\right)}{b^4\,192i-b^2d^2\,1920i+d^4\,1728i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3*sin(c + d*x)^3,x)
```

```
[Out] exp(a*3i - c*1i + b*x*3i - d*x*1i)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/(b^
4*576i + d^4*64i - b^2*d^2*640i) + (exp(- a*6i - b*x*6i)*(9*b*d^2 + 3*b^2*d
 - 9*b^3 - 3*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*2i - b*x*
2i)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*64
0i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(b^4*576
i + d^4*64i - b^2*d^2*640i)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((9*b*d^2
 + 3*b^2*d - 9*b^3 - 3*d^3)/(b^4*576i + d^4*64i - b^2*d^2*640i) + (exp(- a*
```

```
6i - b*x*6i)*(9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3))/(b^4*576i + d^4*64i - b^2
*d^2*640i) - (exp(- a*2i - b*x*2i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(
b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 - 81*b^
2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i)) - exp(a*3i - c*
3i + b*x*3i - d*x*3i)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)/(b^4*192i + d^4*1728
i - b^2*d^2*1920i) + (exp(- a*6i - b*x*6i)*(9*b*d^2 + b^2*d - b^3 - 9*d^3))
/(b^4*192i + d^4*1728i - b^2*d^2*1920i) - (exp(- a*2i - b*x*2i)*(9*b*d^2 -
27*b^2*d - 9*b^3 + 27*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i) - (exp(-
 a*4i - b*x*4i)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(b^4*192i + d^4*1728
i - b^2*d^2*1920i)) + exp(a*3i + c*3i + b*x*3i + d*x*3i)*((9*b*d^2 + b^2*d
- b^3 - 9*d^3)/(b^4*192i + d^4*1728i - b^2*d^2*1920i) + (exp(- a*6i - b*x*6
i)*(9*b*d^2 - b^2*d - b^3 + 9*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i)
- (exp(- a*2i - b*x*2i)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(b^4*192i +
d^4*1728i - b^2*d^2*1920i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 - 27*b^2*d - 9*
b^3 + 27*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i))
```

3.209 $\int \cos^n(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=277

$$-\frac{2^{-1-n}e^{i(a-cn)+i(b-dn)x+in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\,{}_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(2+\frac{b}{d}-n\right);-e^{2i(c+dx}\right.}{b-dn}$$

```
[Out] -2^(-1-n)*exp(I*(-c*n+a)+I*(-d*n+b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*(
d*x+c)))^n*hypergeom([-n, 1/2*(-d*n+b)/d],[1+1/2*b/d-1/2*n],-exp(2*I*(d*x+c
)))/((1+exp(2*I*c+2*I*d*x))^n)/(-d*n+b)-2^(-1-n)*exp(-I*(c*n+a)-I*(d*n+b)*x
+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-n, 1/2*(-d*n-b
)/d],[1+1/2*(-d*n-b)/d],-exp(2*I*(d*x+c)))/((1+exp(2*I*c+2*I*d*x))^n)/(d*n+
b)
```

Rubi [A]
time = 0.41, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4651, 2323, 2285, 2283}

$$-\frac{2^{-n-1}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}\,{}_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(\frac{b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(i(a-cn)+ix(b-dn)+in(c+dx))}{b-dn}-\frac{2^{-n-1}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}\,{}_2F_1\left(-n,-\frac{b+dn}{2d};1-\frac{b+dn}{2d};-e^{2i(c+dx)}\right)\exp(-i(a+cn)-ix(b+dn)+in(c+dx))}{b+dn}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^n*Sin[a + b*x],x]
```

```
[Out] -((2^(-1 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(E^((-I)*(c +
 d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d), (2 + b/d
 - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*(b - d*n))
) - (2^(-1 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*x))*(E^((-I)
*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -1/2*(b + d*n)/d, 1
- (b + d*n)/(2*d), -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*
(b + d*n))
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2285

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2323

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 4651

```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d*
x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \cos^n(c+dx)\sin(a+bx)\,dx = 2^{-1-n}\int \left(ie^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n - ie^{ia+ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)$$

$$= \left(i2^{-1-n}\right)\int e^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\,dx - \left(i2^{-1-n}\right)\int e^{ia+ibx}\left(e^{-i(c+dx}\right.$$

$$= \left(i2^{-1-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)\int e^{-ia-ibx-in(c+dx}$$

$$= -\left(\left(i2^{-1-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)\int e^{i(a-cn)+i(b-}\right.$$

$$= -\frac{2^{-1-n}\exp(i(a-cn)+i(b-dn)x+in(c+dx))\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+\right.}{b-dn}$$

Mathematica [A]
time = 0.93, size = 203, normalized size = 0.73

$$-\frac{2^{-1-n}e^{-ibx}\left(1+e^{2i(c+dx)}\right)^{-n}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^n\left((b-dn)\,{}_2F_1\left(-n,-\frac{b+dn}{2d};-\frac{b+d(-2+n)}{2d};-e^{2i(c+dx)}\right)(\cos(a)-i\sin(a))+e^{2ibx}(b+dn)\,{}_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(2+\frac{b}{d}-n\right);-e^{2i(c+dx)}\right)(\cos(a)+i\sin(a))\right)}{(b-dn)(b+dn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^n*Sin[a + b*x],x]
```

```
[Out] -((2^(-1 - n)*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^n*((b - d*n)*Hype
rgeometric2F1[-n, -1/2*(b + d*n)/d, -1/2*(b + d*(-2 + n))/d, -E^((2*I)*(c +
 d*x))]*(Cos[a] - I*Sin[a]) + E^((2*I)*b*x)*(b + d*n)*Hypergeometric2F1[-n,
 (b - d*n)/(2*d), (2 + b/d - n)/2, -E^((2*I)*(c + d*x))]*(Cos[a] + I*Sin[a]
)))/(E^(I*b*x)*(1 + E^((2*I)*(c + d*x)))^n*(b - d*n)*(b + d*n)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(\cos^n\left(dx+c\right)\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^n*sin(b*x+a),x)
```

```
[Out] int(cos(d*x+c)^n*sin(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^n*sin(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^n*sin(b*x + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**n*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^n*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (c+d\,x)^n\,\sin (a+b\,x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^n*sin(a + b*x),x)
[Out] int(cos(c + d*x)^n*sin(a + b*x), x)
```

3.210 $\int \cos^3(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=91

$$-\frac{\cos(a-3c+(b-3d)x)}{8(b-3d)}-\frac{3\cos(a-c+(b-d)x)}{8(b-d)}-\frac{3\cos(a+c+(b+d)x)}{8(b+d)}-\frac{\cos(a+3c+(b+3d)x)}{8(b+3d)}$$

```
[Out] -1/8*cos(a-3*c+(b-3*d)*x)/(b-3*d)-3/8*cos(a-c+(b-d)*x)/(b-d)-3/8*cos(a+c+(b
+d)*x)/(b+d)-1/8*cos(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4670, 2718}

$$-\frac{\cos(a+x(b-3d)-3c)}{8(b-3d)}-\frac{3\cos(a+x(b-d)-c)}{8(b-d)}-\frac{3\cos(a+x(b+d)+c)}{8(b+d)}-\frac{\cos(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[a + b*x],x]
```

```
[Out] -1/8*Cos[a - 3*c + (b - 3*d)*x]/(b - 3*d) - (3*Cos[a - c + (b - d)*x])/(8*(
b - d)) - (3*Cos[a + c + (b + d)*x])/(8*(b + d)) - Cos[a + 3*c + (b + 3*d)*
x]/(8*(b + 3*d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sin(a+bx)\,dx &= \int \left(\frac{1}{8}\sin(a-3c+(b-3d)x)+\frac{3}{8}\sin(a-c+(b-d)x)+\frac{3}{8}\sin(a+c+(b\\ &= \frac{1}{8}\int \sin(a-3c+(b-3d)x)\,dx+\frac{1}{8}\int \sin(a+3c+(b+3d)x)\,dx+\frac{3}{8}\int \sin\\ &= -\frac{\cos(a-3c+(b-3d)x)}{8(b-3d)}-\frac{3\cos(a-c+(b-d)x)}{8(b-d)}-\frac{3\cos(a+c+(b+d\end{aligned}$$

Mathematica [A]
time = 0.53, size = 87, normalized size = 0.96

$$\frac{1}{8}\left(-\frac{\cos(a-3c+bx-3dx)}{b-3d}-\frac{3\cos(a-c+bx-dx)}{b-d}-\frac{\cos(a+3c+bx+3dx)}{b+3d}-\frac{3\cos(a+c+(b+d)x)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[a + b*x],x]
```

```
[Out] (-(Cos[a - 3*c + b*x - 3*d*x]/(b - 3*d)) - (3*Cos[a - c + b*x - d*x])/(b -
d) - Cos[a + 3*c + b*x + 3*d*x]/(b + 3*d) - (3*Cos[a + c + (b + d)*x])/(b +
 d))/8
```

Maple [A]
time = 0.18, size = 84, normalized size = 0.92

method	result	size
default	$-\frac{\cos(a-3c+(b-3d)x)}{8(b-3d)}-\frac{3\cos(a-c+(b-d)x)}{8(b-d)}-\frac{3\cos(a+c+(b+d)x)}{8(b+d)}-\frac{\cos(a+3c+(b+3d)x)}{8(b+3d)}$	84
risch	$-\frac{\cos(bx-3dx+a-3c)}{8(b-3d)}-\frac{3\cos(bx-dx+a-c)}{8(b-d)}-\frac{3\cos(bx+dx+a+c)}{8(b+d)}-\frac{\cos(bx+3dx+a+3c)}{8(b+3d)}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*cos(a-3*c+(b-3*d)*x)/(b-3*d)-3/8*cos(a-c+(b-d)*x)/(b-d)-3/8*cos(a+c+(b
+d)*x)/(b+d)-1/8*cos(a+3*c+(b+3*d)*x)/(b+3*d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 912 vs. 2(83) = 166.
time = 0.32, size = 912, normalized size = 10.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/16*((b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))*
cos((b + 3*d)*x + a + 6*c) + (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos(3
*c) + 3*d^3*cos(3*c))*cos((b + 3*d)*x + a) + 3*(b^3*cos(3*c) - b^2*d*cos(3*
c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*cos((b + d)*x + a + 4*c) + 3*(b^3*c
os(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*cos((b + d)*x
 + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*c
os(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b
*d^2*cos(3*c) - 9*d^3*cos(3*c))*cos(-(b - d)*x - a - 2*c) + (b^3*cos(3*c) +
 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3*cos(3*c))*cos(-(b - 3*d)*x - a +
```

```
6*c) + (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3*cos(3*c))
*cos(-(b - 3*d)*x - a) + (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3*c)
+ 3*d^3*sin(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) - 3*b^2*d*sin(
3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))*sin((b + 3*d)*x + a) + 3*(b^3*sin(3
*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*sin((b + d)*x + a
 + 4*c) - 3*(b^3*sin(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3
*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*
sin(3*c) - 9*d^3*sin(3*c))*sin(-(b - d)*x - a + 4*c) - 3*(b^3*sin(3*c) + b^
2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9*d^3*sin(3*c))*sin(-(b - d)*x - a - 2*c)
 + (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))*sin(
-(b - 3*d)*x - a + 6*c) - (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c)
 - 3*d^3*sin(3*c))*sin(-(b - 3*d)*x - a))/(b^4*cos(3*c)^2 + b^4*sin(3*c)^2
+ 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 - 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^
2)
```

<u>Fricas</u> [A]
time = 2.10, size = 106, normalized size = 1.16

$$\frac{6\,bd^2\cos\left(bx+a\right)\cos\left(dx+c\right)-\left(b^3-bd^2\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^3+3\,\left(2\,d^3-\left(b^2d-d^3\right)\cos\left(dx+c\right)^2\right)\sin\left(bx+a\right)\sin\left(dx+c\right)}{b^4-10\,b^2d^2+9\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] (6*b*d^2*cos(b*x + a)*cos(d*x + c) - (b^3 - b*d^2)*cos(b*x + a)*cos(d*x + c
)^3 + 3*(2*d^3 - (b^2*d - d^3)*cos(d*x + c)^2)*sin(b*x + a)*sin(d*x + c))/(
b^4 - 10*b^2*d^2 + 9*d^4)
```

<u>Sympy</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(78) = 156$.
time = 2.36, size = 921, normalized size = 10.12

$$\begin{cases} x\sin\left(a\right)\cos^3\left(c\right) & \text{for}\ b=0\wedge d=0\\ -\frac{3x\sin\left(a-3dx\right)\sin^2\left(c+dx\right)\cos\left(c+dx\right)}{8}+\frac{x\sin\left(a-3dx\right)\cos^3\left(c+dx\right)}{8}-\frac{x\sin^3\left(c+dx\right)\cos\left(a-3dx\right)}{8}+\frac{3x\sin\left(c+dx\right)\cos\left(a-3dx\right)\cos^2\left(c+dx\right)}{8}-\frac{\sin\left(a-3dx\right)\sin^3\left(c+dx\right)}{8d}+\frac{\sin^2\left(c+dx\right)\cos\left(a-3dx\right)\cos\left(c+dx\right)}{4d}+\frac{7\cos\left(a-3dx\right)\cos^3\left(c+dx\right)}{24d} & \text{for}\ b=-3d\\ \frac{3x\sin\left(a-dx\right)\sin^2\left(c+dx\right)\cos\left(c+dx\right)}{8}+\frac{3x\sin\left(a-dx\right)\cos^3\left(c+dx\right)}{8}+\frac{3x\sin^3\left(c+dx\right)\cos\left(a-dx\right)}{8}+\frac{3x\sin\left(c+dx\right)\cos\left(a-dx\right)\cos^2\left(c+dx\right)}{8}-\frac{3\sin\left(a-dx\right)\sin^3\left(c+dx\right)}{8d}+\frac{3\sin^2\left(c+dx\right)\cos\left(a-dx\right)\cos\left(c+dx\right)}{4d}+\frac{5\cos\left(a-dx\right)\cos^3\left(c+dx\right)}{8d} & \text{for}\ b=-d\\ \frac{3x\sin\left(a+dx\right)\sin^2\left(c+dx\right)\cos\left(c+dx\right)}{8}+\frac{3x\sin\left(a+dx\right)\cos^3\left(c+dx\right)}{8}-\frac{3x\sin^3\left(c+dx\right)\cos\left(a+dx\right)}{8}-\frac{3x\sin\left(c+dx\right)\cos\left(a+dx\right)\cos^2\left(c+dx\right)}{8}-\frac{3\sin\left(a+dx\right)\sin^3\left(c+dx\right)}{8d}-\frac{3\sin^2\left(c+dx\right)\cos\left(a+dx\right)\cos\left(c+dx\right)}{4d}-\frac{5\cos\left(a+dx\right)\cos^3\left(c+dx\right)}{8d} & \text{for}\ b=d\\ -\frac{3x\sin\left(a+3dx\right)\sin^2\left(c+dx\right)\cos\left(c+dx\right)}{8}+\frac{x\sin\left(a+3dx\right)\cos^3\left(c+dx\right)}{8}+\frac{x\sin^3\left(c+dx\right)\cos\left(a+3dx\right)}{8}-\frac{3x\sin\left(c+dx\right)\cos\left(a+3dx\right)\cos^2\left(c+dx\right)}{8}-\frac{\sin\left(a+3dx\right)\sin^3\left(c+dx\right)}{8d}-\frac{\sin^2\left(c+dx\right)\cos\left(a+3dx\right)\cos\left(c+dx\right)}{4d}-\frac{7\cos\left(a+3dx\right)\cos^3\left(c+dx\right)}{24d} & \text{for}\ b=3d\\ -\frac{b^3\cos\left(a+bx\right)\cos^3\left(c+dx\right)}{b^4-10b^2d^2+9d^4}-\frac{3b^2d\sin\left(a+bx\right)\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{b^4-10b^2d^2+9d^4}+\frac{6bd^2\sin^2\left(c+dx\right)\cos\left(a+bx\right)\cos\left(c+dx\right)}{b^4-10b^2d^2+9d^4}+\frac{7bd^2\cos\left(a+bx\right)\cos^3\left(c+dx\right)}{b^4-10b^2d^2+9d^4}+\frac{6d^3\sin\left(a+bx\right)\sin^3\left(c+dx\right)}{b^4-10b^2d^2+9d^4}+\frac{9d^3\sin\left(a+bx\right)\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{b^4-10b^2d^2+9d^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a)*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*sin(a - 3*d*x)*s
in(c + d*x)**2*cos(c + d*x)/8 + x*sin(a - 3*d*x)*cos(c + d*x)**3/8 - x*sin(
c + d*x)**3*cos(a - 3*d*x)/8 + 3*x*sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)
**2/8 - sin(a - 3*d*x)*sin(c + d*x)**3/(8*d) + sin(c + d*x)**2*cos(a - 3*d*
x)*cos(c + d*x)/(4*d) + 7*cos(a - 3*d*x)*cos(c + d*x)**3/(24*d), Eq(b, -3*d
)), (3*x*sin(a - d*x)*sin(c + d*x)**2*cos(c + d*x)/8 + 3*x*sin(a - d*x)*cos
```

```
(c + d*x)**3/8 + 3*x*sin(c + d*x)**3*cos(a - d*x)/8 + 3*x*sin(c + d*x)*cos(
a - d*x)*cos(c + d*x)**2/8 - 3*sin(a - d*x)*sin(c + d*x)**3/(8*d) + 3*sin(c
 + d*x)**2*cos(a - d*x)*cos(c + d*x)/(4*d) + 5*cos(a - d*x)*cos(c + d*x)**3
/(8*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**2*cos(c + d*x)/8 + 3*x*
sin(a + d*x)*cos(c + d*x)**3/8 - 3*x*sin(c + d*x)**3*cos(a + d*x)/8 - 3*x*s
in(c + d*x)*cos(a + d*x)*cos(c + d*x)**2/8 - 3*sin(a + d*x)*sin(c + d*x)**3
/(8*d) - 3*sin(c + d*x)**2*cos(a + d*x)*cos(c + d*x)/(4*d) - 5*cos(a + d*x)
*cos(c + d*x)**3/(8*d), Eq(b, d)), (-3*x*sin(a + 3*d*x)*sin(c + d*x)**2*cos
(c + d*x)/8 + x*sin(a + 3*d*x)*cos(c + d*x)**3/8 + x*sin(c + d*x)**3*cos(a
+ 3*d*x)/8 - 3*x*sin(c + d*x)*cos(a + 3*d*x)*cos(c + d*x)**2/8 - sin(a + 3*
d*x)*sin(c + d*x)**3/(8*d) - sin(c + d*x)**2*cos(a + 3*d*x)*cos(c + d*x)/(4
*d) - 7*cos(a + 3*d*x)*cos(c + d*x)**3/(24*d), Eq(b, 3*d)), (-b**3*cos(a +
b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*sin(a + b*x)
*sin(c + d*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*b*d**2*sin
(c + d*x)**2*cos(a + b*x)*cos(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 7*b
*d**2*cos(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*d**3*
sin(a + b*x)*sin(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*sin(a
+ b*x)*sin(c + d*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4), True))
```

Giac [A]
time = 0.39, size = 84, normalized size = 0.92

$$-\frac{\cos\left(bx+3\,dx+a+3\,c\right)}{8\,\left(b+3\,d\right)}-\frac{3\,\cos\left(bx+dx+a+c\right)}{8\,\left(b+d\right)}-\frac{3\,\cos\left(bx-dx+a-c\right)}{8\,\left(b-d\right)}-\frac{\cos\left(bx-3\,dx+a-3\,c\right)}{8\,\left(b-3\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*cos(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 3/8*cos(b*x + d*x + a + c)/(b +
 d) - 3/8*cos(b*x - d*x + a - c)/(b - d) - 1/8*cos(b*x - 3*d*x + a - 3*c)/(
b - 3*d)
```

Mupad [B]
time = 1.61, size = 297, normalized size = 3.26

$$-{\mathrm{e}}^{a\,1{}\mathrm{i}-c\,3{}\mathrm{i}+b\,x\,1{}\mathrm{i}-d\,x\,3{}\mathrm{i}}\,\left(\frac{b+3\,d}{16\,b^2-144\,d^2}+\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,\left(b-3\,d\right)}{16\,b^2-144\,d^2}\right)-{\mathrm{e}}^{a\,1{}\mathrm{i}+c\,3{}\mathrm{i}+b\,x\,1{}\mathrm{i}+d\,x\,3{}\mathrm{i}}\,\left(\frac{b-3\,d}{16\,b^2-144\,d^2}+\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,\left(b+3\,d\right)}{16\,b^2-144\,d^2}\right)-{\mathrm{e}}^{a\,1{}\mathrm{i}-c\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}\,\left(\frac{3\,b+3\,d}{16\,b^2-16\,d^2}+\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,\left(3\,b-3\,d\right)}{16\,b^2-16\,d^2}\right)-{\mathrm{e}}^{a\,1{}\mathrm{i}+c\,1{}\mathrm{i}+b\,x\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,\left(\frac{3\,b-3\,d}{16\,b^2-16\,d^2}+\frac{{\mathrm{e}}^{-a\,2{}\mathrm{i}-b\,x\,2{}\mathrm{i}}\,\left(3\,b+3\,d\right)}{16\,b^2-16\,d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*sin(a + b*x),x)
```

```
[Out] - exp(a*1i - c*3i + b*x*1i - d*x*3i)*((b + 3*d)/(16*b^2 - 144*d^2) + (exp(-
 a*2i - b*x*2i)*(b - 3*d))/(16*b^2 - 144*d^2)) - exp(a*1i + c*3i + b*x*1i +
 d*x*3i)*((b - 3*d)/(16*b^2 - 144*d^2) + (exp(- a*2i - b*x*2i)*(b + 3*d))/(
16*b^2 - 144*d^2)) - exp(a*1i - c*1i + b*x*1i - d*x*1i)*((3*b + 3*d)/(16*b^
2 - 16*d^2) + (exp(- a*2i - b*x*2i)*(3*b - 3*d))/(16*b^2 - 16*d^2)) - exp(a
*1i + c*1i + b*x*1i + d*x*1i)*((3*b - 3*d)/(16*b^2 - 16*d^2) + (exp(- a*2i
- b*x*2i)*(3*b + 3*d))/(16*b^2 - 16*d^2))
```

3.211 $\int \cos^2(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=62

$$-\frac{\cos(a+bx)}{2b}-\frac{\cos(a-2c+(b-2d)x)}{4(b-2d)}-\frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}$$

```
[Out] -1/2*cos(b*x+a)/b-1/4*cos(a-2*c+(b-2*d)*x)/(b-2*d)-1/4*cos(a+2*c+(b+2*d)*x)
/(b+2*d)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4670, 2718}

$$-\frac{\cos(a+x(b-2d)-2c)}{4(b-2d)}-\frac{\cos(a+x(b+2d)+2c)}{4(b+2d)}-\frac{\cos(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/2*Cos[a + b*x]/b - Cos[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) - Cos[a + 2*
c + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin(a+bx)\,dx &= \int \left(\frac{1}{2}\sin(a+bx)+\frac{1}{4}\sin(a-2c+(b-2d)x)+\frac{1}{4}\sin(a+2c+(b+2d)x)\right)\\ &= \frac{1}{4}\int \sin(a-2c+(b-2d)x)\,dx+\frac{1}{4}\int \sin(a+2c+(b+2d)x)\,dx+\frac{1}{2}\int \sin\\ &= -\frac{\cos(a+bx)}{2b}-\frac{\cos(a-2c+(b-2d)x)}{4(b-2d)}-\frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}\end{aligned}$$

Mathematica [A]
time = 0.83, size = 71, normalized size = 1.15

$$\frac{1}{4}\left(-\frac{2\cos(a)\cos(bx)}{b}-\frac{\cos(a-2c+bx-2dx)}{b-2d}-\frac{\cos(a+2c+bx+2dx)}{b+2d}+\frac{2\sin(a)\sin(bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[a + b*x],x]
```

```
[Out] ((-2*Cos[a]*Cos[b*x])/b - Cos[a - 2*c + b*x - 2*d*x]/(b - 2*d) - Cos[a + 2*
c + b*x + 2*d*x]/(b + 2*d) + (2*Sin[a]*Sin[b*x])/b)/4
```

Maple [A]
time = 0.21, size = 57, normalized size = 0.92

method	result
default	$-\frac{\cos(bx+a)}{2b}-\frac{\cos(a-2c+(b-2d)x)}{4(b-2d)}-\frac{\cos(a+2c+(b+2d)x)}{4(b+2d)}$
risch	$-\frac{\cos(bx+a)}{2b}-\frac{\cos(bx-2dx+a-2c)}{4(b-2d)}-\frac{\cos(bx+2dx+a+2c)}{4(b+2d)}$
norman	$\frac{\frac{-2b^2+4d^2}{b\left(b^2-4d^2\right)}+\frac{\left(-2b^2+4d^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b\left(b^2-4d^2\right)}-\frac{8d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-4d^2}+\frac{8d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2-4d^2}-\frac{4b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2-4d^2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cos(b*x+a)/b-1/4*cos(a-2*c+(b-2*d)*x)/(b-2*d)-1/4*cos(a+2*c+(b+2*d)*x)
/(b+2*d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. 2(56) = 112.
time = 0.30, size = 414, normalized size = 6.68

$$-\frac{\left(b^2\cos(2c)-2bd\cos(2c)\right)\cos\left((b+2d)x+a+4c\right)+\left(b^2\cos(2c)-2bd\cos(2c)\right)\cos\left((b+2d)x+a\right)+\left(b^2\cos(2c)+2bd\cos(2c)\right)\cos\left(-(b-2d)x-a+4c\right)+\left(b^2\cos(2c)+2bd\cos(2c)\right)\cos\left(-(b-2d)x-a\right)+2\left(b^2\cos(2c)-4d^2\cos(2c)\right)\cos\left(bx+a+2c\right)+2\left(b^2\cos(2c)-4d^2\cos(2c)\right)\cos\left(bx+a-2c\right)+\left(b^2\sin(2c)-2bd\sin(2c)\right)\sin\left((b+2d)x+a+4c\right)-\left(b^2\sin(2c)-2bd\sin(2c)\right)\sin\left((b+2d)x+a\right)+\left(b^2\sin(2c)+2bd\sin(2c)\right)\sin\left(-(b-2d)x-a+4c\right)-\left(b^2\sin(2c)+2bd\sin(2c)\right)\sin\left(-(b-2d)x-a\right)+2\left(b^2\sin(2c)-4d^2\sin(2c)\right)\sin\left(bx+a+2c\right)-2\left(b^2\sin(2c)-4d^2\sin(2c)\right)\sin\left(bx+a-2c\right)}{8\left(b^3\cos(2c)^2+b^3\sin(2c)^2-4\left(b\cos(2c)^2+b\sin(2c)^2\right)d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*((b^2*cos(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a + 4*c) + (b^2*cos
(2*c) - 2*b*d*cos(2*c))*cos((b + 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(2*
c))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*cos(-(b -
 2*d)*x - a) + 2*(b^2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a + 2*c) + 2*(b^
2*cos(2*c) - 4*d^2*cos(2*c))*cos(b*x + a - 2*c) + (b^2*sin(2*c) - 2*b*d*sin
(2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*sin(2*c) - 2*b*d*sin(2*c))*sin((b
+ 2*d)*x + a) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a + 4*c)
```

```
 - (b^2*sin(2*c) + 2*b*d*sin(2*c))*sin(-(b - 2*d)*x - a) + 2*(b^2*sin(2*c)
- 4*d^2*sin(2*c))*sin(b*x + a + 2*c) - 2*(b^2*sin(2*c) - 4*d^2*sin(2*c))*si
n(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c)^2 + b*si
n(2*c)^2)*d^2)
```

Fricas [A]
time = 2.96, size = 66, normalized size = 1.06

$$-\frac{b^2\cos(bx+a)\cos(dx+c)^2+2\,bd\cos(dx+c)\sin(bx+a)\sin(dx+c)-2\,d^2\cos(bx+a)}{b^3-4\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b^2*cos(b*x + a)*cos(d*x + c)^2 + 2*b*d*cos(d*x + c)*sin(b*x + a)*sin(d*x
 + c) - 2*d^2*cos(b*x + a))/(b^3 - 4*b*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(51) = 102.
time = 0.80, size = 408, normalized size = 6.58

$$\begin{cases} x\sin(a)\cos^2(c) & \text{for}\; b=0\wedge d=0 \\ \left(\frac{x\sin^2(c+dx)}{2}+\frac{x\cos^2(c+dx)}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\sin(a) & \text{for}\; b=0 \\ -\frac{x\sin(a-2dx)\sin^2(c+dx)}{4}+\frac{x\sin(a-2dx)\cos^2(c+dx)}{4}+\frac{x\sin(c+dx)\cos(a-2dx)\cos(c+dx)}{2}+\frac{3\sin(a-2dx)\sin(c+dx)\cos(c+dx)}{4d}+\frac{\sin^2(c+dx)\cos(a-2dx)}{2d} & \text{for}\; b=-2d \\ -\frac{x\sin(a+2dx)\sin^2(c+dx)}{4}+\frac{x\sin(a+2dx)\cos^2(c+dx)}{4}-\frac{x\sin(c+dx)\cos(a+2dx)\cos(c+dx)}{2}+\frac{3\sin(a+2dx)\sin(c+dx)\cos(c+dx)}{4d}-\frac{\sin^2(c+dx)\cos(a+2dx)}{2d} & \text{for}\; b=2d \\ -\frac{b^2\cos(a+bx)\cos^2(c+dx)}{b^3-4bd^2}-\frac{2bd\sin(a+bx)\sin(c+dx)\cos(c+dx)}{b^3-4bd^2}+\frac{2d^2\sin^2(c+dx)\cos(a+bx)}{b^3-4bd^2}+\frac{2d^2\cos(a+bx)\cos^2(c+dx)}{b^3-4bd^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a)*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
+ x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a), Eq(b, 0)),
 (-x*sin(a - 2*d*x)*sin(c + d*x)**2/4 + x*sin(a - 2*d*x)*cos(c + d*x)**2/4
+ x*sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/2 + 3*sin(a - 2*d*x)*sin(c + d
*x)*cos(c + d*x)/(4*d) + sin(c + d*x)**2*cos(a - 2*d*x)/(2*d), Eq(b, -2*d))
, (-x*sin(a + 2*d*x)*sin(c + d*x)**2/4 + x*sin(a + 2*d*x)*cos(c + d*x)**2/4
 - x*sin(c + d*x)*cos(a + 2*d*x)*cos(c + d*x)/2 + 3*sin(a + 2*d*x)*sin(c +
d*x)*cos(c + d*x)/(4*d) - sin(c + d*x)**2*cos(a + 2*d*x)/(2*d), Eq(b, 2*d))
, (-b**2*cos(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*sin(a + b*x
)*sin(c + d*x)*cos(c + d*x)/(b**3 - 4*b*d**2) + 2*d**2*sin(c + d*x)**2*cos(
a + b*x)/(b**3 - 4*b*d**2) + 2*d**2*cos(a + b*x)*cos(c + d*x)**2/(b**3 - 4*
b*d**2), True))
```

Giac [A]
time = 0.41, size = 56, normalized size = 0.90

$$-\frac{\cos(bx+2\,dx+a+2\,c)}{4\,(b+2\,d)}-\frac{\cos(bx-2\,dx+a-2\,c)}{4\,(b-2\,d)}-\frac{\cos(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*cos(b*x + 2*d*x + a + 2*c)/(b + 2*d) - 1/4*cos(b*x - 2*d*x + a - 2*c)/
(b - 2*d) - 1/2*cos(b*x + a)/b
```

Mupad [B]
time = 0.77, size = 97, normalized size = 1.56

$$\frac{d\,\left(2\,b\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)-2\,b\,\cos\left(a+2\,c+b\,x+2\,d\,x\right)\right)+b^2\,\cos\left(a-2\,c+b\,x-2\,d\,x\right)+b^2\,\cos\left(a+2\,c+b\,x+2\,d\,x\right)}{16\,b\,d^2-4\,b^3}-\frac{\cos\left(a+b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(a + b*x),x)
```

```
[Out] (d*(2*b*cos(a - 2*c + b*x - 2*d*x) - 2*b*cos(a + 2*c + b*x + 2*d*x)) + b^2*
cos(a - 2*c + b*x - 2*d*x) + b^2*cos(a + 2*c + b*x + 2*d*x))/(16*b*d^2 - 4*
b^3) - cos(a + b*x)/(2*b)
```

3.212 $\int \cos(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=43

$$-\frac{\cos(a-c+(b-d)x)}{2(b-d)}-\frac{\cos(a+c+(b+d)x)}{2(b+d)}$$

```
[Out] -1/2*cos(a-c+(b-d)*x)/(b-d)-1/2*cos(a+c+(b+d)*x)/(b+d)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4670, 2718}

$$-\frac{\cos(a+x(b-d)-c)}{2(b-d)}-\frac{\cos(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[a + b*x],x]
```

```
[Out] -1/2*Cos[a - c + (b - d)*x]/(b - d) - Cos[a + c + (b + d)*x]/(2*(b + d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)\sin(a+bx)\,dx &= \int \left(\frac{1}{2}\sin(a-c+(b-d)x)+\frac{1}{2}\sin(a+c+(b+d)x)\right)\,dx\\
&= \frac{1}{2}\int \sin(a-c+(b-d)x)\,dx+\frac{1}{2}\int \sin(a+c+(b+d)x)\,dx\\
&= -\frac{\cos(a-c+(b-d)x)}{2(b-d)}-\frac{\cos(a+c+(b+d)x)}{2(b+d)}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 43, normalized size = 1.00

$$-\frac{\cos(a-c+(b-d)x)}{2(b-d)}-\frac{\cos(a+c+(b+d)x)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[a + b*x],x]
```

```
[Out] -1/2*Cos[a - c + (b - d)*x]/(b - d) - Cos[a + c + (b + d)*x]/(2*(b + d))
```

Maple [A]
time = 0.12, size = 40, normalized size = 0.93

method	result	size
default	$-\frac{\cos(a-c+(b-d)x)}{2(b-d)}-\frac{\cos(a+c+(b+d)x)}{2(b+d)}$	40
risch	$-\frac{\cos(bx-dx+a-c)}{2(b-d)}-\frac{\cos(bx+dx+a+c)}{2(b+d)}$	41
norman	$\frac{\frac{2b\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2-d^2}+\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2-d^2}-\frac{4d\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-d^2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*cos(a-c+(b-d)*x)/(b-d)-1/2*cos(a+c+(b+d)*x)/(b+d)
```

Maxima [A]
time = 0.26, size = 40, normalized size = 0.93

$$-\frac{\cos\left(bx+dx+a+c\right)}{2\left(b+d\right)}-\frac{\cos\left(-bx+dx-a+c\right)}{2\left(b-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*cos(b*x + d*x + a + c)/(b + d) - 1/2*cos(-b*x + d*x - a + c)/(b - d)
```

Fricas [A]
time = 3.97, size = 42, normalized size = 0.98

$$-\frac{b\cos\left(bx+a\right)\cos\left(dx+c\right)+d\sin\left(bx+a\right)\sin\left(dx+c\right)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*cos(b*x + a)*cos(d*x + c) + d*sin(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(34) = 68$.

time = 0.32, size = 155, normalized size = 3.60

$$\begin{cases} x\sin(a)\cos(c) & \text{for } b = 0 \wedge d = 0 \\ \frac{x\sin(a-dx)\cos(c+dx)}{2} + \frac{x\sin(c+dx)\cos(a-dx)}{2} + \frac{\sin(a-dx)\sin(c+dx)}{2d} & \text{for } b = -d \\ \frac{x\sin(a+dx)\cos(c+dx)}{2} - \frac{x\sin(c+dx)\cos(a+dx)}{2} + \frac{\sin(a+dx)\sin(c+dx)}{2d} & \text{for } b = d \\ -\frac{b\cos(a+bx)\cos(c+dx)}{b^2-d^2} - \frac{d\sin(a+bx)\sin(c+dx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a),x)
```

```
[Out] Piecewise((x*sin(a)*cos(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x)*cos(c + d
*x)/2 + x*sin(c + d*x)*cos(a - d*x)/2 + sin(a - d*x)*sin(c + d*x)/(2*d), Eq
(b, -d)), (x*sin(a + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)*cos(a + d*x)/2 +
sin(a + d*x)*sin(c + d*x)/(2*d), Eq(b, d)), (-b*cos(a + b*x)*cos(c + d*x)/(
b**2 - d**2) - d*sin(a + b*x)*sin(c + d*x)/(b**2 - d**2), True))
```

Giac [A]
time = 0.42, size = 40, normalized size = 0.93

$$-\frac{\cos(bx+dx+a+c)}{2(b+d)} - \frac{\cos(bx-dx+a-c)}{2(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2*cos(b*x + d*x + a + c)/(b + d) - 1/2*cos(b*x - d*x + a - c)/(b - d)
```

Mupad [B]
time = 0.84, size = 85, normalized size = 1.98

$$-\frac{b\left(\frac{\cos(a-c+b\,x-d\,x)}{2}+\frac{\cos(a+c+b\,x+d\,x)}{2}\right)}{b^2-d^2}-\frac{d\left(\frac{\cos(a-c+b\,x-d\,x)}{2}-\frac{\cos(a+c+b\,x+d\,x)}{2}\right)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(a + b*x),x)
```

```
[Out] - (b*(cos(a - c + b*x - d*x)/2 + cos(a + c + b*x + d*x)/2))/(b^2 - d^2) - (
d*(cos(a - c + b*x - d*x)/2 - cos(a + c + b*x + d*x)/2))/(b^2 - d^2)
```

3.213 $\int \sec(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=27

$$-\frac{\cos(a-c)\log(\cos(c+bx))}{b}+x\sin(a-c)$$

```
[Out] -cos(a-c)*ln(cos(b*x+c))/b+x*sin(a-c)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4676, 3556, 8}

$$x\sin(a-c)-\frac{\cos(a-c)\log(\cos(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]*Sin[a + b*x],x]
```

```
[Out] -((Cos[a - c]*Log[Cos[c + b*x]])/b) + x*Sin[a - c]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4676

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \sec(c+bx)\sin(a+bx)\,dx &= \cos(a-c)\int \tan(c+bx)\,dx+\sin(a-c)\int 1\,dx\\ &= -\frac{\cos(a-c)\log(\cos(c+bx))}{b}+x\sin(a-c)\end{aligned}$$

Mathematica [A]
time = 0.17, size = 27, normalized size = 1.00

$$-\frac{\cos(a-c)\log(\cos(c+bx))}{b}+x\sin(a-c)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]*Sin[a + b*x],x]
```

```
[Out] -((Cos[a - c]*Log[Cos[c + b*x]])/b) + x*Sin[a - c]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(27) = 54$.
time = 0.40, size = 161, normalized size = 5.96

method	result
risch	$-ix\,\mathrm{e}^{i(a-c)}+2i\cos\left(a-c\right)x+\frac{2i\cos(a-c)a}{b}-\frac{\ln\left(\mathrm{e}^{2i(bx+a)}+\mathrm{e}^{2i(a-c)}\right)\cos(a-c)}{b}$
default	$\frac{-\frac{(\cos(a)\cos(c)+\sin(a)\sin(c))\ln(-\tan(bx+a)\cos(a)\sin(c)+\tan(bx+a)\sin(a)\cos(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}{\left(\cos^{2}(a)\right)\left(\cos^{2}(c)\right)+\left(\cos^{2}(c)\right)\left(\sin^{2}(a)\right)+\left(\cos^{2}(a)\right)\left(\sin^{2}(c)\right)+\left(\sin^{2}(a)\right)\left(\sin^{2}(c)\right)}+\frac{\frac{(\cos(a)\cos(c)+\sin(a)\sin(c))\ln\left(1+\mathrm{t}\right.}{2}}{\left(\cos^{2}\right.}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-(cos(a)*cos(c)+sin(a)*sin(c))/(cos(a)^2*cos(c)^2+cos(c)^2*sin(a)^2+co
s(a)^2*sin(c)^2+sin(a)^2*sin(c)^2)*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*
sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))+1/(cos(c)^2+sin(c)^2)/(cos(a)^2+
sin(a)^2)*(1/2*(cos(a)*cos(c)+sin(a)*sin(c))*ln(1+tan(b*x+a)^2)+(sin(a)*cos
(c)-cos(a)*sin(c))*arctan(tan(b*x+a))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(27) = 54$.
time = 0.29, size = 73, normalized size = 2.70

$$-\frac{2\,bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\cos\left(2\,bx\right)^{2}+2\,\cos\left(2\,bx\right)\cos\left(2\,c\right)+\cos\left(2\,c\right)^{2}+\sin\left(2\,bx\right)^{2}-2\,\sin\left(2\,bx\right)\sin\left(2\,c\right)+\sin\left(2\,c\right)^{2}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x*sin(-a + c) + cos(-a + c)*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2
*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2))/b
```

Fricas [A]
time = 3.87, size = 31, normalized size = 1.15

$$-\frac{bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(-\cos\left(bx+c\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -(b*x*sin(-a + c) + cos(-a + c)*log(-cos(b*x + c)))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(20) = 40$.
time = 5.17, size = 435, normalized size = 16.11

$$\left(\begin{cases} -x & \text{for}\: c = \frac{\pi}{2} \\ x & \text{for}\: c = -\frac{\pi}{2} \\ 0 & \text{for}\: b = 0 \\ -\frac{2bx\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)-1}-\frac{1}{\tan\left(\frac{c}{2}\right)-1}\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)-1}-\frac{1}{\tan\left(\frac{c}{2}\right)-1}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{\log\left(\tan\left(\frac{bx}{2}\right)+\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{c}{2}\right)+1}\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{\log\left(\tan\left(\frac{bx}{2}\right)+\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{c}{2}\right)+1}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} & \text{otherwise} \end{cases}\right)\cos(a) + \left(\begin{cases} -\frac{\log(\sin(bx))}{b} & \text{for}\: c = \frac{\pi}{2} \\ \frac{\log(\sin(bx))}{b} & \text{for}\: c = -\frac{\pi}{2} \\ \frac{x}{\cos(c)} & \text{for}\: b = 0 \\ -\frac{bx\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{bx}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{2\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{2\log\left(\tan\left(\frac{bx}{2}\right)-\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)-1}-\frac{1}{\tan\left(\frac{c}{2}\right)-1}\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{2\log\left(\tan\left(\frac{bx}{2}\right)+\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{c}{2}\right)+1}\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} & \text{otherwise} \end{cases}\right)\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)*sin(b*x+a),x)
```

```
[Out] Piecewise((-x, Eq(c, pi/2)), (x, Eq(c, -pi/2)), (0, Eq(b, 0)), (-2*b*x*tan(
c/2)/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**
2 + b) + log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - tan(
c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**2 + b) - l
og(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))/(b*tan(c/2)**2
+ b) + log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2
)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(ta
n(c/2) + 1))/(b*tan(c/2)**2 + b), True))*cos(a) + Piecewise((-log(sin(b*x))
/b, Eq(c, pi/2)), (log(sin(b*x))/b, Eq(c, -pi/2)), (x/cos(c), Eq(b, 0)), (-
b*x*tan(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) + 2*log(tan(b
*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(t
an(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b
*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)/(b*tan(c/2)**2
 + b), True))*sin(a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(27) = 54$.
time = 0.44, size = 158, normalized size = 5.85

$$\frac{\frac{4\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)(bx+c)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}+\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)^2+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^2+1\right)\log\left(\tan(bx+c)^2+1\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - ta
n(1/2*c))*(b*x + c)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^
2 + 1) + (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*log(tan(b*x + c)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^2 +
tan(1/2*a)^2 + tan(1/2*c)^2 + 1))/b
```

Mupad [B]
time = 0.89, size = 112, normalized size = 4.15

$$x\left(\frac{e^{-a\,1\mathrm{i}+c\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{e^{a\,1\mathrm{i}-c\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)+x\left(\frac{e^{-a\,1\mathrm{i}+c\,1\mathrm{i}}\,1\mathrm{i}}{2}+\frac{e^{a\,1\mathrm{i}-c\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)-\frac{\ln\left(e^{a\,2\mathrm{i}-c\,2\mathrm{i}}+e^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)\,\left(\frac{e^{-a\,1\mathrm{i}+c\,1\mathrm{i}}}{2}+\frac{e^{a\,1\mathrm{i}-c\,1\mathrm{i}}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(c + b*x),x)
```

```
[Out] x*((exp(c*1i - a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2) + x*((exp(c*1i - a*1i
)*1i)/2 + (exp(a*1i - c*1i)*1i)/2) - (log(exp(a*2i - c*2i) + exp(a*2i + b*x
*2i))*(exp(c*1i - a*1i)/2 + exp(a*1i - c*1i)/2))/b
```

3.214 $\int \sec^2(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=34

$$\frac{\cos(a-c)\sec(c+bx)}{b}+\frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b}$$

```
[Out] cos(a-c)*sec(b*x+c)/b+arctanh(sin(b*x+c))*sin(a-c)/b
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4676, 2686, 8, 3855}

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4676

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \sec^2(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \sec(c+bx)\tan(c+bx)\,dx + \sin(a-c)\int \sec(c+bx)\,dx$$
$$= \frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b} + \frac{\cos(a-c)\text{Subst}(\int 1\,dx, x, \sec(c+bx))}{b}$$
$$= \frac{\cos(a-c)\sec(c+bx)}{b} + \frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 88, normalized size = 2.59

$$\frac{\cos(a-c)\sec(c+bx)}{b} - \frac{2i\text{ArcTan}\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x])/b - ((2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)
/2]*Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Si
n[c])]*Sin[a - c])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(34) = 68.
time = 0.65, size = 346, normalized size = 10.18

method	result
risch	$\frac{e^{i(bx+3a)}+e^{i(bx+a+2c)}}{b\left(e^{2i(bx+a+c)}+e^{2ia}\right)} - \frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\sin(a-c)}{b} + \frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\sin(a-c)}{b}$
default	$\frac{\frac{4(2\sin(a)\cos(c)-2\cos(a)\sin(c))\tan\left(\frac{bx}{2}+\frac{a}{2}\right)+8\cos(a)\cos(c)+8\sin(}{\left(-4\left(\cos^2(a)\right)\left(\cos^2(c)\right)-4\left(\cos^2(c)\right)\left(\sin^2(a)\right)-4\left(\cos^2(a)\right)\left(\sin^2(c)\right)-4\left(\sin^2(a)\right)\left(\sin^2(c)\right)\right)\left(\cos(a)\cos(c)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin(a)\sin(c)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right.\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(4*((2*sin(a)*cos(c)-2*cos(a)*sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a)*cos(c
)+2*sin(a)*sin(c))/(-4*cos(a)^2*cos(c)^2-4*cos(c)^2*sin(a)^2-4*cos(a)^2*sin
(c)^2-4*sin(a)^2*sin(c)^2)/(cos(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c
)*tan(1/2*b*x+1/2*a)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2
*a)*sin(a)*cos(c)-cos(a)*cos(c)-sin(a)*sin(c))+4*(2*sin(a)*cos(c)-2*cos(a)*
sin(c))/(-4*cos(a)^2*cos(c)^2-4*cos(c)^2*sin(a)^2-4*cos(a)^2*sin(c)^2-4*sin
```

```
(a)^2*sin(c)^2)/(-cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin(c)^2-cos
(a)^2*sin(c)^2)^(1/2)*arctan(1/2*(2*(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2*b
*x+1/2*a)-2*sin(a)*cos(c)+2*cos(a)*sin(c))/(-cos(c)^2*sin(a)^2-cos(a)^2*cos
(c)^2-sin(a)^2*sin(c)^2-cos(a)^2*sin(c)^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(34) = 68$.
time = 0.55, size = 387, normalized size = 11.38

$$\frac{2\left(\cos\left(bx+2a\right)+\cos\left(bx+2c\right)\right)\cos\left(2bx+a+2c\right)+2\cos\left(bx+2a\right)\cos\left(a\right)+2\cos\left(bx+2c\right)\cos\left(a\right)+\left(\cos\left(2bx+a+2c\right)^{2}\sin\left(-a+c\right)+2\cos\left(2bx+a+2c\right)\cos\left(a\right)\sin\left(-a+c\right)+\sin\left(2bx+a+2c\right)^{2}\sin\left(-a+c\right)+2\sin\left(2bx+a+2c\right)\sin\left(a\right)\sin\left(-a+c\right)+\left(\cos\left(a\right)^{2}+\sin\left(a\right)^{2}\right)\sin\left(-a+c\right)\right)\log\left(\frac{\cos(bx+2c)^{2}+\cos(c)^{2}-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^{2}+2\cos(bx+2c)\sin(c)+\sin(c)^{2}}{\cos(bx+2c)^{2}+\cos(c)^{2}+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^{2}-2\cos(bx+2c)\sin(c)+\sin(c)^{2}}\right)+2\left(\sin\left(bx+2a\right)+\sin\left(bx+2c\right)\right)\sin\left(2bx+a+2c\right)+2\sin\left(bx+2a\right)\sin\left(a\right)+2\sin\left(bx+2c\right)\sin\left(a\right)}{2\left(b\cos\left(2bx+a+2c\right)^{2}+2b\cos\left(2bx+a+2c\right)\cos\left(a\right)+b\sin\left(2bx+a+2c\right)^{2}+2b\sin\left(2bx+a+2c\right)\sin\left(a\right)+\left(\cos\left(a\right)^{2}+\sin\left(a\right)^{2}\right)b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(cos(b*x + 2*a) + cos(b*x + 2*c))*cos(2*b*x + a + 2*c) + 2*cos(b*x +
 2*a)*cos(a) + 2*cos(b*x + 2*c)*cos(a) + (cos(2*b*x + a + 2*c)^2*sin(-a + c
) + 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(2*b*x + a + 2*c)^2*sin(
-a + c) + 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c) + (cos(a)^2 + sin(a)^2)
*sin(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) +
sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 +
cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*si
n(c) + sin(c)^2)) + 2*(sin(b*x + 2*a) + sin(b*x + 2*c))*sin(2*b*x + a + 2*c
) + 2*sin(b*x + 2*a)*sin(a) + 2*sin(b*x + 2*c)*sin(a))/(b*cos(2*b*x + a + 2
*c)^2 + 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^2 + 2*b*si
n(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.
time = 2.92, size = 69, normalized size = 2.03

$$-\frac{\cos\left(bx+c\right)\log\left(\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right)-\cos\left(bx+c\right)\log\left(-\sin\left(bx+c\right)+1\right)\sin\left(-a+c\right)-2\cos\left(-a+c\right)}{2b\cos\left(bx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(b*x + c)*log(sin(b*x + c) + 1)*sin(-a + c) - cos(b*x + c)*log(-si
n(b*x + c) + 1)*sin(-a + c) - 2*cos(-a + c))/(b*cos(b*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1448 vs. $2(27) = 54$.
time = 91.13, size = 5545, normalized size = 163.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**2*sin(b*x+a),x)

[Out] Piecewise((log(tan(b*x/2))/b, Eq(c, -pi/2) | Eq(c, pi/2)), (0, Eq(b, 0)), (
-2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**3
*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)*
*3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(tan(
b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**3/(b*tan(c/2
)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c
/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 8*log(tan(b*x/2) - tan(c/2)/(tan(c/
2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2
)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2)
 - b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(t
an(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c
/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2
)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*
tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan
(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2)
 + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3*tan(b*x/2)**2/(b
*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4
*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) + tan(c/2)
/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2)**
2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) -
b*tan(b*x/2)**2 + b) - 8*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(
c/2) + 1))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)
**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**
2 + b) - 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan
(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(
c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log
(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)/(b*tan(c
/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan
(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 2*tan(c/2)**4/(b*tan(c/2)**4*tan(
b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b
*x/2) - b*tan(b*x/2)**2 + b) - 4*tan(c/2)**3*tan(b*x/2)/(b*tan(c/2)**4*tan(
b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b
*x/2) - b*tan(b*x/2)**2 + b) - 4*tan(c/2)*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x
/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/
2) - b*tan(b*x/2)**2 + b) + 2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4
- 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 +
b), True))*cos(a) + Piecewise((-1/(b*sin(b*x)), Eq(c, -pi/2) | Eq(c, pi/2))
, (x/cos(c)**2, Eq(b, 0)), (-log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(
tan(c/2) - 1))*tan(c/2)**6*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*t
an(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*t
an(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan
(b*x/2) + b*tan(b*x/2)**2 - b) + log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) -
 1/(tan(c/2) - 1))*tan(c/2)**6/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**6
```

```
 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**4
 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) +
b*tan(b*x/2)**2 - b) + 4*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(
c/2) - 1))*tan(c/2)**5*tan(b*x/2)/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)
**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)
**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2)
 + b*tan(b*x/2)**2 - b) + 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(t
an(c/2) - 1))*tan(c/2)**4*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*ta
n(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*ta
n(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(
b*x/2) + b*tan(b*x/2)**2 - b) - 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1)
 - 1/(tan(c/2) - 1))*tan(c/2)**4/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**
6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**
4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) +
 b*tan(b*x/2)**2 - b) - 8*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan
(c/2) - 1))*tan(c/2)**3*tan(b*x/2)/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2
)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2
)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2
) + b*tan(b*x/2)**2 - b) - 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(
tan(c/2) - 1))*tan(c/2)**2*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*t
an(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*t
an(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan
(b*x/2) + b*tan(b*x/2)**2 - b) + 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1)
 - 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**6...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(34) = 68$.

time = 0.44, size = 248, normalized size = 7.29

$$\frac{2\left(\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)+1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}-\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)-1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}-\frac{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)^2+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^2+1}{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1\right)\left(\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)^2-1\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 2*((tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/
2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1
/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2
*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) - 1))/(tan(1/
2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*a)^2*tan(
1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)/((tan
(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b*x + 1/
2*c)^2 - 1)))/b
```

Mupad [B]

time = 5.35, size = 254, normalized size = 7.47

$$\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}+1\right)}{b\,\left(e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)}+\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}-\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(c + b*x)^2,x)
```

```
[Out] (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) + 1))/(b*(exp(a*2i - c*2i) + exp(a*2i
 + b*x*2i))) + (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) -
(exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i)
)^(1/2))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - (log(exp
(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) + (exp(a*2i)*exp(-c*2i)*(
exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*
2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2))
```

3.215 $\int \sec^3(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=38

$$\frac{\cos(a-c)\sec^2(c+bx)}{2b}+\frac{\sin(a-c)\tan(c+bx)}{b}$$

```
[Out] 1/2*cos(a-c)*sec(b*x+c)^2/b+sin(a-c)*tan(b*x+c)/b
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {4676, 2686, 30, 3852, 8}

$$\frac{\sin(a-c)\tan(bx+c)}{b}+\frac{\cos(a-c)\sec^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x]^2)/(2*b) + (Sin[a - c]*Tan[c + b*x])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4676

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
```

```
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \sec^3(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \sec^2(c+bx)\tan(c+bx)\,dx + \sin(a-c)\int \sec^2(c+bx)\,dx$$
$$= \frac{\cos(a-c)\mathrm{Subst}(\int x\,dx, x, \sec(c+bx))}{b} - \frac{\sin(a-c)\mathrm{Subst}(\int 1\,dx, x, -\tan(c}{b}$$
$$= \frac{\cos(a-c)\sec^2(c+bx)}{2b} + \frac{\sin(a-c)\tan(c+bx)}{b}$$

Mathematica [A]
time = 0.20, size = 34, normalized size = 0.89

$$\frac{\sec(c)\sec^2(c+bx)(\cos(a)+\sin(a-c)\sin(c+2bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^3*Sin[a + b*x],x]
[Out] (Sec[c]*Sec[c + b*x]^2*(Cos[a] + Sin[a - c]*Sin[c + 2*b*x]))/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(36) = 72.
time = 0.97, size = 121, normalized size = 3.18

method	result
risch	$\frac{2\,e^{i(2bx+5a+c)}+e^{i(5a-c)}-e^{i(3a+c)}}{\left(e^{2i(bx+a+c)}+e^{2ia}\right)^2 b}$
default	$\frac{\frac{\cos(a)\cos(c)+\sin(a)\sin(c)}{2(\sin(a)\cos(c)-\cos(a)\sin(c))^2(-\tan(bx+a)\cos(a)\sin(c)+\tan(bx+a)\sin(a)\cos(c)+\cos(a)\cos(c)+\sin(a)\sin(c))^2} - \frac{}{(\sin(a)\cos(c)-\cos(a)\sin(c))^2(-\tan}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/b*(1/2*(cos(a)*cos(c)+sin(a)*sin(c))/(sin(a)*cos(c)-cos(a)*sin(c))^2/(-ta
n(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c)
)^2-1/(sin(a)*cos(c)-cos(a)*sin(c))^2/(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)
*sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(36) = 72.
time = 0.29, size = 391, normalized size = 10.29

$$\frac{(2\cos(2bx+2a+2c)+\cos(2a)-\cos(2c))\cos(4bx+a+5c)+2(2\cos(2bx+2a+2c)+\cos(2a)-\cos(2c))\cos(2bx+a+3c)+(\cos(2a)-\cos(2c))\cos(a+c)+2\cos(2bx+2a+2c)\cos(a+c)+(2\sin(2bx+2a+2c)+\sin(2a)-\sin(2c))\sin(4bx+a+5c)+2(2\sin(2bx+2a+2c)+\sin(2a)-\sin(2c))\sin(2bx+a+3c)+(\sin(2a)-\sin(2c))\sin(a+c)+2\sin(2bx+2a+2c)\sin(a+c)}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2+4b\cos(2bx+a+3c)\cos(a+c)+b\cos(a+c)^2+b\sin(4bx+a+5c)^2+4b\sin(2bx+a+3c)^2+4b\sin(2bx+a+3c)\sin(a+c)+b\sin(a+c)^2+2(2b\cos(2bx+a+3c)+b\cos(a+c))\cos(4bx+a+5c)+2(2b\sin(2bx+a+3c)+b\sin(a+c))\sin(4bx+a+5c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] ((2*cos(2*b*x + 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(4*b*x + a + 5*c) + 2*
(2*cos(2*b*x + 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(2*b*x + a + 3*c) + (co
s(2*a) - cos(2*c))*cos(a + c) + 2*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (2*si
n(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(4*b*x + a + 5*c) + 2*(2*sin
(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(2*b*x + a + 3*c) + (sin(2*a)
 - sin(2*c))*sin(a + c) + 2*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos(4*b*x
 + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 + 4*b*cos(2*b*x + a + 3*c)*cos(a
 + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x + a + 3*c
)^2 + 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(2*b*cos(2*b
*x + a + 3*c) + b*cos(a + c))*cos(4*b*x + a + 5*c) + 2*(2*b*sin(2*b*x + a +
 3*c) + b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Fricas [**A**]
time = 3.60, size = 42, normalized size = 1.11

$$-\frac{2\,\cos\left(bx+c\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-\cos\left(-a+c\right)}{2\,b\cos\left(bx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*cos(b*x + c)*sin(b*x + c)*sin(-a + c) - cos(-a + c))/(b*cos(b*x + c
)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**3*sin(b*x+a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(36) = 72.
time = 0.42, size = 174, normalized size = 4.58

$$\frac{\tan\left(bx+c\right)^{2}\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}-\tan\left(bx+c\right)^{2}\tan\left(\frac{1}{2}\,a\right)^{2}+4\,\tan\left(bx+c\right)^{2}\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)+4\,\tan\left(bx+c\right)\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)-\tan\left(bx+c\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}-4\,\tan\left(bx+c\right)\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(bx+c\right)^{2}+4\,\tan\left(bx+c\right)\tan\left(\frac{1}{2}\,a\right)-4\,\tan\left(bx+c\right)\tan\left(\frac{1}{2}\,c\right)}{2\,\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)^{2}+\tan\left(\frac{1}{2}\,c\right)^{2}+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + c)^2*tan(1/2*a)^2
 + 4*tan(b*x + c)^2*tan(1/2*a)*tan(1/2*c) + 4*tan(b*x + c)*tan(1/2*a)^2*tan
(1/2*c) - tan(b*x + c)^2*tan(1/2*c)^2 - 4*tan(b*x + c)*tan(1/2*a)*tan(1/2*c
)^2 + tan(b*x + c)^2 + 4*tan(b*x + c)*tan(1/2*a) - 4*tan(b*x + c)*tan(1/2*c
))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*b)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(c + b*x)^3,x)
```

```
[Out] \text{Hanged}
```

3.216 $\int \sec^4(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=67

$$\frac{\cos(a-c)\sec^3(c+bx)}{3b}+\frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{2b}+\frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b}$$

```
[Out] 1/3*cos(a-c)*sec(b*x+c)^3/b+1/2*arctanh(sin(b*x+c))*sin(a-c)/b+1/2*sec(b*x+
c)*sin(a-c)*tan(b*x+c)/b
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4676, 2686, 30, 3853, 3855}

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\sec^3(bx+c)}{3b}+\frac{\sin(a-c)\tan(bx+c)\sec(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^4*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x]^3)/(3*b) + (ArcTanh[Sin[c + b*x]]*Sin[a - c])/(2*b
) + (Sec[c + b*x]*Sin[a - c]*Tan[c + b*x])/(2*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4676

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \sec^4(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \sec^3(c+bx)\tan(c+bx)\,dx + \sin(a-c)\int \sec^3(c+bx)\,dx$$
$$= \frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b} + \frac{\cos(a-c)\mathrm{Subst}\left(\int x^2\,dx, x, \sec(c+bx)\right)}{b}$$
$$= \frac{\cos(a-c)\sec^3(c+bx)}{3b} + \frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{2b} + \frac{\sec(c+bx)\sin(a}{2}$$

Mathematica [A]
time = 0.48, size = 64, normalized size = 0.96

$$\frac{12\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\sin(a-c)+\sec^3(c+bx)(4\cos(a-c)+3\sin(a-c)\sin(2(c+bx)))}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^4*Sin[a + b*x],x]
```

```
[Out] (12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c] + Sec[c + b*x]^3*(4*Co
s[a - c] + 3*Sin[a - c]*Sin[2*(c + b*x)]))/(12*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1781 vs. $2(61) = 122$.
time = 2.02, size = 1782, normalized size = 26.60

method	result
risch	$\frac{-3\,e^{i(5bx+7a+4c)}+3\,e^{i(5bx+5a+6c)}+8\,e^{i(3bx+7a+2c)}+8\,e^{i(3bx+5a+4c)}+3\,e^{i(bx+7a)}-3\,e^{i(bx+5a+2c)}}{6b\left(e^{2i(bx+a+c)}+e^{2ia}\right)^3}+\frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\sin(a-c)}{2b}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^4*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(4*(-1/4*(sin(a)*cos(c)-cos(a)*sin(c))*(cos(a)^2*cos(c)^2+2*cos(a)*cos(
c)*sin(a)*sin(c)+sin(a)^2*sin(c)^2)/(cos(a)^4*cos(c)^4+2*sin(a)^2*cos(a)^2*
cos(c)^4+cos(c)^4*sin(a)^4+2*cos(a)^4*sin(c)^2*cos(c)^2+4*cos(c)^2*sin(c)^2
*cos(a)^2*sin(a)^2+2*sin(a)^4*sin(c)^2*cos(c)^2+sin(c)^4*cos(a)^4+2*sin(a)^
```

```
2*cos(a)^2*sin(c)^4+sin(c)^4*sin(a)^4)*tan(1/2*b*x+1/2*a)^5-1/4*(2*cos(a)^4
*cos(c)^4-sin(a)^2*cos(a)^2*cos(c)^4+2*cos(c)^4*sin(a)^4+10*cos(a)^3*cos(c)
^3*sin(a)*sin(c)-10*sin(a)^3*cos(a)*sin(c)*cos(c)^3-cos(a)^4*sin(c)^2*cos(c
)^2+28*cos(c)^2*sin(c)^2*cos(a)^2*sin(a)^2-sin(a)^4*sin(c)^2*cos(c)^2-10*si
n(a)*cos(a)^3*sin(c)^3*cos(c)+10*cos(c)*sin(c)^3*cos(a)*sin(a)^3+2*sin(c)^4
*cos(a)^4-sin(a)^2*cos(a)^2*sin(c)^4+2*sin(c)^4*sin(a)^4)/(cos(a)*cos(c)+si
n(a)*sin(c))/(cos(a)^4*cos(c)^4+2*sin(a)^2*cos(a)^2*cos(c)^4+cos(c)^4*sin(a
)^4+2*cos(a)^4*sin(c)^2*cos(c)^2+4*cos(c)^2*sin(c)^2*cos(a)^2*sin(a)^2+2*si
n(a)^4*sin(c)^2*cos(c)^2+sin(c)^4*cos(a)^4+2*sin(a)^2*cos(a)^2*sin(c)^4+sin
(c)^4*sin(a)^4)*tan(1/2*b*x+1/2*a)^4+1/6*(sin(a)*cos(c)-cos(a)*sin(c))*(6*c
os(a)^4*cos(c)^4-7*sin(a)^2*cos(a)^2*cos(c)^4+2*cos(c)^4*sin(a)^4+38*cos(a)
^3*cos(c)^3*sin(a)*sin(c)-22*sin(a)^3*cos(a)*sin(c)*cos(c)^3-7*cos(a)^4*sin
(c)^2*cos(c)^2+76*cos(c)^2*sin(c)^2*cos(a)^2*sin(a)^2-7*sin(a)^4*sin(c)^2*c
os(c)^2-22*sin(a)*cos(a)^3*sin(c)^3*cos(c)+38*cos(c)*sin(c)^3*cos(a)*sin(a)
^3+2*sin(c)^4*cos(a)^4-7*sin(a)^2*cos(a)^2*sin(c)^4+6*sin(c)^4*sin(a)^4)/(c
os(a)*cos(c)+sin(a)*sin(c))^2/(cos(a)^4*cos(c)^4+2*sin(a)^2*cos(a)^2*cos(c)
^4+cos(c)^4*sin(a)^4+2*cos(a)^4*sin(c)^2*cos(c)^2+4*cos(c)^2*sin(c)^2*cos(a
)^2*sin(a)^2+2*sin(a)^4*sin(c)^2*cos(c)^2+sin(c)^4*cos(a)^4+2*sin(a)^2*cos(
a)^2*sin(c)^4+sin(c)^4*sin(a)^4)*tan(1/2*b*x+1/2*a)^3-1/2*(4*cos(a)^2*cos(c
)^3*sin(a)-cos(c)^3*sin(a)^3-4*cos(a)^3*cos(c)^2*sin(c)+11*cos(c)^2*sin(c)*
cos(a)*sin(a)^2-11*cos(c)*sin(c)^2*cos(a)^2*sin(a)+4*cos(c)*sin(c)^2*sin(a)
^3+sin(c)^3*cos(a)^3-4*sin(c)^3*cos(a)*sin(a)^2)/(cos(a)*cos(c)+sin(a)*sin(
c))*(sin(a)*cos(c)-cos(a)*sin(c))/(cos(a)^4*cos(c)^4+2*sin(a)^2*cos(a)^2*co
s(c)^4+cos(c)^4*sin(a)^4+2*cos(a)^4*sin(c)^2*cos(c)^2+4*cos(c)^2*sin(c)^2*c
os(a)^2*sin(a)^2+2*sin(a)^4*sin(c)^2*cos(c)^2+sin(c)^4*cos(a)^4+2*sin(a)^2*
cos(a)^2*sin(c)^4+sin(c)^4*sin(a)^4)*tan(1/2*b*x+1/2*a)^2-1/4*(3*cos(a)^2*c
os(c)^2-2*cos(c)^2*sin(a)^2+10*cos(a)*cos(c)*sin(a)*sin(c)-2*cos(a)^2*sin(c
)^2+3*sin(a)^2*sin(c)^2)*(sin(a)*cos(c)-cos(a)*sin(c))/(cos(a)^4*cos(c)^4+2
*sin(a)^2*cos(a)^2*cos(c)^4+cos(c)^4*sin(a)^4+2*cos(a)^4*sin(c)^2*cos(c)^2+
4*cos(c)^2*sin(c)^2*cos(a)^2*sin(a)^2+2*sin(a)^4*sin(c)^2*cos(c)^2+sin(c)^4
*cos(a)^4+2*sin(a)^2*cos(a)^2*sin(c)^4+sin(c)^4*sin(a)^4)*tan(1/2*b*x+1/2*a
)-1/12*(2*cos(c)^3*cos(a)^3-cos(c)^3*cos(a)*sin(a)^2+8*cos(c)^2*sin(c)*cos(
a)^2*sin(a)-cos(c)^2*sin(c)*sin(a)^3-cos(c)*sin(c)^2*cos(a)^3+8*cos(c)*sin(
c)^2*cos(a)*sin(a)^2-sin(c)^3*cos(a)^2*sin(a)+2*sin(c)^3*sin(a)^3)/(cos(a)^
4*cos(c)^4+2*sin(a)^2*cos(a)^2*cos(c)^4+cos(c)^4*sin(a)^4+2*cos(a)^4*sin(c)
^2*cos(c)^2+4*cos(c)^2*sin(c)^2*cos(a)^2*sin(a)^2+2*sin(a)^4*sin(c)^2*cos(c
)^2+sin(c)^4*cos(a)^4+2*sin(a)^2*cos(a)^2*sin(c)^4+sin(c)^4*sin(a)^4))/(cos
(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(1/2*b*x+1/2*a)^2+2*tan(1/
2*b*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*sin(a)*cos(c)-cos(a)*cos(c)
-sin(a)*sin(c))^3-(sin(a)*cos(c)-cos(a)*sin(c))/(cos(a)^4*cos(c)^4+2*sin(a)
^2*cos(a)^2*cos(c)^4+cos(c)^4*sin(a)^4+2*cos(a)^4*sin(c)^2*cos(c)^2+4*cos(c
)^2*sin(c)^2*cos(a)^2*sin(a)^2+2*sin(a)^4*sin(c)^2*cos(c)^2+sin(c)^4*cos(a)
^4+2*sin(a)^2*cos(a)^2*sin(c)^4+sin(c)^4*sin(a)^4)/(-cos(c)^2*sin(a)^2-cos(
a)^2*cos(c)^2-sin(a)^2*sin(c)^2-cos(a)^2*sin(c)^2)^(1/2)*arctan(1/2*(2*(cos
(a)*cos(c)+sin(a)*sin(c))*tan(1/2*b*x+1/2*a)-2*sin(a)*cos(c)+2*cos(a)*sin(c
```

```
))/(-cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin(c)^2-cos(a)^2*sin(c)^
2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1424 vs. $2(61) = 122$.
time = 0.59, size = 1424, normalized size = 21.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/12*(2*(3*cos(5*b*x + 2*a + 4*c) - 3*cos(5*b*x + 6*c) - 8*cos(3*b*x + 2*a
 + 2*c) - 8*cos(3*b*x + 4*c) - 3*cos(b*x + 2*a) + 3*cos(b*x + 2*c))*cos(6*b
*x + a + 6*c) + 6*(3*cos(4*b*x + a + 4*c) + 3*cos(2*b*x + a + 2*c) + cos(a)
)*cos(5*b*x + 2*a + 4*c) - 6*(3*cos(4*b*x + a + 4*c) + 3*cos(2*b*x + a + 2*
c) + cos(a))*cos(5*b*x + 6*c) - 6*(8*cos(3*b*x + 2*a + 2*c) + 8*cos(3*b*x +
 4*c) + 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*cos(4*b*x + a + 4*c) - 16*(3*c
os(2*b*x + a + 2*c) + cos(a))*cos(3*b*x + 2*a + 2*c) - 16*(3*cos(2*b*x + a
+ 2*c) + cos(a))*cos(3*b*x + 4*c) - 18*(cos(b*x + 2*a) - cos(b*x + 2*c))*co
s(2*b*x + a + 2*c) - 6*cos(b*x + 2*a)*cos(a) + 6*cos(b*x + 2*c)*cos(a) - 3*
(cos(6*b*x + a + 6*c)^2*sin(-a + c) + 9*cos(4*b*x + a + 4*c)^2*sin(-a + c)
+ 9*cos(2*b*x + a + 2*c)^2*sin(-a + c) + 6*cos(2*b*x + a + 2*c)*cos(a)*sin(
-a + c) + sin(6*b*x + a + 6*c)^2*sin(-a + c) + 9*sin(4*b*x + a + 4*c)^2*sin
(-a + c) + 9*sin(2*b*x + a + 2*c)^2*sin(-a + c) + 6*sin(2*b*x + a + 2*c)*si
n(a)*sin(-a + c) + 2*(3*cos(4*b*x + a + 4*c)*sin(-a + c) + 3*cos(2*b*x + a
+ 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(6*b*x + a + 6*c) + 6*(3*cos(2*
b*x + a + 2*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(4*b*x + a + 4*c) + 2*(
3*sin(4*b*x + a + 4*c)*sin(-a + c) + 3*sin(2*b*x + a + 2*c)*sin(-a + c) + s
in(a)*sin(-a + c))*sin(6*b*x + a + 6*c) + 6*(3*sin(2*b*x + a + 2*c)*sin(-a
+ c) + sin(a)*sin(-a + c))*sin(4*b*x + a + 4*c) + (cos(a)^2 + sin(a)^2)*sin
(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(
b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(
c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c)
 + sin(c)^2)) + 2*(3*sin(5*b*x + 2*a + 4*c) - 3*sin(5*b*x + 6*c) - 8*sin(3*
b*x + 2*a + 2*c) - 8*sin(3*b*x + 4*c) - 3*sin(b*x + 2*a) + 3*sin(b*x + 2*c)
)*sin(6*b*x + a + 6*c) + 6*(3*sin(4*b*x + a + 4*c) + 3*sin(2*b*x + a + 2*c)
 + sin(a))*sin(5*b*x + 2*a + 4*c) - 6*(3*sin(4*b*x + a + 4*c) + 3*sin(2*b*x
 + a + 2*c) + sin(a))*sin(5*b*x + 6*c) - 6*(8*sin(3*b*x + 2*a + 2*c) + 8*si
n(3*b*x + 4*c) + 3*sin(b*x + 2*a) - 3*sin(b*x + 2*c))*sin(4*b*x + a + 4*c)
- 16*(3*sin(2*b*x + a + 2*c) + sin(a))*sin(3*b*x + 2*a + 2*c) - 16*(3*sin(2
*b*x + a + 2*c) + sin(a))*sin(3*b*x + 4*c) - 18*(sin(b*x + 2*a) - sin(b*x +
 2*c))*sin(2*b*x + a + 2*c) - 6*sin(b*x + 2*a)*sin(a) + 6*sin(b*x + 2*c)*si
n(a))/(b*cos(6*b*x + a + 6*c)^2 + 9*b*cos(4*b*x + a + 4*c)^2 + 9*b*cos(2*b*
x + a + 2*c)^2 + 6*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(6*b*x + a + 6*c)^2
```

```
 + 9*b*sin(4*b*x + a + 4*c)^2 + 9*b*sin(2*b*x + a + 2*c)^2 + 6*b*sin(2*b*x
+ a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b + 2*(3*b*cos(4*b*x + a + 4*c) +
 3*b*cos(2*b*x + a + 2*c) + b*cos(a))*cos(6*b*x + a + 6*c) + 6*(3*b*cos(2*b
*x + a + 2*c) + b*cos(a))*cos(4*b*x + a + 4*c) + 2*(3*b*sin(4*b*x + a + 4*c
) + 3*b*sin(2*b*x + a + 2*c) + b*sin(a))*sin(6*b*x + a + 6*c) + 6*(3*b*sin(
2*b*x + a + 2*c) + b*sin(a))*sin(4*b*x + a + 4*c))
```

Fricas [A]
time = 2.17, size = 94, normalized size = 1.40

$$-\frac{3 \cos\left(bx+c\right)^{3} \log\left(\sin\left(bx+c\right)+1\right) \sin\left(-a+c\right) - 3 \cos\left(bx+c\right)^{3} \log\left(-\sin\left(bx+c\right)+1\right) \sin\left(-a+c\right) + 6 \cos\left(bx+c\right) \sin\left(bx+c\right) \sin\left(-a+c\right) - 4 \cos\left(-a+c\right)}{12\, b \cos\left(bx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(3*cos(b*x + c)^3*log(sin(b*x + c) + 1)*sin(-a + c) - 3*cos(b*x + c)^
3*log(-sin(b*x + c) + 1)*sin(-a + c) + 6*cos(b*x + c)*sin(b*x + c)*sin(-a +
 c) - 4*cos(-a + c))/(b*cos(b*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**4*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. 2(61) = 122.
time = 0.44, size = 495, normalized size = 7.39

$$\frac{\frac{3 \left(\tan\left(\frac{1}{2}a\right)^{2} \tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^{2} + \tan\left(\frac{1}{2}a\right) - \tan\left(\frac{1}{2}c\right)\right) \log\left(\left|\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)+1\right|\right)}{\tan\left(\frac{1}{2}a\right)^{2} \tan\left(\frac{1}{2}c\right)^{2} + \tan\left(\frac{1}{2}a\right)^{2} + \tan\left(\frac{1}{2}c\right)^{2} + 1} - \frac{3 \left(\tan\left(\frac{1}{2}a\right)^{2} \tan\left(\frac{1}{2}c\right) - \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^{2} + \tan\left(\frac{1}{2}a\right) - \tan\left(\frac{1}{2}c\right)\right) \log\left(\left|\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)-1\right|\right)}{\tan\left(\frac{1}{2}a\right)^{2} \tan\left(\frac{1}{2}c\right)^{2} + \tan\left(\frac{1}{2}a\right)^{2} + \tan\left(\frac{1}{2}c\right)^{2} + 1} + \frac{2 \left(3 \tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)^{5} \tan\left(\frac{1}{2}a\right)^{2} \tan\left(\frac{1}{2}c\right) - 3 \tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)^{5} \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^{2} - 3 \tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)^{4} \tan\left(\frac{1}{2}a\right)^{2} \tan\left(\frac{1}{2}c\right)^{2} + 3 \tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)^{5} \tan\left(\frac{1}{2}a\right) + 3 \tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)^{4} \tan\left(\frac{1}{2}a\right)^{2} - 3 \tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)^{5} \tan\left(\frac{1}{2}c\right) - 12 \tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)^{4} \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right) + 3 \tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)^{4} \tan\left(\frac{1}{2}c\right)^{2} - 3 \tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)^{4} - 3 \tan\left(\frac{1}{2}bx+\frac{1}{2}c\right) \tan\left(\frac{1}{2}a\right)^{2} \tan\left(\frac{1}{2}c\right) + 3 \tan\left(\frac{1}{2}bx+\frac{1}{2}c\right) \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right)^{2} - \tan\left(\frac{1}{2}a\right)^{2} \tan\left(\frac{1}{2}c\right)^{2} - 3 \tan\left(\frac{1}{2}bx+\frac{1}{2}c\right) \tan\left(\frac{1}{2}a\right) + \tan\left(\frac{1}{2}a\right)^{2} + 3 \tan\left(\frac{1}{2}bx+\frac{1}{2}c\right) \tan\left(\frac{1}{2}c\right) - 4 \tan\left(\frac{1}{2}a\right) \tan\left(\frac{1}{2}c\right) + \tan\left(\frac{1}{2}c\right)^{2} - 1\right)}{\left(\tan\left(\frac{1}{2}a\right)^{2} \tan\left(\frac{1}{2}c\right)^{2} + \tan\left(\frac{1}{2}a\right)^{2} + \tan\left(\frac{1}{2}c\right)^{2} + 1\right) \left(\tan\left(\frac{1}{2}bx+\frac{1}{2}c\right)^{2} - 1\right)^{3}}}{3\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^4*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*(3*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - ta
n(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2 + t
an(1/2*a)^2 + tan(1/2*c)^2 + 1) - 3*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*t
an(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) - 1))/(
tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(3*tan(1/2
*b*x + 1/2*c)^5*tan(1/2*a)^2*tan(1/2*c) - 3*tan(1/2*b*x + 1/2*c)^5*tan(1/2*
a)*tan(1/2*c)^2 - 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2*tan(1/2*c)^2 + 3*ta
n(1/2*b*x + 1/2*c)^5*tan(1/2*a) + 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2 - 3
```

```
*tan(1/2*b*x + 1/2*c)^5*tan(1/2*c) - 12*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)*t
an(1/2*c) + 3*tan(1/2*b*x + 1/2*c)^4*tan(1/2*c)^2 - 3*tan(1/2*b*x + 1/2*c)^
4 - 3*tan(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c) + 3*tan(1/2*b*x + 1/2*c)
*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 - 3*tan(1/2*b*x + 1/2*
c)*tan(1/2*a) + tan(1/2*a)^2 + 3*tan(1/2*b*x + 1/2*c)*tan(1/2*c) - 4*tan(1/
2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1)/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a
)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b*x + 1/2*c)^2 - 1)^3))/b
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(c + b*x)^4,x)
[Out] \text{Hanged}
```

3.217 $\int \sec^5(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{\cos(a-c)\sec^4(c+bx)}{4b}+\frac{\sin(a-c)\tan(c+bx)}{b}+\frac{\sin(a-c)\tan^3(c+bx)}{3b}$$

```
[Out] 1/4*cos(a-c)*sec(b*x+c)^4/b+sin(a-c)*tan(b*x+c)/b+1/3*sin(a-c)*tan(b*x+c)^3
/b
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {4676, 2686, 30, 3852}

$$\frac{\sin(a-c)\tan^3(bx+c)}{3b}+\frac{\sin(a-c)\tan(bx+c)}{b}+\frac{\cos(a-c)\sec^4(bx+c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^5*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x]^4)/(4*b) + (Sin[a - c]*Tan[c + b*x])/b + (Sin[a -
c]*Tan[c + b*x]^3)/(3*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4676

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \sec^5(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \sec^4(c+bx)\tan(c+bx)\,dx + \sin(a-c)\int \sec^4(c+bx)\,dx$$
$$= \frac{\cos(a-c)\text{Subst}\left(\int x^3\,dx, x, \sec(c+bx)\right)}{b} - \frac{\sin(a-c)\text{Subst}\left(\int \left(1+x^2\right)\,dx, x\right.}{b}$$
$$= \frac{\cos(a-c)\sec^4(c+bx)}{4b} + \frac{\sin(a-c)\tan(c+bx)}{b} + \frac{\sin(a-c)\tan^3(c+bx)}{3b}$$

Mathematica [**A**]
time = 0.37, size = 48, normalized size = 0.81

$$\frac{\sec(c)\sec^4(c+bx)(3\cos(a)+\sin(a-c)(4\sin(c+2bx)+\sin(3c+4bx)))}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^5*Sin[a + b*x],x]
```

```
[Out] (Sec[c]*Sec[c + b*x]^4*(3*Cos[a] + Sin[a - c]*(4*Sin[c + 2*b*x] + Sin[3*c +
 4*b*x])))/(12*b)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(55) = 110$.
time = 2.68, size = 324, normalized size = 5.49

method	result
risch	$\frac{4\,e^{i(4bx+9a+3c)}+\frac{8\,e^{i(2bx+9a+c)}}{3}-\frac{8\,e^{i(2bx+7a+3c)}}{3}+\frac{2\,e^{i(9a-c)}}{3}-\frac{2\,e^{i(7a+c)}}{3}}{\left(e^{2i(bx+a+c)}+e^{2ia}\right)^4 b}$
default	$-\frac{1}{(\sin(a)\cos(c)-\cos(a)\sin(c))^4(-\tan(bx+a)\cos(a)\sin(c)+\tan(bx+a)\sin(a)\cos(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}+\frac{}{2(\sin(a)\cos(c)-\cos(a)\sin(c))^4(-\tan}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^5*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-1/(sin(a)*cos(c)-cos(a)*sin(c))^4/(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+
a)*sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))+1/2*(3*cos(a)*cos(c)+3*sin(a)
*sin(c))/(sin(a)*cos(c)-cos(a)*sin(c))^4/(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x
+a)*sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))^2-1/3*(cos(c)^2*sin(a)^2+3*c
os(a)^2*cos(c)^2+4*cos(a)*cos(c)*sin(a)*sin(c)+3*sin(a)^2*sin(c)^2+cos(a)^2
*sin(c)^2)/(sin(a)*cos(c)-cos(a)*sin(c))^4/(-tan(b*x+a)*cos(a)*sin(c)+tan(b
*x+a)*sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))^3+1/4*(cos(a)*cos(c)+sin(a
)*sin(c))*(cos(a)^2*cos(c)^2+cos(c)^2*sin(a)^2+cos(a)^2*sin(c)^2+sin(a)^2*s
```

```
in(c)^2)/(sin(a)*cos(c)-cos(a)*sin(c))^4/(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x
+a)*sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))^4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(55) = 110$.
time = 0.29, size = 1074, normalized size = 18.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 2/3*((6*cos(4*b*x + 2*a + 4*c) + 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4
*c) + cos(2*a) - cos(2*c))*cos(8*b*x + a + 9*c) + 4*(6*cos(4*b*x + 2*a + 4*
c) + 4*cos(2*b*x + 2*a + 2*c) - 4*cos(2*b*x + 4*c) + cos(2*a) - cos(2*c))*c
os(6*b*x + a + 7*c) + 6*(4*cos(2*b*x + a + 3*c) + cos(a + c))*cos(4*b*x + 2
*a + 4*c) + 6*(6*cos(4*b*x + 2*a + 4*c) + 4*cos(2*b*x + 2*a + 2*c) - 4*cos(
2*b*x + 4*c) + cos(2*a) - cos(2*c))*cos(4*b*x + a + 5*c) + 4*(4*cos(2*b*x +
 2*a + 2*c) + cos(2*a) - cos(2*c))*cos(2*b*x + a + 3*c) - 4*(4*cos(2*b*x +
a + 3*c) + cos(a + c))*cos(2*b*x + 4*c) + (cos(2*a) - cos(2*c))*cos(a + c)
+ 4*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (6*sin(4*b*x + 2*a + 4*c) + 4*sin(2
*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) - sin(2*c))*sin(8*b*x + a
 + 9*c) + 4*(6*sin(4*b*x + 2*a + 4*c) + 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*
b*x + 4*c) + sin(2*a) - sin(2*c))*sin(6*b*x + a + 7*c) + 6*(4*sin(2*b*x + a
 + 3*c) + sin(a + c))*sin(4*b*x + 2*a + 4*c) + 6*(6*sin(4*b*x + 2*a + 4*c)
+ 4*sin(2*b*x + 2*a + 2*c) - 4*sin(2*b*x + 4*c) + sin(2*a) - sin(2*c))*sin(
4*b*x + a + 5*c) + 4*(4*sin(2*b*x + 2*a + 2*c) + sin(2*a) - sin(2*c))*sin(2
*b*x + a + 3*c) - 4*(4*sin(2*b*x + a + 3*c) + sin(a + c))*sin(2*b*x + 4*c)
+ (sin(2*a) - sin(2*c))*sin(a + c) + 4*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(
b*cos(8*b*x + a + 9*c)^2 + 16*b*cos(6*b*x + a + 7*c)^2 + 36*b*cos(4*b*x + a
 + 5*c)^2 + 16*b*cos(2*b*x + a + 3*c)^2 + 8*b*cos(2*b*x + a + 3*c)*cos(a +
c) + b*cos(a + c)^2 + b*sin(8*b*x + a + 9*c)^2 + 16*b*sin(6*b*x + a + 7*c)^
2 + 36*b*sin(4*b*x + a + 5*c)^2 + 16*b*sin(2*b*x + a + 3*c)^2 + 8*b*sin(2*b
*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(4*b*cos(6*b*x + a + 7*c) + 6
*b*cos(4*b*x + a + 5*c) + 4*b*cos(2*b*x + a + 3*c) + b*cos(a + c))*cos(8*b*
x + a + 9*c) + 8*(6*b*cos(4*b*x + a + 5*c) + 4*b*cos(2*b*x + a + 3*c) + b*c
os(a + c))*cos(6*b*x + a + 7*c) + 12*(4*b*cos(2*b*x + a + 3*c) + b*cos(a +
c))*cos(4*b*x + a + 5*c) + 2*(4*b*sin(6*b*x + a + 7*c) + 6*b*sin(4*b*x + a
+ 5*c) + 4*b*sin(2*b*x + a + 3*c) + b*sin(a + c))*sin(8*b*x + a + 9*c) + 8*
(6*b*sin(4*b*x + a + 5*c) + 4*b*sin(2*b*x + a + 3*c) + b*sin(a + c))*sin(6*
b*x + a + 7*c) + 12*(4*b*sin(2*b*x + a + 3*c) + b*sin(a + c))*sin(4*b*x + a
 + 5*c))
```

Fricas [A]

time = 3.02, size = 53, normalized size = 0.90

$$-\frac{4\left(2\cos\left(bx+c\right)^{3}+\cos\left(bx+c\right)\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-3\cos\left(-a+c\right)}{12\,b\cos\left(bx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(4*(2*cos(b*x + c)^3 + cos(b*x + c))*sin(b*x + c)*sin(-a + c) - 3*cos
(-a + c))/(b*cos(b*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**5*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(55) = 110.
time = 0.43, size = 327, normalized size = 5.54

$$\frac{3\tan\left(bx+c\right)^{4}\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-3\tan\left(bx+c\right)^{4}\tan\left(\frac{1}{2}a\right)^{2}+12\tan\left(bx+c\right)^{4}\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+8\tan\left(bx+c\right)^{3}\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-3\tan\left(bx+c\right)^{4}\tan\left(\frac{1}{2}c\right)^{2}-8\tan\left(bx+c\right)^{3}\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}+6\tan\left(bx+c\right)^{2}\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+3\tan\left(bx+c\right)^{4}+8\tan\left(bx+c\right)^{3}\tan\left(\frac{1}{2}a\right)-6\tan\left(bx+c\right)^{2}\tan\left(\frac{1}{2}a\right)^{2}-8\tan\left(bx+c\right)^{3}\tan\left(\frac{1}{2}c\right)+24\tan\left(bx+c\right)^{2}\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+24\tan\left(bx+c\right)\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-6\tan\left(bx+c\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-24\tan\left(bx+c\right)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}+6\tan\left(bx+c\right)^{2}+24\tan\left(bx+c\right)\tan\left(\frac{1}{2}a\right)-24\tan\left(bx+c\right)\tan\left(\frac{1}{2}c\right)}{12\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^5*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*(3*tan(b*x + c)^4*tan(1/2*a)^2*tan(1/2*c)^2 - 3*tan(b*x + c)^4*tan(1/2
*a)^2 + 12*tan(b*x + c)^4*tan(1/2*a)*tan(1/2*c) + 8*tan(b*x + c)^3*tan(1/2*
a)^2*tan(1/2*c) - 3*tan(b*x + c)^4*tan(1/2*c)^2 - 8*tan(b*x + c)^3*tan(1/2*
a)*tan(1/2*c)^2 + 6*tan(b*x + c)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 3*tan(b*x +
c)^4 + 8*tan(b*x + c)^3*tan(1/2*a) - 6*tan(b*x + c)^2*tan(1/2*a)^2 - 8*tan(
b*x + c)^3*tan(1/2*c) + 24*tan(b*x + c)^2*tan(1/2*a)*tan(1/2*c) + 24*tan(b*
x + c)*tan(1/2*a)^2*tan(1/2*c) - 6*tan(b*x + c)^2*tan(1/2*c)^2 - 24*tan(b*x
 + c)*tan(1/2*a)*tan(1/2*c)^2 + 6*tan(b*x + c)^2 + 24*tan(b*x + c)*tan(1/2*
a) - 24*tan(b*x + c)*tan(1/2*c))/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2
 + tan(1/2*c)^2 + 1)*b)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(c + b*x)^5,x)
```

```
[Out] \text{Hanged}
```

3.218 $\int \sec^6(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=94

$$\frac{\cos(a-c)\sec^5(c+bx)}{5b}+\frac{3\tanh^{-1}(\sin(c+bx))\sin(a-c)}{8b}+\frac{3\sec(c+bx)\sin(a-c)\tan(c+bx)}{8b}+\frac{\sec^3(c+bx}{}$$

```
[Out] 1/5*cos(a-c)*sec(b*x+c)^5/b+3/8*arctanh(sin(b*x+c))*sin(a-c)/b+3/8*sec(b*x+
c)*sin(a-c)*tan(b*x+c)/b+1/4*sec(b*x+c)^3*sin(a-c)*tan(b*x+c)/b
```

Rubi [A]

time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4676, 2686, 30, 3853, 3855}

$$\frac{3\sin(a-c)\tanh^{-1}(\sin(bx+c))}{8b}+\frac{\cos(a-c)\sec^5(bx+c)}{5b}+\frac{\sin(a-c)\tan(bx+c)\sec^3(bx+c)}{4b}+\frac{3\sin(a-c)\tan(bx+c)\sec(bx+c)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + b*x]^6*Sin[a + b*x],x]
```

```
[Out] (Cos[a - c]*Sec[c + b*x]^5)/(5*b) + (3*ArcTanh[Sin[c + b*x]]*Sin[a - c])/(8
*b) + (3*Sec[c + b*x]*Sin[a - c]*Tan[c + b*x])/(8*b) + (Sec[c + b*x]^3*Sin[
a - c]*Tan[c + b*x])/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4676

```
Int[Sec[w_]^(n_.)*Sin[v_], x_Symbol] :> Dist[Cos[v - w], Int[Tan[w]*Sec[w]^
(n - 1), x], x] + Dist[Sin[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \sec^6(c+bx)\sin(a+bx)\,dx = \cos(a-c)\int \sec^5(c+bx)\tan(c+bx)\,dx + \sin(a-c)\int \sec^5(c+bx)\,dx$$
$$= \frac{\sec^3(c+bx)\sin(a-c)\tan(c+bx)}{4b} + \frac{\cos(a-c)\text{Subst}\left(\int x^4\,dx, x, \sec(c+bx\right.}{b}$$
$$= \frac{\cos(a-c)\sec^5(c+bx)}{5b} + \frac{3\sec(c+bx)\sin(a-c)\tan(c+bx)}{8b} + \frac{\sec^3(c+bx}{}$$
$$= \frac{\cos(a-c)\sec^5(c+bx)}{5b} + \frac{3\tanh^{-1}(\sin(c+bx))\sin(a-c)}{8b} + \frac{3\sec(c+bx)\,\text{si}}{}$$

Mathematica [A]
time = 1.05, size = 78, normalized size = 0.83

$$\frac{480\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\sin(a-c)+2\sec^5(c+bx)(64\cos(a-c)+5\sin(a-c)(14\sin(2(c+bx))+3\sin(4(c+bx))))}{640b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + b*x]^6*Sin[a + b*x],x]
```

```
[Out] (480*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c] + 2*Sec[c + b*x]^5*(6
4*Cos[a - c] + 5*Sin[a - c]*(14*Sin[2*(c + b*x)] + 3*Sin[4*(c + b*x)])))/(6
40*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6741 vs. 2(86) = 172.
time = 6.90, size = 6742, normalized size = 71.72

method	result
risch	$\frac{-15\,e^{i(9bx+11a+8c)}+15\,e^{i(9bx+9a+10c)}-70\,e^{i(7bx+11a+6c)}+70\,e^{i(7bx+9a+8c)}+128\,e^{i(5bx+11a+4c)}+128\,e^{i(5bx+9a+6c)}+70\,e^{i(3bx+11a+2}}{40b\left(e^{2i(bx+a+c)}+e^{2ia}\right)^5}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+c)^6*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3096 vs. $2(86) = 172$.
time = 0.69, size = 3096, normalized size = 32.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/80*(2*(15*cos(9*b*x + 2*a + 8*c) - 15*cos(9*b*x + 10*c) + 70*cos(7*b*x +
 2*a + 6*c) - 70*cos(7*b*x + 8*c) - 128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*
b*x + 6*c) - 70*cos(3*b*x + 2*a + 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*x +
 2*a) + 15*cos(b*x + 2*c))*cos(10*b*x + a + 10*c) + 30*(5*cos(8*b*x + a + 8
*c) + 10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a +
 2*c) + cos(a))*cos(9*b*x + 2*a + 8*c) - 30*(5*cos(8*b*x + a + 8*c) + 10*co
s(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c) + cos
(a))*cos(9*b*x + 10*c) + 10*(70*cos(7*b*x + 2*a + 6*c) - 70*cos(7*b*x + 8*c
) - 128*cos(5*b*x + 2*a + 4*c) - 128*cos(5*b*x + 6*c) - 70*cos(3*b*x + 2*a
+ 2*c) + 70*cos(3*b*x + 4*c) - 15*cos(b*x + 2*a) + 15*cos(b*x + 2*c))*cos(8
*b*x + a + 8*c) + 140*(10*cos(6*b*x + a + 6*c) + 10*cos(4*b*x + a + 4*c) +
5*cos(2*b*x + a + 2*c) + cos(a))*cos(7*b*x + 2*a + 6*c) - 140*(10*cos(6*b*x
 + a + 6*c) + 10*cos(4*b*x + a + 4*c) + 5*cos(2*b*x + a + 2*c) + cos(a))*co
s(7*b*x + 8*c) - 20*(128*cos(5*b*x + 2*a + 4*c) + 128*cos(5*b*x + 6*c) + 70
*cos(3*b*x + 2*a + 2*c) - 70*cos(3*b*x + 4*c) + 15*cos(b*x + 2*a) - 15*cos(
b*x + 2*c))*cos(6*b*x + a + 6*c) - 256*(10*cos(4*b*x + a + 4*c) + 5*cos(2*b
*x + a + 2*c) + cos(a))*cos(5*b*x + 2*a + 4*c) - 256*(10*cos(4*b*x + a + 4*
c) + 5*cos(2*b*x + a + 2*c) + cos(a))*cos(5*b*x + 6*c) - 100*(14*cos(3*b*x
+ 2*a + 2*c) - 14*cos(3*b*x + 4*c) + 3*cos(b*x + 2*a) - 3*cos(b*x + 2*c))*c
os(4*b*x + a + 4*c) - 140*(5*cos(2*b*x + a + 2*c) + cos(a))*cos(3*b*x + 2*a
 + 2*c) + 140*(5*cos(2*b*x + a + 2*c) + cos(a))*cos(3*b*x + 4*c) - 150*(cos
(b*x + 2*a) - cos(b*x + 2*c))*cos(2*b*x + a + 2*c) - 30*cos(b*x + 2*a)*cos(
a) + 30*cos(b*x + 2*c)*cos(a) - 15*(cos(10*b*x + a + 10*c)^2*sin(-a + c) +
25*cos(8*b*x + a + 8*c)^2*sin(-a + c) + 100*cos(6*b*x + a + 6*c)^2*sin(-a +
 c) + 100*cos(4*b*x + a + 4*c)^2*sin(-a + c) + 25*cos(2*b*x + a + 2*c)^2*si
n(-a + c) + 10*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(10*b*x + a + 1
0*c)^2*sin(-a + c) + 25*sin(8*b*x + a + 8*c)^2*sin(-a + c) + 100*sin(6*b*x
+ a + 6*c)^2*sin(-a + c) + 100*sin(4*b*x + a + 4*c)^2*sin(-a + c) + 25*sin(
2*b*x + a + 2*c)^2*sin(-a + c) + 10*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c)
 + 2*(5*cos(8*b*x + a + 8*c)*sin(-a + c) + 10*cos(6*b*x + a + 6*c)*sin(-a +
 c) + 10*cos(4*b*x + a + 4*c)*sin(-a + c) + 5*cos(2*b*x + a + 2*c)*sin(-a +
 c) + cos(a)*sin(-a + c))*cos(10*b*x + a + 10*c) + 10*(10*cos(6*b*x + a + 6
*c)*sin(-a + c) + 10*cos(4*b*x + a + 4*c)*sin(-a + c) + 5*cos(2*b*x + a + 2
*c)*sin(-a + c) + cos(a)*sin(-a + c))*cos(8*b*x + a + 8*c) + 20*(10*cos(4*b
*x + a + 4*c)*sin(-a + c) + 5*cos(2*b*x + a + 2*c)*sin(-a + c) + cos(a)*sin
(-a + c))*cos(6*b*x + a + 6*c) + 20*(5*cos(2*b*x + a + 2*c)*sin(-a + c) + c
```

```
os(a)*sin(-a + c))*cos(4*b*x + a + 4*c) + 2*(5*sin(8*b*x + a + 8*c)*sin(-a
+ c) + 10*sin(6*b*x + a + 6*c)*sin(-a + c) + 10*sin(4*b*x + a + 4*c)*sin(-a
 + c) + 5*sin(2*b*x + a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c))*sin(10*b*x
 + a + 10*c) + 10*(10*sin(6*b*x + a + 6*c)*sin(-a + c) + 10*sin(4*b*x + a +
 4*c)*sin(-a + c) + 5*sin(2*b*x + a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c)
)*sin(8*b*x + a + 8*c) + 20*(10*sin(4*b*x + a + 4*c)*sin(-a + c) + 5*sin(2*
b*x + a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c))*sin(6*b*x + a + 6*c) + 20*
(5*sin(2*b*x + a + 2*c)*sin(-a + c) + sin(a)*sin(-a + c))*sin(4*b*x + a + 4
*c) + (cos(a)^2 + sin(a)^2)*sin(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 -
 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin
(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2
*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) + 2*(15*sin(9*b*x + 2*a + 8*c)
 - 15*sin(9*b*x + 10*c) + 70*sin(7*b*x + 2*a + 6*c) - 70*sin(7*b*x + 8*c) -
 128*sin(5*b*x + 2*a + 4*c) - 128*sin(5*b*x + 6*c) - 70*sin(3*b*x + 2*a + 2
*c) + 70*sin(3*b*x + 4*c) - 15*sin(b*x + 2*a) + 15*sin(b*x + 2*c))*sin(10*b
*x + a + 10*c) + 30*(5*sin(8*b*x + a + 8*c) + 10*sin(6*b*x + a + 6*c) + 10*
sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*sin(9*b*x + 2*a + 8
*c) - 30*(5*sin(8*b*x + a + 8*c) + 10*sin(6*b*x + a + 6*c) + 10*sin(4*b*x +
 a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*sin(9*b*x + 10*c) + 10*(70*sin
(7*b*x + 2*a + 6*c) - 70*sin(7*b*x + 8*c) - 128*sin(5*b*x + 2*a + 4*c) - 12
8*sin(5*b*x + 6*c) - 70*sin(3*b*x + 2*a + 2*c) + 70*sin(3*b*x + 4*c) - 15*s
in(b*x + 2*a) + 15*sin(b*x + 2*c))*sin(8*b*x + a + 8*c) + 140*(10*sin(6*b*x
 + a + 6*c) + 10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*si
n(7*b*x + 2*a + 6*c) - 140*(10*sin(6*b*x + a + 6*c) + 10*sin(4*b*x + a + 4*
c) + 5*sin(2*b*x + a + 2*c) + sin(a))*sin(7*b*x + 8*c) - 20*(128*sin(5*b*x
+ 2*a + 4*c) + 128*sin(5*b*x + 6*c) + 70*sin(3*b*x + 2*a + 2*c) - 70*sin(3*
b*x + 4*c) + 15*sin(b*x + 2*a) - 15*sin(b*x + 2*c))*sin(6*b*x + a + 6*c) -
256*(10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a))*sin(5*b*x +
 2*a + 4*c) - 256*(10*sin(4*b*x + a + 4*c) + 5*sin(2*b*x + a + 2*c) + sin(a
))*sin(5*b*x + 6*c) - 100*(14*sin(3*b*x + 2*a + 2*c) - 14*sin(3*b*x + 4*c)
+ 3*sin(b*x + 2*a) - 3*sin(b*x + 2*c))*sin(4*b*...
```

Fricas [A]
time = 2.95, size = 107, normalized size = 1.14

$$-\frac{15 \cos\left(bx+c\right)^{5} \log\left(\sin\left(bx+c\right)+1\right) \sin\left(-a+c\right) - 15 \cos\left(bx+c\right)^{5} \log\left(-\sin\left(bx+c\right)+1\right) \sin\left(-a+c\right) + 10 \left(3 \cos\left(bx+c\right)^{3} + 2 \cos\left(bx+c\right)\right) \sin\left(bx+c\right) \sin\left(-a+c\right) - 16 \cos\left(-a+c\right)}{80 \, b \cos\left(bx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/80*(15*cos(b*x + c)^5*log(sin(b*x + c) + 1)*sin(-a + c) - 15*cos(b*x + c
)^5*log(-sin(b*x + c) + 1)*sin(-a + c) + 10*(3*cos(b*x + c)^3 + 2*cos(b*x +
 c))*sin(b*x + c)*sin(-a + c) - 16*cos(-a + c))/(b*cos(b*x + c)^5)
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)**6*sin(b*x+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 756 vs. 2(86) = 172.
time = 0.44, size = 756, normalized size = 8.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+c)^6*sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/20*(15*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) -
tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) + 1))/(tan(1/2*a)^2*tan(1/2*c)^2 +
 tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 15*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a
)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x + 1/2*c) - 1)
)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(25*tan
(1/2*b*x + 1/2*c)^9*tan(1/2*a)^2*tan(1/2*c) - 25*tan(1/2*b*x + 1/2*c)^9*tan
(1/2*a)*tan(1/2*c)^2 - 20*tan(1/2*b*x + 1/2*c)^8*tan(1/2*a)^2*tan(1/2*c)^2
+ 25*tan(1/2*b*x + 1/2*c)^9*tan(1/2*a) + 20*tan(1/2*b*x + 1/2*c)^8*tan(1/2*
a)^2 - 25*tan(1/2*b*x + 1/2*c)^9*tan(1/2*c) - 80*tan(1/2*b*x + 1/2*c)^8*tan
(1/2*a)*tan(1/2*c) - 10*tan(1/2*b*x + 1/2*c)^7*tan(1/2*a)^2*tan(1/2*c) + 20
*tan(1/2*b*x + 1/2*c)^8*tan(1/2*c)^2 + 10*tan(1/2*b*x + 1/2*c)^7*tan(1/2*a)
*tan(1/2*c)^2 - 20*tan(1/2*b*x + 1/2*c)^8 - 10*tan(1/2*b*x + 1/2*c)^7*tan(1
/2*a) + 10*tan(1/2*b*x + 1/2*c)^7*tan(1/2*c) - 40*tan(1/2*b*x + 1/2*c)^4*ta
n(1/2*a)^2*tan(1/2*c)^2 + 40*tan(1/2*b*x + 1/2*c)^4*tan(1/2*a)^2 - 160*tan(
1/2*b*x + 1/2*c)^4*tan(1/2*a)*tan(1/2*c) + 10*tan(1/2*b*x + 1/2*c)^3*tan(1/
2*a)^2*tan(1/2*c) + 40*tan(1/2*b*x + 1/2*c)^4*tan(1/2*c)^2 - 10*tan(1/2*b*x
 + 1/2*c)^3*tan(1/2*a)*tan(1/2*c)^2 - 40*tan(1/2*b*x + 1/2*c)^4 + 10*tan(1/
2*b*x + 1/2*c)^3*tan(1/2*a) - 10*tan(1/2*b*x + 1/2*c)^3*tan(1/2*c) - 25*tan
(1/2*b*x + 1/2*c)*tan(1/2*a)^2*tan(1/2*c) + 25*tan(1/2*b*x + 1/2*c)*tan(1/2
*a)*tan(1/2*c)^2 - 4*tan(1/2*a)^2*tan(1/2*c)^2 - 25*tan(1/2*b*x + 1/2*c)*ta
n(1/2*a) + 4*tan(1/2*a)^2 + 25*tan(1/2*b*x + 1/2*c)*tan(1/2*c) - 16*tan(1/2
*a)*tan(1/2*c) + 4*tan(1/2*c)^2 - 4)/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*
a)^2 + tan(1/2*c)^2 + 1)*(tan(1/2*b*x + 1/2*c)^2 - 1)^5))/b
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/cos(c + b*x)^6,x)
```

```
[Out] \text{Hanged}
```

3.219 $\int \cos^n(c+dx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=386

$$-\frac{i2^{-2-n}e^{-i(2a+cn)-i(2b+dn)x+in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^{n}\,{}_2F_1\left(\frac{1}{2}\left(-\frac{2b}{d}-n\right),-n;\frac{1}{2}\left(2-\frac{2b}{d}-\right.\right.}{2b+dn}$$

```
[Out] -I*2^(-2-n)*exp(-I*(c*n+2*a)-I*(d*n+2*b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+ex
p(I*(d*x+c)))^n*hypergeom([-n, -b/d-1/2*n],[1-b/d-1/2*n],-exp(2*I*(d*x+c)))
/((1+exp(2*I*c+2*I*d*x))^n)/(d*n+2*b)+I*2^(-2-n)*exp(I*(-c*n+2*a)+I*(-d*n+2
*b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-n, b/d-1/
2*n],[1+b/d-1/2*n],-exp(2*I*(d*x+c)))/((1+exp(2*I*c+2*I*d*x))^n)/(-d*n+2*b)
+I*2^(-1-n)*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-n, -1/2*n],[1-1/
2*n],-exp(2*I*(d*x+c)))/d/((1+exp(2*I*(d*x+c)))^n)/n
```

Rubi [A]
time = 0.49, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {4651, 2320, 2057, 371, 2323, 2285, 2283}

$$-\frac{i2^{-n-2}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(-\frac{2b}{d}-n\right),-n;\frac{1}{2}\left(-\frac{2b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(-i(2a+cn)-ix(2b+dn)+in(c+dx))}{2b+dn}+\frac{i2^{-n-2}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(\frac{2b}{d}-n\right),-n;\frac{1}{2}\left(\frac{2b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(i(2a-cn)+ix(2b-dn)+in(c+dx))}{2b-dn}+\frac{i2^{-n-1}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2i(c+dx)}\right)^{-n}{}_2F_1\left(-n,-\frac{n}{2};1-\frac{n}{2};-e^{2i(c+dx)}\right)}{dn}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^n*Sin[a + b*x]^2,x]
```

```
[Out] ((-I)*2^(-2 - n)*E^((-I)*(2*a + c*n) - I*(2*b + d*n)*x + I*n*(c + d*x))*(E^
((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((-2*b)/d - n)/2, -
n, (2 - (2*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x
))^n*(2*b + d*n)) + (I*2^(-2 - n)*E^(I*(2*a - c*n) + I*(2*b - d*n)*x + I*n*
(c + d*x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((2*b
)/d - n)/2, -n, (2 + (2*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*
c + (2*I)*d*x))^n*(2*b - d*n)) + (I*2^(-1 - n)*(E^((-I)*(c + d*x)) + E^(I*(
c + d*x)))^n*Hypergeometric2F1[-n, -1/2*n, 1 - n/2, -E^((2*I)*(c + d*x))])/
(d*(1 + E^((2*I)*(c + d*x)))^n*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2057

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
```

racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])), Int[x^(m + j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && PosQ[n - j]

Rule 2283

Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hypergeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1, Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])

Rule 2285

Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> Int[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] && !LinearMatchQ[{u, v}, x]

Rule 2320

Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 2323

Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] && !IntegerQ[n] && LinearQ[{v, w}, x]

Rule 4651

Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] :> Dist[1/2^(p + q), Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a, b, c, d, q}, x] && IGtQ[p, 0] && !IntegerQ[q]

Rubi steps

$$\int \cos^n(c+dx)\sin^2(a+bx)\,dx = 2^{-2-n}\int \left(2\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n - e^{-2ia-2ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n - e^{2ia}\right.$$

$$= -\left(2^{-2-n}\int e^{-2ia-2ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n dx\right) - 2^{-2-n}\int e^{2ia+2ibx}\left(e^{-i(\right.$$

$$= -\frac{(i2^{-1-n})\,\text{Subst}\left(\int \frac{\left(\frac{1}{x}+x\right)^n}{x}dx, x, e^{i(c+dx)}\right)}{d} - \left(2^{-2-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right.\right.$$

$$= -\left(\left(2^{-2-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)\int e^{i(2a-cn)+i(2}\right.$$

$$= -\frac{i2^{-2-n}\exp(-i(2a+cn)-i(2b+dn)x+in(c+dx))\left(1+e^{2ic+2idx}\right)^{-n}(}{2b+}$$

Mathematica [A]
time = 2.19, size = 249, normalized size = 0.65

$$-\frac{i2^{-2-n}e^{-2i(a+bx)}\left(1+e^{2i(c+dx)}\right)^{-n}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^n\left(dn(-2b+dn)\,{}_2F_1\left(-\frac{b}{d}-\frac{n}{2},-n;1-\frac{b}{d}-\frac{n}{2};-e^{2i(c+dx)}\right)+e^{2i(a+bx)}(2b+dn)\left(de^{2i(a+bx)}n\,{}_2F_1\left(\frac{b}{d}-\frac{n}{2},-n;1+\frac{b}{d}-\frac{n}{2};-e^{2i(c+dx)}\right)+2(2b-dn)\,{}_2F_1\left(-n,-\frac{n}{2};1-\frac{n}{2};-e^{2i(c+dx)}\right)\right)\right)}{-4b^2dn+d^3n^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^n*Sin[a + b*x]^2,x]
```

```
[Out] ((-I)*2^(-2 - n)*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x)))^n*(d*n*(-2*b +
 d*n)*Hypergeometric2F1[-(b/d) - n/2, -n, 1 - b/d - n/2, -E^((2*I)*(c + d*x
))] + E^((2*I)*(a + b*x))*(2*b + d*n)*(d*E^((2*I)*(a + b*x))*n*Hypergeometr
ic2F1[b/d - n/2, -n, 1 + b/d - n/2, -E^((2*I)*(c + d*x))] + 2*(2*b - d*n)*H
ypergeometric2F1[-n, -1/2*n, 1 - n/2, -E^((2*I)*(c + d*x))])))/(E^((2*I)*(a
 + b*x))*(1 + E^((2*I)*(c + d*x)))^n*(-4*b^2*d*n + d^3*n^3))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(\cos^n(dx+c)\right)\left(\sin^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^n*sin(b*x+a)^2,x)
```

```
[Out] int(cos(d*x+c)^n*sin(b*x+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^n*sin(b*x + a)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*cos(d*x + c)^n, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**n*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^n*sin(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^n\,\sin\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^n*sin(a + b*x)^2,x)
```

```
[Out] int(cos(c + d*x)^n*sin(a + b*x)^2, x)
```

3.220 $\int \cos(c+dx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=68

$$-\frac{\sin(2a-c+(2b-d)x)}{4(2b-d)}+\frac{\sin(c+dx)}{2d}-\frac{\sin(2a+c+(2b+d)x)}{4(2b+d)}$$

```
[Out] -1/4*sin(2*a-c+(2*b-d)*x)/(2*b-d)+1/2*sin(d*x+c)/d-1/4*sin(2*a+c+(2*b+d)*x)
/(2*b+d)
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4670, 2717}

$$-\frac{\sin(2a+x(2b-d)-c)}{4(2b-d)}-\frac{\sin(2a+x(2b+d)+c)}{4(2b+d)}+\frac{\sin(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[a + b*x]^2,x]
```

```
[Out] -1/4*Sin[2*a - c + (2*b - d)*x]/(2*b - d) + Sin[c + d*x]/(2*d) - Sin[2*a +
c + (2*b + d)*x]/(4*(2*b + d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^2(a+bx)\,dx &= \int \left(-\frac{1}{4}\cos(2a-c+(2b-d)x)+\frac{1}{2}\cos(c+dx)-\frac{1}{4}\cos(2a+c+(2b+d\right.\\ &= -\left(\frac{1}{4}\int \cos(2a-c+(2b-d)x)\,dx\right)-\frac{1}{4}\int \cos(2a+c+(2b+d)x)\,dx+\\ &= -\frac{\sin(2a-c+(2b-d)x)}{4(2b-d)}+\frac{\sin(c+dx)}{2d}-\frac{\sin(2a+c+(2b+d)x)}{4(2b+d)}\end{aligned}$$

Mathematica [A]
time = 0.86, size = 76, normalized size = 1.12

$$\frac{1}{4}\left(\frac{2\cos(dx)\sin(c)}{d}+\frac{2\cos(c)\sin(dx)}{d}-\frac{\sin(2a-c+2bx-dx)}{2b-d}-\frac{\sin(2a+c+2bx+dx)}{2b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((2*Cos[d*x]*Sin[c])/d + (2*Cos[c]*Sin[d*x])/d - Sin[2*a - c + 2*b*x - d*x]
/(2*b - d) - Sin[2*a + c + 2*b*x + d*x]/(2*b + d))/4
```

Maple [A]
time = 0.23, size = 63, normalized size = 0.93

method	result
default	$-\frac{\sin(2a-c+(2b-d)x)}{4(2b-d)}+\frac{\sin(dx+c)}{2d}-\frac{\sin(2a+c+(2b+d)x)}{4(2b+d)}$
risch	$\frac{2\sin(dx+c)b^2}{d(2b-d)(2b+d)}-\frac{d\sin(dx+c)}{2(2b-d)(2b+d)}-\frac{\sin(2bx-dx+2a-c)b}{2(2b-d)(2b+d)}-\frac{d\sin(2bx-dx+2a-c)}{4(2b-d)(2b+d)}-\frac{\sin(2bx+dx+2a+c)b}{2(2b-d)(2b+d)}+\frac{d\sin(2bx+dx+2a}{4(2b-d)(2b+}$
norman	$\frac{-\frac{4b\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b^2-d^2}+\frac{4b\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b^2-d^2}+\frac{4b\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4b^2-d^2}-\frac{4b\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4b^2-d^2}+\frac{4b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(4b^2-d^2\right)}+\frac{4b^2\left(\tan^4\left(\frac{bx}{2}+\right.\right.}{d\left(4b\right.}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*sin(2*a-c+(2*b-d)*x)/(2*b-d)+1/2*sin(d*x+c)/d-1/4*sin(2*a+c+(2*b+d)*x)
/(2*b+d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(62) = 124$.
time = 0.29, size = 371, normalized size = 5.46

$$-\frac{(2bd\sin(c)-d^2\sin(c))\cos((2b+d)x+2a+2c)-(2bd\sin(c)-d^2\sin(c))\cos((2b+d)x+2a)-(2bd\sin(c)+d^2\sin(c))\cos(-(2b-d)x-2a+2c)+(2bd\sin(c)+d^2\sin(c))\cos(-(2b-d)x-2a)-2(4b^2\sin(c)-d^2\sin(c))\cos(dx+2c)+2(4b^2\sin(c)-d^2\sin(c))\cos(dx)-(2bd\cos(c)-d^2\cos(c))\sin((2b+d)x+2a+2c)-(2bd\cos(c)-d^2\cos(c))\sin((2b+d)x+2a)+(2bd\cos(c)+d^2\cos(c))\sin(-(2b-d)x-2a+2c)+(2bd\cos(c)+d^2\cos(c))\sin(-(2b-d)x-2a)+2(4b^2\cos(c)-d^2\cos(c))\sin(dx+2c)+2(4b^2\cos(c)-d^2\cos(c))\sin(dx)}{8\left((\cos(c)^2+\sin(c)^2)d^3-4(b^2\cos(c)^2+b^2\sin(c)^2)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((2*b*d*sin(c) - d^2*sin(c))*cos((2*b + d)*x + 2*a + 2*c) - (2*b*d*sin
(c) - d^2*sin(c))*cos((2*b + d)*x + 2*a) - (2*b*d*sin(c) + d^2*sin(c))*cos(
-(2*b - d)*x - 2*a + 2*c) + (2*b*d*sin(c) + d^2*sin(c))*cos(-(2*b - d)*x -
2*a) - 2*(4*b^2*sin(c) - d^2*sin(c))*cos(d*x + 2*c) + 2*(4*b^2*sin(c) - d^2
*sin(c))*cos(d*x) - (2*b*d*cos(c) - d^2*cos(c))*sin((2*b + d)*x + 2*a + 2*c
) - (2*b*d*cos(c) - d^2*cos(c))*sin((2*b + d)*x + 2*a) + (2*b*d*cos(c) + d^
2*cos(c))*sin(-(2*b - d)*x - 2*a + 2*c) + (2*b*d*cos(c) + d^2*cos(c))*sin(-
```

```
(2*b - d)*x - 2*a) + 2*(4*b^2*cos(c) - d^2*cos(c))*sin(d*x + 2*c) + 2*(4*b^
2*cos(c) - d^2*cos(c))*sin(d*x))/((cos(c)^2 + sin(c)^2)*d^3 - 4*(b^2*cos(c)
^2 + b^2*sin(c)^2)*d)
```

Fricas [A]
time = 3.62, size = 70, normalized size = 1.03

$$-\frac{2\,bd\cos\left(bx+a\right)\cos\left(dx+c\right)\sin\left(bx+a\right)-\left(d^2\cos\left(bx+a\right)^2+2\,b^2-d^2\right)\sin\left(dx+c\right)}{4\,b^2d-d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b*d*cos(b*x + a)*cos(d*x + c)*sin(b*x + a) - (d^2*cos(b*x + a)^2 + 2*b^
2 - d^2)*sin(d*x + c))/(4*b^2*d - d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(49) = 98.
time = 0.79, size = 410, normalized size = 6.03

$$\begin{cases} x\sin^2\left(a\right)\cos\left(c\right) & \text{for}\: b=0\wedge d=0 \\ \frac{x\sin^2\left(a-\frac{dx}{2}\right)\cos\left(c+dx\right)}{4}+\frac{x\sin\left(a-\frac{dx}{2}\right)\sin\left(c+dx\right)\cos\left(a-\frac{dx}{2}\right)}{2}-\frac{x\cos^2\left(a-\frac{dx}{2}\right)\cos\left(c+dx\right)}{4}+\frac{3\sin\left(a-\frac{dx}{2}\right)\cos\left(a-\frac{dx}{2}\right)\cos\left(c+dx\right)}{2d}+\frac{\sin\left(c+dx\right)\cos^2\left(a-\frac{dx}{2}\right)}{d} & \text{for}\: b=-\frac{d}{2} \\ \frac{x\sin^2\left(a+\frac{dx}{2}\right)\cos\left(c+dx\right)}{4}-\frac{x\sin\left(a+\frac{dx}{2}\right)\sin\left(c+dx\right)\cos\left(a+\frac{dx}{2}\right)}{2}-\frac{x\cos^2\left(a+\frac{dx}{2}\right)\cos\left(c+dx\right)}{4}+\frac{\sin^2\left(a+\frac{dx}{2}\right)\sin\left(c+dx\right)}{d}+\frac{\sin\left(a+\frac{dx}{2}\right)\cos\left(a+\frac{dx}{2}\right)\cos\left(c+dx\right)}{2d} & \text{for}\: b=\frac{d}{2} \\ \left(\frac{x\sin^2\left(a+bx\right)}{2}+\frac{x\cos^2\left(a+bx\right)}{2}-\frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{2b}\right)\cos\left(c\right) & \text{for}\: d=0 \\ \frac{2b^2\sin^2\left(a+bx\right)\sin\left(c+dx\right)}{4b^2d-d^3}+\frac{2b^2\sin\left(c+dx\right)\cos^2\left(a+bx\right)}{4b^2d-d^3}-\frac{2bd\sin\left(a+bx\right)\cos\left(a+bx\right)\cos\left(c+dx\right)}{4b^2d-d^3}-\frac{d^2\sin^2\left(a+bx\right)\sin\left(c+dx\right)}{4b^2d-d^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a)**2*cos(c), Eq(b, 0) & Eq(d, 0)), (x*sin(a - d*x/2)**2*c
os(c + d*x)/4 + x*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)/2 - x*cos(a -
d*x/2)**2*cos(c + d*x)/4 + 3*sin(a - d*x/2)*cos(a - d*x/2)*cos(c + d*x)/(2*
d) + sin(c + d*x)*cos(a - d*x/2)**2/d, Eq(b, -d/2)), (x*sin(a + d*x/2)**2*c
os(c + d*x)/4 - x*sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)/2 - x*cos(a +
d*x/2)**2*cos(c + d*x)/4 + sin(a + d*x/2)**2*sin(c + d*x)/d + sin(a + d*x/2
)*cos(a + d*x/2)*cos(c + d*x)/(2*d), Eq(b, d/2)), ((x*sin(a + b*x)**2/2 + x
*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a + b*x)/(2*b))*cos(c), Eq(d, 0)), (2
*b**2*sin(a + b*x)**2*sin(c + d*x)/(4*b**2*d - d**3) + 2*b**2*sin(c + d*x)*
cos(a + b*x)**2/(4*b**2*d - d**3) - 2*b*d*sin(a + b*x)*cos(a + b*x)*cos(c +
 d*x)/(4*b**2*d - d**3) - d**2*sin(a + b*x)**2*sin(c + d*x)/(4*b**2*d - d**
3), True))
```

Giac [A]
time = 0.44, size = 61, normalized size = 0.90

$$-\frac{\sin\left(2\,bx+dx+2\,a+c\right)}{4\,\left(2\,b+d\right)}-\frac{\sin\left(2\,bx-dx+2\,a-c\right)}{4\,\left(2\,b-d\right)}+\frac{\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*sin(2*b*x + d*x + 2*a + c)/(2*b + d) - 1/4*sin(2*b*x - d*x + 2*a - c)/
(2*b - d) + 1/2*sin(d*x + c)/d
```

Mupad [B]
time = 0.81, size = 105, normalized size = 1.54

$$\frac{\sin\left(c+d\,x\right)}{2\,d}-\frac{b\,\left(2\,d\,\sin\left(2\,a+c+2\,b\,x+d\,x\right)+2\,d\,\sin\left(2\,a-c+2\,b\,x-d\,x\right)\right)-d^2\,\sin\left(2\,a+c+2\,b\,x+d\,x\right)+d^2\,\sin\left(2\,a-c+2\,b\,x-d\,x\right)}{16\,b^2\,d-4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(a + b*x)^2,x)
```

```
[Out] sin(c + d*x)/(2*d) - (b*(2*d*sin(2*a + c + 2*b*x + d*x) + 2*d*sin(2*a - c +
 2*b*x - d*x)) - d^2*sin(2*a + c + 2*b*x + d*x) + d^2*sin(2*a - c + 2*b*x -
 d*x))/(16*b^2*d - 4*d^3)
```

3.221 $\int \cos^2(c+dx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=88

$$\frac{x}{4}-\frac{\sin(2a+2bx)}{8b}-\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)}+\frac{\sin(2c+2dx)}{8d}-\frac{\sin(2(a+c)+2(b+d)x)}{16(b+d)}$$

```
[Out] 1/4*x-1/8*sin(2*b*x+2*a)/b-1/16*sin(2*a-2*c+2*(b-d)*x)/(b-d)+1/8*sin(2*d*x+
2*c)/d-1/16*sin(2*a+2*c+2*(b+d)*x)/(b+d)
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4670, 2717}

$$-\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}-\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}-\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] x/4 - Sin[2*a + 2*b*x]/(8*b) - Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) +
Sin[2*c + 2*d*x]/(8*d) - Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{4}-\frac{1}{4}\cos(2a+2bx)-\frac{1}{8}\cos(2(a-c)+2(b-d)x)+\frac{1}{4}\cos(2c+2dx\right.\\ &= \frac{x}{4}-\frac{1}{8}\int \cos(2(a-c)+2(b-d)x)\,dx-\frac{1}{8}\int \cos(2(a+c)+2(b+d)x)\,d\\ &= \frac{x}{4}-\frac{\sin(2a+2bx)}{8b}-\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)}+\frac{\sin(2c+2dx)}{8d}-\frac{\sin(2(\end{aligned}$$

Mathematica [A]
time = 0.81, size = 108, normalized size = 1.23

$$\frac{(-2b^2d+2d^3)\sin(2(a+bx))-bd(b+d)\sin(2(a-c+(b-d)x))+b(b-d)(4d(b+d)x+2(b+d)\sin(2(c+dx))-d\sin(2(a+c+(b+d)x)))}{16b(b-d)d(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] ((-2*b^2*d + 2*d^3)*Sin[2*(a + b*x)] - b*d*(b + d)*Sin[2*(a - c + (b - d)*x
)] + b*(b - d)*(4*d*(b + d)*x + 2*(b + d)*Sin[2*(c + d*x)] - d*Sin[2*(a + c
 + (b + d)*x)]))/(16*b*(b - d)*d*(b + d))
```

Maple [A]
time = 0.18, size = 83, normalized size = 0.94

method	result
default	$\frac{x}{4}-\frac{\sin(2bx+2a)}{8b}+\frac{\sin(2dx+2c)}{8d}-\frac{\sin((2b-2d)x+2a-2c)}{16(b-d)}-\frac{\sin((2b+2d)x+2a+2c)}{16(b+d)}$
risch	$\frac{x}{4}-\frac{\sin(2bx+2a)}{8b}+\frac{\sin(2dx+2c)b^2}{8(b-d)d(b+d)}-\frac{d\sin(2dx+2c)}{8(b-d)(b+d)}-\frac{\sin(2bx-2dx+2a-2c)b}{16(b-d)(b+d)}-\frac{d\sin(2bx-2dx+2a-2c)}{16(b-d)(b+d)}-\frac{\sin(2bx+2dx+2}{16(b-d)(b+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x-1/8*sin(2*b*x+2*a)/b+1/8*sin(2*d*x+2*c)/d-1/16/(b-d)*sin((2*b-2*d)*x+
2*a-2*c)-1/16/(b+d)*sin((2*b+2*d)*x+2*a+2*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(78) = 156.
time = 0.32, size = 620, normalized size = 7.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)
^2)*d)*x - (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c) +
 (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) + (b^2*d*sin(2*c)
 + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) - (b^2*d*sin(2*c) + b*d^2*
sin(2*c))*cos(-2*(b - d)*x - 2*a) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2
*b*x + 2*a + 2*c) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a - 2*c
) - 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) + 2*(b^3*sin(2*c) - b*d^2*
sin(2*c))*cos(2*d*x + 4*c) + (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d
)*x + 2*a + 4*c) + (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)*x + 2*a)
 - (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c) - (b^2*d
```

```
*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) + 2*(b^2*d*cos(2*c) - d
^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) + 2*(b^2*d*cos(2*c) - d^3*cos(2*c))*sin
(2*b*x + 2*a - 2*c) - 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x) - 2*(b^3
*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2 + b*sin(2*c)^2
)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)
```

Fricas [A]
time = 2.93, size = 108, normalized size = 1.23

$$\frac{\left(2\,bd^2\cos\left(bx+a\right)^2+b^3-2\,bd^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(b^3d-bd^3\right)x-\left(2\,b^2d\cos\left(bx+a\right)\cos\left(dx+c\right)^2-d^3\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{4\left(b^3d-bd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*b*d^2*cos(b*x + a)^2 + b^3 - 2*b*d^2)*cos(d*x + c)*sin(d*x + c) + (
b^3*d - b*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - d^3*cos(b*x + a))
*sin(b*x + a))/(b^3*d - b*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(76) = 152$.
time = 1.87, size = 1027, normalized size = 11.67

$$\begin{cases} x\sin^2(a)\cos^2(c) & \text{for } b=0 \wedge d=0 \\ \left(\frac{x\sin^2(c+dx)}{2}+\frac{x\cos^2(c+dx)}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\sin^2(a) & \text{for } b=0 \\ \frac{x\sin^2(a-dx)\sin^2(c+dx)}{8}+\frac{3x\sin^2(a-dx)\cos^2(c+dx)}{8}+\frac{x\sin(a-dx)\sin(c+dx)\cos(a-dx)\cos(c+dx)}{2}+\frac{3x\sin^2(c+dx)\cos^2(a-dx)}{8}+\frac{x\cos^2(a-dx)\cos^2(c+dx)}{8}+\frac{\sin^2(a-dx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{3\sin(a-dx)\sin^2(c+dx)\cos(a-dx)}{8d}+\frac{\sin(a-dx)\cos(a-dx)\cos^2(c+dx)}{8d} & \text{for } b=-d \\ \frac{x\sin^2(a+dx)\sin^2(c+dx)}{8}+\frac{3x\sin^2(a+dx)\cos^2(c+dx)}{8}-\frac{x\sin(a+dx)\sin(c+dx)\cos(a+dx)\cos(c+dx)}{2}+\frac{3x\sin^2(c+dx)\cos^2(a+dx)}{8}+\frac{x\cos^2(a+dx)\cos^2(c+dx)}{8}+\frac{\sin(a+dx)\sin^2(c+dx)\cos(a+dx)}{8d}-\frac{5\sin(a+dx)\cos(a+dx)\cos^2(c+dx)}{8d}+\frac{\sin(c+dx)\cos^2(a+dx)\cos(c+dx)}{2d} & \text{for } b=d \\ \left(\frac{x\sin^2(a+bx)}{2}+\frac{x\cos^2(a+bx)}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)\cos^2(c) & \text{for } d=0 \\ \frac{b^3dx\sin^2(a+bx)\sin^2(c+dx)}{4b^3d-4bd^3}+\frac{b^3dx\sin^2(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3}+\frac{b^3dx\sin^2(c+dx)\cos^2(a+bx)}{4b^3d-4bd^3}+\frac{b^3dx\cos^2(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3}+\frac{b^3\sin^2(a+bx)\sin(c+dx)\cos(c+dx)}{4b^3d-4bd^3}+\frac{b^3\sin(c+dx)\cos^2(a+bx)\cos(c+dx)}{4b^3d-4bd^3}-\frac{2b^2d\sin(a+bx)\cos(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3}-\frac{bd^3x\sin^2(a+bx)\sin^2(c+dx)}{4b^3d-4bd^3}-\frac{bd^3x\sin^2(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3}-\frac{bd^3x\sin^2(c+dx)\cos^2(a+bx)}{4b^3d-4bd^3}-\frac{bd^3x\cos^2(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3}-\frac{2bd^2\sin^2(a+bx)\sin(c+dx)\cos(c+dx)}{4b^3d-4bd^3}+\frac{d^3\sin(a+bx)\sin^2(c+dx)\cos(a+bx)}{4b^3d-4bd^3}+\frac{d^3\sin(a+bx)\cos(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a)**2*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2
/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**2, Eq(b
, 0)), (x*sin(a - d*x)**2*sin(c + d*x)**2/8 + 3*x*sin(a - d*x)**2*cos(c + d
*x)**2/8 + x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + 3*x*si
n(c + d*x)**2*cos(a - d*x)**2/8 + x*cos(a - d*x)**2*cos(c + d*x)**2/8 + sin
(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*sin(a - d*x)*sin(c + d*x)*
*2*cos(a - d*x)/(8*d) + sin(a - d*x)*cos(a - d*x)*cos(c + d*x)**2/(8*d), Eq
(b, -d)), (x*sin(a + d*x)**2*sin(c + d*x)**2/8 + 3*x*sin(a + d*x)**2*cos(c
+ d*x)**2/8 - x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x)/2 + 3*x
*sin(c + d*x)**2*cos(a + d*x)**2/8 + x*cos(a + d*x)**2*cos(c + d*x)**2/8 +
sin(a + d*x)*sin(c + d*x)**2*cos(a + d*x)/(8*d) - 5*sin(a + d*x)*cos(a + d*
x)*cos(c + d*x)**2/(8*d) + sin(c + d*x)*cos(a + d*x)**2*cos(c + d*x)/(2*d),
 Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(
a + b*x)/(2*b))*cos(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)**2*sin(c + d*x
)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c + d*x)**2/(4*b*
*3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2/(4*b**3*d - 4*b
*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b
```

```
**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d - 4*b*d**3) + b**3*
sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*b*d**3) - 2*b**2*d*
sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*
sin(a + b*x)**2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(a + b*
x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(c + d*x)**2*cos(
a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cos(a + b*x)**2*cos(c + d*x)**
2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x
)/(4*b**3*d - 4*b*d**3) + d**3*sin(a + b*x)*sin(c + d*x)**2*cos(a + b*x)/(4
*b**3*d - 4*b*d**3) + d**3*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**
3*d - 4*b*d**3), True))
```

Giac [A]
time = 0.44, size = 80, normalized size = 0.91

$$\frac{1}{4}\,x - \frac{\sin\left(2\,bx + 2\,dx + 2\,a + 2\,c\right)}{16\,(b + d)} - \frac{\sin\left(2\,bx - 2\,dx + 2\,a - 2\,c\right)}{16\,(b - d)} - \frac{\sin\left(2\,bx + 2\,a\right)}{8\,b} + \frac{\sin\left(2\,dx + 2\,c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*x - 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) - 1/16*sin(2*b*x - 2*d*
x + 2*a - 2*c)/(b - d) - 1/8*sin(2*b*x + 2*a)/b + 1/8*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 0.93, size = 177, normalized size = 2.01

$$-\frac{b\,d^2\,\sin\left(2\,a-2\,c+2\,b\,x-2\,d\,x\right)-2\,b^3\,\sin\left(2\,c+2\,d\,x\right)-2\,d^3\,\sin\left(2\,a+2\,b\,x\right)-b\,d^2\,\sin\left(2\,a+2\,c+2\,b\,x+2\,d\,x\right)+b^2\,d\,\sin\left(2\,a-2\,c+2\,b\,x-2\,d\,x\right)+b^2\,d\,\sin\left(2\,a+2\,c+2\,b\,x+2\,d\,x\right)+2\,b^2\,d\,\sin\left(2\,a+2\,b\,x\right)+2\,b\,d^2\,\sin\left(2\,c+2\,d\,x\right)+4\,b\,d^3\,x-4\,b^3\,d\,x}{16\,b\,d\,\left(b^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(a + b*x)^2,x)
```

```
[Out] -(b*d^2*sin(2*a - 2*c + 2*b*x - 2*d*x) - 2*b^3*sin(2*c + 2*d*x) - 2*d^3*sin
(2*a + 2*b*x) - b*d^2*sin(2*a + 2*c + 2*b*x + 2*d*x) + b^2*d*sin(2*a - 2*c
+ 2*b*x - 2*d*x) + b^2*d*sin(2*a + 2*c + 2*b*x + 2*d*x) + 2*b^2*d*sin(2*a +
 2*b*x) + 2*b*d^2*sin(2*c + 2*d*x) + 4*b*d^3*x - 4*b^3*d*x)/(16*b*d*(b^2 -
d^2))
```

3.222 $\int \cos^3(c+dx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=144

$$-\frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)}-\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sin(c+dx)}{8d}+\frac{\sin(3c+3dx)}{24d}-\frac{3\sin(2a+c+(2b$$

```
[Out] -1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)+
3/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d-3/16*sin(2*a+c+(2*b+d)*x)/(2*b+d)-1/
16*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Rubi [A]

time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4670, 2717}

$$-\frac{\sin(2a+x(2b-3d)-3c)}{16(2b-3d)}-\frac{3\sin(2a+x(2b-d)-c)}{16(2b-d)}-\frac{3\sin(2a+x(2b+d)+c)}{16(2b+d)}-\frac{\sin(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3\sin(c+dx)}{8d}+\frac{\sin(3c+3dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/16*Sin[2*a - 3*c + (2*b - 3*d)*x]/(2*b - 3*d) - (3*Sin[2*a - c + (2*b -
d)*x])/(16*(2*b - d)) + (3*Sin[c + d*x])/(8*d) + Sin[3*c + 3*d*x]/(24*d) -
(3*Sin[2*a + c + (2*b + d)*x])/(16*(2*b + d)) - Sin[2*a + 3*c + (2*b + 3*d)
*x]/(16*(2*b + 3*d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}
\int \cos^3(c+dx)\sin^2(a+bx)\,dx &= \int \left(-\frac{1}{16}\cos(2a-3c+(2b-3d)x)-\frac{3}{16}\cos(2a-c+(2b-d)x)+\frac{3}{8}\cos\right. \\
&= -\left(\frac{1}{16}\int \cos(2a-3c+(2b-3d)x)\,dx\right)-\frac{1}{16}\int \cos(2a+3c+(2b+3d) \\
&= -\frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)}-\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sin(c+dx)}{8d}
\end{aligned}$$

Mathematica [A]
time = 1.85, size = 158, normalized size = 1.10

$$\frac{1}{48}\left(\frac{18\cos(dx)\sin(c)}{d}+\frac{2\cos(3dx)\sin(3c)}{d}+\frac{18\cos(c)\sin(dx)}{d}+\frac{2\cos(3c)\sin(3dx)}{d}-\frac{3\sin(2a-3c+2bx-3dx)}{2b-3d}-\frac{9\sin(2a-c+2bx-dx)}{2b-d}-\frac{9\sin(2a+c+2bx+dx)}{2b+d}-\frac{3\sin(2a+3c+2bx+3dx)}{2b+3d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((18*Cos[d*x]*Sin[c])/d + (2*Cos[3*d*x]*Sin[3*c])/d + (18*Cos[c]*Sin[d*x])/
d + (2*Cos[3*c]*Sin[3*d*x])/d - (3*Sin[2*a - 3*c + 2*b*x - 3*d*x])/(2*b - 3
*d) - (9*Sin[2*a - c + 2*b*x - d*x])/(2*b - d) - (9*Sin[2*a + c + 2*b*x + d
*x])/(2*b + d) - (3*Sin[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d))/48
```

Maple [A]
time = 0.25, size = 133, normalized size = 0.92

method	result
default	$-\frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)}-\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sin(dx+c)}{8d}+\frac{\sin(3dx+3c)}{24d}-\frac{3\sin(2a+c+(2b+d)x)}{16(2b+d)}-\frac{\sin(2a+3c+(2b+}{16(2b+3d)}$
risch	$\frac{3\sin(dx+c)b^2}{2d(2b-d)(2b+d)}-\frac{3d\sin(dx+c)}{8(2b-d)(2b+d)}-\frac{\sin(2bx-3dx+2a-3c)b}{8(2b-3d)(2b+3d)}-\frac{3d\sin(2bx-3dx+2a-3c)}{16(2b-3d)(2b+3d)}-\frac{3\sin(2bx-dx+2a-c)b}{8(2b-d)(2b+d)}-\frac{3d\sin(2b}{16(2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)-3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)+
3/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d-3/16*sin(2*a+c+(2*b+d)*x)/(2*b+d)-1/
16*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(132) = 264$.
time = 0.35, size = 1362, normalized size = 9.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*
sin(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*d^2
*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*cos((2*b + 3*d)*x + 2*a) + 9
*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c
))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c)
- 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c))*cos((2*b + d)*x + 2*a - 2*c) - 9*(8*b
^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*co
s(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18
```

```
*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) - 3*(8*b^3*
d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(-
(2*b - 3*d)*x - 2*a + 6*c) + 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*
b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(-(2*b - 3*d)*x - 2*a) + 2*(16*b^4*sin(
3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x) - 2*(16*b^4*sin(3*c
) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x + 6*c) - 18*(16*b^4*sin
(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x + 4*c) + 18*(16*b^4*s
in(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x - 2*c) - 3*(8*b^3*d
*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((2
*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*
d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((2*b + 3*d)*x + 2*a) - 9*(8*b^3*d*cos(3*
c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*cos(3*c))*sin((2*b + d)
*x + 2*a + 4*c) - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3
*c) + 9*d^4*cos(3*c))*sin((2*b + d)*x + 2*a - 2*c) + 9*(8*b^3*d*cos(3*c) +
4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) - 9*d^4*cos(3*c))*sin(-(2*b - d)*x -
 2*a + 4*c) + 9*(8*b^3*d*cos(3*c) + 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c)
- 9*d^4*cos(3*c))*sin(-(2*b - d)*x - 2*a - 2*c) + 3*(8*b^3*d*cos(3*c) + 12*
b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) - 3*d^4*cos(3*c))*sin(-(2*b - 3*d)*x -
2*a + 6*c) + 3*(8*b^3*d*cos(3*c) + 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) -
 3*d^4*cos(3*c))*sin(-(2*b - 3*d)*x - 2*a) + 2*(16*b^4*cos(3*c) - 40*b^2*d^
2*cos(3*c) + 9*d^4*cos(3*c))*sin(3*d*x) + 2*(16*b^4*cos(3*c) - 40*b^2*d^2*c
os(3*c) + 9*d^4*cos(3*c))*sin(3*d*x + 6*c) + 18*(16*b^4*cos(3*c) - 40*b^2*d
^2*cos(3*c) + 9*d^4*cos(3*c))*sin(d*x + 4*c) + 18*(16*b^4*cos(3*c) - 40*b^2
*d^2*cos(3*c) + 9*d^4*cos(3*c))*sin(d*x - 2*c))/(9*(cos(3*c)^2 + sin(3*c)^2
)*d^5 - 40*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^3 + 16*(b^4*cos(3*c)^2 + b^4
*sin(3*c)^2)*d)
```

Fricas [A]
time = 1.71, size = 174, normalized size = 1.21

$$\frac{6\left(6\,bd^3\cos\left(bx+a\right)\cos\left(dx+c\right)-\left(4\,b^3d-bd^3\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^3\right)\sin\left(bx+a\right)-\left(18\,d^4\cos\left(bx+a\right)^2-16\,b^4+40\,b^2d^2-18\,d^4-\left(8\,b^4-38\,b^2d^2+9\,d^4+9\left(4\,b^2d^2-d^4\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{3\left(16\,b^4d-40\,b^2d^3+9\,d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(6*(6*b*d^3*cos(b*x + a)*cos(d*x + c) - (4*b^3*d - b*d^3)*cos(b*x + a)*
cos(d*x + c)^3)*sin(b*x + a) - (18*d^4*cos(b*x + a)^2 - 16*b^4 + 40*b^2*d^2
 - 18*d^4 - (8*b^4 - 38*b^2*d^2 + 9*d^4 + 9*(4*b^2*d^2 - d^4)*cos(b*x + a)^
2)*cos(d*x + c)^2)*sin(d*x + c))/(16*b^4*d - 40*b^2*d^3 + 9*d^5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2006 vs. $2(116) = 232$.
time = 6.64, size = 2006, normalized size = 13.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a)**2*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-3*x*sin(a - 3*d*x
/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 + x*sin(a - 3*d*x/2)**2*cos(c + d*x)
**3/16 - x*sin(a - 3*d*x/2)*sin(c + d*x)**3*cos(a - 3*d*x/2)/8 + 3*x*sin(a
- 3*d*x/2)*sin(c + d*x)*cos(a - 3*d*x/2)*cos(c + d*x)**2/8 + 3*x*sin(c + d*
x)**2*cos(a - 3*d*x/2)**2*cos(c + d*x)/16 - x*cos(a - 3*d*x/2)**2*cos(c + d
*x)**3/16 + 7*sin(a - 3*d*x/2)**2*sin(c + d*x)**3/(16*d) - 3*sin(a - 3*d*x/
2)*sin(c + d*x)**2*cos(a - 3*d*x/2)*cos(c + d*x)/(4*d) + 5*sin(a - 3*d*x/2)
*cos(a - 3*d*x/2)*cos(c + d*x)**3/(8*d) + 11*sin(c + d*x)**3*cos(a - 3*d*x/
2)**2/(48*d) + sin(c + d*x)*cos(a - 3*d*x/2)**2*cos(c + d*x)**2/d, Eq(b, -3
*d/2)), (3*x*sin(a - d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 + 3*x*sin(a
- d*x/2)**2*cos(c + d*x)**3/16 + 3*x*sin(a - d*x/2)*sin(c + d*x)**3*cos(a -
 d*x/2)/8 + 3*x*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)*cos(c + d*x)**2/
8 - 3*x*sin(c + d*x)**2*cos(a - d*x/2)**2*cos(c + d*x)/16 - 3*x*cos(a - d*x
/2)**2*cos(c + d*x)**3/16 - 17*sin(a - d*x/2)**2*sin(c + d*x)**3/(48*d) + 7
*sin(a - d*x/2)*sin(c + d*x)**2*cos(a - d*x/2)*cos(c + d*x)/(4*d) + 13*sin(
a - d*x/2)*cos(a - d*x/2)*cos(c + d*x)**3/(8*d) + 49*sin(c + d*x)**3*cos(a
- d*x/2)**2/(48*d) + sin(c + d*x)*cos(a - d*x/2)**2*cos(c + d*x)**2/d, Eq(b
, -d/2)), (3*x*sin(a + d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 + 3*x*sin(
a + d*x/2)**2*cos(c + d*x)**3/16 - 3*x*sin(a + d*x/2)*sin(c + d*x)**3*cos(a
 + d*x/2)/8 - 3*x*sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)*cos(c + d*x)**
2/8 - 3*x*sin(c + d*x)**2*cos(a + d*x/2)**2*cos(c + d*x)/16 - 3*x*cos(a + d
*x/2)**2*cos(c + d*x)**3/16 + 31*sin(a + d*x/2)**2*sin(c + d*x)**3/(48*d) +
 sin(a + d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/d + sin(a + d*x/2)*sin(c +
d*x)**2*cos(a + d*x/2)*cos(c + d*x)/(4*d) + 3*sin(a + d*x/2)*cos(a + d*x/2)
*cos(c + d*x)**3/(8*d) + sin(c + d*x)**3*cos(a + d*x/2)**2/(48*d), Eq(b, d/
2)), (-3*x*sin(a + 3*d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 + x*sin(a +
3*d*x/2)**2*cos(c + d*x)**3/16 + x*sin(a + 3*d*x/2)*sin(c + d*x)**3*cos(a +
 3*d*x/2)/8 - 3*x*sin(a + 3*d*x/2)*sin(c + d*x)*cos(a + 3*d*x/2)*cos(c + d*
x)**2/8 + 3*x*sin(c + d*x)**2*cos(a + 3*d*x/2)**2*cos(c + d*x)/16 - x*cos(a
 + 3*d*x/2)**2*cos(c + d*x)**3/16 + 7*sin(a + 3*d*x/2)**2*sin(c + d*x)**3/(
16*d) + 3*sin(a + 3*d*x/2)*sin(c + d*x)**2*cos(a + 3*d*x/2)*cos(c + d*x)/(4
*d) - 5*sin(a + 3*d*x/2)*cos(a + 3*d*x/2)*cos(c + d*x)**3/(8*d) + 11*sin(c
+ d*x)**3*cos(a + 3*d*x/2)**2/(48*d) + sin(c + d*x)*cos(a + 3*d*x/2)**2*cos
(c + d*x)**2/d, Eq(b, 3*d/2)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2
- sin(a + b*x)*cos(a + b*x)/(2*b))*cos(c)**3, Eq(d, 0)), (16*b**4*sin(a + b
*x)**2*sin(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 24*b**4*sin(
a + b*x)**2*sin(c + d*x)*cos(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d*
*5) + 16*b**4*sin(c + d*x)**3*cos(a + b*x)**2/(48*b**4*d - 120*b**2*d**3 +
27*d**5) + 24*b**4*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)**2/(48*b**4*d
- 120*b**2*d**3 + 27*d**5) - 24*b**3*d*sin(a + b*x)*cos(a + b*x)*cos(c + d*
x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 40*b**2*d**2*sin(a + b*x)**2*
sin(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 78*b**2*d**2*sin(a
+ b*x)**2*sin(c + d*x)*cos(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5
```

```
) - 40*b**2*d**2*sin(c + d*x)**3*cos(a + b*x)**2/(48*b**4*d - 120*b**2*d**3
 + 27*d**5) - 42*b**2*d**2*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)**2/(48
*b**4*d - 120*b**2*d**3 + 27*d**5) + 36*b*d**3*sin(a + b*x)*sin(c + d*x)**2
*cos(a + b*x)*cos(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 42*b*d**
3*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27
*d**5) + 18*d**4*sin(a + b*x)**2*sin(c + d*x)**3/(48*b**4*d - 120*b**2*d**3
 + 27*d**5) + 27*d**4*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)**2/(48*b**4
*d - 120*b**2*d**3 + 27*d**5), True))
```

Giac [A]
time = 0.42, size = 129, normalized size = 0.90

$$-\frac{\sin\left(2\,bx+3\,dx+2\,a+3\,c\right)}{16\,\left(2\,b+3\,d\right)}-\frac{3\,\sin\left(2\,bx+dx+2\,a+c\right)}{16\,\left(2\,b+d\right)}-\frac{3\,\sin\left(2\,bx-dx+2\,a-c\right)}{16\,\left(2\,b-d\right)}-\frac{\sin\left(2\,bx-3\,dx+2\,a-3\,c\right)}{16\,\left(2\,b-3\,d\right)}+\frac{\sin\left(3\,dx+3\,c\right)}{24\,d}+\frac{3\,\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*sin(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) - 3/16*sin(2*b*x + d*x + 2
*a + c)/(2*b + d) - 3/16*sin(2*b*x - d*x + 2*a - c)/(2*b - d) - 1/16*sin(2*
b*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*sin(3*d*x + 3*c)/d + 3/8*sin(d*
x + c)/d
```

Mupad [B]
time = 1.97, size = 495, normalized size = 3.44

$$-\mathrm{e}^{a\,2\mathrm{i}-c\,1\mathrm{i}+b\,x\,2\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(24\,b^2-6\,d^2\right)}{b^2\,d\,128\mathrm{i}-d^3\,32\mathrm{i}}+\frac{3\,d\,\left(2\,b+d\right)}{b^2\,d\,128\mathrm{i}-d^3\,32\mathrm{i}}-\frac{3\,d\,\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(2\,b-d\right)}{b^2\,d\,128\mathrm{i}-d^3\,32\mathrm{i}}\right)+\mathrm{e}^{a\,2\mathrm{i}+c\,1\mathrm{i}+b\,x\,2\mathrm{i}+d\,x\,1\mathrm{i}}\,\left(-\frac{3\,d\,\left(2\,b-d\right)}{b^2\,d\,128\mathrm{i}-d^3\,32\mathrm{i}}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(24\,b^2-6\,d^2\right)}{b^2\,d\,128\mathrm{i}-d^3\,32\mathrm{i}}+\frac{3\,d\,\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(2\,b+d\right)}{b^2\,d\,128\mathrm{i}-d^3\,32\mathrm{i}}\right)-\mathrm{e}^{a\,2\mathrm{i}-c\,3\mathrm{i}+b\,x\,2\mathrm{i}-d\,x\,3\mathrm{i}}\,\left(\frac{3\,d\,\left(2\,b+3\,d\right)}{b^2\,d\,384\mathrm{i}-d^3\,864\mathrm{i}}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(8\,b^2-18\,d^2\right)}{b^2\,d\,384\mathrm{i}-d^3\,864\mathrm{i}}-\frac{3\,d\,\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(2\,b-3\,d\right)}{b^2\,d\,384\mathrm{i}-d^3\,864\mathrm{i}}\right)+\mathrm{e}^{a\,2\mathrm{i}+c\,3\mathrm{i}+b\,x\,2\mathrm{i}+d\,x\,3\mathrm{i}}\,\left(-\frac{3\,d\,\left(2\,b-3\,d\right)}{b^2\,d\,384\mathrm{i}-d^3\,864\mathrm{i}}+\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(8\,b^2-18\,d^2\right)}{b^2\,d\,384\mathrm{i}-d^3\,864\mathrm{i}}+\frac{3\,d\,\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(2\,b+3\,d\right)}{b^2\,d\,384\mathrm{i}-d^3\,864\mathrm{i}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*sin(a + b*x)^2,x)
```

```
[Out] exp(a*2i + c*1i + b*x*2i + d*x*1i)*((exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))
/(b^2*d*128i - d^3*32i) - (3*d*(2*b - d))/(b^2*d*128i - d^3*32i) + (3*d*exp
(- a*4i - b*x*4i)*(2*b + d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*1i + b*
x*2i - d*x*1i)*((exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(b^2*d*128i - d^3*3
2i) + (3*d*(2*b + d))/(b^2*d*128i - d^3*32i) - (3*d*exp(- a*4i - b*x*4i)*(2
*b - d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*3i + b*x*2i - d*x*3i)*((3*d
*(2*b + 3*d))/(b^2*d*384i - d^3*864i) + (exp(- a*2i - b*x*2i)*(8*b^2 - 18*d
^2))/(b^2*d*384i - d^3*864i) - (3*d*exp(- a*4i - b*x*4i)*(2*b - 3*d))/(b^2*
d*384i - d^3*864i)) + exp(a*2i + c*3i + b*x*2i + d*x*3i)*((exp(- a*2i - b*x
*2i)*(8*b^2 - 18*d^2))/(b^2*d*384i - d^3*864i) - (3*d*(2*b - 3*d))/(b^2*d*3
84i - d^3*864i) + (3*d*exp(- a*4i - b*x*4i)*(2*b + 3*d))/(b^2*d*384i - d^3*
864i))
```

3.223 $\int \cos^n(c+dx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=568

$$\frac{2^{-3-n}e^{i(3a-cn)+i(3b-dn)x+in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n {}_2F_1\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-n;\frac{1}{2}\left(2+\frac{3b}{d}-n\right);-\right.}{3b-dn}$$

```
[Out] 2^(-3-n)*exp(I*(-c*n+3*a)+I*(-d*n+3*b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(
I*(d*x+c)))^n*hypergeom([-n, 3/2*b/d-1/2*n],[1+3/2*b/d-1/2*n],-exp(2*I*(d*x
+c)))/((1+exp(2*I*c+2*I*d*x))^n)/(-d*n+3*b)-3*2^(-3-n)*exp(I*(-c*n+a)+I*(-d
*n+b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-n, 1/2*
(-d*n+b)/d],[1+1/2*b/d-1/2*n],-exp(2*I*(d*x+c)))/((1+exp(2*I*c+2*I*d*x))^n)
/(-d*n+b)-3*2^(-3-n)*exp(-I*(c*n+a)-I*(d*n+b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c
))+exp(I*(d*x+c)))^n*hypergeom([-n, 1/2*(-d*n-b)/d],[1+1/2*(-d*n-b)/d],-exp
(2*I*(d*x+c)))/((1+exp(2*I*c+2*I*d*x))^n)/(d*n+b)+2^(-3-n)*exp(-I*(c*n+3*a)
-I*(d*n+3*b)*x+I*n*(d*x+c))*(exp(-I*(d*x+c))+exp(I*(d*x+c)))^n*hypergeom([-
n, 1/2*(-d*n-3*b)/d],[1-3/2*b/d-1/2*n],-exp(2*I*(d*x+c)))/((1+exp(2*I*c+2*I
*d*x))^n)/(d*n+3*b)
```

Rubi [A]
time = 0.84, antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {4651, 2323, 2285, 2283}

$$\frac{2^{-n-3}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}{}_2F_1\left(\frac{1}{2}\left(\frac{3b}{d}-n\right),-n;\frac{1}{2}\left(\frac{3b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(i(3a-cn)+ix(3b-dn)+in(c+dx))}{3b-dn}-\frac{3\,2^{-n-3}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}{}_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(\frac{b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(i(a-cn)+ix(b-dn)+in(c+dx))}{b-dn}-\frac{3\,2^{-n-3}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}{}_2F_1\left(-n,-\frac{b+dn}{2d};1-\frac{b+dn}{2d};-e^{2i(c+dx)}\right)\exp(-i(a+cn)-ix(b+dn)+in(c+dx))}{b+dn}+\frac{2^{-n-3}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\left(1+e^{2ic+2idx}\right)^{-n}{}_2F_1\left(-n,-\frac{3b+dn}{2d};\frac{1}{2}\left(-\frac{3b}{d}-n+2\right);-e^{2i(c+dx)}\right)\exp(-i(3a+cn)-ix(3b+dn)+in(c+dx))}{3b+dn}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^n*Sin[a + b*x]^3,x]
```

```
[Out] (2^(-3 - n)*E^(I*(3*a - c*n) + I*(3*b - d*n)*x + I*n*(c + d*x))*(E^((-I)*(c
 + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[((3*b)/d - n)/2, -n, (2 + (
3*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n*(3*b
 - d*n)) - (3*2^(-3 - n)*E^(I*(a - c*n) + I*(b - d*n)*x + I*n*(c + d*x))*(E
^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, (b - d*n)/(2*d
), (2 + b/d - n)/2, -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2*I)*d*x))^n
*(b - d*n)) - (3*2^(-3 - n)*E^((-I)*(a + c*n) - I*(b + d*n)*x + I*n*(c + d*
x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1[-n, -1/2*(b
+ d*n)/d, 1 - (b + d*n)/(2*d), -E^((2*I)*(c + d*x))])/((1 + E^((2*I)*c + (2
*I)*d*x))^n*(b + d*n)) + (2^(-3 - n)*E^((-I)*(3*a + c*n) - I*(3*b + d*n)*x
+ I*n*(c + d*x))*(E^((-I)*(c + d*x)) + E^(I*(c + d*x)))^n*Hypergeometric2F1
[-n, -1/2*(3*b + d*n)/d, (2 - (3*b)/d - n)/2, -E^((2*I)*(c + d*x))])/((1 +
E^((2*I)*c + (2*I)*d*x))^n*(3*b + d*n))
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
```

```
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2285

```
Int[((a_) + (b_.)*(F_)^((e_.)*(v_)))^(p_)*(G_)^((h_.)*(u_)), x_Symbol] :> I
nt[G^(h*ExpandToSum[u, x])*(a + b*F^(e*ExpandToSum[v, x]))^p, x] /; FreeQ[{
F, G, a, b, e, h, p}, x] && LinearQ[{u, v}, x] &&  !LinearMatchQ[{u, v}, x]
```

Rule 2323

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 4651

```
Int[Cos[(c_.) + (d_.)*(x_)]^(q_.)*Sin[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> Dist[1/2^(p + q), Int[ExpandIntegrand[(E^((-I)*(c + d*x)) + E^(I*(c + d*
x)))^q, (I/E^(I*(a + b*x)) - I*E^(I*(a + b*x)))^p, x], x], x] /; FreeQ[{a,
b, c, d, q}, x] && IGtQ[p, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \cos^n(c+dx)\sin^3(a+bx)\,dx = 2^{-3-n}\int \left(3ie^{-ia-ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n - 3ie^{ia+ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)\right.$$

$$= -\left(\left(i2^{-3-n}\right)\int e^{-3ia-3ibx}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\,dx\right) + \left(i2^{-3-n}\right)\int e^{3ia+3il}$$

$$= -\left(\left(i2^{-3-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)\int e^{-3ia-3ibx-}\right.$$

$$= \left(i2^{-3-n}e^{in(c+dx)}\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c+dx)}+e^{i(c+dx)}\right)^n\right)\int e^{i(3a-cn)+i(3b-c}$$

$$= \frac{2^{-3-n}\exp(i(3a-cn)+i(3b-dn)x+in(c+dx))\left(1+e^{2ic+2idx}\right)^{-n}\left(e^{-i(c\right.}}{3b-dn}$$

Mathematica [A]
time = 32.93, size = 355, normalized size = 0.62

$$2^{-3-n}e^{i(-3a+dnx)}\left(1+e^{2i(c+dx)}\right)^{-n}\left(e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)\right)^n\left(\frac{e^{i(6a+3bx-dnx)}\,{}_2F_1\left(\frac{3b}{2d}-\frac{n}{2},-n;1+\frac{3b}{2d}-\frac{n}{2};-e^{2i(c+dx)}\right)}{3b-dn}-\frac{3e^{i(4a+bx-dnx)}\,{}_2F_1\left(-n,\frac{b-dn}{2d};\frac{1}{2}\left(2+\frac{b}{d}-n\right);-e^{2i(c+dx)}\right)}{b-dn}-\frac{3e^{2ia-i(b+dn)x}\,{}_2F_1\left(-n,-\frac{b+dn}{2d};-\frac{b+d(-2+n)}{2d};-e^{2i(c+dx)}\right)}{b+dn}+\frac{e^{-i(3b+dn)x}\,{}_2F_1\left(-n,-\frac{3b+dn}{2d};1-\frac{3b}{2d}-\frac{n}{2};-e^{2i(c+dx)}\right)}{3b+dn}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^n*Sin[a + b*x]^3,x]
```

```
[Out] (2^(-3 - n)*E^(I*(-3*a + d*n*x))*((1 + E^((2*I)*(c + d*x)))/E^(I*(c + d*x))
)^n*((E^(I*(6*a + 3*b*x - d*n*x))*Hypergeometric2F1[(3*b)/(2*d) - n/2, -n,
1 + (3*b)/(2*d) - n/2, -E^((2*I)*(c + d*x))])/(3*b - d*n) - (3*E^(I*(4*a +
b*x - d*n*x))*Hypergeometric2F1[-n, (b - d*n)/(2*d), (2 + b/d - n)/2, -E^((
2*I)*(c + d*x))])/(b - d*n) - (3*E^((2*I)*a - I*(b + d*n)*x)*Hypergeometric
2F1[-n, -1/2*(b + d*n)/d, -1/2*(b + d*(-2 + n))/d, -E^((2*I)*(c + d*x))])/(
b + d*n) + Hypergeometric2F1[-n, -1/2*(3*b + d*n)/d, 1 - (3*b)/(2*d) - n/2,
 -E^((2*I)*(c + d*x))]/(E^(I*(3*b + d*n)*x)*(3*b + d*n))))/(1 + E^((2*I)*(c
 + d*x)))^n
```

Maple [**F**]
time = 0.08, size = 0, normalized size = 0.00

$$\int \left(\cos^{n}\left(dx+c\right)\right)\left(\sin^{3}\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^n*sin(b*x+a)^3,x)
```

```
[Out] int(cos(d*x+c)^n*sin(b*x+a)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^n*sin(b*x + a)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*cos(d*x + c)^n*sin(b*x + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**n*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^n*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^n*sin(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^n\,\sin\left(a+b\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^n*sin(a + b*x)^3,x)
```

```
[Out] int(cos(c + d*x)^n*sin(a + b*x)^3, x)
```

3.224 $\int \cos(c+dx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=97

$$-\frac{3\cos(a-c+(b-d)x)}{8(b-d)}+\frac{\cos(3a-c+(3b-d)x)}{8(3b-d)}-\frac{3\cos(a+c+(b+d)x)}{8(b+d)}+\frac{\cos(3a+c+(3b+d)x)}{8(3b+d)}$$

```
[Out] -3/8*cos(a-c+(b-d)*x)/(b-d)+1/8*cos(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*cos(a+c+(b
+d)*x)/(b+d)+1/8*cos(3*a+c+(3*b+d)*x)/(3*b+d)
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4670, 2718}

$$-\frac{3\cos(a+x(b-d)-c)}{8(b-d)}+\frac{\cos(3a+x(3b-d)-c)}{8(3b-d)}-\frac{3\cos(a+x(b+d)+c)}{8(b+d)}+\frac{\cos(3a+x(3b+d)+c)}{8(3b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Cos[a - c + (b - d)*x])/(8*(b - d)) + Cos[3*a - c + (3*b - d)*x]/(8*(3*
b - d)) - (3*Cos[a + c + (b + d)*x])/(8*(b + d)) + Cos[3*a + c + (3*b + d)*
x]/(8*(3*b + d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sin^3(a+bx)\,dx &= \int \left(\frac{3}{8}\sin(a-c+(b-d)x)-\frac{1}{8}\sin(3a-c+(3b-d)x)+\frac{3}{8}\sin(a+c+(b\right.\\ &= -\left(\frac{1}{8}\int \sin(3a-c+(3b-d)x)\,dx\right)-\frac{1}{8}\int \sin(3a+c+(3b+d)x)\,dx+\frac{3}{8}\\ &= -\frac{3\cos(a-c+(b-d)x)}{8(b-d)}+\frac{\cos(3a-c+(3b-d)x)}{8(3b-d)}-\frac{3\cos(a+c+(b+d}{8(b+d)}\end{aligned}$$

Mathematica [**A**]
time = 0.56, size = 90, normalized size = 0.93

$$\frac{1}{8}\left(-\frac{3\cos(a-c+bx-dx)}{b-d}+\frac{\cos(3a-c+3bx-dx)}{3b-d}+\frac{\cos(3a+c+3bx+dx)}{3b+d}-\frac{3\cos(a+c+(b+d)x)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sin[a + b*x]^3,x]
```

```
[Out] ((-3*Cos[a - c + b*x - d*x])/(b - d) + Cos[3*a - c + 3*b*x - d*x]/(3*b - d)
 + Cos[3*a + c + 3*b*x + d*x]/(3*b + d) - (3*Cos[a + c + (b + d)*x])/(b + d
))/8
```

Maple [**A**]
time = 0.18, size = 90, normalized size = 0.93

method	result
default	$-\frac{3\cos(a-c+(b-d)x)}{8(b-d)}+\frac{\cos(3a-c+(3b-d)x)}{24b-8d}-\frac{3\cos(a+c+(b+d)x)}{8(b+d)}+\frac{\cos(3a+c+(3b+d)x)}{24b+8d}$
risch	$-\frac{27\cos(bx-dx+a-c)b^3}{8(b+d)(3b+d)(-b+d)(-3b+d)}-\frac{27\cos(bx-dx+a-c)b^2d}{8(b+d)(3b+d)(-b+d)(-3b+d)}+\frac{3\cos(bx-dx+a-c)b\,d^2}{8(b+d)(3b+d)(-b+d)(-3b+d)}+\frac{3\cos(bx-dx+a-c}{8(b+d)(3b+d)(-b+d)(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*cos(a-c+(b-d)*x)/(b-d)+1/8*cos(3*a-c+(3*b-d)*x)/(3*b-d)-3/8*cos(a+c+(b
+d)*x)/(b+d)+1/8*cos(3*a+c+(3*b+d)*x)/(3*b+d)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 785 vs. $2(89) = 178$.
time = 0.32, size = 785, normalized size = 8.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((3*b^3*cos(c) - b^2*d*cos(c) - 3*b*d^2*cos(c) + d^3*cos(c))*cos((3*b
+ d)*x + 3*a + 2*c) + (3*b^3*cos(c) - b^2*d*cos(c) - 3*b*d^2*cos(c) + d^3*c
os(c))*cos((3*b + d)*x + 3*a) + (3*b^3*cos(c) + b^2*d*cos(c) - 3*b*d^2*cos(
c) - d^3*cos(c))*cos(-(3*b - d)*x - 3*a + 2*c) + (3*b^3*cos(c) + b^2*d*cos(
c) - 3*b*d^2*cos(c) - d^3*cos(c))*cos(-(3*b - d)*x - 3*a) - 3*(9*b^3*cos(c)
 - 9*b^2*d*cos(c) - b*d^2*cos(c) + d^3*cos(c))*cos((b + d)*x + a + 2*c) - 3
*(9*b^3*cos(c) - 9*b^2*d*cos(c) - b*d^2*cos(c) + d^3*cos(c))*cos((b + d)*x
+ a) - 3*(9*b^3*cos(c) + 9*b^2*d*cos(c) - b*d^2*cos(c) - d^3*cos(c))*cos(-(
b - d)*x - a + 2*c) - 3*(9*b^3*cos(c) + 9*b^2*d*cos(c) - b*d^2*cos(c) - d^3
```

```
*cos(c))*cos(-(b - d)*x - a) + (3*b^3*sin(c) - b^2*d*sin(c) - 3*b*d^2*sin(c
) + d^3*sin(c))*sin((3*b + d)*x + 3*a + 2*c) - (3*b^3*sin(c) - b^2*d*sin(c)
 - 3*b*d^2*sin(c) + d^3*sin(c))*sin((3*b + d)*x + 3*a) + (3*b^3*sin(c) + b^
2*d*sin(c) - 3*b*d^2*sin(c) - d^3*sin(c))*sin(-(3*b - d)*x - 3*a + 2*c) - (
3*b^3*sin(c) + b^2*d*sin(c) - 3*b*d^2*sin(c) - d^3*sin(c))*sin(-(3*b - d)*x
 - 3*a) - 3*(9*b^3*sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d^3*sin(c))*sin
((b + d)*x + a + 2*c) + 3*(9*b^3*sin(c) - 9*b^2*d*sin(c) - b*d^2*sin(c) + d
^3*sin(c))*sin((b + d)*x + a) - 3*(9*b^3*sin(c) + 9*b^2*d*sin(c) - b*d^2*si
n(c) - d^3*sin(c))*sin(-(b - d)*x - a + 2*c) + 3*(9*b^3*sin(c) + 9*b^2*d*si
n(c) - b*d^2*sin(c) - d^3*sin(c))*sin(-(b - d)*x - a))/(9*b^4*cos(c)^2 + 9*
b^4*sin(c)^2 + (cos(c)^2 + sin(c)^2)*d^4 - 10*(b^2*cos(c)^2 + b^2*sin(c)^2)
*d^2)
```

Fricas [A]
time = 3.76, size = 116, normalized size = 1.20

$$-\frac{(7b^2d-d^3-(b^2d-d^3)\cos(bx+a)^2)\sin(bx+a)\sin(dx+c)-3\left((b^3-bd^2)\cos(bx+a)^3-(3b^3-bd^2)\cos(bx+a)\right)\cos(dx+c)}{9b^4-10b^2d^2+d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -((7*b^2*d - d^3 - (b^2*d - d^3)*cos(b*x + a)^2)*sin(b*x + a)*sin(d*x + c)
- 3*((b^3 - b*d^2)*cos(b*x + a)^3 - (3*b^3 - b*d^2)*cos(b*x + a))*cos(d*x +
 c))/(9*b^4 - 10*b^2*d^2 + d^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(76) = 152$.
time = 2.27, size = 937, normalized size = 9.66

$$\begin{cases} x\sin^3(a)\cos(c) & \text{for } b=0 \wedge d=0 \\ \frac{3x\sin^3(a-dx)\cos(c+dx)}{8}+\frac{3x\sin^2(a-dx)\sin(c+dx)\cos(a-dx)}{8}+\frac{3x\sin(a-dx)\cos^2(a-dx)\cos(c+dx)}{8}+\frac{3x\sin(c+dx)\cos^3(a-dx)}{8}-\frac{\sin^3(a-dx)\sin(c+dx)}{8d}+\frac{3\sin^2(a-dx)\cos(a-dx)\cos(c+dx)}{4d}+\frac{3\cos^3(a-dx)\cos(c+dx)}{8d} & \text{for } b=-d \\ \frac{x\sin^3\left(a-\frac{dx}{3}\right)\cos(c+dx)}{8}+\frac{3x\sin^2\left(a-\frac{dx}{3}\right)\sin(c+dx)\cos\left(a-\frac{dx}{3}\right)}{8}-\frac{3x\sin\left(a-\frac{dx}{3}\right)\cos^2\left(a-\frac{dx}{3}\right)\cos(c+dx)}{8}-\frac{x\sin(c+dx)\cos^3\left(a-\frac{dx}{3}\right)}{8}+\frac{7\sin^3\left(a-\frac{dx}{3}\right)\sin(c+dx)}{8d}+\frac{3\sin\left(a-\frac{dx}{3}\right)\sin(c+dx)\cos^2\left(a-\frac{dx}{3}\right)}{4d}-\frac{3\cos^3\left(a-\frac{dx}{3}\right)\cos(c+dx)}{8d} & \text{for } b=-\frac{d}{3} \\ \frac{x\sin^3\left(a+\frac{dx}{3}\right)\cos(c+dx)}{8}-\frac{3x\sin^2\left(a+\frac{dx}{3}\right)\sin(c+dx)\cos\left(a+\frac{dx}{3}\right)}{8}-\frac{3x\sin\left(a+\frac{dx}{3}\right)\cos^2\left(a+\frac{dx}{3}\right)\cos(c+dx)}{8}+\frac{x\sin(c+dx)\cos^3\left(a+\frac{dx}{3}\right)}{8}+\frac{7\sin^3\left(a+\frac{dx}{3}\right)\sin(c+dx)}{8d}+\frac{3\sin\left(a+\frac{dx}{3}\right)\sin(c+dx)\cos^2\left(a+\frac{dx}{3}\right)}{4d}+\frac{3\cos^3\left(a+\frac{dx}{3}\right)\cos(c+dx)}{8d} & \text{for } b=\frac{d}{3} \\ \frac{3x\sin^3(a+dx)\cos(c+dx)}{8}-\frac{3x\sin^2(a+dx)\sin(c+dx)\cos(a+dx)}{8}+\frac{3x\sin(a+dx)\cos^2(a+dx)\cos(c+dx)}{8}-\frac{3x\sin(c+dx)\cos^3(a+dx)}{8}+\frac{5\sin^3(a+dx)\sin(c+dx)}{8d}+\frac{3\sin(a+dx)\sin(c+dx)\cos^2(a+dx)}{4d}+\frac{3\cos^3(a+dx)\cos(c+dx)}{8d} & \text{for } b=d \\ -\frac{9b^3\sin^2(a+bx)\cos(a+bx)\cos(c+dx)}{9b^4-10b^2d^2+d^4}-\frac{6b^3\cos^3(a+bx)\cos(c+dx)}{9b^4-10b^2d^2+d^4}-\frac{7b^2d\sin^3(a+bx)\sin(c+dx)}{9b^4-10b^2d^2+d^4}-\frac{6b^2d\sin(a+bx)\sin(c+dx)\cos^2(a+bx)}{9b^4-10b^2d^2+d^4}+\frac{3bd^2\sin^2(a+bx)\cos(a+bx)\cos(c+dx)}{9b^4-10b^2d^2+d^4}+\frac{d^3\sin^3(a+bx)\sin(c+dx)}{9b^4-10b^2d^2+d^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sin(a)**3*cos(c), Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - d*x)**3*c
os(c + d*x)/8 + 3*x*sin(a - d*x)**2*sin(c + d*x)*cos(a - d*x)/8 + 3*x*sin(a
 - d*x)*cos(a - d*x)**2*cos(c + d*x)/8 + 3*x*sin(c + d*x)*cos(a - d*x)**3/8
 - sin(a - d*x)**3*sin(c + d*x)/(8*d) + 3*sin(a - d*x)**2*cos(a - d*x)*cos(
c + d*x)/(4*d) + 3*cos(a - d*x)**3*cos(c + d*x)/(8*d), Eq(b, -d)), (x*sin(a
 - d*x/3)**3*cos(c + d*x)/8 + 3*x*sin(a - d*x/3)**2*sin(c + d*x)*cos(a - d*
x/3)/8 - 3*x*sin(a - d*x/3)*cos(a - d*x/3)**2*cos(c + d*x)/8 - x*sin(c + d*
x)*cos(a - d*x/3)**3/8 + 7*sin(a - d*x/3)**3*sin(c + d*x)/(8*d) + 3*sin(a -
```

```
 d*x/3)*sin(c + d*x)*cos(a - d*x/3)**2/(4*d) - 3*cos(a - d*x/3)**3*cos(c +
d*x)/(8*d), Eq(b, -d/3)), (x*sin(a + d*x/3)**3*cos(c + d*x)/8 - 3*x*sin(a +
 d*x/3)**2*sin(c + d*x)*cos(a + d*x/3)/8 - 3*x*sin(a + d*x/3)*cos(a + d*x/3
)**2*cos(c + d*x)/8 + x*sin(c + d*x)*cos(a + d*x/3)**3/8 + 7*sin(a + d*x/3)
**3*sin(c + d*x)/(8*d) + 3*sin(a + d*x/3)*sin(c + d*x)*cos(a + d*x/3)**2/(4
*d) + 3*cos(a + d*x/3)**3*cos(c + d*x)/(8*d), Eq(b, d/3)), (3*x*sin(a + d*x
)**3*cos(c + d*x)/8 - 3*x*sin(a + d*x)**2*sin(c + d*x)*cos(a + d*x)/8 + 3*x
*sin(a + d*x)*cos(a + d*x)**2*cos(c + d*x)/8 - 3*x*sin(c + d*x)*cos(a + d*x
)**3/8 + 5*sin(a + d*x)**3*sin(c + d*x)/(8*d) + 3*sin(a + d*x)*sin(c + d*x)
*cos(a + d*x)**2/(4*d) + 3*cos(a + d*x)**3*cos(c + d*x)/(8*d), Eq(b, d)), (
-9*b**3*sin(a + b*x)**2*cos(a + b*x)*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 +
d**4) - 6*b**3*cos(a + b*x)**3*cos(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4)
- 7*b**2*d*sin(a + b*x)**3*sin(c + d*x)/(9*b**4 - 10*b**2*d**2 + d**4) - 6*
b**2*d*sin(a + b*x)*sin(c + d*x)*cos(a + b*x)**2/(9*b**4 - 10*b**2*d**2 + d
**4) + 3*b*d**2*sin(a + b*x)**2*cos(a + b*x)*cos(c + d*x)/(9*b**4 - 10*b**2
*d**2 + d**4) + d**3*sin(a + b*x)**3*sin(c + d*x)/(9*b**4 - 10*b**2*d**2 +
d**4), True))
```

Giac [A]
time = 0.42, size = 89, normalized size = 0.92

$$\frac{\cos\left(3\,bx+dx+3\,a+c\right)}{8\,(3\,b+d)}+\frac{\cos\left(3\,bx-dx+3\,a-c\right)}{8\,(3\,b-d)}-\frac{3\,\cos\left(bx+dx+a+c\right)}{8\,(b+d)}-\frac{3\,\cos\left(bx-dx+a-c\right)}{8\,(b-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*cos(3*b*x + d*x + 3*a + c)/(3*b + d) + 1/8*cos(3*b*x - d*x + 3*a - c)/(
3*b - d) - 3/8*cos(b*x + d*x + a + c)/(b + d) - 3/8*cos(b*x - d*x + a - c)/
(b - d)
```

Mupad [B]
time = 1.61, size = 471, normalized size = 4.86

$$-\mathrm{e}^{a\,3\mathrm{i}-c\,1\mathrm{i}+b\,x\,3\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\frac{-3\,b^3-b^2\,d+3\,b\,d^2+d^3}{144\,b^4-160\,b^2\,d^2+16\,d^4}+\frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,\left(-3\,b^3+b^2\,d+3\,b\,d^2-d^3\right)}{144\,b^4-160\,b^2\,d^2+16\,d^4}-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(-27\,b^3-27\,b^2\,d+3\,b\,d^2+3\,d^3\right)}{144\,b^4-160\,b^2\,d^2+16\,d^4}-\frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(-27\,b^3+27\,b^2\,d+3\,b\,d^2-3\,d^3\right)}{144\,b^4-160\,b^2\,d^2+16\,d^4}\right)-\mathrm{e}^{a\,3\mathrm{i}+c\,1\mathrm{i}+b\,x\,3\mathrm{i}+d\,x\,1\mathrm{i}}\,\left(\frac{-3\,b^3+b^2\,d+3\,b\,d^2-d^3}{144\,b^4-160\,b^2\,d^2+16\,d^4}+\frac{\mathrm{e}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,\left(-3\,b^3-b^2\,d+3\,b\,d^2+d^3\right)}{144\,b^4-160\,b^2\,d^2+16\,d^4}-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(-27\,b^3+27\,b^2\,d+3\,b\,d^2-3\,d^3\right)}{144\,b^4-160\,b^2\,d^2+16\,d^4}-\frac{\mathrm{e}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(-27\,b^3-27\,b^2\,d+3\,b\,d^2+3\,d^3\right)}{144\,b^4-160\,b^2\,d^2+16\,d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*sin(a + b*x)^3,x)
```

```
[Out] - exp(a*3i - c*1i + b*x*3i - d*x*1i)*((3*b*d^2 - b^2*d - 3*b^3 + d^3)/(144*
b^4 + 16*d^4 - 160*b^2*d^2) + (exp(- a*6i - b*x*6i)*(3*b*d^2 + b^2*d - 3*b^
3 - d^3))/(144*b^4 + 16*d^4 - 160*b^2*d^2) - (exp(- a*2i - b*x*2i)*(3*b*d^2
 - 27*b^2*d - 27*b^3 + 3*d^3))/(144*b^4 + 16*d^4 - 160*b^2*d^2) - (exp(- a*
4i - b*x*4i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(144*b^4 + 16*d^4 - 160
*b^2*d^2)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((3*b*d^2 + b^2*d - 3*b^3 -
 d^3)/(144*b^4 + 16*d^4 - 160*b^2*d^2) + (exp(- a*6i - b*x*6i)*(3*b*d^2 - b
```

```
^2*d - 3*b^3 + d^3))/(144*b^4 + 16*d^4 - 160*b^2*d^2) - (exp(- a*2i - b*x*2
i)*(3*b*d^2 + 27*b^2*d - 27*b^3 - 3*d^3))/(144*b^4 + 16*d^4 - 160*b^2*d^2)
- (exp(- a*4i - b*x*4i)*(3*b*d^2 - 27*b^2*d - 27*b^3 + 3*d^3))/(144*b^4 + 1
6*d^4 - 160*b^2*d^2))
```

3.225 $\int \cos^2(c+dx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=138

$$-\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}-\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}+\frac{\cos(3a-2c+(3b-2d)x)}{16(3b-2d)}-\frac{3\cos(a+2c+(b}{16(b+2d)}$$

```
[Out] -3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b-3/16*cos(a-2*c+(b-2*d)*x)/(b-2*d)+1
/16*cos(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)-3/16*cos(a+2*c+(b+2*d)*x)/(b+2*d)+1/
16*cos(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

Rubi [A]

time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4670, 2718}

$$-\frac{3\cos(a+x(b-2d)-2c)}{16(b-2d)}+\frac{\cos(3a+x(3b-2d)-2c)}{16(3b-2d)}-\frac{3\cos(a+x(b+2d)+2c)}{16(b+2d)}+\frac{\cos(3a+x(3b+2d)+2c)}{16(3b+2d)}-\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Cos[a + b*x])/(8*b) + Cos[3*a + 3*b*x]/(24*b) - (3*Cos[a - 2*c + (b - 2
*d)*x])/(16*(b - 2*d)) + Cos[3*a - 2*c + (3*b - 2*d)*x]/(16*(3*b - 2*d)) -
(3*Cos[a + 2*c + (b + 2*d)*x])/(16*(b + 2*d)) + Cos[3*a + 2*c + (3*b + 2*d)
*x]/(16*(3*b + 2*d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sin^3(a+bx)\,dx &= \int \left(\frac{3}{8}\sin(a+bx)-\frac{1}{8}\sin(3a+3bx)+\frac{3}{16}\sin(a-2c+(b-2d)x)-\frac{1}{16}\mathrm{s}\right.\\ &= -\left(\frac{1}{16}\int \sin(3a-2c+(3b-2d)x)\,dx\right)-\frac{1}{16}\int \sin(3a+2c+(3b+2d)x\\ &= -\frac{3\cos(a+bx)}{8b}+\frac{\cos(3a+3bx)}{24b}-\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}+\frac{\cos(3a-}{16}\end{aligned}$$

Mathematica [A]
time = 1.65, size = 153, normalized size = 1.11

$$\frac{1}{48}\left(-\frac{18\cos(a)\cos(bx)}{b}+\frac{2\cos(3a)\cos(3bx)}{b}-\frac{9\cos(a-2c+bx-2dx)}{b-2d}+\frac{3\cos(3a-2c+3bx-2dx)}{3b-2d}-\frac{9\cos(a+2c+bx+2dx)}{b+2d}+\frac{3\cos(3a+2c+3bx+2dx)}{3b+2d}+\frac{18\sin(a)\sin(bx)}{b}-\frac{2\sin(3a)\sin(3bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] ((-18*Cos[a]*Cos[b*x])/b + (2*Cos[3*a]*Cos[3*b*x])/b - (9*Cos[a - 2*c + b*x
 - 2*d*x])/(b - 2*d) + (3*Cos[3*a - 2*c + 3*b*x - 2*d*x])/(3*b - 2*d) - (9*
Cos[a + 2*c + b*x + 2*d*x])/(b + 2*d) + (3*Cos[3*a + 2*c + 3*b*x + 2*d*x])/
(3*b + 2*d) + (18*Sin[a]*Sin[b*x])/b - (2*Sin[3*a]*Sin[3*b*x])/b)/48
```

Maple [A]
time = 0.22, size = 127, normalized size = 0.92

method	result
default	$-\frac{3\cos(bx+a)}{8b}+\frac{\cos(3bx+3a)}{24b}-\frac{3\cos(a-2c+(b-2d)x)}{16(b-2d)}+\frac{\cos(3a-2c+(3b-2d)x)}{48b-32d}-\frac{3\cos(a+2c+(b+2d)x)}{16(b+2d)}+\frac{\cos(3a+2c+(3b-}{48b+32d}$
risch	$-\frac{3\cos(bx+a)}{8b}-\frac{27\cos(bx-2dx+a-2c)b^3}{16(b+2d)(3b+2d)(3b-2d)(b-2d)}-\frac{27\cos(bx-2dx+a-2c)b^2d}{8(b+2d)(3b+2d)(3b-2d)(b-2d)}+\frac{3\cos(bx-2dx+a-2c)b\,d^2}{4(b+2d)(3b+2d)(3b-2d)(b-2d)}+\frac{}{2(b+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*cos(b*x+a)/b+1/24*cos(3*b*x+3*a)/b-3/16*cos(a-2*c+(b-2*d)*x)/(b-2*d)+1
/16*cos(3*a-2*c+(3*b-2*d)*x)/(3*b-2*d)-3/16*cos(a+2*c+(b+2*d)*x)/(b+2*d)+1/
16*cos(3*a+2*c+(3*b+2*d)*x)/(3*b+2*d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. 2(126) = 252.
time = 0.33, size = 1360, normalized size = 9.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(3*b^4*cos(2*c) - 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3*
cos(2*c))*cos((3*b + 2*d)*x + 3*a + 4*c) + 3*(3*b^4*cos(2*c) - 2*b^3*d*cos(
2*c) - 12*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((3*b + 2*d)*x + 3*a) + 3
*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c) - 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c
))*cos(-(3*b - 2*d)*x - 3*a + 4*c) + 3*(3*b^4*cos(2*c) + 2*b^3*d*cos(2*c) -
 12*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(3*b - 2*d)*x - 3*a) - 9*(9*b
^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) + 8*b*d^3*cos(2*c))*co
s((b + 2*d)*x + a + 4*c) - 9*(9*b^4*cos(2*c) - 18*b^3*d*cos(2*c) - 4*b^2*d^
```

```
2*cos(2*c) + 8*b*d^3*cos(2*c))*cos((b + 2*d)*x + a) - 9*(9*b^4*cos(2*c) + 1
8*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) - 8*b*d^3*cos(2*c))*cos(-(b - 2*d)*x
- a + 4*c) - 9*(9*b^4*cos(2*c) + 18*b^3*d*cos(2*c) - 4*b^2*d^2*cos(2*c) - 8
*b*d^3*cos(2*c))*cos(-(b - 2*d)*x - a) + 2*(9*b^4*cos(2*c) - 40*b^2*d^2*cos
(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a + 2*c) + 2*(9*b^4*cos(2*c) - 40*b^
2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(3*b*x + 3*a - 2*c) - 18*(9*b^4*cos(2*
c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a + 2*c) - 18*(9*b^4*
cos(2*c) - 40*b^2*d^2*cos(2*c) + 16*d^4*cos(2*c))*cos(b*x + a - 2*c) + 3*(3
*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*
sin((3*b + 2*d)*x + 3*a + 4*c) - 3*(3*b^4*sin(2*c) - 2*b^3*d*sin(2*c) - 12*
b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((3*b + 2*d)*x + 3*a) + 3*(3*b^4*si
n(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(3
*b - 2*d)*x - 3*a + 4*c) - 3*(3*b^4*sin(2*c) + 2*b^3*d*sin(2*c) - 12*b^2*d^
2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(3*b - 2*d)*x - 3*a) - 9*(9*b^4*sin(2*c
) - 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c) + 8*b*d^3*sin(2*c))*sin((b + 2*d
)*x + a + 4*c) + 9*(9*b^4*sin(2*c) - 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c)
 + 8*b*d^3*sin(2*c))*sin((b + 2*d)*x + a) - 9*(9*b^4*sin(2*c) + 18*b^3*d*si
n(2*c) - 4*b^2*d^2*sin(2*c) - 8*b*d^3*sin(2*c))*sin(-(b - 2*d)*x - a + 4*c)
 + 9*(9*b^4*sin(2*c) + 18*b^3*d*sin(2*c) - 4*b^2*d^2*sin(2*c) - 8*b*d^3*sin
(2*c))*sin(-(b - 2*d)*x - a) + 2*(9*b^4*sin(2*c) - 40*b^2*d^2*sin(2*c) + 16
*d^4*sin(2*c))*sin(3*b*x + 3*a + 2*c) - 2*(9*b^4*sin(2*c) - 40*b^2*d^2*sin(
2*c) + 16*d^4*sin(2*c))*sin(3*b*x + 3*a - 2*c) - 18*(9*b^4*sin(2*c) - 40*b^
2*d^2*sin(2*c) + 16*d^4*sin(2*c))*sin(b*x + a + 2*c) + 18*(9*b^4*sin(2*c) -
 40*b^2*d^2*sin(2*c) + 16*d^4*sin(2*c))*sin(b*x + a - 2*c))/(9*b^5*cos(2*c)
^2 + 9*b^5*sin(2*c)^2 + 16*(b*cos(2*c)^2 + b*sin(2*c)^2)*d^4 - 40*(b^3*cos(
2*c)^2 + b^3*sin(2*c)^2)*d^2)
```

Fricas [**A**]
time = 3.26, size = 179, normalized size = 1.30

$$-\frac{2\left(b^{2} d^{2}-4 d^{4}\right) \cos \left(b x+a\right)^{3}+6\left(7 b^{3} d-4 b d^{3}-\left(b^{3} d-4 b d^{3}\right) \cos \left(b x+a\right)^{2}\right) \cos \left(d x+c\right) \sin \left(b x+a\right) \sin \left(d x+c\right)-9\left(\left(b^{4}-4 b^{2} d^{2}\right) \cos \left(b x+a\right)^{3}-\left(3 b^{4}-4 b^{2} d^{2}\right) \cos \left(b x+a\right)\right) \cos \left(d x+c\right)^{2}-6\left(7 b^{2} d^{2}-4 d^{4}\right) \cos \left(b x+a\right)}{3\left(9 b^{5}-40 b^{3} d^{2}+16 b d^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(b^2*d^2 - 4*d^4)*cos(b*x + a)^3 + 6*(7*b^3*d - 4*b*d^3 - (b^3*d -
4*b*d^3)*cos(b*x + a)^2)*cos(d*x + c)*sin(b*x + a)*sin(d*x + c) - 9*((b^4 -
 4*b^2*d^2)*cos(b*x + a)^3 - (3*b^4 - 4*b^2*d^2)*cos(b*x + a))*cos(d*x + c)
^2 - 6*(7*b^2*d^2 - 4*d^4)*cos(b*x + a))/(9*b^5 - 40*b^3*d^2 + 16*b*d^4)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2030 vs. $2(116) = 232$.
time = 6.34, size = 2030, normalized size = 14.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*sin(b*x+a)**3,x)

[Out] Piecewise((x*sin(a)**3*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2
/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*sin(a)**3, Eq(b
, 0)), (-3*x*sin(a - 2*d*x)**3*sin(c + d*x)**2/16 + 3*x*sin(a - 2*d*x)**3*c
os(c + d*x)**2/16 + 3*x*sin(a - 2*d*x)**2*sin(c + d*x)*cos(a - 2*d*x)*cos(c
 + d*x)/8 - 3*x*sin(a - 2*d*x)*sin(c + d*x)**2*cos(a - 2*d*x)**2/16 + 3*x*s
in(a - 2*d*x)*cos(a - 2*d*x)**2*cos(c + d*x)**2/16 + 3*x*sin(c + d*x)*cos(a
 - 2*d*x)**3*cos(c + d*x)/8 + 13*sin(a - 2*d*x)**3*sin(c + d*x)*cos(c + d*x
)/(16*d) + sin(a - 2*d*x)**2*sin(c + d*x)**2*cos(a - 2*d*x)/(2*d) + 7*sin(a
 - 2*d*x)*sin(c + d*x)*cos(a - 2*d*x)**2*cos(c + d*x)/(8*d) + 49*sin(c + d*
x)**2*cos(a - 2*d*x)**3/(96*d) - 17*cos(a - 2*d*x)**3*cos(c + d*x)**2/(96*d
), Eq(b, -2*d)), (-x*sin(a - 2*d*x/3)**3*sin(c + d*x)**2/16 + x*sin(a - 2*d
*x/3)**3*cos(c + d*x)**2/16 + 3*x*sin(a - 2*d*x/3)**2*sin(c + d*x)*cos(a -
2*d*x/3)*cos(c + d*x)/8 + 3*x*sin(a - 2*d*x/3)*sin(c + d*x)**2*cos(a - 2*d*
x/3)**2/16 - 3*x*sin(a - 2*d*x/3)*cos(a - 2*d*x/3)**2*cos(c + d*x)**2/16 -
x*sin(c + d*x)*cos(a - 2*d*x/3)**3*cos(c + d*x)/8 + 15*sin(a - 2*d*x/3)**3*
sin(c + d*x)*cos(c + d*x)/(16*d) + 3*sin(a - 2*d*x/3)**2*sin(c + d*x)**2*co
s(a - 2*d*x/3)/(2*d) - 9*sin(a - 2*d*x/3)*sin(c + d*x)*cos(a - 2*d*x/3)**2*
cos(c + d*x)/(8*d) + 11*sin(c + d*x)**2*cos(a - 2*d*x/3)**3/(32*d) + 21*cos
(a - 2*d*x/3)**3*cos(c + d*x)**2/(32*d), Eq(b, -2*d/3)), (-x*sin(a + 2*d*x/
3)**3*sin(c + d*x)**2/16 + x*sin(a + 2*d*x/3)**3*cos(c + d*x)**2/16 - 3*x*s
in(a + 2*d*x/3)**2*sin(c + d*x)*cos(a + 2*d*x/3)*cos(c + d*x)/8 + 3*x*sin(a
 + 2*d*x/3)*sin(c + d*x)**2*cos(a + 2*d*x/3)**2/16 - 3*x*sin(a + 2*d*x/3)*c
os(a + 2*d*x/3)**2*cos(c + d*x)**2/16 + x*sin(c + d*x)*cos(a + 2*d*x/3)**3*
cos(c + d*x)/8 + 15*sin(a + 2*d*x/3)**3*sin(c + d*x)*cos(c + d*x)/(16*d) -
3*sin(a + 2*d*x/3)**2*sin(c + d*x)**2*cos(a + 2*d*x/3)/(2*d) - 9*sin(a + 2*
d*x/3)*sin(c + d*x)*cos(a + 2*d*x/3)**2*cos(c + d*x)/(8*d) - 11*sin(c + d*x
)**2*cos(a + 2*d*x/3)**3/(32*d) - 21*cos(a + 2*d*x/3)**3*cos(c + d*x)**2/(3
2*d), Eq(b, 2*d/3)), (-3*x*sin(a + 2*d*x)**3*sin(c + d*x)**2/16 + 3*x*sin(a
 + 2*d*x)**3*cos(c + d*x)**2/16 - 3*x*sin(a + 2*d*x)**2*sin(c + d*x)*cos(a
+ 2*d*x)*cos(c + d*x)/8 - 3*x*sin(a + 2*d*x)*sin(c + d*x)**2*cos(a + 2*d*x)
2/16 + 3*x*sin(a + 2*d*x)*cos(a + 2*d*x)2*cos(c + d*x)**2/16 - 3*x*sin(
c + d*x)*cos(a + 2*d*x)**3*cos(c + d*x)/8 + 13*sin(a + 2*d*x)**3*sin(c + d*
x)*cos(c + d*x)/(16*d) - sin(a + 2*d*x)**2*sin(c + d*x)**2*cos(a + 2*d*x)/(
2*d) + 7*sin(a + 2*d*x)*sin(c + d*x)*cos(a + 2*d*x)**2*cos(c + d*x)/(8*d) -
 49*sin(c + d*x)**2*cos(a + 2*d*x)**3/(96*d) + 17*cos(a + 2*d*x)**3*cos(c +
 d*x)**2/(96*d), Eq(b, 2*d)), (-27*b**4*sin(a + b*x)**2*cos(a + b*x)*cos(c
+ d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 18*b**4*cos(a + b*x)**3*c
os(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 42*b**3*d*sin(a + b*
x)**3*sin(c + d*x)*cos(c + d*x)/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 36*
b**3*d*sin(a + b*x)*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(27*b**5 - 12
0*b**3*d**2 + 48*b*d**4) + 42*b**2*d**2*sin(a + b*x)**2*sin(c + d*x)**2*cos
(a + b*x)/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) + 78*b**2*d**2*sin(a + b*x)
2*cos(a + b*x)*cos(c + d*x)2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) + 40

```
*b**2*d**2*sin(c + d*x)**2*cos(a + b*x)**3/(27*b**5 - 120*b**3*d**2 + 48*b*
d**4) + 40*b**2*d**2*cos(a + b*x)**3*cos(c + d*x)**2/(27*b**5 - 120*b**3*d*
*2 + 48*b*d**4) + 24*b*d**3*sin(a + b*x)**3*sin(c + d*x)*cos(c + d*x)/(27*b
**5 - 120*b**3*d**2 + 48*b*d**4) - 24*d**4*sin(a + b*x)**2*sin(c + d*x)**2*
cos(a + b*x)/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 24*d**4*sin(a + b*x)**
2*cos(a + b*x)*cos(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) - 16*d
**4*sin(c + d*x)**2*cos(a + b*x)**3/(27*b**5 - 120*b**3*d**2 + 48*b*d**4) -
 16*d**4*cos(a + b*x)**3*cos(c + d*x)**2/(27*b**5 - 120*b**3*d**2 + 48*b*d*
*4), True))
```

Giac [A]
time = 0.42, size = 124, normalized size = 0.90

$$\frac{\cos(3\,bx+2\,dx+3\,a+2\,c)}{16\,(3\,b+2\,d)}+\frac{\cos(3\,bx-2\,dx+3\,a-2\,c)}{16\,(3\,b-2\,d)}+\frac{\cos(3\,bx+3\,a)}{24\,b}-\frac{3\,\cos(bx+2\,dx+a+2\,c)}{16\,(b+2\,d)}-\frac{3\,\cos(bx-2\,dx+a-2\,c)}{16\,(b-2\,d)}-\frac{3\,\cos(bx+a)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*cos(3*b*x + 2*d*x + 3*a + 2*c)/(3*b + 2*d) + 1/16*cos(3*b*x - 2*d*x +
3*a - 2*c)/(3*b - 2*d) + 1/24*cos(3*b*x + 3*a)/b - 3/16*cos(b*x + 2*d*x + a
 + 2*c)/(b + 2*d) - 3/16*cos(b*x - 2*d*x + a - 2*c)/(b - 2*d) - 3/8*cos(b*x
 + a)/b
```

Mupad [B]
time = 2.03, size = 438, normalized size = 3.17

$$-\frac{81\,b^4\cos(a-2\,c+b\,x-2\,d\,x)+81\,b^4\cos(a+2\,c+b\,x+2\,d\,x)+162\,b^4\cos(a+b\,x)+288\,d^4\cos(a+b\,x)-9\,b^4\cos(3\,a-2\,c+3\,b\,x-2\,d\,x)-9\,b^4\cos(3\,a+2\,c+3\,b\,x+2\,d\,x)-18\,b^4\cos(3\,a+3\,b\,x)-32\,d^4\cos(3\,a+3\,b\,x)+24\,b\,d^3\cos(3\,a-2\,c+3\,b\,x-2\,d\,x)-24\,b\,d^3\cos(3\,a+2\,c+3\,b\,x+2\,d\,x)-6\,b^3\,d\cos(3\,a-2\,c+3\,b\,x-2\,d\,x)+6\,b^3\,d\cos(3\,a+2\,c+3\,b\,x+2\,d\,x)-36\,b^2\,d^2\cos(a-2\,c+b\,x-2\,d\,x)-36\,b^2\,d^2\cos(a+2\,c+b\,x+2\,d\,x)-720\,b^2\,d^2\cos(a+b\,x)+36\,b^2\,d^2\cos(3\,a-2\,c+3\,b\,x-2\,d\,x)+36\,b^2\,d^2\cos(3\,a+2\,c+3\,b\,x+2\,d\,x)+80\,b^2\,d^2\cos(3\,a+3\,b\,x)-72\,b\,d^3\cos(a-2\,c+b\,x-2\,d\,x)+72\,b\,d^3\cos(a+2\,c+b\,x+2\,d\,x)+162\,b^3\,d\cos(a-2\,c+b\,x-2\,d\,x)-162\,b^3\,d\cos(a+2\,c+b\,x+2\,d\,x)}{48\,(9\,b^5-40\,b^3\,d^2+16\,b\,d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*sin(a + b*x)^3,x)
```

```
[Out] -(81*b^4*cos(a - 2*c + b*x - 2*d*x) + 81*b^4*cos(a + 2*c + b*x + 2*d*x) + 1
62*b^4*cos(a + b*x) + 288*d^4*cos(a + b*x) - 9*b^4*cos(3*a - 2*c + 3*b*x -
2*d*x) - 9*b^4*cos(3*a + 2*c + 3*b*x + 2*d*x) - 18*b^4*cos(3*a + 3*b*x) - 3
2*d^4*cos(3*a + 3*b*x) + 24*b*d^3*cos(3*a - 2*c + 3*b*x - 2*d*x) - 24*b*d^3
*cos(3*a + 2*c + 3*b*x + 2*d*x) - 6*b^3*d*cos(3*a - 2*c + 3*b*x - 2*d*x) +
6*b^3*d*cos(3*a + 2*c + 3*b*x + 2*d*x) - 36*b^2*d^2*cos(a - 2*c + b*x - 2*d
*x) - 36*b^2*d^2*cos(a + 2*c + b*x + 2*d*x) - 720*b^2*d^2*cos(a + b*x) + 36
*b^2*d^2*cos(3*a - 2*c + 3*b*x - 2*d*x) + 36*b^2*d^2*cos(3*a + 2*c + 3*b*x
+ 2*d*x) + 80*b^2*d^2*cos(3*a + 3*b*x) - 72*b*d^3*cos(a - 2*c + b*x - 2*d*x
) + 72*b*d^3*cos(a + 2*c + b*x + 2*d*x) + 162*b^3*d*cos(a - 2*c + b*x - 2*d
*x) - 162*b^3*d*cos(a + 2*c + b*x + 2*d*x))/(48*(16*b*d^4 + 9*b^5 - 40*b^3*
d^2))
```

3.226 $\int \cos^3(c+dx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=195

$$-\frac{3\cos(a-3c+(b-3d)x)}{32(b-3d)}-\frac{9\cos(a-c+(b-d)x)}{32(b-d)}+\frac{\cos(3(a-c)+3(b-d)x)}{96(b-d)}+\frac{3\cos(3a-c+(3b-d)x)}{32(3b-d)}$$

```
[Out] -3/32*cos(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cos(a-c+(b-d)*x)/(b-d)+1/96*cos(3*a
-3*c+3*(b-d)*x)/(b-d)+3/32*cos(3*a-c+(3*b-d)*x)/(3*b-d)-9/32*cos(a+c+(b+d)*
x)/(b+d)+1/96*cos(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*cos(3*a+c+(3*b+d)*x)/(3*b+d
)-3/32*cos(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A]

time = 0.10, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4670, 2718}

$$-\frac{3\cos(a+x(b-3d)-3c)}{32(b-3d)}-\frac{9\cos(a+x(b-d)-c)}{32(b-d)}+\frac{\cos(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\cos(3a+x(3b-d)-c)}{32(3b-d)}-\frac{9\cos(a+x(b+d)+c)}{32(b+d)}+\frac{\cos(3(a+c)+3x(b+d))}{96(b+d)}+\frac{3\cos(3a+x(3b+d)+c)}{32(3b+d)}-\frac{3\cos(a+x(b+3d)+3c)}{32(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Cos[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) - (9*Cos[a - c + (b - d)*x])
/(32*(b - d)) + Cos[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Cos[3*a - c
+ (3*b - d)*x])/(32*(3*b - d)) - (9*Cos[a + c + (b + d)*x])/(32*(b + d)) +
Cos[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Cos[3*a + c + (3*b + d)*x])/
(32*(3*b + d)) - (3*Cos[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4670

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)\sin^3(a+bx)\,dx &= \int \left(\frac{3}{32}\sin(a-3c+(b-3d)x)+\frac{9}{32}\sin(a-c+(b-d)x)-\frac{1}{32}\sin(3(a-\right.\\ &= -\left(\frac{1}{32}\int \sin(3(a-c)+3(b-d)x)\,dx\right)-\frac{1}{32}\int \sin(3(a+c)+3(b+d)x)\\ &= -\frac{3\cos(a-3c+(b-3d)x)}{32(b-3d)}-\frac{9\cos(a-c+(b-d)x)}{32(b-d)}+\frac{\cos(3(a-c)+3(\right.}{96(b-d)}\end{aligned}$$

Mathematica [A]
time = 1.60, size = 176, normalized size = 0.90

$$\frac{1}{96}\left(-\frac{9\cos(a-3c+bx-3dx)}{b-3d}-\frac{27\cos(a-c+bx-dx)}{b-d}+\frac{\cos(3(a-c+bx-dx))}{b-d}+\frac{9\cos(3a-c+3bx-dx)}{3b-d}+\frac{9\cos(3a+c+3bx+dx)}{3b+d}-\frac{9\cos(a+3c+bx+3dx)}{b+3d}-\frac{27\cos(a+c+(b+d)x)}{b+d}+\frac{\cos(3(a+c+(b+d)x))}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] ((-9*Cos[a - 3*c + b*x - 3*d*x])/(b - 3*d) - (27*Cos[a - c + b*x - d*x])/(b
 - d) + Cos[3*(a - c + b*x - d*x)]/(b - d) + (9*Cos[3*a - c + 3*b*x - d*x])
/(3*b - d) + (9*Cos[3*a + c + 3*b*x + d*x])/(3*b + d) - (9*Cos[a + 3*c + b*
x + 3*d*x])/(b + 3*d) - (27*Cos[a + c + (b + d)*x])/(b + d) + Cos[3*(a + c
+ (b + d)*x)]/(b + d))/96
```

Maple [A]
time = 0.33, size = 184, normalized size = 0.94

method	result
default	$-\frac{3\cos(a-3c+(b-3d)x)}{32(b-3d)}-\frac{9\cos(a-c+(b-d)x)}{32(b-d)}-\frac{9\cos(a+c+(b+d)x)}{32(b+d)}-\frac{3\cos(a+3c+(b+3d)x)}{32(b+3d)}+\frac{\cos((3b-3d)x+3a-3c)}{96b-96d}+$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/32*cos(a-3*c+(b-3*d)*x)/(b-3*d)-9/32*cos(a-c+(b-d)*x)/(b-d)-9/32*cos(a+c
+(b+d)*x)/(b+d)-3/32*cos(a+3*c+(b+3*d)*x)/(b+3*d)+1/96/(b-d)*cos((3*b-3*d)*
x+3*a-3*c)+3/32*cos(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*cos(3*a+c+(3*b+d)*x)/(3*b
+d)+1/96/(b+d)*cos((3*b+3*d)*x+3*a+3*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2612 vs. $2(179) = 358$.
time = 0.41, size = 2612, normalized size = 13.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/192*(9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*b^2*d^
3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos((3*b + d)*x + 3*a + 4*
c) + 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*b^2*d^3*
cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*cos((3*b + d)*x + 3*a - 2*c)
 + 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2*d^3*co
s(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*cos(-(3*b - d)*x - 3*a + 4*c)
```

+ 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2*d^3*cos
(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*cos(-(3*b - d)*x - 3*a - 2*c) -
 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b^2*d^3*c
os(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*cos((b + 3*d)*x + a + 6*c) - 9*(
9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b^2*d^3*cos(3
*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*cos((b + 3*d)*x + a) + (9*b^5*cos(3*
c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4
*cos(3*c) - 9*d^5*cos(3*c))*cos(3*(b + d)*x + 3*a + 6*c) + (9*b^5*cos(3*c)
- 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*co
s(3*c) - 9*d^5*cos(3*c))*cos(3*(b + d)*x + 3*a) - 27*(9*b^5*cos(3*c) - 9*b^
4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c)
 - 9*d^5*cos(3*c))*cos((b + d)*x + a + 4*c) - 27*(9*b^5*cos(3*c) - 9*b^4*d*
cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) - 9
*d^5*cos(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5*cos(3*c) + 9*b^4*d*cos(
3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5
*cos(3*c))*cos(-(b - d)*x - a + 4*c) - 27*(9*b^5*cos(3*c) + 9*b^4*d*cos(3*c
) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*co
s(3*c))*cos(-(b - d)*x - a - 2*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3*c) - 82
*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*cos(3*c)
)*cos(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3*c) - 82*b
^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*
cos(-3*(b - d)*x - 3*a) - 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 10*b^3*d^
2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))*cos(-(b
 - 3*d)*x - a + 6*c) - 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 10*b^3*d^2*c
os(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))*cos(-(b -
3*d)*x - a) + 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10
*b^2*d^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*sin((3*b + d)*x + 3
*a + 4*c) - 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b
^2*d^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*sin((3*b + d)*x + 3*a
 - 2*c) + 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2
*d^3*sin(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*sin(-(3*b - d)*x - 3*a
+ 4*c) - 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*
d^3*sin(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*sin(-(3*b - d)*x - 3*a -
 2*c) - 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^
2*d^3*sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*sin((b + 3*d)*x + a + 6*c
) + 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^
3*sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*sin((b + 3*d)*x + a) + (9*b^5
*sin(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) +
9*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*sin(3*(b + d)*x + 3*a + 6*c) - (9*b^5*si
n(3*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b
*d^4*sin(3*c) - 9*d^5*sin(3*c))*sin(3*(b + d)*x + 3*a) - 27*(9*b^5*sin(3*c)
 - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*s
in(3*c) - 9*d^5*sin(3*c))*sin((b + d)*x + a + 4*c) + 27*(9*b^5*sin(3*c) - 9
*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3
*c) - 9*d^5*sin(3*c))*sin((b + d)*x + a - 2*c) - 27*(9*b^5*sin(3*c) + 9*b^4

```
*d*sin(3*c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c)
+ 9*d^5*sin(3*c))*sin(-(b - d)*x - a + 4*c) + 27*(9*b^5*sin(3*c) + 9*b^4*d*
sin(3*c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9
*d^5*sin(3*c))*sin(-(b - d)*x - a - 2*c) + (9*b^5*sin(3*c) + 9*b^4*d*sin(3*
c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*s
in(3*c))*sin(-3*(b - d)*x - 3*a + 6*c) - (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c)
 - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin
(3*c))*sin(-3*(b - d)*x - 3*a) - 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10
*b^3*d^2*sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*
sin(-(b - 3*d)*x - a + 6*c) + 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^
3*d^2*sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*sin
(-(b - 3*d)*x - a))/(9*b^6*cos(3*c)^2 + 9*b^6*sin(3*c)^2 - 9*(cos(3*c)^2 +
sin(3*c)^2)*d^6 + 91*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^4 - 91*(b^4*cos(3*
c)^2 + b^4*sin(3*c)^2)*d^2)
```

Fricas [**A**]
time = 2.77, size = 264, normalized size = 1.35

$$\frac{\left((9\,b^5-82\,b^3d^2+9\,bd^4)\cos\left(bx+a\right)^3-3\,(9\,b^5-28\,b^3d^2+3\,bd^4)\cos\left(bx+a\right)\right)\cos\left(dx+c\right)^3+\left(122\,b^2d^3-18\,d^5-2\,(b^2d^3-9\,d^5)\cos\left(bx+a\right)^2-\left(63\,b^4d-88\,b^2d^3+9\,d^5-(9\,b^4d-82\,b^2d^3+9\,d^5)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)^2\right)\sin\left(bx+a\right)\sin\left(dx+c\right)-6\left((b^3d^2-9\,bd^4)\cos\left(bx+a\right)^3-3\,(7\,b^3d^2-3\,bd^4)\cos\left(bx+a\right)\right)\cos\left(dx+c\right)}{3\,(9\,b^6-91\,b^4d^2+91\,b^2d^4-9\,d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(((9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cos(b*x + a)^3 - 3*(9*b^5 - 28*b^3*d^2
 + 3*b*d^4)*cos(b*x + a))*cos(d*x + c)^3 + (122*b^2*d^3 - 18*d^5 - 2*(b^2*d
^3 - 9*d^5)*cos(b*x + a)^2 - (63*b^4*d - 88*b^2*d^3 + 9*d^5 - (9*b^4*d - 82
*b^2*d^3 + 9*d^5)*cos(b*x + a)^2)*cos(d*x + c)^2)*sin(b*x + a)*sin(d*x + c)
 - 6*((b^3*d^2 - 9*b*d^4)*cos(b*x + a)^3 - 3*(7*b^3*d^2 - 3*b*d^4)*cos(b*x
+ a))*cos(d*x + c))/(9*b^6 - 91*b^4*d^2 + 91*b^2*d^4 - 9*d^6)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3582 vs. 2(172) = 344.
time = 20.86, size = 3582, normalized size = 18.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((x*sin(a)**3*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (-9*x*sin(a - 3*d*x
)**3*sin(c + d*x)**2*cos(c + d*x)/32 + 3*x*sin(a - 3*d*x)**3*cos(c + d*x)**
3/32 - 3*x*sin(a - 3*d*x)**2*sin(c + d*x)**3*cos(a - 3*d*x)/32 + 9*x*sin(a
- 3*d*x)**2*sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/32 - 9*x*sin(a - 3*
d*x)*sin(c + d*x)**2*cos(a - 3*d*x)**2*cos(c + d*x)/32 + 3*x*sin(a - 3*d*x)
*cos(a - 3*d*x)**2*cos(c + d*x)**3/32 - 3*x*sin(c + d*x)**3*cos(a - 3*d*x)*
*3/32 + 9*x*sin(c + d*x)*cos(a - 3*d*x)**3*cos(c + d*x)**2/32 - sin(a - 3*d
```

```
*x)**3*sin(c + d*x)**3/(12*d) - 13*sin(a - 3*d*x)**3*sin(c + d*x)*cos(c + d
*x)**2/(320*d) + 3*sin(a - 3*d*x)**2*sin(c + d*x)**2*cos(a - 3*d*x)*cos(c +
 d*x)/(20*d) + 101*sin(a - 3*d*x)**2*cos(a - 3*d*x)*cos(c + d*x)**3/(320*d)
 - 27*sin(a - 3*d*x)*sin(c + d*x)**3*cos(a - 3*d*x)**2/(320*d) + 51*sin(c +
 d*x)**2*cos(a - 3*d*x)**3*cos(c + d*x)/(320*d) + cos(a - 3*d*x)**3*cos(c +
 d*x)**3/(5*d), Eq(b, -3*d)), (3*x*sin(a - d*x)**3*sin(c + d*x)**2*cos(c +
d*x)/16 + 5*x*sin(a - d*x)**3*cos(c + d*x)**3/16 + 3*x*sin(a - d*x)**2*sin(
c + d*x)**3*cos(a - d*x)/16 + 9*x*sin(a - d*x)**2*sin(c + d*x)*cos(a - d*x)
*cos(c + d*x)**2/16 + 9*x*sin(a - d*x)*sin(c + d*x)**2*cos(a - d*x)**2*cos(
c + d*x)/16 + 3*x*sin(a - d*x)*cos(a - d*x)**2*cos(c + d*x)**3/16 + 5*x*sin
(c + d*x)**3*cos(a - d*x)**3/16 + 3*x*sin(c + d*x)*cos(a - d*x)**3*cos(c +
d*x)**2/16 - 5*sin(a - d*x)**3*sin(c + d*x)**3/(16*d) + 3*sin(a - d*x)**2*s
in(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/(4*d) + 11*sin(a - d*x)**2*cos(a -
 d*x)*cos(c + d*x)**3/(16*d) - 3*sin(a - d*x)*sin(c + d*x)**3*cos(a - d*x)*
*2/(16*d) + sin(c + d*x)**2*cos(a - d*x)**3*cos(c + d*x)/(2*d) + 19*cos(a -
 d*x)**3*cos(c + d*x)**3/(48*d), Eq(b, -d)), (3*x*sin(a - d*x/3)**3*sin(c +
 d*x)**2*cos(c + d*x)/32 + 3*x*sin(a - d*x/3)**3*cos(c + d*x)**3/32 + 9*x*s
in(a - d*x/3)**2*sin(c + d*x)**3*cos(a - d*x/3)/32 + 9*x*sin(a - d*x/3)**2*
sin(c + d*x)*cos(a - d*x/3)*cos(c + d*x)**2/32 - 9*x*sin(a - d*x/3)*sin(c +
 d*x)**2*cos(a - d*x/3)**2*cos(c + d*x)/32 - 9*x*sin(a - d*x/3)*cos(a - d*x
/3)**2*cos(c + d*x)**3/32 - 3*x*sin(c + d*x)**3*cos(a - d*x/3)**3/32 - 3*x*
sin(c + d*x)*cos(a - d*x/3)**3*cos(c + d*x)**2/32 + 3*sin(a - d*x/3)**3*sin
(c + d*x)**3/(4*d) + 351*sin(a - d*x/3)**3*sin(c + d*x)*cos(c + d*x)**2/(32
0*d) - 9*sin(a - d*x/3)**2*sin(c + d*x)**2*cos(a - d*x/3)*cos(c + d*x)/(20*
d) - 183*sin(a - d*x/3)**2*cos(a - d*x/3)*cos(c + d*x)**3/(320*d) + 9*sin(a
 - d*x/3)*sin(c + d*x)**3*cos(a - d*x/3)**2/(320*d) - 33*sin(c + d*x)**2*co
s(a - d*x/3)**3*cos(c + d*x)/(320*d) - cos(a - d*x/3)**3*cos(c + d*x)**3/(1
0*d), Eq(b, -d/3)), (3*x*sin(a + d*x/3)**3*sin(c + d*x)**2*cos(c + d*x)/32
+ 3*x*sin(a + d*x/3)**3*cos(c + d*x)**3/32 - 9*x*sin(a + d*x/3)**2*sin(c +
d*x)**3*cos(a + d*x/3)/32 - 9*x*sin(a + d*x/3)**2*sin(c + d*x)*cos(a + d*x/
3)*cos(c + d*x)**2/32 - 9*x*sin(a + d*x/3)*sin(c + d*x)**2*cos(a + d*x/3)**
2*cos(c + d*x)/32 - 9*x*sin(a + d*x/3)*cos(a + d*x/3)**2*cos(c + d*x)**3/32
 + 3*x*sin(c + d*x)**3*cos(a + d*x/3)**3/32 + 3*x*sin(c + d*x)*cos(a + d*x/
3)**3*cos(c + d*x)**2/32 + 3*sin(a + d*x/3)**3*sin(c + d*x)**3/(4*d) + 351*
sin(a + d*x/3)**3*sin(c + d*x)*cos(c + d*x)**2/(320*d) + 9*sin(a + d*x/3)**
2*sin(c + d*x)**2*cos(a + d*x/3)*cos(c + d*x)/(20*d) + 183*sin(a + d*x/3)**
2*cos(a + d*x/3)*cos(c + d*x)**3/(320*d) + 9*sin(a + d*x/3)*sin(c + d*x)**3
*cos(a + d*x/3)**2/(320*d) + 33*sin(c + d*x)**2*cos(a + d*x/3)**3*cos(c + d
*x)/(320*d) + cos(a + d*x/3)**3*cos(c + d*x)**3/(10*d), Eq(b, d/3)), (3*x*s
in(a + d*x)**3*sin(c + d*x)**2*cos(c + d*x)/16 + 5*x*sin(a + d*x)**3*cos(c
+ d*x)**3/16 - 3*x*sin(a + d*x)**2*sin(c + d*x)**3*cos(a + d*x)/16 - 9*x*si
n(a + d*x)**2*sin(c + d*x)*cos(a + d*x)*cos(c + d*x)**2/16 + 9*x*sin(a + d*
x)*sin(c + d*x)**2*cos(a + d*x)**2*cos(c + d*x)/16 + 3*x*sin(a + d*x)*cos(a
 + d*x)**2*cos(c + d*x)**3/16 - 5*x*sin(c + d*x)**3*cos(a + d*x)**3/16 - 3*
x*sin(c + d*x)*cos(a + d*x)**3*cos(c + d*x)**2/16 + 7*sin(a + d*x)**3*sin(c
```

```
 + d*x)**3/(48*d) + 11*sin(a + d*x)**3*sin(c + d*x)*cos(c + d*x)**2/(16*d)
- 3*sin(a + d*x)**2*sin(c + d*x)**2*cos(a + d*x)*cos(c + d*x)/(4*d) + sin(a
 + d*x)*sin(c + d*x)**3*cos(a + d*x)**2/(2*d) + 3*sin(c + d*x)**2*cos(a + d
*x)**3*cos(c + d*x)/(16*d) + cos(a + d*x)**3*cos(c + d*x)**3/(16*d), Eq(b,
d)), (-9*x*sin(a + 3*d*x)**3*sin(c + d*x)**2*cos(c + d*x)/32 + 3*x*sin(a +
3*d*x)**3*cos(c + d*x)**3/32 + 3*x*sin(a + 3*d*x)**2*sin(c + d*x)**3*cos(a
+ 3*d*x)/32 - 9*x*sin(a + 3*d*x)**2*sin(c + d*x)*cos(a + 3*d*x)*cos(c + d*x
)**2/32 - 9*x*sin(a + 3*d*x)*sin(c + d*x)**2*cos(a + 3*d*x)**2*cos(c + d*x)
/32 + 3*x*sin(a + 3*d*x)*cos(a + 3*d*x)**2*cos(c + d*x)**3/32 + 3*x*sin(c +
 d*x)**3*cos(a + 3*d*x)**3/32 - 9*x*sin(c + d*x)*cos(a + 3*d*x)**3*cos(c +
d*x)**2/32 - sin(a + 3*d*x)**3*sin(c + d*x)**3/(12*d) - 13*sin(a + 3*d*x)**
3*sin(c + d*x)*cos(c + d*x)**2/(320*d) - 3*sin(a + 3*d*x)**2*sin(c + d*x)**
2*cos(a + 3*d*x)*cos(c + d*x)/(20*d) - 101*sin(a + 3*d*x)**2*cos(a + 3*d*x)
*cos(c + d*x)**3/(320*d) - 27*sin(a + 3*d*x)*sin(c + d*x)**3*cos(a + 3*d*x)
**2/(320*d) - 51*sin(c + d*x)**2*cos(a + 3*d*x)...
```

Giac [A]
time = 0.41, size = 181, normalized size = 0.93

$$\frac{\cos\left(3\,bx+3\,dx+3\,a+3\,c\right)}{96\,\left(b+d\right)}+\frac{3\,\cos\left(3\,bx+dx+3\,a+c\right)}{32\,\left(3\,b+d\right)}+\frac{3\,\cos\left(3\,bx-dx+3\,a-c\right)}{32\,\left(3\,b-d\right)}+\frac{\cos\left(3\,bx-3\,dx+3\,a-3\,c\right)}{96\,\left(b-d\right)}-\frac{3\,\cos\left(bx+3\,dx+a+3\,c\right)}{32\,\left(b+3\,d\right)}-\frac{9\,\cos\left(bx+dx+a+c\right)}{32\,\left(b+d\right)}-\frac{9\,\cos\left(bx-dx+a-c\right)}{32\,\left(b-d\right)}-\frac{3\,\cos\left(bx-3\,dx+a-3\,c\right)}{32\,\left(b-3\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/96*cos(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*cos(3*b*x + d*x + 3*a +
c)/(3*b + d) + 3/32*cos(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*cos(3*b*x -
 3*d*x + 3*a - 3*c)/(b - d) - 3/32*cos(b*x + 3*d*x + a + 3*c)/(b + 3*d) - 9
/32*cos(b*x + d*x + a + c)/(b + d) - 9/32*cos(b*x - d*x + a - c)/(b - d) -
3/32*cos(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

Mupad [B]
time = 4.14, size = 951, normalized size = 4.88

$$-{\mathrm{e}}^{a\,3\mathrm{i}-c\,1\mathrm{i}+b\,x\,3\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\frac{-9\,b^3-3\,b^2\,d+9\,b\,d^2+3\,d^3}{576\,b^4-640\,b^2\,d^2+64\,d^4}+\frac{{\mathrm{e}}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,\left(-9\,b^3+3\,b^2\,d+9\,b\,d^2-3\,d^3\right)}{576\,b^4-640\,b^2\,d^2+64\,d^4}-\frac{{\mathrm{e}}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(-81\,b^3-81\,b^2\,d+9\,b\,d^2+9\,d^3\right)}{576\,b^4-640\,b^2\,d^2+64\,d^4}-\frac{{\mathrm{e}}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(-81\,b^3+81\,b^2\,d+9\,b\,d^2-9\,d^3\right)}{576\,b^4-640\,b^2\,d^2+64\,d^4}\right)-{\mathrm{e}}^{a\,3\mathrm{i}+c\,1\mathrm{i}+b\,x\,3\mathrm{i}+d\,x\,1\mathrm{i}}\,\left(\frac{-9\,b^3+3\,b^2\,d+9\,b\,d^2-3\,d^3}{576\,b^4-640\,b^2\,d^2+64\,d^4}+\frac{{\mathrm{e}}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,\left(-9\,b^3-3\,b^2\,d+9\,b\,d^2+3\,d^3\right)}{576\,b^4-640\,b^2\,d^2+64\,d^4}-\frac{{\mathrm{e}}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(-81\,b^3+81\,b^2\,d+9\,b\,d^2-9\,d^3\right)}{576\,b^4-640\,b^2\,d^2+64\,d^4}-\frac{{\mathrm{e}}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(-81\,b^3-81\,b^2\,d+9\,b\,d^2+9\,d^3\right)}{576\,b^4-640\,b^2\,d^2+64\,d^4}\right)-{\mathrm{e}}^{a\,3\mathrm{i}-c\,3\mathrm{i}+b\,x\,3\mathrm{i}-d\,x\,3\mathrm{i}}\,\left(\frac{-b^3-b^2\,d+9\,b\,d^2+9\,d^3}{192\,b^4-1920\,b^2\,d^2+1728\,d^4}+\frac{{\mathrm{e}}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,\left(-b^3+b^2\,d+9\,b\,d^2-9\,d^3\right)}{192\,b^4-1920\,b^2\,d^2+1728\,d^4}-\frac{{\mathrm{e}}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(-9\,b^3-27\,b^2\,d+9\,b\,d^2+27\,d^3\right)}{192\,b^4-1920\,b^2\,d^2+1728\,d^4}-\frac{{\mathrm{e}}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(-9\,b^3+27\,b^2\,d+9\,b\,d^2-27\,d^3\right)}{192\,b^4-1920\,b^2\,d^2+1728\,d^4}\right)-{\mathrm{e}}^{a\,3\mathrm{i}+c\,3\mathrm{i}+b\,x\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,\left(\frac{-b^3+b^2\,d+9\,b\,d^2-9\,d^3}{192\,b^4-1920\,b^2\,d^2+1728\,d^4}+\frac{{\mathrm{e}}^{-a\,6\mathrm{i}-b\,x\,6\mathrm{i}}\,\left(-b^3-b^2\,d+9\,b\,d^2+9\,d^3\right)}{192\,b^4-1920\,b^2\,d^2+1728\,d^4}-\frac{{\mathrm{e}}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(-9\,b^3+27\,b^2\,d+9\,b\,d^2-27\,d^3\right)}{192\,b^4-1920\,b^2\,d^2+1728\,d^4}-\frac{{\mathrm{e}}^{-a\,4\mathrm{i}-b\,x\,4\mathrm{i}}\,\left(-9\,b^3-27\,b^2\,d+9\,b\,d^2+27\,d^3\right)}{192\,b^4-1920\,b^2\,d^2+1728\,d^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*sin(a + b*x)^3,x)
```

```
[Out] - exp(a*3i - c*1i + b*x*3i - d*x*1i)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/(
576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- a*6i - b*x*6i)*(9*b*d^2 + 3*b^2*d
- 9*b^3 - 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (exp(- a*2i - b*x*2i)*
(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (
exp(- a*4i - b*x*4i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d
^4 - 640*b^2*d^2)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((9*b*d^2 + 3*b^2*d
 - 9*b^3 - 3*d^3)/(576*b^4 + 64*d^4 - 640*b^2*d^2) + (exp(- a*6i - b*x*6i)*
(9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3))/(576*b^4 + 64*d^4 - 640*b^2*d^2) - (ex
```

```
p(- a*2i - b*x*2i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(576*b^4 + 64*d^4
 - 640*b^2*d^2) - (exp(- a*4i - b*x*4i)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^
3))/(576*b^4 + 64*d^4 - 640*b^2*d^2)) - exp(a*3i - c*3i + b*x*3i - d*x*3i)*
((9*b*d^2 - b^2*d - b^3 + 9*d^3)/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) + (exp
(- a*6i - b*x*6i)*(9*b*d^2 + b^2*d - b^3 - 9*d^3))/(192*b^4 + 1728*d^4 - 19
20*b^2*d^2) - (exp(- a*2i - b*x*2i)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/
(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 27*b
^2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2)) - exp(a*3i + c
*3i + b*x*3i + d*x*3i)*((9*b*d^2 + b^2*d - b^3 - 9*d^3)/(192*b^4 + 1728*d^4
 - 1920*b^2*d^2) + (exp(- a*6i - b*x*6i)*(9*b*d^2 - b^2*d - b^3 + 9*d^3))/(
192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- a*2i - b*x*2i)*(9*b*d^2 + 27*b^
2*d - 9*b^3 - 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b^2*d^2) - (exp(- a*4i -
b*x*4i)*(9*b*d^2 - 27*b^2*d - 9*b^3 + 27*d^3))/(192*b^4 + 1728*d^4 - 1920*b
^2*d^2))
```

3.227 $\int \cos(a+bx)\csc(c+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\cos(a-c)\log(\sin(c+bx))}{b}-x\sin(a-c)$$

```
[Out] cos(a-c)*ln(sin(b*x+c))/b-x*sin(a-c)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {4677, 3556, 8}

$$\frac{\cos(a-c)\log(\sin(bx+c))}{b}-x\sin(a-c)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[c + b*x],x]
```

```
[Out] (Cos[a - c]*Log[Sin[c + b*x]])/b - x*Sin[a - c]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4677

```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Dist[Cos[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] - Dist[Sin[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\csc(c+bx)\,dx &= \cos(a-c)\int \cot(c+bx)\,dx-\sin(a-c)\int 1\,dx\\ &= \frac{\cos(a-c)\log(\sin(c+bx))}{b}-x\sin(a-c)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 58, normalized size = 2.15

$$\frac{-2i\text{ArcTan}(\tan(c+bx))\cos(a-c)+\cos(a-c)\left(2ibx+\log\left(\sin^2(c+bx)\right)\right)-2bx\sin(a-c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[c + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[Tan[c + b*x]]*Cos[a - c] + Cos[a - c]*((2*I)*b*x + Log[Sin[c
 + b*x]^2]) - 2*b*x*Sin[a - c])/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(27) = 54.
time = 0.38, size = 162, normalized size = 6.00

method	result
risch	$ix\,\mathrm{e}^{i(a-c)}-2i\cos\left(a-c\right)x-\frac{2i\cos(a-c)a}{b}+\frac{\ln\left(\mathrm{e}^{2i(bx+a)}-\mathrm{e}^{2i(a-c)}\right)\cos(a-c)}{b}$
default	$\frac{\frac{\frac{(-\cos(a)\cos(c)-\sin(a)\sin(c))\ln\left(1+\tan^2(bx+a)\right)}{2}+(-\sin(a)\cos(c)+\cos(a)\sin(c))\arctan(\tan(bx+a))}{\left(\cos^2(c)+\sin^2(c)\right)\left(\cos^2(a)+\sin^2(a)\right)}+\frac{(\cos(a)\cos(c)+\sin(a)\sin(c))\ln(\tan(bx+a)\cos(}{\left(\cos^2(a)\right)\left(\cos^2(c)\right)+\left(\cos^2(c)\right)\left(\mathrm{s}\right.}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(a)^2)*(1/2*(-cos(a)*cos(c)-sin(a)*
sin(c))*ln(1+tan(b*x+a)^2)+(-sin(a)*cos(c)+cos(a)*sin(c))*arctan(tan(b*x+a)
))+(cos(a)*cos(c)+sin(a)*sin(c))/(cos(a)^2*cos(c)^2+cos(c)^2*sin(a)^2+cos(a
)^2*sin(c)^2+sin(a)^2*sin(c)^2)*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(
a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(27) = 54.
time = 0.29, size = 106, normalized size = 3.93

$$\frac{2\,bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\cos\left(bx\right)^2+2\,\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^2+\sin\left(bx\right)^2-2\,\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^2\right)+\cos\left(-a+c\right)\log\left(\cos\left(bx\right)^2-2\,\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^2+\sin\left(bx\right)^2+2\,\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*sin(-a + c) + cos(-a + c)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + c
os(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + cos(-a + c)*log(cos(
b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + si
n(c)^2))/b
```

Fricas [A]
time = 2.65, size = 30, normalized size = 1.11

$$\frac{bx\sin\left(-a+c\right)+\cos\left(-a+c\right)\log\left(\frac{1}{2}\sin\left(bx+c\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c),x, algorithm="fricas")
```

```
[Out] (b*x*sin(-a + c) + cos(-a + c)*log(1/2*sin(b*x + c)))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(20) = 40.
time = 4.00, size = 335, normalized size = 12.41

$$-\left(\begin{cases}0 & \text{for}\: b=0\wedge\left(b=0\vee c=0\right)\\ x & \text{for}\: c=0\\ -\frac{bx\tan^{2}\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}+\frac{bx}{b\tan^{2}\left(\frac{c}{2}\right)+b}-\frac{2\log\left(\tan\left(\frac{c}{2}\right)+\tan\left(\frac{bx}{2}\right)\right)\tan\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}-\frac{2\log\left(\tan\left(\frac{bx}{2}\right)-\frac{1}{\tan\left(\frac{c}{2}\right)}\right)\tan\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}+\frac{2\log\left(\tan^{2}\left(\frac{bx}{2}\right)+1\right)\tan\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b} & \text{otherwise}\end{cases}\right)\sin\left(a\right)+\left(\begin{cases}\tilde{\infty}x & \text{for}\: b=0\wedge c=0\\ \frac{x}{\sin\left(c\right)} & \text{for}\: b=0\\ \frac{\log\left(\sin\left(bx\right)\right)}{b} & \text{for}\: c=0\\ \frac{2bx\tan\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}-\frac{\log\left(\tan\left(\frac{c}{2}\right)+\tan\left(\frac{bx}{2}\right)\right)\tan^{2}\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}+\frac{\log\left(\tan\left(\frac{c}{2}\right)+\tan\left(\frac{bx}{2}\right)\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}-\frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{1}{\tan\left(\frac{c}{2}\right)}\right)\tan^{2}\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}+\frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{1}{\tan\left(\frac{c}{2}\right)}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}+\frac{\log\left(\tan^{2}\left(\frac{bx}{2}\right)+1\right)\tan^{2}\left(\frac{c}{2}\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b}-\frac{\log\left(\tan^{2}\left(\frac{bx}{2}\right)+1\right)}{b\tan^{2}\left(\frac{c}{2}\right)+b} & \text{otherwise}\end{cases}\right)\cos\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c),x)
```

```
[Out] -Piecewise((0, Eq(b, 0) & (Eq(b, 0) | Eq(c, 0))), (x, Eq(c, 0)), (-b*x*tan(
c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) - 2*log(tan(c/2) + ta
n(b*x/2))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - 1/tan(c/2))*tan
(c/2)/(b*tan(c/2)**2 + b) + 2*log(tan(b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**
2 + b), True))*sin(a) + Piecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (x/sin(c),
Eq(b, 0)), (log(sin(b*x))/b, Eq(c, 0)), (2*b*x*tan(c/2)/(b*tan(c/2)**2 + b)
 - log(tan(c/2) + tan(b*x/2))*tan(c/2)**2/(b*tan(c/2)**2 + b) + log(tan(c/2
) + tan(b*x/2))/(b*tan(c/2)**2 + b) - log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)
**2/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - 1/tan(c/2))/(b*tan(c/2)**2 + b)
+ log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**
2 + 1)/(b*tan(c/2)**2 + b), True))*cos(a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(27) = 54.
time = 0.44, size = 482, normalized size = 17.85

$$-\frac{\frac{4\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)\left(bx+a\right)}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1}+\frac{\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}a\right)^{2}+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^{2}+1\right)\log\left(\tan\left(bx+a\right)^{2}+1\right)}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1}-\frac{2\left(\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)^{4}-2\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)^{2}+8\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)^{3}-2\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{4}+\tan\left(\frac{1}{2}a\right)^{4}-8\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)+20\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-8\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}c\right)^{4}-2\tan\left(\frac{1}{2}a\right)^{2}+8\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-2\tan\left(\frac{1}{2}c\right)^{2}+1\right)\log\left(\left|\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)^{2}+4\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-2\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-\tan\left(bx+a\right)\tan\left(\frac{1}{2}c\right)^{2}+2\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(bx+a\right)-2\tan\left(\frac{1}{2}a\right)+2\tan\left(\frac{1}{2}c\right)\right|\right)}{\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)^{4}+4\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}a\right)^{4}+4\tan\left(\frac{1}{2}a\right)^{3}\tan\left(\frac{1}{2}c\right)+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}c\right)^{4}+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+1}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c),x, algorithm="giac")
```

```
[Out] -1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - t
an(1/2*c))*(b*x + a)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)
^2 + 1) + (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*
c) - tan(1/2*c)^2 + 1)*log(tan(b*x + a)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^2 +
```

```
 tan(1/2*a)^2 + tan(1/2*c)^2 + 1) - 2*(tan(1/2*a)^4*tan(1/2*c)^4 - 2*tan(1/
2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*
c)^4 + tan(1/2*a)^4 - 8*tan(1/2*a)^3*tan(1/2*c) + 20*tan(1/2*a)^2*tan(1/2*c
)^2 - 8*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*c)^4 - 2*tan(1/2*a)^2 + 8*tan(1/2
*a)*tan(1/2*c) - 2*tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(
1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c)
 - 2*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan
(1/2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(
1/2*c)^4 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^4 + 4*tan(1/2*a)^3*tan(
1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + 4*tan(1/2*a)*tan(1/2*c)
 + 1))/b
```

Mupad [B]
time = 0.70, size = 115, normalized size = 4.26

$$-x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}-\frac{e^{a\,1i-c\,1i}\,1i}{2}\right)-x\left(\frac{e^{-a\,1i+c\,1i}\,1i}{2}+\frac{e^{a\,1i-c\,1i}\,1i}{2}\right)+\frac{\ln\left(-e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)\,\left(\frac{e^{-a\,1i+c\,1i}}{2}+\frac{e^{a\,1i-c\,1i}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(c + b*x),x)
```

```
[Out] (log(exp(a*2i + b*x*2i) - exp(a*2i - c*2i))*(exp(c*1i - a*1i)/2 + exp(a*1i
- c*1i)/2))/b - x*((exp(c*1i - a*1i)*1i)/2 + (exp(a*1i - c*1i)*1i)/2) - x*(
(exp(c*1i - a*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2)
```

3.228 $\int \cos(a+bx)\csc^2(c+bx)\,dx$

Optimal. Leaf size=35

$$-\frac{\cos(a-c)\csc(c+bx)}{b}+\frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b}$$

```
[Out] -cos(a-c)*csc(b*x+c)/b+arctanh(cos(b*x+c))*sin(a-c)/b
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {4677, 2686, 8, 3855}

$$\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\cos(a-c)\csc(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[c + b*x]^2,x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4677

```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Dist[Cos[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] - Dist[Sin[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$
\begin{aligned}
\int \cos(a+bx)\csc^2(c+bx)\,dx &= \cos(a-c)\int \cot(c+bx)\csc(c+bx)\,dx - \sin(a-c)\int \csc(c+bx)\,dx \\
&= \frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b} - \frac{\cos(a-c)\text{Subst}(\int 1\,dx, x, \csc(c+bx))}{b} \\
&= -\frac{\cos(a-c)\csc(c+bx)}{b} + \frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 90, normalized size = 2.57

$$
-\frac{\cos(a-c)\csc(c+bx)}{b} + \frac{2i\text{ArcTan}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[c + b*x]^2,x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + ((2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]
*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]
*Sin[c])]*Sin[a - c])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(35) = 70$.
time = 0.67, size = 408, normalized size = 11.66

method	result
risch	$\frac{i\left(e^{i(bx+3a)}+e^{i(bx+a+2c)}\right)}{b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)} + \frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\sin(a-c)}{b} - \frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\sin(a-c)}{b}$
default	$-\frac{2\left(-\frac{\left(\left(\cos^2(a)\right)\left(\cos^2(c)\right)+2\cos(a)\cos(c)\sin(a)\sin(c)+\left(\sin^2(a)\right)\left(\sin^2(c)\right)\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\left(\left(\cos^2(a)\right)\left(\cos^2(c)\right)+\left(\cos^2(c)\right)\left(\sin^2(a)\right)+\left(\cos^2(a)\right)\left(\sin^2(c)\right)+\left(\sin^2(a)\right)\left(\sin^2(c)\right)\right)(\sin(a)\cos(c)-\cos(a)\sin(c))}+\frac{\cdots}{\left(\cos^2(a)\right)\left(\cos^2(c)\right)+\cdots}\right)}{\cos(c)\sin(a)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\sin(c)\cos(a)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\cos(a)\cos(c)+2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\sin(a)\sin(c)\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-2*(-(cos(a)^2*cos(c)^2+2*cos(a)*cos(c)*sin(a)*sin(c)+sin(a)^2*sin(c)^
2)/(cos(a)^2*cos(c)^2+cos(c)^2*sin(a)^2+cos(a)^2*sin(c)^2+sin(a)^2*sin(c)^2
)/(sin(a)*cos(c)-cos(a)*sin(c))*tan(1/2*b*x+1/2*a)+1/(cos(a)^2*cos(c)^2+cos
(c)^2*sin(a)^2+cos(a)^2*sin(c)^2+sin(a)^2*sin(c)^2)*(cos(a)*cos(c)+sin(a)*s
in(c)))/(cos(c)*sin(a)*tan(1/2*b*x+1/2*a)^2-sin(c)*cos(a)*tan(1/2*b*x+1/2*a
)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*cos(c)+2*tan(1/2*b*x+1/2*a)*sin(a)*sin(c)-s
```

```
in(a)*cos(c)+cos(a)*sin(c))-2*(sin(a)*cos(c)-cos(a)*sin(c))/(cos(a)^2*cos(c
)^2+cos(c)^2*sin(a)^2+cos(a)^2*sin(c)^2+sin(a)^2*sin(c)^2)/(-cos(c)^2*sin(a
)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin(c)^2-cos(a)^2*sin(c)^2)^(1/2)*arctan(1/2
*(2*(sin(a)*cos(c)-cos(a)*sin(c))*tan(1/2*b*x+1/2*a)+2*cos(a)*cos(c)+2*sin(
a)*sin(c))/(-cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin(c)^2-cos(a)^2
*sin(c)^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(35) = 70$.
time = 0.30, size = 450, normalized size = 12.86

$$\frac{2\left(\sin(bx+2a)+\sin(bx+2c)\right)\cos(2bx+a+2c)-\left(\cos(2bx+a+2c)^{2}\sin(-a+c)-2\cos(2bx+a+2c)\cos(a)\sin(-a+c)+\sin(2bx+a+2c)^{2}\sin(-a+c)-2\sin(2bx+a+2c)\sin(a)\sin(-a+c)+\left(\cos(a)^{2}+\sin(a)^{2}\right)\sin(-a+c)\right)\log\left(\cos(bx)^{2}+2\cos(bx)\cos(c)+\cos(c)^{2}+\sin(bx)^{2}-2\sin(bx)\sin(c)+\sin(c)^{2}\right)+\left(\cos(2bx+a+2c)^{2}\sin(-a+c)-2\cos(2bx+a+2c)\cos(a)\sin(-a+c)+\sin(2bx+a+2c)^{2}\sin(-a+c)-2\sin(2bx+a+2c)\sin(a)\sin(-a+c)+\left(\cos(a)^{2}+\sin(a)^{2}\right)\sin(-a+c)\right)\log\left(\cos(bx)^{2}-2\cos(bx)\cos(c)+\cos(c)^{2}+\sin(bx)^{2}+2\sin(bx)\sin(c)+\sin(c)^{2}\right)-2\left(\cos(bx+2a)+\cos(bx+2c)\right)\sin(2bx+a+2c)-2\cos(a)\sin(bx+2a)-2\cos(a)\sin(bx+2c)+2\cos(bx+2a)\sin(a)+2\cos(bx+2c)\sin(a)}{2\left(b\cos(2bx+a+2c)^{2}-2b\cos(2bx+a+2c)\cos(a)+b\sin(2bx+a+2c)^{2}-2b\sin(2bx+a+2c)\sin(a)+\left(\cos(a)^{2}+\sin(a)^{2}\right)b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(sin(b*x + 2*a) + sin(b*x + 2*c))*cos(2*b*x + a + 2*c) - (cos(2*b*x
+ a + 2*c)^2*sin(-a + c) - 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(
2*b*x + a + 2*c)^2*sin(-a + c) - 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c)
+ (cos(a)^2 + sin(a)^2)*sin(-a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + c
os(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^2) + (cos(2*b*x + a + 2*c
)^2*sin(-a + c) - 2*cos(2*b*x + a + 2*c)*cos(a)*sin(-a + c) + sin(2*b*x + a
 + 2*c)^2*sin(-a + c) - 2*sin(2*b*x + a + 2*c)*sin(a)*sin(-a + c) + (cos(a)
^2 + sin(a)^2)*sin(-a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 +
 sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 2*(cos(b*x + 2*a) + cos(b*x +
 2*c))*sin(2*b*x + a + 2*c) - 2*cos(a)*sin(b*x + 2*a) - 2*cos(a)*sin(b*x +
2*c) + 2*cos(b*x + 2*a)*sin(a) + 2*cos(b*x + 2*c)*sin(a))/(b*cos(2*b*x + a
+ 2*c)^2 - 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^2 - 2*b
*sin(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(35) = 70$.
time = 1.97, size = 71, normalized size = 2.03

$$-\frac{\log\left(\frac{1}{2}\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)\sin\left(-a+c\right)-\log\left(-\frac{1}{2}\cos\left(bx+c\right)+\frac{1}{2}\right)\sin\left(bx+c\right)\sin\left(-a+c\right)+2\cos\left(-a+c\right)}{2\,b\sin\left(bx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(log(1/2*cos(b*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) - log(-1/2*cos(b
*x + c) + 1/2)*sin(b*x + c)*sin(-a + c) + 2*cos(-a + c))/(b*sin(b*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1572 vs. $2(27) = 54$.
time = 60.01, size = 3264, normalized size = 93.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)**2,x)
```

```
[Out] -Piecewise((0, Eq(b, 0) & Eq(c, 0)), (log(tan(b*x/2))/b, Eq(c, 0)), (0, Eq(
b, 0)), (-log(tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*
tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b
*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(c/2)
**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 -
 b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + lo
g(tan(c/2) + tan(b*x/2))*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)
**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) -
 b*tan(b*x/2)) + 2*log(tan(c/2) + tan(b*x/2))*tan(c/2)**2*tan(b*x/2)/(b*tan
(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c
/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(c/2) + tan(b*x/2))
*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2
)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)
) - log(tan(c/2) + tan(b*x/2))*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c
/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2
) - b*tan(b*x/2)) - log(tan(c/2) + tan(b*x/2))*tan(b*x/2)/(b*tan(c/2)**4*ta
n(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x
/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)
**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*
tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(t
an(b*x/2) - 1/tan(c/2))*tan(c/2)**3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)
 + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 -
 b*tan(c/2) - b*tan(b*x/2)) - log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3/(b*t
an(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan
(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - 2*log(tan(b*x/2) - 1/tan
(c/2))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan
(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(
b*x/2)) - log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**
4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan
(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2) - 1/tan(c/2))*tan(
c/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**
3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) + log(tan(b*x/2)
- 1/tan(c/2))*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/
2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2
)) + tan(c/2)**4*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b
*x/2)**2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*
x/2)) - 2*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)*
*2 - b*tan(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2))
- 2*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*ta
n(c/2)**3 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)) - tan(b*x
/2)/(b*tan(c/2)**4*tan(b*x/2) + b*tan(c/2)**3*tan(b*x/2)**2 - b*tan(c/2)**3
```

```
 + b*tan(c/2)*tan(b*x/2)**2 - b*tan(c/2) - b*tan(b*x/2)), True))*sin(a) + P
iecewise((zoo*x, Eq(b, 0) & Eq(c, 0)), (x/sin(c)**2, Eq(b, 0)), (-1/(b*sin(
b*x)), Eq(c, 0)), (4*log(tan(c/2) + tan(b*x/2))*tan(c/2)**4*tan(b*x/2)/(2*b
*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 +
 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2))
 + 4*log(tan(c/2) + tan(b*x/2))*tan(c/2)**3*tan(b*x/2)**2/(2*b*tan(c/2)**5*
tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)
**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) - 4*log(tan(
c/2) + tan(b*x/2))*tan(c/2)**3/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**
4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan
(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) - 4*log(tan(c/2) + tan(b*x/2))*tan(c/2)
**2*tan(b*x/2)/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2
- 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*t
an(c/2)*tan(b*x/2)) - 4*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**4*tan(b*x/2)
/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)
**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*
x/2)) - 4*log(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3*tan(b*x/2)**2/(2*b*tan(c
/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*t
an(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) + 4*l
og(tan(b*x/2) - 1/tan(c/2))*tan(c/2)**3/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*t
an(c/2)**4*tan(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2
- 2*b*tan(c/2)**2 - 2*b*tan(c/2)*tan(b*x/2)) + 4*log(tan(b*x/2) - 1/tan(c/2
))*tan(c/2)**2*tan(b*x/2)/(2*b*tan(c/2)**5*tan(b*x/2) + 2*b*tan(c/2)**4*tan
(b*x/2)**2 - 2*b*tan(c/2)**4 + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)
**2 - 2*b*tan(c/2)*tan(b*x/2)) + tan(c/2)**6*tan(b*x/2)/(2*b*tan(c/2)**5*ta
n(b*x/2) + 2*b*tan(c/2)**4*tan(b*x/2)**2 - 2*b*...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(35) = 70$.

time = 0.54, size = 893, normalized size = 25.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - t
an(1/2*c))*log(abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(1
/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)
 - 2*tan(1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/
2*c) - 2*tan(1/2*c)^2)/abs(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c) -
 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)
^2 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c)
+ 4*tan(1/2*a)*tan(1/2*c) + 2))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 +
 tan(1/2*c)^2 + 1) - (tan(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)^4 - 2*ta
```

```
n(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)^2 + 8*tan(1/2*b*x + 1/2*a)*tan(1
/2*a)^3*tan(1/2*c)^3 - 2*tan(1/2*a)^4*tan(1/2*c)^3 - 2*tan(1/2*b*x + 1/2*a)
*tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*b*x + 1/
2*a)*tan(1/2*a)^4 - 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^3*tan(1/2*c) + 2*tan(
1/2*a)^4*tan(1/2*c) + 20*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^2 - 1
2*tan(1/2*a)^3*tan(1/2*c)^2 - 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^
3 + 12*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*b*x + 1/2*a)*tan(1/2*c)^4 - 2*ta
n(1/2*a)*tan(1/2*c)^4 - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2 + 2*tan(1/2*a)^
3 + 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - 12*tan(1/2*a)^2*tan(1/2*
c) - 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c)^2 + 12*tan(1/2*a)*tan(1/2*c)^2 - 2*t
an(1/2*c)^3 + tan(1/2*b*x + 1/2*a) - 2*tan(1/2*a) + 2*tan(1/2*c))/((tan(1/2
*b*x + 1/2*a)^2*tan(1/2*a)^2*tan(1/2*c) - tan(1/2*b*x + 1/2*a)^2*tan(1/2*a)
*tan(1/2*c)^2 + tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*b*
x + 1/2*a)^2*tan(1/2*a) - tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2 - tan(1/2*b*x +
 1/2*a)^2*tan(1/2*c) + 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - tan(1
/2*a)^2*tan(1/2*c) - tan(1/2*b*x + 1/2*a)*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2
*c)^2 + tan(1/2*b*x + 1/2*a) - tan(1/2*a) + tan(1/2*c))*(tan(1/2*a)^2*tan(1
/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))))/b
```

Mupad [B]
time = 5.21, size = 252, normalized size = 7.20

$$-\frac{\ln\left(\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}-1\right)-\frac{\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}-1\right)\,1\mathrm{i}}{\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}}}\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-1\right)}{2\,b\,\sqrt{-\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}}}+\frac{\ln\left(\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}-1\right)+\frac{\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}-1\right)\,1\mathrm{i}}{\sqrt{-\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{-c\,2\mathrm{i}}}}\right)\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-1\right)}{2\,b\,\sqrt{-\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}}}+\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+1\right)\,1\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(c + b*x)^2,x)
```

```
[Out] (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) + (exp(a*2i)*exp(-c*2
i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i
- c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - (log(exp(a*1i)*exp(b*x*1i)*
(exp(a*2i)*exp(-c*2i) - 1) - (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) -
1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*
2i - c*2i))^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) + 1)*1i)/(b*(exp
(a*2i - c*2i) - exp(a*2i + b*x*2i)))
```

3.229 $\int \cos(a+bx)\csc^3(c+bx)\,dx$

Optimal. Leaf size=38

$$-\frac{\cos(a-c)\csc^2(c+bx)}{2b}+\frac{\cot(c+bx)\sin(a-c)}{b}$$

```
[Out] -1/2*cos(a-c)*csc(b*x+c)^2/b+cot(b*x+c)*sin(a-c)/b
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4677, 2686, 30, 3852, 8}

$$\frac{\sin(a-c)\cot(bx+c)}{b}-\frac{\cos(a-c)\csc^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Csc[c + b*x]^3,x]
```

```
[Out] -1/2*(Cos[a - c]*Csc[c + b*x]^2)/b + (Cot[c + b*x]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4677

```
Int[Cos[v_]*Csc[w_]^(n_.), x_Symbol] :> Dist[Cos[v - w], Int[Cot[w]*Csc[w]^
(n - 1), x], x] - Dist[Sin[v - w], Int[Csc[w]^(n - 1), x], x] /; GtQ[n, 0]
```

```
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \cos(a+bx)\csc^3(c+bx)\,dx = \cos(a-c)\int \cot(c+bx)\csc^2(c+bx)\,dx - \sin(a-c)\int \csc^2(c+bx)\,dx$$
$$= -\frac{\cos(a-c)\text{Subst}(\int x\,dx, x, \csc(c+bx))}{b} + \frac{\sin(a-c)\text{Subst}(\int 1\,dx, x, \cot(c}{b}$$
$$= -\frac{\cos(a-c)\csc^2(c+bx)}{2b} + \frac{\cot(c+bx)\sin(a-c)}{b}$$

Mathematica [A]
time = 0.23, size = 35, normalized size = 0.92

$$-\frac{\csc(c)\csc^2(c+bx)(\sin(a)-\cos(c+2bx)\sin(a-c))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Csc[c + b*x]^3,x]
[Out] -1/2*(Csc[c]*Csc[c + b*x]^2*(Sin[a] - Cos[c + 2*b*x]*Sin[a - c]))/b
```

Maple [A]
time = 0.86, size = 55, normalized size = 1.45

method	result	size
default	$-\frac{1}{2b(\cos(a)\cos(c)+\sin(a)\sin(c))(\tan(bx+a)\cos(a)\cos(c)+\tan(bx+a)\sin(a)\sin(c)+\cos(a)\sin(c)-\sin(a)\cos(c))^2}$	55
risch	$-\frac{-2\,e^{i(2bx+5a+c)}+e^{i(5a-c)}-e^{i(3a+c)}}{\left(-e^{2i(bx+a+c)}+e^{2ia}\right)^2 b}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/sin(b*x+c)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/b/(cos(a)*cos(c)+sin(a)*sin(c))/(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*s
in(a)*sin(c)+cos(a)*sin(c)-sin(a)*cos(c))^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(36) = 72.
time = 0.27, size = 395, normalized size = 10.39

$$\frac{(2\cos(2bx+2a+2c)-\cos(2a)+\cos(2c))\cos(4bx+a+5c)-2(2\cos(2bx+2a+2c)-\cos(2a)+\cos(2c))\cos(2bx+a+3c)-(\cos(2a)-\cos(2c))\cos(a+c)+2\cos(2bx+2a+2c)\cos(a+c)+(2\sin(2bx+2a+2c)-\sin(2a)+\sin(2c))\sin(4bx+a+5c)-2(2\sin(2bx+2a+2c)-\sin(2a)+\sin(2c))\sin(2bx+a+3c)-(\sin(2a)-\sin(2c))\sin(a+c)+2\sin(2bx+2a+2c)\sin(a+c)}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2-4b\cos(2bx+a+3c)\cos(a+c)+b\cos(a+c)^2+b\sin(4bx+a+5c)^2+4b\sin(2bx+a+3c)^2-4b\sin(2bx+a+3c)\sin(a+c)+b\sin(a+c)^2-2(2b\cos(2bx+a+3c)-b\cos(a+c))\cos(4bx+a+5c)-2(2b\sin(2bx+a+3c)-b\sin(a+c))\sin(4bx+a+5c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] ((2*cos(2*b*x + 2*a + 2*c) - cos(2*a) + cos(2*c))*cos(4*b*x + a + 5*c) - 2*
(2*cos(2*b*x + 2*a + 2*c) - cos(2*a) + cos(2*c))*cos(2*b*x + a + 3*c) - (co
s(2*a) - cos(2*c))*cos(a + c) + 2*cos(2*b*x + 2*a + 2*c)*cos(a + c) + (2*si
n(2*b*x + 2*a + 2*c) - sin(2*a) + sin(2*c))*sin(4*b*x + a + 5*c) - 2*(2*sin
(2*b*x + 2*a + 2*c) - sin(2*a) + sin(2*c))*sin(2*b*x + a + 3*c) - (sin(2*a)
 - sin(2*c))*sin(a + c) + 2*sin(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos(4*b*x
 + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 - 4*b*cos(2*b*x + a + 3*c)*cos(a
 + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x + a + 3*c
)^2 - 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 - 2*(2*b*cos(2*b
*x + a + 3*c) - b*cos(a + c))*cos(4*b*x + a + 5*c) - 2*(2*b*sin(2*b*x + a +
 3*c) - b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Fricas [**A**]
time = 2.12, size = 45, normalized size = 1.18

$$\frac{2\cos\left(bx+c\right)\sin\left(bx+c\right)\sin\left(-a+c\right)+\cos\left(-a+c\right)}{2\left(b\cos\left(bx+c\right)^{2}-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cos(b*x + c)*sin(b*x + c)*sin(-a + c) + cos(-a + c))/(b*cos(b*x + c)
^2 - b)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(36) = 72.
time = 0.46, size = 327, normalized size = 8.61

$$-\frac{\tan\left(\frac{1}{2}a\right)^{6}\tan\left(\frac{1}{2}c\right)^{6}+3\tan\left(\frac{1}{2}a\right)^{6}\tan\left(\frac{1}{2}c\right)^{4}+3\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)^{6}+3\tan\left(\frac{1}{2}a\right)^{6}\tan\left(\frac{1}{2}c\right)^{2}+9\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)^{4}+3\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{6}+\tan\left(\frac{1}{2}a\right)^{6}+9\tan\left(\frac{1}{2}a\right)^{4}\tan\left(\frac{1}{2}c\right)^{2}+9\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{4}+\tan\left(\frac{1}{2}c\right)^{6}+3\tan\left(\frac{1}{2}a\right)^{4}+9\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+3\tan\left(\frac{1}{2}c\right)^{4}+3\tan\left(\frac{1}{2}a\right)^{2}+3\tan\left(\frac{1}{2}c\right)^{2}+1}{2\left(\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)^{2}+4\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-2\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-\tan\left(bx+a\right)\tan\left(\frac{1}{2}c\right)^{2}+2\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(bx+a\right)-2\tan\left(\frac{1}{2}a\right)+2\tan\left(\frac{1}{2}c\right)\right)^{2}\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}a\right)^{2}+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^{2}+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/sin(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a
)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^
```

```
4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^
2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2
*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1)/
((tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c)^2 - tan(b*x + a)*tan(1/2*a)^2 + 4*ta
n(b*x + a)*tan(1/2*a)*tan(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)
*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) + 2
*tan(1/2*c))^2*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan
(1/2*c) - tan(1/2*c)^2 + 1)*b)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/sin(c + b*x)^3,x)
[Out] \text{Hanged}
```

3.230 $\int \sin(a+bx)\tan^3(c+bx)\,dx$

Optimal. Leaf size=72

$$-\frac{3\tanh^{-1}(\sin(c+bx))\cos(a-c)}{2b}+\frac{\sec(c+bx)\sin(a-c)}{b}+\frac{\sin(a+bx)}{b}+\frac{\cos(a-c)\sec(c+bx)\tan(c+bx)}{2b}$$

```
[Out] -3/2*arctanh(sin(b*x+c))*cos(a-c)/b+sec(b*x+c)*sin(a-c)/b+sin(b*x+a)/b+1/2*
cos(a-c)*sec(b*x+c)*tan(b*x+c)/b
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {4672, 4675, 2717, 3855, 2686, 8, 2691}

$$\frac{\sin(a-c)\sec(bx+c)}{b}-\frac{3\cos(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\tan(bx+c)\sec(bx+c)}{2b}+\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[c + b*x]^3,x]
```

```
[Out] (-3*ArcTanh[Sin[c + b*x]]*Cos[a - c])/(2*b) + (Sec[c + b*x]*Sin[a - c])/b +
 Sin[a + b*x]/b + (Cos[a - c]*Sec[c + b*x]*Tan[c + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4672

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4675

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\int \sin(a+bx)\tan^3(c+bx)\,dx = \cos(a-c)\int \sec(c+bx)\tan^2(c+bx)\,dx - \int \cos(a+bx)\tan^2(c+bx)\,dx$$
$$= \frac{\cos(a-c)\sec(c+bx)\tan(c+bx)}{2b} - \frac{1}{2}\cos(a-c)\int \sec(c+bx)\,dx + \sin(a$$
$$= -\frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{2b} + \frac{\cos(a-c)\sec(c+bx)\tan(c+bx)}{2b} - \cos$$
$$= -\frac{3\tanh^{-1}(\sin(c+bx))\cos(a-c)}{2b} + \frac{\sec(c+bx)\sin(a-c)}{b} + \frac{\sin(a+bx)}{b} +$$

Mathematica [A]
time = 0.42, size = 70, normalized size = 0.97

$$\frac{-12\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+\sec^2(c+bx)(2\sin(a-2c-bx)+5\sin(a+bx)+\sin(a+2c+3bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[c + b*x]^3,x]
```

```
[Out] (-12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Cos[a - c] + Sec[c + b*x]^2*(2*S
in[a - 2*c - b*x] + 5*Sin[a + b*x] + Sin[a + 2*c + 3*b*x]))/(4*b)
```

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 186, normalized size = 2.58

method	result
risch	$-\frac{i e^{i(bx+a)}}{2b}+\frac{i e^{-i(bx+a)}}{2b}-\frac{i\left(3\,e^{i(3bx+5a+2c)}-e^{i(3bx+3a+4c)}+e^{i(bx+5a)}-3\,e^{i(bx+3a+2c)}\right)}{2b\left(e^{2i(bx+a+c)}+e^{2ia}\right)^{2}}+\frac{3\ln\left(e^{i(bx+a)}-i e^{i(a-c)}\right)\cos(a-c)}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*exp(I*(b*x+a))/b+1/2*I/b*exp(-I*(b*x+a))-1/2*I/b/(exp(2*I*(b*x+a+c))
+exp(2*I*a))^2*(3*exp(I*(3*b*x+5*a+2*c))-exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+
5*a))-3*exp(I*(b*x+3*a+2*c)))+3/2*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*cos(a
-c)-3/2*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*cos(a-c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(68) = 136$.
time = 0.57, size = 1027, normalized size = 14.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(sin(5*b*x + a + 4*c) + 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*cos(
6*b*x + 2*a + 4*c) - 2*(5*sin(4*b*x + 2*a + 2*c) - 2*sin(4*b*x + 4*c) + 2*s
in(2*b*x + 2*a) - 5*sin(2*b*x + 2*c))*cos(5*b*x + a + 4*c) + 10*(2*sin(3*b*
x + a + 2*c) + sin(b*x + a))*cos(4*b*x + 2*a + 2*c) - 4*(2*sin(3*b*x + a +
2*c) + sin(b*x + a))*cos(4*b*x + 4*c) - 4*(2*sin(2*b*x + 2*a) - 5*sin(2*b*x
 + 2*c))*cos(3*b*x + a + 2*c) - 3*(cos(5*b*x + a + 4*c)^2*cos(-a + c) + 4*c
os(3*b*x + a + 2*c)^2*cos(-a + c) + 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos
(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x + a + 4*c)^2
+ 4*cos(-a + c)*sin(3*b*x + a + 2*c)^2 + 4*cos(-a + c)*sin(3*b*x + a + 2*c)
*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2 + 2*(2*cos(3*b*x + a + 2*c)*cos(
-a + c) + cos(b*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) + 2*(2*cos(-a + c)
*sin(3*b*x + a + 2*c) + cos(-a + c)*sin(b*x + a))*sin(5*b*x + a + 4*c))*log
((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2
+ 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(
c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2))
 - 2*(cos(5*b*x + a + 4*c) + 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*sin(6*b
*x + 2*a + 4*c) + 2*(5*cos(4*b*x + 2*a + 2*c) - 2*cos(4*b*x + 4*c) + 2*cos(
2*b*x + 2*a) - 5*cos(2*b*x + 2*c) - 1)*sin(5*b*x + a + 4*c) - 10*(2*cos(3*b
*x + a + 2*c) + cos(b*x + a))*sin(4*b*x + 2*a + 2*c) + 4*(2*cos(3*b*x + a +
 2*c) + cos(b*x + a))*sin(4*b*x + 4*c) + 4*(2*cos(2*b*x + 2*a) - 5*cos(2*b*
x + 2*c) - 1)*sin(3*b*x + a + 2*c) - 4*cos(b*x + a)*sin(2*b*x + 2*a) + 10*c
os(b*x + a)*sin(2*b*x + 2*c) + 4*cos(2*b*x + 2*a)*sin(b*x + a) - 10*cos(2*b
*x + 2*c)*sin(b*x + a) - 2*sin(b*x + a))/(b*cos(5*b*x + a + 4*c)^2 + 4*b*co
```

```
s(3*b*x + a + 2*c)^2 + 4*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x +
a)^2 + b*sin(5*b*x + a + 4*c)^2 + 4*b*sin(3*b*x + a + 2*c)^2 + 4*b*sin(3*b*
x + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2 + 2*(2*b*cos(3*b*x + a + 2*c)
+ b*cos(b*x + a))*cos(5*b*x + a + 4*c) + 2*(2*b*sin(3*b*x + a + 2*c) + b*si
n(b*x + a))*sin(5*b*x + a + 4*c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(68) = 136$.
time = 2.59, size = 376, normalized size = 5.22

$$-\frac{\frac{3\sqrt{2}\left(2\left(\cos(-2a+2c)+1\right)\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-2\left(\cos(-2a+2c)^2+\cos(-2a+2c)\right)\cos(bx+a)^2+\cos(-2a+2c)^2-1\right)\log\left(-\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)+\frac{2\sqrt{2}\left((\cos(-2a+2c)+1)\sin(bx+a)+\cos(bx+a)\sin(-2a+2c)\right)}{\sqrt{\cos(-2a+2c)+1}}-\cos(-2a+2c)-3}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)+1}\right)}{\sqrt{\cos(-2a+2c)+1}}-4\left(4\cos(bx+a)^2\cos(-2a+2c)-3\cos(-2a+2c)+5\right)\sin(bx+a)-4\left(4\cos(bx+a)^3-5\cos(bx+a)\right)\sin(-2a+2c)}{8\left(2b\cos(bx+a)^2\cos(-2a+2c)-2b\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-b\cos(-2a+2c)+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(3*sqrt(2)*(2*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a)*sin(-2*a
 + 2*c) - 2*(cos(-2*a + 2*c)^2 + cos(-2*a + 2*c))*cos(b*x + a)^2 + cos(-2*a
 + 2*c)^2 - 1)*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(
b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a) +
cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) -
 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*
a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a + 2*c) + 1) - 4*(4*cos(b*x +
 a)^2*cos(-2*a + 2*c) - 3*cos(-2*a + 2*c) + 5)*sin(b*x + a) - 4*(4*cos(b*x
+ a)^3 - 5*cos(b*x + a))*sin(-2*a + 2*c))/(2*b*cos(b*x + a)^2*cos(-2*a + 2*
c) - 2*b*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(a+bx)\tan^3(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*tan(b*x + c)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(b*x + c)^3, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(c + b*x)^3,x)
[Out] \text{Hanged}
```

3.231 $\int \sin(a+bx)\tan^2(c+bx)\,dx$

Optimal. Leaf size=44

$$\frac{\cos(a+bx)}{b}+\frac{\cos(a-c)\sec(c+bx)}{b}+\frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b}$$

```
[Out] cos(b*x+a)/b+cos(a-c)*sec(b*x+c)/b+arctanh(sin(b*x+c))*sin(a-c)/b
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4672, 4675, 2718, 3855, 2686, 8}

$$\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}+\frac{\cos(a-c)\sec(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[c + b*x]^2,x]
```

```
[Out] Cos[a + b*x]/b + (Cos[a - c]*Sec[c + b*x])/b + (ArcTanh[Sin[c + b*x]]*Sin[a
 - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4672

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
```

```
w, x] && NeQ[w, v]
```

Rule 4675

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\tan^2(c+bx)\,dx &= \cos(a-c)\int \sec(c+bx)\tan(c+bx)\,dx - \int \cos(a+bx)\tan(c+bx)\,dx\\ &= \frac{\cos(a-c)\mathrm{Subst}(\int 1\,dx, x, \sec(c+bx))}{b} + \sin(a-c)\int \sec(c+bx)\,dx - \int\\ &= \frac{\cos(a+bx)}{b} + \frac{\cos(a-c)\sec(c+bx)}{b} + \frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 109, normalized size = 2.48

$$\frac{\cos(a)\cos(bx)}{b} + \frac{\cos(a-c)\sec(c+bx)}{b} - \frac{2i\mathrm{ArcTan}\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b} - \frac{\sin(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[c + b*x]^2,x]
```

```
[Out] (Cos[a]*Cos[b*x])/b + (Cos[a - c]*Sec[c + b*x])/b - ((2*I)*ArcTan[((I*Cos[c
] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/
2] - I*Cos[(b*x)/2]*Sin[c])]*Sin[a - c])/b - (Sin[a]*Sin[b*x])/b
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 143, normalized size = 3.25

method	result	size
risch	$\frac{e^{i(bx+a)}}{2b} + \frac{e^{-i(bx+a)}}{2b} + \frac{e^{i(bx+3a)}+e^{i(bx+a+2c)}}{b\left(e^{2i(bx+a+c)}+e^{2ia}\right)} + \frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\sin(a-c)}{b} - \frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\sin(a-c)}{b}$	143

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+1/b/(exp(2*I*(b*x+a+c))+exp(2*I*
a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))+ln(exp(I*(b*x+a))+I*exp(I*(a-c)))
/b*sin(a-c)-ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(44) = 88$.
time = 0.55, size = 520, normalized size = 11.82

$$\frac{(\cos(3bx+a+2c)+\cos(bx+a))\cos(4bx+2a+2c)+(3\cos(2bx+2a)+3\cos(2bx+2c)+1)\cos(3bx+a+2c)+3\cos(2bx+2a)\cos(bx+a)+3\cos(2bx+2c)\cos(bx+a)+(\cos(3bx+a+2c)^2\sin(-a+c)+2\cos(3bx+a+2c)\cos(bx+a)\sin(-a+c)+\cos(bx+a)^2\sin(-a+c)+\sin(3bx+a+2c)^2\sin(-a+c)+2\sin(3bx+a+2c)\sin(bx+a)\sin(-a+c)+\sin(bx+a)^2\sin(-a+c))\log\left(\frac{\cos(bx+2c)^2+\cos(c)^2-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2+2\cos(bx+2c)\sin(c)+\sin(c)^2}{\cos(bx+2c)^2+\cos(c)^2+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2-2\cos(bx+2c)\sin(c)+\sin(c)^2}\right)+(\sin(3bx+a+2c)+\sin(bx+a))\sin(4bx+2a+2c)+3(\sin(2bx+2a)+\sin(2bx+2c))\sin(3bx+a+2c)+3\sin(2bx+2a)\sin(bx+a)+3\sin(2bx+2c)\sin(bx+a)+\cos(bx+a)}{2\,(b\cos(3bx+a+2c)^2+2b\cos(3bx+a+2c)\cos(bx+a)+b\cos(bx+a)^2+b\sin(3bx+a+2c)^2+2b\sin(3bx+a+2c)\sin(bx+a)+b\sin(bx+a)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((cos(3*b*x + a + 2*c) + cos(b*x + a))*cos(4*b*x + 2*a + 2*c) + (3*cos(
2*b*x + 2*a) + 3*cos(2*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 3*cos(2*b*x +
 2*a)*cos(b*x + a) + 3*cos(2*b*x + 2*c)*cos(b*x + a) + (cos(3*b*x + a + 2*c
)^2*sin(-a + c) + 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a + c) + cos(b*x
 + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c)^2*sin(-a + c) + 2*sin(3*b*x + a
+ 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c))*log((cos(b*x
+ 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*
x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x
 + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) + (sin(3*
b*x + a + 2*c) + sin(b*x + a))*sin(4*b*x + 2*a + 2*c) + 3*(sin(2*b*x + 2*a)
 + sin(2*b*x + 2*c))*sin(3*b*x + a + 2*c) + 3*sin(2*b*x + 2*a)*sin(b*x + a)
 + 3*sin(2*b*x + 2*c)*sin(b*x + a) + cos(b*x + a))/(b*cos(3*b*x + a + 2*c)^
2 + 2*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x
+ a + 2*c)^2 + 2*b*sin(3*b*x + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(44) = 88$.
time = 1.75, size = 315, normalized size = 7.16

$$-\frac{4(\cos(-2a+2c)+1)\cos(bx+a)^2-4\cos(bx+a)\sin(bx+a)\sin(-2a+2c)+\frac{\sqrt{2}\left((\cos(-2a+2c)+1)\cos(bx+a)\sin(-2a+2c)+\left(\cos(-2a+2c)^2-1\right)\sin(bx+a)\right)\log\left(-\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)+\frac{2\sqrt{2}((\cos(-2a+2c)+1)\sin(bx+a)+\cos(bx+a)\sin(-2a+2c))}{\sqrt{\cos(-2a+2c)+1}}-\cos(-2a+2c)-3}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)+1}\right)}{\sqrt{\cos(-2a+2c)+1}}+4\cos(-2a+2c)+4}{4(b\sin(bx+a)\sin(-2a+2c)-(b\cos(-2a+2c)+b)\cos(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)^2 - 4*cos(b*x + a)*sin(b*x + a)*
sin(-2*a + 2*c) + sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(-2*a + 2*
c) + (cos(-2*a + 2*c)^2 - 1)*sin(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2*a
+ 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a
 + 2*c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2
*c) + 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b
*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a
+ 2*c) + 1) + 4*cos(-2*a + 2*c) + 4)/(b*sin(b*x + a)*sin(-2*a + 2*c) - (b*c
os(-2*a + 2*c) + b)*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(a+bx)\tan^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)*tan(b*x+c)**2,x)

[Out] Integral(sin(a + b*x)*tan(b*x + c)**2, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(b*x+a)*tan(b*x+c)^2,x, algorithm="giac")

[Out] integrate(sin(b*x + a)*tan(b*x + c)^2, x)

Mupad [B]
time = 5.31, size = 294, normalized size = 6.68

$$\frac{e^{-a\,1i-b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}+1\right)\,1i}{b\,\left(e^{a\,2i-c\,2i}\,1i+e^{a\,2i+b\,x\,2i}\,1i\right)}+\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}-\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(a + b*x)*tan(c + b*x)^2,x)

[Out] exp(- a*1i - b*x*1i)/(2*b) + exp(a*1i + b*x*1i)/(2*b) + (exp(a*1i + b*x*1i)
*(exp(a*2i - c*2i) + 1)*1i)/(b*(exp(a*2i - c*2i)*1i + exp(a*2i + b*x*2i)*1i
)) + (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) - (exp(a*2i)
*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(
exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - (log(exp(a*1i)*exp
(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) + (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*
exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) - 1))/
(2*b*(-exp(a*2i - c*2i))^(1/2))

3.232 $\int \sin(a+bx)\tan(c+bx)\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sin(a+bx)}{b}$$

```
[Out] arctanh(sin(b*x+c))*cos(a-c)/b-sin(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {4672, 2717, 3855}

$$\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[c + b*x],x]
```

```
[Out] (ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - Sin[a + b*x]/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4672

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \sin(a+bx)\tan(c+bx)\,dx&=\cos(a-c)\int \sec(c+bx)\,dx-\int \cos(a+bx)\,dx\\&=\frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sin(a+bx)}{b}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 94, normalized size = 3.24

$$-\frac{2i\text{ArcTan}\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b}-\frac{\cos(bx)\sin(a)}{b}-\frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[c + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)
/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Cos[b
*x]*Sin[a])/b - (Cos[a]*Sin[b*x])/b
```

Maple [C] Result contains complex when optimal does not.
time = 0.10, size = 99, normalized size = 3.41

method	result	size
risch	$\frac{i e^{i(bx+a)}}{2b}-\frac{i e^{-i(bx+a)}}{2b}+\frac{\ln\left(e^{i(bx+a)}+i e^{i(a-c)}\right)\cos(a-c)}{b}-\frac{\ln\left(e^{i(bx+a)}-i e^{i(a-c)}\right)\cos(a-c)}{b}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))+ln(exp(I*(b*x+a))+I*exp(I*(a
-c)))/b*cos(a-c)-ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*cos(a-c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(29) = 58.
time = 0.53, size = 131, normalized size = 4.52

$$-\frac{\cos(-a+c)\log\left(\frac{\cos(bx+2c)^2+\cos(c)^2-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2+2\cos(bx+2c)\sin(c)+\sin(c)^2}{\cos(bx+2c)^2+\cos(c)^2+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2-2\cos(bx+2c)\sin(c)+\sin(c)^2}\right)+2\sin(bx+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(-a + c)*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c
) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^
2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c
)*sin(c) + sin(c)^2)) + 2*sin(b*x + a))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(29) = 58.
time = 2.02, size = 188, normalized size = 6.48

$$\frac{\sqrt{2}\sqrt{\cos(-2a+2c)+1}\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\frac{2\sqrt{2}\,((\cos(-2a+2c)+1)\sin(bx+a)+\cos(bx+a)\sin(-2a+2c))}{\sqrt{\cos(-2a+2c)+1}}-\cos(-2a+2c)-3}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)+1}\right)-4\sin(bx+a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(cos(-2*a + 2*c) + 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c
) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*
c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) +
 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x +
a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1)) - 4*sin(b*x + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin (a + bx) \tan (bx + c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*tan(b*x + c), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(b*x + c), x)
```

Mupad [B]
time = 5.49, size = 227, normalized size = 7.83

$$-\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b}+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(c + b*x),x)
```

```
[Out] (exp(a*1i + b*x*1i)*1i)/(2*b) - (exp(- a*1i - b*x*1i)*1i)/(2*b) + (log(- ex
p(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1) - (exp(a*2i)*exp(-c*2i)*(exp
(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i)
+ 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log((exp(a*2i)*exp(-c*2i)*(exp(a*2i)*
exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b*x*1i)*(e
xp(a*2i)*exp(-c*2i) + 1))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/
2))
```

3.233 $\int \cot(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=29

$$-\frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b}+\frac{\sin(a+bx)}{b}$$

```
[Out] -arctanh(cos(b*x+c))*sin(a-c)/b+sin(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4674, 2717, 3855}

$$\frac{\sin(a+bx)}{b}-\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + b*x]*Sin[a + b*x],x]
```

```
[Out] -((ArcTanh[Cos[c + b*x]]*Sin[a - c])/b) + Sin[a + b*x]/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4674

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cot(c+bx)\sin(a+bx)\,dx&=\sin(a-c)\int \csc(c+bx)\,dx+\int \cos(a+bx)\,dx\\&=-\frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b}+\frac{\sin(a+bx)}{b}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 93, normalized size = 3.21

$$\frac{\cos(bx)\sin(a)}{b} - \frac{2i\text{ArcTan}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b} + \frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + b*x]*Sin[a + b*x],x]
```

```
[Out] (Cos[b*x]*Sin[a])/b - ((2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/
2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*S
in[a - c])/b + (Cos[a]*Sin[b*x])/b
```

Maple [C] Result contains complex when optimal does not.
time = 0.11, size = 95, normalized size = 3.28

method	result	size
risch	$-\frac{i e^{i(bx+a)}}{2b}+\frac{i e^{-i(bx+a)}}{2b}-\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\sin(a-c)}{b}+\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\sin(a-c)}{b}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*exp(I*(b*x+a))/b+1/2*I/b*exp(-I*(b*x+a))-ln(exp(I*(b*x+a))+exp(I*(a-
c)))/b*sin(a-c)+ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*sin(a-c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(29) = 58.
time = 0.29, size = 105, normalized size = 3.62

$$\frac{\log\left(\cos(bx)^2+2\cos(bx)\cos(c)+\cos(c)^2+\sin(bx)^2-2\sin(bx)\sin(c)+\sin(c)^2\right)\sin(-a+c)-\log\left(\cos(bx)^2-2\cos(bx)\cos(c)+\cos(c)^2+\sin(bx)^2+2\sin(bx)\sin(c)+\sin(c)^2\right)\sin(-a+c)+2\sin(bx+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x
)*sin(c) + sin(c)^2)*sin(-a + c) - log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos
(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*sin(-a + c) + 2*sin(b*x
+ a))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(29) = 58.
time = 2.08, size = 197, normalized size = 6.79

$$\frac{\frac{\sqrt{2}\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)+\frac{2\sqrt{2}\left((\cos(-2a+2c)+1)\cos(bx+a)-\sin(bx+a)\sin(-2a+2c)\right)}{\sqrt{\cos(-2a+2c)+1}}-\cos(-2a+2c)+3}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)-1}\right)\sin(-2a+2c)}{\sqrt{\cos(-2a+2c)+1}}+4\sin(bx+a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x
 + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a) - sin
(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) + 3)
/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a +
 2*c) - cos(-2*a + 2*c) - 1))*sin(-2*a + 2*c)/sqrt(cos(-2*a + 2*c) + 1) + 4
*sin(b*x + a))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin{(a + bx)} \cot{(bx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*cot(b*x + c), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(29) = 58.
time = 0.41, size = 226, normalized size = 7.79

$$-\frac{2\left(\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^2\right)\log\left(\left|\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right)-1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}c\right)}-\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}bx\right)+\tan\left(\frac{1}{2}c\right)\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}+\frac{\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)^2-\tan\left(\frac{1}{2}bx\right)-2\tan\left(\frac{1}{2}a\right)}{\left(\tan\left(\frac{1}{2}bx\right)^2+1\right)\left(\tan\left(\frac{1}{2}a\right)^2+1\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -2*((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*tan(1
/2*c) - tan(1/2*c)^2)*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/(tan(1/2*a)^2*t
an(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1/2*c)) - (tan(1
/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log
(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2
+ tan(1/2*c)^2 + 1) + (tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*b*x) - 2*tan(1/2
*a))/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 + 1)))/b
```

Mupad [**B**]
time = 4.85, size = 233, normalized size = 8.03

$$\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b}-\frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b}-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + b*x)*sin(a + b*x),x)
```

```
[Out] (exp(- a*1i - b*x*1i)*1i)/(2*b) - (exp(a*1i + b*x*1i)*1i)/(2*b) - (log(- ex
p(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) - (exp(a*2i)*exp(-c*2i)*(exp
(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i)
 - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) + (log((exp(a*2i)*exp(-c*2i)*(exp(a*
2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b*x*1
i)*(exp(a*2i)*exp(-c*2i) - 1))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*
2i))^(1/2))
```

3.234 $\int \cot^2(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b}+\frac{\cos(a+bx)}{b}-\frac{\csc(c+bx)\sin(a-c)}{b}$$

```
[Out] -arctanh(cos(b*x+c))*cos(a-c)/b+cos(b*x+a)/b-csc(b*x+c)*sin(a-c)/b
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4674, 4673, 2718, 3855, 2686, 8}

$$-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a-c)\csc(bx+c)}{b}+\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) + Cos[a + b*x]/b - (Csc[c + b*x]*Si
n[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4673

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
```

```
w, x] && NeQ[w, v]
```

Rule 4674

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\int \cot^2(c+bx)\sin(a+bx)\,dx = \sin(a-c)\int \cot(c+bx)\csc(c+bx)\,dx + \int \cos(a+bx)\cot(c+bx)\,dx$$
$$= \cos(a-c)\int \csc(c+bx)\,dx - \frac{\sin(a-c)\text{Subst}(\int 1\,dx, x, \csc(c+bx))}{b} - \int \text{s}$$
$$= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b} + \frac{\cos(a+bx)}{b} - \frac{\csc(c+bx)\sin(a-c)}{b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.12, size = 111, normalized size = 2.41

$$-\frac{2i\text{ArcTan}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b} + \frac{\cos(a)\cos(bx)}{b} - \frac{\csc(c+bx)\sin(a-c)}{b} - \frac{\sin(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + b*x]^2*Sin[a + b*x],x]
[Out] ((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)
/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b + (Cos[a
]*Cos[b*x])/b - (Csc[c + b*x]*Sin[a - c])/b - (Sin[a]*Sin[b*x])/b
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 143, normalized size = 3.11

method	result	size
risch	$\frac{e^{i(bx+a)}}{2b} + \frac{e^{-i(bx+a)}}{2b} + \frac{e^{i(bx+3a)}-e^{i(bx+a+2c)}}{b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)} + \frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\cos(a-c)}{b} - \frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos(a-c)}{b}$	143

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+c)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
[Out] 1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+1/b/(-exp(2*I*(b*x+a+c))+exp(2*I
*a))*(exp(I*(b*x+3*a))-exp(I*(b*x+a+2*c)))+ln(exp(I*(b*x+a))-exp(I*(a-c)))/
b*cos(a-c)-ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(46) = 92$.
time = 0.30, size = 612, normalized size = 13.30

$$\frac{\left(\cos\left(3bx+a+2c\right)-\cos\left(bx+a\right)\right)\cos\left(4bx+2a+2c\right)-\left(3\cos\left(2bx+2a\right)-3\cos\left(2bx+2c\right)+1\right)\cos\left(3bx+a+2c\right)+3\cos\left(2bx+2a\right)\cos\left(bx+a\right)-3\cos\left(2bx+2c\right)\cos\left(bx+a\right)-\left(\cos\left(3bx+a+2c\right)^{2}\cos\left(-a+c\right)-2\cos\left(3bx+a+2c\right)\cos\left(bx+a\right)\cos\left(-a+c\right)+\cos\left(bx+a\right)^{2}\cos\left(-a+c\right)+\cos\left(-a+c\right)\sin\left(3bx+a+2c\right)^{2}-2\cos\left(-a+c\right)\sin\left(3bx+a+2c\right)\sin\left(bx+a\right)+\cos\left(-a+c\right)\sin\left(bx+a\right)^{2}\right)\log\left(\cos\left(bx\right)^{2}+2\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^{2}+\sin\left(bx\right)^{2}-2\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^{2}\right)+\left(\cos\left(3bx+a+2c\right)^{2}\cos\left(-a+c\right)-2\cos\left(3bx+a+2c\right)\cos\left(bx+a\right)\cos\left(-a+c\right)+\cos\left(bx+a\right)^{2}\cos\left(-a+c\right)+\cos\left(-a+c\right)\sin\left(3bx+a+2c\right)^{2}-2\cos\left(-a+c\right)\sin\left(3bx+a+2c\right)\sin\left(bx+a\right)+\cos\left(-a+c\right)\sin\left(bx+a\right)^{2}\right)\log\left(\cos\left(bx\right)^{2}-2\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^{2}+\sin\left(bx\right)^{2}+2\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^{2}\right)+\left(\sin\left(3bx+a+2c\right)-\sin\left(bx+a\right)\right)\sin\left(4bx+2a+2c\right)-3\left(\sin\left(2bx+2a\right)-\sin\left(2bx+2c\right)\right)\sin\left(3bx+a+2c\right)+3\sin\left(2bx+2a\right)\sin\left(bx+a\right)-3\sin\left(2bx+2c\right)\sin\left(bx+a\right)+\cos\left(bx+a\right)}{2\left(b\cos\left(3bx+a+2c\right)^{2}-2b\cos\left(3bx+a+2c\right)\cos\left(bx+a\right)+b\cos\left(bx+a\right)^{2}+b\sin\left(3bx+a+2c\right)^{2}-2b\sin\left(3bx+a+2c\right)\sin\left(bx+a\right)+b\sin\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*((cos(3*b*x + a + 2*c) - cos(b*x + a))*cos(4*b*x + 2*a + 2*c) - (3*cos(
2*b*x + 2*a) - 3*cos(2*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 3*cos(2*b*x +
 2*a)*cos(b*x + a) - 3*cos(2*b*x + 2*c)*cos(b*x + a) - (cos(3*b*x + a + 2*c
)^2*cos(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos(-a + c) + cos(b*x
 + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b*x + a + 2*c)^2 - 2*cos(-a + c)*si
n(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2)*log(cos(b*x)^
2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)^
2) + (cos(3*b*x + a + 2*c)^2*cos(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x +
 a)*cos(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b*x + a +
2*c)^2 - 2*cos(-a + c)*sin(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(
b*x + a)^2)*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*
sin(b*x)*sin(c) + sin(c)^2) + (sin(3*b*x + a + 2*c) - sin(b*x + a))*sin(4*b
*x + 2*a + 2*c) - 3*(sin(2*b*x + 2*a) - sin(2*b*x + 2*c))*sin(3*b*x + a + 2
*c) + 3*sin(2*b*x + 2*a)*sin(b*x + a) - 3*sin(2*b*x + 2*c)*sin(b*x + a) + c
os(b*x + a))/(b*cos(3*b*x + a + 2*c)^2 - 2*b*cos(3*b*x + a + 2*c)*cos(b*x +
 a) + b*cos(b*x + a)^2 + b*sin(3*b*x + a + 2*c)^2 - 2*b*sin(3*b*x + a + 2*c
)*sin(b*x + a) + b*sin(b*x + a)^2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(46) = 92$.
time = 2.21, size = 316, normalized size = 6.87

$$\frac{4\left(\cos\left(-2a+2c\right)+1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+\frac{\sqrt{2}\left(\left(\cos(-2a+2c)+1\right)\cos(bx+a)\sin(-2a+2c)+\left(\cos(-2a+2c)^{2}+2\cos(-2a+2c)+1\right)\sin(bx+a)\right)\log\left(-\frac{2\cos(bx+a)^{2}\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\frac{2\sqrt{2}\left((\cos(-2a+2c)+1)\cos(bx+a)-\sin(bx+a)\sin(-2a+2c)\right)}{\sqrt{\cos\left(-2a+2c\right)+1}}-\cos(-2a+2c)+3}{2\cos(bx+a)^{2}\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)-1}\right)}{\sqrt{\cos\left(-2a+2c\right)+1}}+4\left(\cos\left(bx+a\right)^{2}+1\right)\sin\left(-2a+2c\right)}{4\left(b\cos\left(bx+a\right)\sin\left(-2a+2c\right)+\left(b\cos\left(-2a+2c\right)+b\right)\sin\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*((cos(-2*a
 + 2*c) + 1)*cos(b*x + a)*sin(-2*a + 2*c) + (cos(-2*a + 2*c)^2 + 2*cos(-2*a
 + 2*c) + 1)*sin(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b
*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos
(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-
2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x
+ a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-2*a + 2*c) + 1) + 4*
```

```
(cos(b*x + a)^2 + 1)*sin(-2*a + 2*c))/(b*cos(b*x + a)*sin(-2*a + 2*c) + (b*
cos(-2*a + 2*c) + b)*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(a+bx)\cot^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)**2*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*cot(b*x + c)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(46) = 92.
time = 0.45, size = 577, normalized size = 12.54

$$-\frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right)-1\right|\right)}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}c\right)^{3}+\tan\left(\frac{1}{2}c\right)}-\frac{\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}a\right)^{2}+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^{2}+1\right)\log\left(\left|\tan\left(\frac{1}{2}bx\right)+\tan\left(\frac{1}{2}c\right)\right|\right)}{\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}+\tan\left(\frac{1}{2}a\right)^{2}+\tan\left(\frac{1}{2}c\right)^{2}+1}+\frac{\tan\left(\frac{1}{2}bx\right)^{3}\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}bx\right)^{3}\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{4}-\tan\left(\frac{1}{2}bx\right)^{3}\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)+6\tan\left(\frac{1}{2}bx\right)^{3}\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}bx\right)^{3}\tan\left(\frac{1}{2}c\right)^{3}+6\tan\left(\frac{1}{2}bx\right)^{2}\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{3}+3\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{4}-\tan\left(\frac{1}{2}bx\right)^{3}\tan\left(\frac{1}{2}a\right)+\tan\left(\frac{1}{2}bx\right)^{3}\tan\left(\frac{1}{2}c\right)-6\tan\left(\frac{1}{2}bx\right)^{2}\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-3\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)-2\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{2}-4\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)^{2}-3\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right)^{3}+2\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^{3}-\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)+3\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right)-2\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+4\tan\left(\frac{1}{2}c\right)^{2}}{\left(\tan\left(\frac{1}{2}bx\right)^{4}\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}bx\right)^{3}\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}bx\right)^{3}+\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right)^{2}-\tan\left(\frac{1}{2}bx\right)-\tan\left(\frac{1}{2}c\right)\right)\left(\tan\left(\frac{1}{2}a\right)^{2}\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}c\right)\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -((tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^2*tan(1/2*c) + 4*tan(1/2*a)*tan(1
/2*c)^2 - tan(1/2*c)^3 + tan(1/2*c))*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/
(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1
/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan
(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + (tan(1/2*b*x)^3*tan(1/2*a)^2
*tan(1/2*c)^3 - tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^4 - tan(1/2*b*x)^3*tan
(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*b
*x)^3*tan(1/2*c)^3 + 6*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^3 + 3*tan(1/2*b
*x)*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^4 - tan(
1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^3*tan(1/2*c) - 6*tan(1/2*b*x)^2*tan(1/
2*a)*tan(1/2*c) - 3*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(1/2*b*x)*t
an(1/2*a)*tan(1/2*c)^2 - 4*tan(1/2*a)^2*tan(1/2*c)^2 - 3*tan(1/2*b*x)*tan(1
/2*c)^3 + 2*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*b*x)*tan(1/2*a) + 3*tan(1/2*b
*x)*tan(1/2*c) - 2*tan(1/2*a)*tan(1/2*c) + 4*tan(1/2*c)^2)/((tan(1/2*b*x)^4
*tan(1/2*c) + tan(1/2*b*x)^3*tan(1/2*c)^2 - tan(1/2*b*x)^3 + tan(1/2*b*x)*t
an(1/2*c)^2 - tan(1/2*b*x) - tan(1/2*c))*(tan(1/2*a)^2*tan(1/2*c) + tan(1/2
*c))))/b
```

Mupad [B]
time = 5.26, size = 290, normalized size = 6.30

$$\frac{e^{-a\,1i-b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}-1\right)\,1i}{b\,\left(e^{a\,2i-c\,2i}\,1i-e^{a\,2i+b\,x\,2i}\,1i\right)}-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + b*x)^2*sin(a + b*x),x)
```

```
[Out] exp(- a*1i - b*x*1i)/(2*b) + exp(a*1i + b*x*1i)/(2*b) + (exp(a*1i + b*x*1i)
*(exp(a*2i - c*2i) - 1)*1i)/(b*(exp(a*2i - c*2i)*1i - exp(a*2i + b*x*2i)*1i
)) - (log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i) - (exp(a*2
i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*
(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) + (log((exp(a*2i)*exp(
-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*
1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i))*(exp(a*2i - c*2i) + 1))/(2*
b*exp(a*2i - c*2i)^(1/2))
```

3.235 $\int \cot^3(c+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=74

$$-\frac{\cos(a-c)\csc(c+bx)}{b}+\frac{3\tanh^{-1}(\cos(c+bx))\sin(a-c)}{2b}-\frac{\cot(c+bx)\csc(c+bx)\sin(a-c)}{2b}-\frac{\sin(a+bx)}{b}$$

```
[Out] -cos(a-c)*csc(b*x+c)/b+3/2*arctanh(cos(b*x+c))*sin(a-c)/b-1/2*cot(b*x+c)*cs
c(b*x+c)*sin(a-c)/b-sin(b*x+a)/b
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {4674, 4673, 2717, 3855, 2686, 8, 2691}

$$-\frac{\cos(a-c)\csc(bx+c)}{b}+\frac{3\sin(a-c)\tanh^{-1}(\cos(bx+c))}{2b}-\frac{\sin(a-c)\cot(bx+c)\csc(bx+c)}{2b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + (3*ArcTanh[Cos[c + b*x]]*Sin[a - c])/(2*b)
 - (Cot[c + b*x]*Csc[c + b*x]*Sin[a - c])/(2*b) - Sin[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4673

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4674

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cot^3(c+bx)\sin(a+bx)\,dx &= \sin(a-c)\int \cot^2(c+bx)\csc(c+bx)\,dx + \int \cos(a+bx)\cot^2(c+bx)\,dx\\ &= -\frac{\cot(c+bx)\csc(c+bx)\sin(a-c)}{2b} + \cos(a-c)\int \cot(c+bx)\csc(c+bx)\\ &= \frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{2b} - \frac{\cot(c+bx)\csc(c+bx)\sin(a-c)}{2b} - \cos(\\ &= -\frac{\cos(a-c)\csc(c+bx)}{b} + \frac{3\tanh^{-1}(\cos(c+bx))\sin(a-c)}{2b} - \cot(c+bx)\end{aligned}$$

Mathematica [A]
time = 0.41, size = 71, normalized size = 0.96

$$\frac{12\tanh^{-1}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\sin(a-c)+\csc^2(c+bx)(2\sin(a-2c-bx)-5\sin(a+bx)+\sin(a+2c+3bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (12*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Sin[a - c] + Csc[c + b*x]^2*(2*Si
n[a - 2*c - b*x] - 5*Sin[a + b*x] + Sin[a + 2*c + 3*b*x]))/(4*b)
```

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 184, normalized size = 2.49

method	result
risch	$\frac{i e^{i(bx+a)}}{2b} - \frac{i e^{-i(bx+a)}}{2b} + \frac{i\left(-3\,e^{i(3bx+5a+2c)} - e^{i(3bx+3a+4c)} + e^{i(bx+5a)} + 3\,e^{i(bx+3a+2c)}\right)}{2b\left(-e^{2i(bx+a+c)} + e^{2ia}\right)^2} + \frac{3\ln\left(e^{i(bx+a)} + e^{i(a-c)}\right)\sin(a-c)}{2b} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+c)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))+1/2*I/b/(-exp(2*I*(b*x+a+c))
+exp(2*I*a))^2*(-3*exp(I*(3*b*x+5*a+2*c))-exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x
+5*a))+3*exp(I*(b*x+3*a+2*c)))+3/2*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*sin(a-
c)-3/2*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*sin(a-c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1254 vs. $2(70) = 140$.
time = 0.32, size = 1254, normalized size = 16.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(sin(5*b*x + a + 4*c) - 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*cos(6
*b*x + 2*a + 4*c) + 2*(5*sin(4*b*x + 2*a + 2*c) + 2*sin(4*b*x + 4*c) - 2*si
n(2*b*x + 2*a) - 5*sin(2*b*x + 2*c))*cos(5*b*x + a + 4*c) + 10*(2*sin(3*b*x
 + a + 2*c) - sin(b*x + a))*cos(4*b*x + 2*a + 2*c) + 4*(2*sin(3*b*x + a + 2
*c) - sin(b*x + a))*cos(4*b*x + 4*c) + 4*(2*sin(2*b*x + 2*a) + 5*sin(2*b*x
+ 2*c))*cos(3*b*x + a + 2*c) - 3*(cos(5*b*x + a + 4*c)^2*sin(-a + c) + 4*co
s(3*b*x + a + 2*c)^2*sin(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(
-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*sin(-a + c) +
 4*sin(3*b*x + a + 2*c)^2*sin(-a + c) - 4*sin(3*b*x + a + 2*c)*sin(b*x + a)
*sin(-a + c) + sin(b*x + a)^2*sin(-a + c) - 2*(2*cos(3*b*x + a + 2*c)*sin(-
a + c) - cos(b*x + a)*sin(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*sin(3*b*x +
a + 2*c)*sin(-a + c) - sin(b*x + a)*sin(-a + c))*sin(5*b*x + a + 4*c))*log(
cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c)
+ sin(c)^2) + 3*(cos(5*b*x + a + 4*c)^2*sin(-a + c) + 4*cos(3*b*x + a + 2*c
)^2*sin(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a + c) + cos(b*x
 + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*sin(-a + c) + 4*sin(3*b*x + a
+ 2*c)^2*sin(-a + c) - 4*sin(3*b*x + a + 2*c)*sin(b*x + a)*sin(-a + c) + si
n(b*x + a)^2*sin(-a + c) - 2*(2*cos(3*b*x + a + 2*c)*sin(-a + c) - cos(b*x
+ a)*sin(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*sin(3*b*x + a + 2*c)*sin(-a +
 c) - sin(b*x + a)*sin(-a + c))*sin(5*b*x + a + 4*c))*log(cos(b*x)^2 - 2*co
s(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 2*(
cos(5*b*x + a + 4*c) - 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*sin(6*b*x + 2
*a + 4*c) - 2*(5*cos(4*b*x + 2*a + 2*c) + 2*cos(4*b*x + 4*c) - 2*cos(2*b*x
```

```
+ 2*a) - 5*cos(2*b*x + 2*c) + 1)*sin(5*b*x + a + 4*c) - 10*(2*cos(3*b*x + a
 + 2*c) - cos(b*x + a))*sin(4*b*x + 2*a + 2*c) - 4*(2*cos(3*b*x + a + 2*c)
- cos(b*x + a))*sin(4*b*x + 4*c) - 4*(2*cos(2*b*x + 2*a) + 5*cos(2*b*x + 2*
c) - 1)*sin(3*b*x + a + 2*c) - 4*cos(b*x + a)*sin(2*b*x + 2*a) - 10*cos(b*x
 + a)*sin(2*b*x + 2*c) + 4*cos(2*b*x + 2*a)*sin(b*x + a) + 10*cos(2*b*x + 2
*c)*sin(b*x + a) - 2*sin(b*x + a))/(b*cos(5*b*x + a + 4*c)^2 + 4*b*cos(3*b*
x + a + 2*c)^2 - 4*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 +
 b*sin(5*b*x + a + 4*c)^2 + 4*b*sin(3*b*x + a + 2*c)^2 - 4*b*sin(3*b*x + a
+ 2*c)*sin(b*x + a) + b*sin(b*x + a)^2 - 2*(2*b*cos(3*b*x + a + 2*c) - b*co
s(b*x + a))*cos(5*b*x + a + 4*c) - 2*(2*b*sin(3*b*x + a + 2*c) - b*sin(b*x
+ a))*sin(5*b*x + a + 4*c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(70) = 140$.
time = 2.30, size = 372, normalized size = 5.03

$$\frac{\frac{3\sqrt{2}\left(2\left(\cos(-2a+2c)^2-1\right)\cos(bx+a)\sin(bx+a)+\left(2\cos(bx+a)^2\cos(-2a+2c)-\cos(-2a+2c)-1\right)\sin(-2a+2c)\right)\log\left(-\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\frac{2\sqrt{2}\left((\cos(-2a+2c)+1)\cos(bx+a)-\sin(bx+a)\sin(-2a+2c)\right)}{\sqrt{\cos(-2a+2c)+1}}-\cos(-2a+2c)+3}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)-1}\right)}{\sqrt{\cos(-2a+2c)+1}}-4\left(4\cos(bx+a)^2\cos(-2a+2c)-3\cos(-2a+2c)-5\right)\sin(bx+a)-4\left(4\cos(bx+a)^3-5\cos(bx+a)\right)\sin(-2a+2c)}{8\left(2b\cos(bx+a)^2\cos(-2a+2c)-2b\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-b\cos(-2a+2c)-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(2)*(2*(cos(-2*a + 2*c)^2 - 1)*cos(b*x + a)*sin(b*x + a) + (2*co
s(b*x + a)^2*cos(-2*a + 2*c) - cos(-2*a + 2*c) - 1)*sin(-2*a + 2*c))*log(-(
2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2
*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a
 + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2
*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a +
 2*c) - 1))/sqrt(cos(-2*a + 2*c) + 1) - 4*(4*cos(b*x + a)^2*cos(-2*a + 2*c)
 - 3*cos(-2*a + 2*c) - 5)*sin(b*x + a) - 4*(4*cos(b*x + a)^3 - 5*cos(b*x +
a))*sin(-2*a + 2*c))/(2*b*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*b*cos(b*x + a)
*sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(a+bx)\cot^3(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+c)**3*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*cot(b*x + c)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 870 vs. $2(70) = 140$.
time = 0.45, size = 870, normalized size = 11.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(b*x+c)^3*sin(b*x+a),x, algorithm="giac")

[Out] 1/4*(12*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2)*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1/2*c)) - 12*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c))*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 8*(tan(1/2*b*x)*tan(1/2*a)^2 - tan(1/2*b*x) - 2*tan(1/2*a))/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 + 1)) - (2*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^7 + tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^7 + tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^8 - 4*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^4 + 6*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^5 - 5*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^5 + 2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^6 - 4*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^6 - tan(1/2*b*x)^2*tan(1/2*c)^7 - 2*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^7 - 6*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^3 + 5*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^3 + 4*tan(1/2*b*x)^3*tan(1/2*c)^4 - 22*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^4 + 4*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^4 + 5*tan(1/2*b*x)^2*tan(1/2*c)^5 - 14*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^5 + 2*tan(1/2*a)^2*tan(1/2*c)^5 + 4*tan(1/2*b*x)*tan(1/2*c)^6 + 2*tan(1/2*a)*tan(1/2*c)^6 - 2*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c) + 2*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^2 - 4*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^2 - 5*tan(1/2*b*x)^2*tan(1/2*c)^3 + 14*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^3 - 4*tan(1/2*b*x)*tan(1/2*c)^4 + 12*tan(1/2*a)*tan(1/2*c)^4 - 2*tan(1/2*c)^5 + tan(1/2*b*x)^2*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*c) + 2*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c) + 4*tan(1/2*b*x)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*c)^3)/((tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*c)^2)*(tan(1/2*b*x)^2*tan(1/2*c) + tan(1/2*b*x)*tan(1/2*c)^2 - tan(1/2*b*x) - tan(1/2*c))^2))/b

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(c + b*x)^3*sin(a + b*x),x)

[Out] \text{Hanged}

3.236 $\int \sin(a+bx)\tan(c+dx)\,dx$

Optimal. Leaf size=143

$$\frac{ie^{-i(a+bx)}}{2b}+\frac{ie^{i(a+bx)}}{2b}-\frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}-\frac{ie^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}$$

```
[Out] 1/2*I/b/exp(I*(b*x+a))+1/2*I*exp(I*(b*x+a))/b-I*hypergeom([1, -1/2*b/d],[1-
1/2*b/d],-exp(2*I*(d*x+c)))/b/exp(I*(b*x+a))-I*exp(I*(b*x+a))*hypergeom([1,
 1/2*b/d],[1+1/2*b/d],-exp(2*I*(d*x+c)))/b
```

Rubi [A]
time = 0.08, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4653, 2225, 2283}

$$-\frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}-\frac{ie^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2i(c+dx)}\right)}{b}+\frac{ie^{-i(a+bx)}}{2b}+\frac{ie^{i(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]*Tan[c + d*x],x]
```

```
[Out] (I/2)/(b*E^(I*(a + b*x))) + ((I/2)*E^(I*(a + b*x)))/b - (I*Hypergeometric2F
1[1, -1/2*b/d, 1 - b/(2*d), -E^((2*I)*(c + d*x))])/(b*E^(I*(a + b*x))) - (I
*E^(I*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c +
d*x))])/b
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4653

```
Int[Sin[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(E^
(I*(a + b*x))*2) - E^(I*(a + b*x))/2 - 1/(E^(I*(a + b*x))*(1 + E^(2*I*(c +
d*x)))) + E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(a+bx)\tan(c+dx)\,dx = \int \left(\frac{1}{2}e^{-i(a+bx)} - \frac{1}{2}e^{i(a+bx)} - \frac{e^{-i(a+bx)}}{1+e^{2i(c+dx)}} + \frac{e^{i(a+bx)}}{1+e^{2i(c+dx)}}\right) dx$$
$$= \frac{1}{2}\int e^{-i(a+bx)}\,dx - \frac{1}{2}\int e^{i(a+bx)}\,dx - \int \frac{e^{-i(a+bx)}}{1+e^{2i(c+dx)}}\,dx + \int \frac{e^{i(a+bx)}}{1+e^{2i(c+dx)}}\,dx$$
$$= \frac{ie^{-i(a+bx)}}{2b} + \frac{ie^{i(a+bx)}}{2b} - \frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b} - \frac{ie^{i(a+bx)}\,{}_2$$

Mathematica [A]
time = 1.45, size = 116, normalized size = 0.81

$$-\frac{ie^{-i(a+bx)}\left(-1-e^{2i(a+bx)}+2\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)+2e^{2i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d};-e^{2i(c+dx)}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]*Tan[c + d*x],x]
```

```
[Out] ((-1/2*I)*(-1 - E^((2*I)*(a + b*x)) + 2*Hypergeometric2F1[1, -1/2*b/d, 1 -
b/(2*d), -E^((2*I)*(c + d*x))] + 2*E^((2*I)*(a + b*x))*Hypergeometric2F1[1,
 b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))]))/(b*E^(I*(a + b*x)))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \sin\left(bx+a\right)\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(d*x+c),x)
```

```
[Out] int(sin(b*x+a)*tan(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sin(b*x + a)*tan(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="fricas")
[Out] integral(sin(b*x + a)*tan(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin{(a+bx)}\tan{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(d*x+c),x)
[Out] Integral(sin(a + b*x)*tan(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(d*x+c),x, algorithm="giac")
[Out] integrate(sin(b*x + a)*tan(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,\mathrm{tan}\left(c+d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(c + d*x),x)
[Out] int(sin(a + b*x)*tan(c + d*x), x)
```

3.237 $\int \cot(c+dx)\sin(a+bx)\,dx$

Optimal. Leaf size=139

$$-\frac{ie^{-i(a+bx)}}{2b}-\frac{ie^{i(a+bx)}}{2b}+\frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b}+\frac{ie^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d};e^{2i(c+dx)}\right)}{b}$$

```
[Out] -1/2*I/b/exp(I*(b*x+a))-1/2*I*exp(I*(b*x+a))/b+I*hypergeom([1, -1/2*b/d],[1
-1/2*b/d],exp(2*I*(d*x+c)))/b/exp(I*(b*x+a))+I*exp(I*(b*x+a))*hypergeom([1,
 1/2*b/d],[1+1/2*b/d],exp(2*I*(d*x+c)))/b
```

Rubi [A]
time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4655, 2225, 2283}

$$\frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b}+\frac{ie^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2i(c+dx)}\right)}{b}-\frac{ie^{-i(a+bx)}}{2b}-\frac{ie^{i(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sin[a + b*x],x]
```

```
[Out] (-1/2*I)/(b*E^(I*(a + b*x))) - ((I/2)*E^(I*(a + b*x)))/b + (I*Hypergeometri
c2F1[1, -1/2*b/d, 1 - b/(2*d), E^((2*I)*(c + d*x))])/(b*E^(I*(a + b*x))) +
(I*E^(I*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c +
 d*x))])/b
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4655

```
Int[Cot[(c_.) + (d_.)*(x_)]*Sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Int[-E^((
-I)*(a + b*x))/2 + E^(I*(a + b*x))/2 + 1/(E^(I*(a + b*x))*(1 - E^(2*I*(c +
d*x)))) - E^(I*(a + b*x))/(1 - E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\sin(a+bx)\,dx &= \int \left(-\frac{1}{2}e^{-i(a+bx)}+\frac{1}{2}e^{i(a+bx)}+\frac{e^{-i(a+bx)}}{1-e^{2i(c+dx)}}-\frac{e^{i(a+bx)}}{1-e^{2i(c+dx)}}\right)dx \\ &= -\left(\frac{1}{2}\int e^{-i(a+bx)}\,dx\right)+\frac{1}{2}\int e^{i(a+bx)}\,dx+\int \frac{e^{-i(a+bx)}}{1-e^{2i(c+dx)}}\,dx-\int \frac{e^{i(a+bx}}{1-e^{2i(c+} \\ &= -\frac{ie^{-i(a+bx)}}{2b}-\frac{ie^{i(a+bx)}}{2b}+\frac{ie^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b}+\frac{ie^{i(a+bx)}}{}\end{aligned}$$

Mathematica [A]
time = 3.14, size = 260, normalized size = 1.87

$$\frac{-\cos(a)\cos(bx)\cot(c)-\frac{ie^{-i(a-2c+bx)}\left(be^{2idx}\,{}_2F_1\left(1,1-\frac{b}{2d};2-\frac{b}{2d};e^{2i(c+dx)}\right)-(b-2d)\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)\right)}{(b-2d)(-1+e^{2ic})}-\frac{ie^{i(a+2c+bx)}\left(be^{2idx}\,{}_2F_1\left(1,1+\frac{b}{2d};2+\frac{b}{2d};e^{2i(c+dx)}\right)-(b+2d)\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d};e^{2i(c+dx)}\right)\right)}{(b+2d)(-1+e^{2ic})}+\cot(c)\sin(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sin[a + b*x],x]
```

```
[Out] (-(Cos[a]*Cos[b*x]*Cot[c]) - (I*(b*E^((2*I)*d*x)*Hypergeometric2F1[1, 1 - b
/(2*d), 2 - b/(2*d), E^((2*I)*(c + d*x))] - (b - 2*d)*Hypergeometric2F1[1,
-1/2*b/d, 1 - b/(2*d), E^((2*I)*(c + d*x))]))/((b - 2*d)*E^(I*(a - 2*c + b*
x))*(-1 + E^((2*I)*c))) - (I*E^(I*(a + 2*c + b*x))*(b*E^((2*I)*d*x)*Hyperge
ometric2F1[1, 1 + b/(2*d), 2 + b/(2*d), E^((2*I)*(c + d*x))] - (b + 2*d)*Hy
pergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d*x))]))/((b + 2*d)*
(-1 + E^((2*I)*c))) + Cot[c]*Sin[a]*Sin[b*x])/b
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \cot\left(dx+c\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*sin(b*x+a),x)
```

```
[Out] int(cot(d*x+c)*sin(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)*sin(b*x + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(cot(d*x + c)*sin(b*x + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sin(a + bx)\cot(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*sin(b*x+a),x)
```

```
[Out] Integral(sin(a + b*x)*cot(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)*sin(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c + d\,x)\,\sin\left(a + b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*sin(a + b*x),x)
```

```
[Out] int(cot(c + d*x)*sin(a + b*x), x)
```

3.238 $\int \cos(a+bx)\cos^3(c+dx)\,dx$

Optimal. Leaf size=91

$$\frac{\sin(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)}+\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(a+3c+(b+3d)x)}{8(b+3d)}$$

```
[Out] 1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sin(a-c+(b-d)*x)/(b-d)+3/8*sin(a+c+(b+
d)*x)/(b+d)+1/8*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4666, 2717}

$$\frac{\sin(a+x(b-3d)-3c)}{8(b-3d)}+\frac{3\sin(a+x(b-d)-c)}{8(b-d)}+\frac{3\sin(a+x(b+d)+c)}{8(b+d)}+\frac{\sin(a+x(b+3d)+3c)}{8(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cos[c + d*x]^3,x]
```

```
[Out] Sin[a - 3*c + (b - 3*d)*x]/(8*(b - 3*d)) + (3*Sin[a - c + (b - d)*x])/(8*(b
 - d)) + (3*Sin[a + c + (b + d)*x])/(8*(b + d)) + Sin[a + 3*c + (b + 3*d)*x
]/(8*(b + 3*d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4666

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cos^3(c+dx)\,dx &= \int \left(\frac{1}{8}\cos(a-3c+(b-3d)x)+\frac{3}{8}\cos(a-c+(b-d)x)+\frac{3}{8}\cos(a+c+\right.\\ &= \frac{1}{8}\int \cos(a-3c+(b-3d)x)\,dx+\frac{1}{8}\int \cos(a+3c+(b+3d)x)\,dx+\frac{3}{8}\int\\ &= \frac{\sin(a-3c+(b-3d)x)}{8(b-3d)}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)}+\frac{3\sin(a+c+(b+d)}{8(b+d)}\end{aligned}$$

Mathematica [A]
time = 0.55, size = 85, normalized size = 0.93

$$\frac{1}{8}\left(\frac{\sin(a-3c+bx-3dx)}{b-3d}+\frac{3\sin(a-c+bx-dx)}{b-d}+\frac{\sin(a+3c+bx+3dx)}{b+3d}+\frac{3\sin(a+c+(b+d)x)}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cos[c + d*x]^3,x]
```

```
[Out] (Sin[a - 3*c + b*x - 3*d*x]/(b - 3*d) + (3*Sin[a - c + b*x - d*x])/(b - d)
+ Sin[a + 3*c + b*x + 3*d*x]/(b + 3*d) + (3*Sin[a + c + (b + d)*x])/(b + d)
)/8
```

Maple [A]
time = 0.22, size = 84, normalized size = 0.92

method	result
default	$\frac{\sin(a-3c+(b-3d)x)}{8b-24d}+\frac{3\sin(a-c+(b-d)x)}{8(b-d)}+\frac{3\sin(a+c+(b+d)x)}{8(b+d)}+\frac{\sin(a+3c+(b+3d)x)}{8b+24d}$
risch	$\frac{\sin(bx-3dx+a-3c)b}{8(b-3d)(b+3d)}+\frac{3\sin(bx-3dx+a-3c)d}{8(b-3d)(b+3d)}+\frac{3\sin(bx-dx+a-c)b}{8(b-d)(b+d)}+\frac{3\sin(bx-dx+a-c)d}{8(b-d)(b+d)}+\frac{3\sin(bx+dx+a+c)b}{8(b-d)(b+d)}-\frac{3\sin(bx}{8(b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cos(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*sin(a-3*c+(b-3*d)*x)/(b-3*d)+3/8*sin(a-c+(b-d)*x)/(b-d)+3/8*sin(a+c+(b+
d)*x)/(b+d)+1/8*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 914 vs. 2(83) = 166.
time = 0.34, size = 914, normalized size = 10.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) + 3*d^3*sin(3*c))*
cos((b + 3*d)*x + a + 6*c) - (b^3*sin(3*c) - 3*b^2*d*sin(3*c) - b*d^2*sin(3
*c) + 3*d^3*sin(3*c))*cos((b + 3*d)*x + a) + 3*(b^3*sin(3*c) - b^2*d*sin(3*
c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x + a + 4*c) - 3*(b^3*s
in(3*c) - b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) + 9*d^3*sin(3*c))*cos((b + d)*x
 + a - 2*c) - 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b*d^2*sin(3*c) - 9*d^3*s
in(3*c))*cos(-(b - d)*x - a + 4*c) + 3*(b^3*sin(3*c) + b^2*d*sin(3*c) - 9*b
*d^2*sin(3*c) - 9*d^3*sin(3*c))*cos(-(b - d)*x - a - 2*c) - (b^3*sin(3*c) +
 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))*cos(-(b - 3*d)*x - a +
```

```
6*c) + (b^3*sin(3*c) + 3*b^2*d*sin(3*c) - b*d^2*sin(3*c) - 3*d^3*sin(3*c))
*cos(-(b - 3*d)*x - a) - (b^3*cos(3*c) - 3*b^2*d*cos(3*c) - b*d^2*cos(3*c)
+ 3*d^3*cos(3*c))*sin((b + 3*d)*x + a + 6*c) - (b^3*cos(3*c) - 3*b^2*d*cos(
3*c) - b*d^2*cos(3*c) + 3*d^3*cos(3*c))*sin((b + 3*d)*x + a) - 3*(b^3*cos(3
*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3*c))*sin((b + d)*x + a
 + 4*c) - 3*(b^3*cos(3*c) - b^2*d*cos(3*c) - 9*b*d^2*cos(3*c) + 9*d^3*cos(3
*c))*sin((b + d)*x + a - 2*c) + 3*(b^3*cos(3*c) + b^2*d*cos(3*c) - 9*b*d^2*
cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 3*(b^3*cos(3*c) + b^
2*d*cos(3*c) - 9*b*d^2*cos(3*c) - 9*d^3*cos(3*c))*sin(-(b - d)*x - a - 2*c)
 + (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c) - 3*d^3*cos(3*c))*sin(
-(b - 3*d)*x - a + 6*c) + (b^3*cos(3*c) + 3*b^2*d*cos(3*c) - b*d^2*cos(3*c)
 - 3*d^3*cos(3*c))*sin(-(b - 3*d)*x - a))/(b^4*cos(3*c)^2 + b^4*sin(3*c)^2
+ 9*(cos(3*c)^2 + sin(3*c)^2)*d^4 - 10*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^
2)
```

Fricas [**A**]
time = 2.01, size = 109, normalized size = 1.20

$$-\frac{\left(6\,bd^{2}\cos\left(dx+c\right)-\left(b^{3}-bd^{2}\right)\cos\left(dx+c\right)^{3}\right)\sin\left(bx+a\right)-3\,\left(2\,d^{3}\cos\left(bx+a\right)-\left(b^{2}d-d^{3}\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{b^{4}-10\,b^{2}d^{2}+9\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -((6*b*d^2*cos(d*x + c) - (b^3 - b*d^2)*cos(d*x + c)^3)*sin(b*x + a) - 3*(2
*d^3*cos(b*x + a) - (b^2*d - d^3)*cos(b*x + a)*cos(d*x + c)^2)*sin(d*x + c)
)/(b^4 - 10*b^2*d^2 + 9*d^4)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(76) = 152$.
time = 2.29, size = 921, normalized size = 10.12

$$\begin{cases} x\cos\left(a\right)\cos^{3}\left(c\right) & \text{for}\: b=0\wedge d=0 \\ \frac{x\sin\left(a-3dx\right)\sin^{3}\left(c+dx\right)}{8}-\frac{3x\sin\left(a-3dx\right)\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}-\frac{3x\sin^{2}\left(c+dx\right)\cos\left(a-3dx\right)\cos\left(c+dx\right)}{8}+\frac{x\cos\left(a-3dx\right)\cos^{3}\left(c+dx\right)}{8}-\frac{\sin\left(a-3dx\right)\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{4d}-\frac{7\sin\left(a-3dx\right)\cos^{3}\left(c+dx\right)}{24d}-\frac{\sin^{3}\left(c+dx\right)\cos\left(a-3dx\right)}{8d} & \text{for}\: b=-3d \\ -\frac{3x\sin\left(a-dx\right)\sin^{3}\left(c+dx\right)}{8}-\frac{3x\sin\left(a-dx\right)\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}+\frac{3x\sin^{2}\left(c+dx\right)\cos\left(a-dx\right)\cos\left(c+dx\right)}{8}+\frac{3x\cos\left(a-dx\right)\cos^{3}\left(c+dx\right)}{8}-\frac{3\sin\left(a-dx\right)\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{4d}-\frac{5\sin\left(a-dx\right)\cos^{3}\left(c+dx\right)}{8d}-\frac{3\sin^{3}\left(c+dx\right)\cos\left(a-dx\right)}{8d} & \text{for}\: b=-d \\ \frac{3x\sin\left(a+dx\right)\sin^{3}\left(c+dx\right)}{8}+\frac{3x\sin\left(a+dx\right)\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}+\frac{3x\sin^{2}\left(c+dx\right)\cos\left(a+dx\right)\cos\left(c+dx\right)}{8}+\frac{3x\cos\left(a+dx\right)\cos^{3}\left(c+dx\right)}{8}+\frac{3\sin\left(a+dx\right)\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{4d}+\frac{5\sin\left(a+dx\right)\cos^{3}\left(c+dx\right)}{8d}-\frac{3\sin^{3}\left(c+dx\right)\cos\left(a+dx\right)}{8d} & \text{for}\: b=d \\ -\frac{x\sin\left(a+3dx\right)\sin^{3}\left(c+dx\right)}{8}+\frac{3x\sin\left(a+3dx\right)\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{8}-\frac{3x\sin^{2}\left(c+dx\right)\cos\left(a+3dx\right)\cos\left(c+dx\right)}{8}+\frac{x\cos\left(a+3dx\right)\cos^{3}\left(c+dx\right)}{8}+\frac{\sin\left(a+3dx\right)\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{4d}+\frac{7\sin\left(a+3dx\right)\cos^{3}\left(c+dx\right)}{24d}-\frac{\sin^{3}\left(c+dx\right)\cos\left(a+3dx\right)}{8d} & \text{for}\: b=3d \\ \frac{b^{3}\sin\left(a+bx\right)\cos^{3}\left(c+dx\right)}{b^{4}-10b^{2}d^{2}+9d^{4}}-\frac{3b^{2}d\sin\left(c+dx\right)\cos\left(a+bx\right)\cos^{2}\left(c+dx\right)}{b^{4}-10b^{2}d^{2}+9d^{4}}-\frac{6bd^{2}\sin\left(a+bx\right)\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)}{b^{4}-10b^{2}d^{2}+9d^{4}}-\frac{7bd^{2}\sin\left(a+bx\right)\cos^{3}\left(c+dx\right)}{b^{4}-10b^{2}d^{2}+9d^{4}}+\frac{6d^{3}\sin^{3}\left(c+dx\right)\cos\left(a+bx\right)}{b^{4}-10b^{2}d^{2}+9d^{4}}+\frac{9d^{3}\sin\left(c+dx\right)\cos\left(a+bx\right)\cos^{2}\left(c+dx\right)}{b^{4}-10b^{2}d^{2}+9d^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)**3,x)
```

```
[Out] Piecewise((x*cos(a)*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (x*sin(a - 3*d*x)*sin(
c + d*x)**3/8 - 3*x*sin(a - 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*sin
(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/8 + x*cos(a - 3*d*x)*cos(c + d*x)*
*3/8 - sin(a - 3*d*x)*sin(c + d*x)**2*cos(c + d*x)/(4*d) - 7*sin(a - 3*d*x)
*cos(c + d*x)**3/(24*d) - sin(c + d*x)**3*cos(a - 3*d*x)/(8*d), Eq(b, -3*d)
), (-3*x*sin(a - d*x)*sin(c + d*x)**3/8 - 3*x*sin(a - d*x)*sin(c + d*x)*cos
```

```
(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a - d*x)*cos(c + d*x)/8 + 3*x*cos(
a - d*x)*cos(c + d*x)**3/8 - 3*sin(a - d*x)*sin(c + d*x)**2*cos(c + d*x)/(4
*d) - 5*sin(a - d*x)*cos(c + d*x)**3/(8*d) - 3*sin(c + d*x)**3*cos(a - d*x)
/(8*d), Eq(b, -d)), (3*x*sin(a + d*x)*sin(c + d*x)**3/8 + 3*x*sin(a + d*x)*
sin(c + d*x)*cos(c + d*x)**2/8 + 3*x*sin(c + d*x)**2*cos(a + d*x)*cos(c + d
*x)/8 + 3*x*cos(a + d*x)*cos(c + d*x)**3/8 + 3*sin(a + d*x)*sin(c + d*x)**2
*cos(c + d*x)/(4*d) + 5*sin(a + d*x)*cos(c + d*x)**3/(8*d) - 3*sin(c + d*x)
**3*cos(a + d*x)/(8*d), Eq(b, d)), (-x*sin(a + 3*d*x)*sin(c + d*x)**3/8 + 3
*x*sin(a + 3*d*x)*sin(c + d*x)*cos(c + d*x)**2/8 - 3*x*sin(c + d*x)**2*cos(
a + 3*d*x)*cos(c + d*x)/8 + x*cos(a + 3*d*x)*cos(c + d*x)**3/8 + sin(a + 3*
d*x)*sin(c + d*x)**2*cos(c + d*x)/(4*d) + 7*sin(a + 3*d*x)*cos(c + d*x)**3/
(24*d) - sin(c + d*x)**3*cos(a + 3*d*x)/(8*d), Eq(b, 3*d)), (b**3*sin(a + b
*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) - 3*b**2*d*sin(c + d*x)*
cos(a + b*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4) - 6*b*d**2*sin(
a + b*x)*sin(c + d*x)**2*cos(c + d*x)/(b**4 - 10*b**2*d**2 + 9*d**4) - 7*b*
d**2*sin(a + b*x)*cos(c + d*x)**3/(b**4 - 10*b**2*d**2 + 9*d**4) + 6*d**3*s
in(c + d*x)**3*cos(a + b*x)/(b**4 - 10*b**2*d**2 + 9*d**4) + 9*d**3*sin(c +
 d*x)*cos(a + b*x)*cos(c + d*x)**2/(b**4 - 10*b**2*d**2 + 9*d**4), True))
```

Giac [A]
time = 0.39, size = 84, normalized size = 0.92

$$\frac{\sin\left(bx+3\,dx+a+3\,c\right)}{8\,(b+3\,d)}+\frac{3\,\sin\left(bx+dx+a+c\right)}{8\,(b+d)}+\frac{3\,\sin\left(bx-dx+a-c\right)}{8\,(b-d)}+\frac{\sin\left(bx-3\,dx+a-3\,c\right)}{8\,(b-3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 3/8*sin(b*x + d*x + a + c)/(b +
d) + 3/8*sin(b*x - d*x + a - c)/(b - d) + 1/8*sin(b*x - 3*d*x + a - 3*c)/(b
 - 3*d)
```

Mupad [B]
time = 1.95, size = 313, normalized size = 3.44

$$\mathrm{e}^{a\,1\mathrm{i}-c\,3\mathrm{i}+b\,x\,1\mathrm{i}-d\,x\,3\mathrm{i}}\,\left(\frac{b+3\,d}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(b-3\,d\right)}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}\right)+\mathrm{e}^{a\,1\mathrm{i}+c\,3\mathrm{i}+b\,x\,1\mathrm{i}+d\,x\,3\mathrm{i}}\,\left(\frac{b-3\,d}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(b+3\,d\right)}{b^2\,16\mathrm{i}-d^2\,144\mathrm{i}}\right)+\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}+b\,x\,1\mathrm{i}-d\,x\,1\mathrm{i}}\,\left(\frac{3\,b+3\,d}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(3\,b-3\,d\right)}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}\right)+\mathrm{e}^{a\,1\mathrm{i}+c\,1\mathrm{i}+b\,x\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,\left(\frac{3\,b-3\,d}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}-\frac{\mathrm{e}^{-a\,2\mathrm{i}-b\,x\,2\mathrm{i}}\,\left(3\,b+3\,d\right)}{b^2\,16\mathrm{i}-d^2\,16\mathrm{i}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cos(c + d*x)^3,x)
```

```
[Out] exp(a*1i - c*3i + b*x*1i - d*x*3i)*((b + 3*d)/(b^2*16i - d^2*144i) - (exp(-
 a*2i - b*x*2i)*(b - 3*d))/(b^2*16i - d^2*144i)) + exp(a*1i + c*3i + b*x*1i
 + d*x*3i)*((b - 3*d)/(b^2*16i - d^2*144i) - (exp(- a*2i - b*x*2i)*(b + 3*d
))/(b^2*16i - d^2*144i)) + exp(a*1i - c*1i + b*x*1i - d*x*1i)*((3*b + 3*d)/
(b^2*16i - d^2*16i) - (exp(- a*2i - b*x*2i)*(3*b - 3*d))/(b^2*16i - d^2*16i
)) + exp(a*1i + c*1i + b*x*1i + d*x*1i)*((3*b - 3*d)/(b^2*16i - d^2*16i) -
(exp(- a*2i - b*x*2i)*(3*b + 3*d))/(b^2*16i - d^2*16i))
```

3.239 $\int \cos(a+bx)\cos^2(c+dx)\,dx$

Optimal. Leaf size=62

$$\frac{\sin(a+bx)}{2b}+\frac{\sin(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\sin(a+2c+(b+2d)x)}{4(b+2d)}$$

```
[Out] 1/2*sin(b*x+a)/b+1/4*sin(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sin(a+2*c+(b+2*d)*x)/
(b+2*d)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {4666, 2717}

$$\frac{\sin(a+x(b-2d)-2c)}{4(b-2d)}+\frac{\sin(a+x(b+2d)+2c)}{4(b+2d)}+\frac{\sin(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cos[c + d*x]^2,x]
```

```
[Out] Sin[a + b*x]/(2*b) + Sin[a - 2*c + (b - 2*d)*x]/(4*(b - 2*d)) + Sin[a + 2*c
 + (b + 2*d)*x]/(4*(b + 2*d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4666

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{array}{l}\int \cos(a+bx)\cos^2(c+dx)\,dx = \int \left(\frac{1}{2}\cos(a+bx)+\frac{1}{4}\cos(a-2c+(b-2d)x)+\frac{1}{4}\cos(a+2c+(b+2d)x\right. \\ \quad =\frac{1}{4}\int \cos(a-2c+(b-2d)x)\,dx+\frac{1}{4}\int \cos(a+2c+(b+2d)x)\,dx+\frac{1}{2}\int \\ \quad =\frac{\sin(a+bx)}{2b}+\frac{\sin(a-2c+(b-2d)x)}{4(b-2d)}+\frac{\sin(a+2c+(b+2d)x)}{4(b+2d)}\end{array}$$

Mathematica [A]
time = 0.81, size = 69, normalized size = 1.11

$$\frac{1}{4}\left(\frac{2\cos(bx)\sin(a)}{b}+\frac{2\cos(a)\sin(bx)}{b}+\frac{\sin(a-2c+bx-2dx)}{b-2d}+\frac{\sin(a+2c+bx+2dx)}{b+2d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cos[c + d*x]^2,x]
```

```
[Out] ((2*Cos[b*x]*Sin[a])/b + (2*Cos[a]*Sin[b*x])/b + Sin[a - 2*c + b*x - 2*d*x]
/(b - 2*d) + Sin[a + 2*c + b*x + 2*d*x]/(b + 2*d))/4
```

Maple [A]
time = 0.23, size = 57, normalized size = 0.92

method	result
default	$\frac{\sin(bx+a)}{2b}+\frac{\sin(a-2c+(b-2d)x)}{4b-8d}+\frac{\sin(a+2c+(b+2d)x)}{4b+8d}$
risch	$\frac{\sin(bx+a)}{2b}+\frac{\sin(bx-2dx+a-2c)b}{4(b-2d)(b+2d)}+\frac{\sin(bx-2dx+a-2c)d}{2(b-2d)(b+2d)}+\frac{\sin(bx+2dx+a+2c)b}{4(b-2d)(b+2d)}-\frac{\sin(bx+2dx+a+2c)d}{2(b-2d)(b+2d)}$
norman	$\frac{-\frac{4d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-4d^2}+\frac{4d\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2-4d^2}+\frac{4d\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-4d^2}-\frac{4d\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2-4d^2}+\frac{2\left(b^2-2d^2\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\left(b^2-4d^2\right)b}+2\left(b^2-\right.}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cos(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(b*x+a)/b+1/4*sin(a-2*c+(b-2*d)*x)/(b-2*d)+1/4*sin(a+2*c+(b+2*d)*x)/
(b+2*d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. 2(56) = 112.
time = 0.34, size = 416, normalized size = 6.71

$$-\frac{(b^2\sin(2c)-2bd\sin(2c))\cos((b+2d)x+a+4c)-(b^2\sin(2c)-2bd\sin(2c))\cos((b+2d)x+a)-(b^2\sin(2c)+2bd\sin(2c))\cos(-(b-2d)x-a+4c)+(b^2\sin(2c)+2bd\sin(2c))\cos(-(b-2d)x-a)+2(b^2\sin(2c)-4d^2\sin(2c))\cos(bx+a+2c)-2(b^2\sin(2c)-4d^2\sin(2c))\cos(bx+a-2c)-(b^2\cos(2c)-2bd\cos(2c))\sin((b+2d)x+a+4c)-(b^2\cos(2c)-2bd\cos(2c))\sin((b+2d)x+a)+(b^2\cos(2c)+2bd\cos(2c))\sin(-(b-2d)x-a+4c)+(b^2\cos(2c)+2bd\cos(2c))\sin(-(b-2d)x-a)-2(b^2\cos(2c)-4d^2\cos(2c))\sin(bx+a+2c)-2(b^2\cos(2c)-4d^2\cos(2c))\sin(bx+a-2c)}{8\left(b^3\cos(2c)^2+b^3\sin(2c)^2-4\left(b\cos(2c)^2+b\sin(2c)^2\right)d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((b^2*sin(2*c) - 2*b*d*sin(2*c))*cos((b + 2*d)*x + a + 4*c) - (b^2*sin
(2*c) - 2*b*d*sin(2*c))*cos((b + 2*d)*x + a) - (b^2*sin(2*c) + 2*b*d*sin(2*
c))*cos(-(b - 2*d)*x - a + 4*c) + (b^2*sin(2*c) + 2*b*d*sin(2*c))*cos(-(b -
 2*d)*x - a) + 2*(b^2*sin(2*c) - 4*d^2*sin(2*c))*cos(b*x + a + 2*c) - 2*(b^
2*sin(2*c) - 4*d^2*sin(2*c))*cos(b*x + a - 2*c) - (b^2*cos(2*c) - 2*b*d*cos
(2*c))*sin((b + 2*d)*x + a + 4*c) - (b^2*cos(2*c) - 2*b*d*cos(2*c))*sin((b
+ 2*d)*x + a) + (b^2*cos(2*c) + 2*b*d*cos(2*c))*sin(-(b - 2*d)*x - a + 4*c)
```

```
 + (b^2*cos(2*c) + 2*b*d*cos(2*c))*sin(-(b - 2*d)*x - a) - 2*(b^2*cos(2*c)
- 4*d^2*cos(2*c))*sin(b*x + a + 2*c) - 2*(b^2*cos(2*c) - 4*d^2*cos(2*c))*si
n(b*x + a - 2*c))/(b^3*cos(2*c)^2 + b^3*sin(2*c)^2 - 4*(b*cos(2*c)^2 + b*si
n(2*c)^2)*d^2)
```

Fricas [A]
time = 2.39, size = 63, normalized size = 1.02

$$-\frac{2\,bd\cos\left(bx+a\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(b^2\cos\left(dx+c\right)^2-2\,d^2\right)\sin\left(bx+a\right)}{b^3-4\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(2*b*d*cos(b*x + a)*cos(d*x + c)*sin(d*x + c) - (b^2*cos(d*x + c)^2 - 2*d^
2)*sin(b*x + a))/(b^3 - 4*b*d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(49) = 98.
time = 0.80, size = 408, normalized size = 6.58

$$\begin{cases} x\cos\left(a\right)\cos^2\left(c\right) & \text{for}\: b=0\wedge d=0 \\ \left(\frac{x\sin^2\left(c+dx\right)}{2}+\frac{x\cos^2\left(c+dx\right)}{2}+\frac{\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}\right)\cos\left(a\right) & \text{for}\: b=0 \\ -\frac{x\sin\left(a-2dx\right)\sin\left(c+dx\right)\cos\left(c+dx\right)}{2}-\frac{x\sin^2\left(c+dx\right)\cos\left(a-2dx\right)}{4}+\frac{x\cos\left(a-2dx\right)\cos^2\left(c+dx\right)}{4}-\frac{\sin\left(a-2dx\right)\sin^2\left(c+dx\right)}{2d}+\frac{3\sin\left(c+dx\right)\cos\left(a-2dx\right)\cos\left(c+dx\right)}{4d} & \text{for}\: b=-2d \\ \frac{x\sin\left(a+2dx\right)\sin\left(c+dx\right)\cos\left(c+dx\right)}{2}-\frac{x\sin^2\left(c+dx\right)\cos\left(a+2dx\right)}{4}+\frac{x\cos\left(a+2dx\right)\cos^2\left(c+dx\right)}{4}+\frac{\sin\left(a+2dx\right)\sin^2\left(c+dx\right)}{2d}+\frac{3\sin\left(c+dx\right)\cos\left(a+2dx\right)\cos\left(c+dx\right)}{4d} & \text{for}\: b=2d \\ \frac{b^2\sin\left(a+bx\right)\cos^2\left(c+dx\right)}{b^3-4bd^2}-\frac{2bd\sin\left(c+dx\right)\cos\left(a+bx\right)\cos\left(c+dx\right)}{b^3-4bd^2}-\frac{2d^2\sin\left(a+bx\right)\sin^2\left(c+dx\right)}{b^3-4bd^2}-\frac{2d^2\sin\left(a+bx\right)\cos^2\left(c+dx\right)}{b^3-4bd^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)**2,x)
```

```
[Out] Piecewise((x*cos(a)*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2/2
+ x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*cos(a), Eq(b, 0)),
 (-x*sin(a - 2*d*x)*sin(c + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)**2*cos(a -
 2*d*x)/4 + x*cos(a - 2*d*x)*cos(c + d*x)**2/4 - sin(a - 2*d*x)*sin(c + d*x
)**2/(2*d) + 3*sin(c + d*x)*cos(a - 2*d*x)*cos(c + d*x)/(4*d), Eq(b, -2*d))
, (x*sin(a + 2*d*x)*sin(c + d*x)*cos(c + d*x)/2 - x*sin(c + d*x)**2*cos(a +
 2*d*x)/4 + x*cos(a + 2*d*x)*cos(c + d*x)**2/4 + sin(a + 2*d*x)*sin(c + d*x
)**2/(2*d) + 3*sin(c + d*x)*cos(a + 2*d*x)*cos(c + d*x)/(4*d), Eq(b, 2*d)),
 (b**2*sin(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*d**2) - 2*b*d*sin(c + d*x)*
cos(a + b*x)*cos(c + d*x)/(b**3 - 4*b*d**2) - 2*d**2*sin(a + b*x)*sin(c + d
*x)**2/(b**3 - 4*b*d**2) - 2*d**2*sin(a + b*x)*cos(c + d*x)**2/(b**3 - 4*b*
d**2), True))
```

Giac [A]
time = 0.39, size = 56, normalized size = 0.90

$$\frac{\sin\left(bx+2\,dx+a+2\,c\right)}{4\,\left(b+2\,d\right)}+\frac{\sin\left(bx-2\,dx+a-2\,c\right)}{4\,\left(b-2\,d\right)}+\frac{\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*sin(b*x + 2*d*x + a + 2*c)/(b + 2*d) + 1/4*sin(b*x - 2*d*x + a - 2*c)/(
b - 2*d) + 1/2*sin(b*x + a)/b
```

Mupad [B]
time = 1.02, size = 98, normalized size = 1.58

$$\frac{\sin\left(a+b\,x\right)}{2\,b}-\frac{d\,\left(2\,b\,\sin\left(a-2\,c+b\,x-2\,d\,x\right)-2\,b\,\sin\left(a+2\,c+b\,x+2\,d\,x\right)\right)+b^2\,\sin\left(a-2\,c+b\,x-2\,d\,x\right)+b^2\,\sin\left(a+2\,c+b\,x+2\,d\,x\right)}{16\,b\,d^2-4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cos(c + d*x)^2,x)
```

```
[Out] sin(a + b*x)/(2*b) - (d*(2*b*sin(a - 2*c + b*x - 2*d*x) - 2*b*sin(a + 2*c +
 b*x + 2*d*x)) + b^2*sin(a - 2*c + b*x - 2*d*x) + b^2*sin(a + 2*c + b*x + 2
*d*x))/(16*b*d^2 - 4*b^3)
```

3.240 $\int \cos(a+bx)\cos(c+dx)\,dx$

Optimal. Leaf size=43

$$\frac{\sin(a-c+(b-d)x)}{2(b-d)}+\frac{\sin(a+c+(b+d)x)}{2(b+d)}$$

```
[Out] 1/2*sin(a-c+(b-d)*x)/(b-d)+1/2*sin(a+c+(b+d)*x)/(b+d)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4666, 2717}

$$\frac{\sin(a+x(b-d)-c)}{2(b-d)}+\frac{\sin(a+x(b+d)+c)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cos[c + d*x],x]
```

```
[Out] Sin[a - c + (b - d)*x]/(2*(b - d)) + Sin[a + c + (b + d)*x]/(2*(b + d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4666

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cos(c+dx)\,dx &= \int \left(\frac{1}{2}\cos(a-c+(b-d)x)+\frac{1}{2}\cos(a+c+(b+d)x)\right)\,dx\\ &= \frac{1}{2}\int \cos(a-c+(b-d)x)\,dx+\frac{1}{2}\int \cos(a+c+(b+d)x)\,dx\\ &= \frac{\sin(a-c+(b-d)x)}{2(b-d)}+\frac{\sin(a+c+(b+d)x)}{2(b+d)}\end{aligned}$$

Mathematica [A]
time = 0.20, size = 43, normalized size = 1.00

$$\frac{\sin(a-c+(b-d)x)}{2(b-d)}+\frac{\sin(a+c+(b+d)x)}{2(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cos[c + d*x],x]
```

```
[Out] Sin[a - c + (b - d)*x]/(2*(b - d)) + Sin[a + c + (b + d)*x]/(2*(b + d))
```

Maple [A]
time = 0.13, size = 40, normalized size = 0.93

method	result	size
default	$\frac{\sin(a-c+(b-d)x)}{2b-2d}+\frac{\sin(a+c+(b+d)x)}{2b+2d}$	40
risch	$\frac{\sin(bx-dx+a-c)b}{2(b-d)(b+d)}+\frac{\sin(bx-dx+a-c)d}{2(b-d)(b+d)}+\frac{\sin(bx+dx+a+c)b}{2(b-d)(b+d)}-\frac{\sin(bx+dx+a+c)d}{2(b-d)(b+d)}$	108
norman	$\frac{\frac{2b\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b^2-d^2}-\frac{2d\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-d^2}-\frac{2b\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2-d^2}+\frac{2d\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2-d^2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	147

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cos(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sin(a-c+(b-d)*x)/(b-d)+1/2*sin(a+c+(b+d)*x)/(b+d)
```

Maxima [A]
time = 0.28, size = 40, normalized size = 0.93

$$\frac{\sin\left(bx+dx+a+c\right)}{2\left(b+d\right)}-\frac{\sin\left(-bx+dx-a+c\right)}{2\left(b-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*sin(b*x + d*x + a + c)/(b + d) - 1/2*sin(-b*x + d*x - a + c)/(b - d)
```

Fricas [A]
time = 2.74, size = 42, normalized size = 0.98

$$\frac{b\cos\left(dx+c\right)\sin\left(bx+a\right)-d\cos\left(bx+a\right)\sin\left(dx+c\right)}{b^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="fricas")
```

```
[Out] (b*cos(d*x + c)*sin(b*x + a) - d*cos(b*x + a)*sin(d*x + c))/(b^2 - d^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(32) = 64$.

time = 0.32, size = 153, normalized size = 3.56

$$\begin{cases} x\cos(a)\cos(c) & \text{for } b = 0 \wedge d = 0 \\ -\frac{x\sin(a-dx)\sin(c+dx)}{2} + \frac{x\cos(a-dx)\cos(c+dx)}{2} + \frac{\sin(c+dx)\cos(a-dx)}{2d} & \text{for } b = -d \\ \frac{x\sin(a+dx)\sin(c+dx)}{2} + \frac{x\cos(a+dx)\cos(c+dx)}{2} + \frac{\sin(c+dx)\cos(a+dx)}{2d} & \text{for } b = d \\ \frac{b\sin(a+bx)\cos(c+dx)}{b^2-d^2} - \frac{d\sin(c+dx)\cos(a+bx)}{b^2-d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c),x)
```

```
[Out] Piecewise((x*cos(a)*cos(c), Eq(b, 0) & Eq(d, 0)), (-x*sin(a - d*x)*sin(c +
d*x)/2 + x*cos(a - d*x)*cos(c + d*x)/2 + sin(c + d*x)*cos(a - d*x)/(2*d), E
q(b, -d)), (x*sin(a + d*x)*sin(c + d*x)/2 + x*cos(a + d*x)*cos(c + d*x)/2 +
 sin(c + d*x)*cos(a + d*x)/(2*d), Eq(b, d)), (b*sin(a + b*x)*cos(c + d*x)/(
b**2 - d**2) - d*sin(c + d*x)*cos(a + b*x)/(b**2 - d**2), True))
```

Giac [A]
time = 0.39, size = 40, normalized size = 0.93

$$\frac{\sin(bx + dx + a + c)}{2(b + d)} + \frac{\sin(bx - dx + a - c)}{2(b - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cos(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*sin(b*x + d*x + a + c)/(b + d) + 1/2*sin(b*x - d*x + a - c)/(b - d)
```

Mupad [B]
time = 1.30, size = 84, normalized size = 1.95

$$\frac{b\left(\frac{\sin(a+c+b\,x+d\,x)}{2} + \frac{\sin(a-c+b\,x-d\,x)}{2}\right)}{b^2 - d^2} - \frac{d\left(\frac{\sin(a+c+b\,x+d\,x)}{2} - \frac{\sin(a-c+b\,x-d\,x)}{2}\right)}{b^2 - d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cos(c + d*x),x)
```

```
[Out] (b*(sin(a + c + b*x + d*x)/2 + sin(a - c + b*x - d*x)/2))/(b^2 - d^2) - (d*
(sin(a + c + b*x + d*x)/2 - sin(a - c + b*x - d*x)/2))/(b^2 - d^2)
```

3.241 $\int \cos(a+bx)\sec(c+bx)\,dx$

Optimal. Leaf size=26

$$x\cos(a-c)+\frac{\log(\cos(c+bx))\sin(a-c)}{b}$$

[Out] x*cos(a-c)+ln(cos(b*x+c))*sin(a-c)/b

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {4679, 3556, 8}

$$\frac{\sin(a-c)\log(\cos(bx+c))}{b}+x\cos(a-c)$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]*Sec[c + b*x],x]

[Out] x*Cos[a - c] + (Log[Cos[c + b*x]]*Sin[a - c])/b

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4679

```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Dist[-Sin[v - w], Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Dist[Cos[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
 && FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sec(c+bx)\,dx &= \cos(a-c)\int 1\,dx-\sin(a-c)\int \tan(c+bx)\,dx\\ &= x\cos(a-c)+\frac{\log(\cos(c+bx))\sin(a-c)}{b}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 26, normalized size = 1.00

$$x\cos(a-c)+\frac{\log(\cos(c+bx))\sin(a-c)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sec[c + b*x],x]
```

```
[Out] x*Cos[a - c] + (Log[Cos[c + b*x]]*Sin[a - c])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(26) = 52.
time = 0.41, size = 161, normalized size = 6.19

method	result
risch	$x\,\mathrm{e}^{i(a-c)}-2i\sin\left(a-c\right)x-\frac{2i\sin(a-c)a}{b}+\frac{\ln\left(\mathrm{e}^{2i(bx+a)}+\mathrm{e}^{2i(a-c)}\right)\sin(a-c)}{b}$
default	$\frac{\frac{\frac{(-\sin(a)\cos(c)+\cos(a)\sin(c))\ln\left(1+\tan^{2}(bx+a)\right)}{2}+(\cos(a)\cos(c)+\sin(a)\sin(c))\arctan(\tan(bx+a))}{\left(\cos^{2}(c)+\sin^{2}(c)\right)\left(\cos^{2}(a)+\sin^{2}(a)\right)}+\frac{(\sin(a)\cos(c)-\cos(a)\sin(c))\ln(-\tan(bx+a)\,\mathrm{co}}{\left(\cos^{2}(a)\right)\left(\cos^{2}(c)\right)+\left(\cos^{2}(c)\right)\left(\right.}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sec(b*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/(cos(c)^2+sin(c)^2)/(cos(a)^2+sin(a)^2)*(1/2*(-sin(a)*cos(c)+cos(a)*
sin(c))*ln(1+tan(b*x+a)^2)+(cos(a)*cos(c)+sin(a)*sin(c))*arctan(tan(b*x+a))
)+(sin(a)*cos(c)-cos(a)*sin(c))/(cos(a)^2*cos(c)^2+cos(c)^2*sin(a)^2+cos(a)
^2*sin(c)^2+sin(a)^2*sin(c)^2)*ln(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*sin(
a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(26) = 52.
time = 0.29, size = 74, normalized size = 2.85

$$\frac{2\,bx\cos\left(-a+c\right)-\log\left(\cos\left(2\,bx\right)^{2}+2\,\cos\left(2\,bx\right)\cos\left(2\,c\right)+\cos\left(2\,c\right)^{2}+\sin\left(2\,bx\right)^{2}-2\,\sin\left(2\,bx\right)\sin\left(2\,c\right)+\sin\left(2\,c\right)^{2}\right)\sin\left(-a+c\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x*cos(-a + c) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c
)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2)*sin(-a + c))/b
```

Fricas [A]
time = 2.14, size = 31, normalized size = 1.19

$$\frac{bx\cos\left(-a+c\right)-\log\left(-\cos\left(bx+c\right)\right)\sin\left(-a+c\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="fricas")
```

```
[Out] (b*x*cos(-a + c) - log(-cos(b*x + c))*sin(-a + c))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(20) = 40$.
time = 5.13, size = 435, normalized size = 16.73

$$-\left(\begin{cases} -x & \text{for}\ c = \frac{\pi}{2} \\ x & \text{for}\ c = -\frac{\pi}{2} \\ 0 & \text{for}\ b = 0 \\ -\frac{2bx\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)-1}-\frac{1}{\tan\left(\frac{c}{2}\right)-1}\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{\log\left(\tan\left(\frac{bx}{2}\right)-\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)-1}-\frac{1}{\tan\left(\frac{c}{2}\right)-1}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{\log\left(\tan\left(\frac{bx}{2}\right)+\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{c}{2}\right)+1}\right)\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{\log\left(\tan\left(\frac{bx}{2}\right)+\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{c}{2}\right)+1}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} & \text{otherwise} \end{cases}\right)\sin(a) + \left(\begin{cases} -\frac{\log(\sin(bx))}{b} & \text{for}\ c = \frac{\pi}{2} \\ \frac{\log(\sin(bx))}{b} & \text{for}\ c = -\frac{\pi}{2} \\ \frac{x}{\cos(c)} & \text{for}\ b = 0 \\ -\frac{bx\tan^2\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{bx}{b\tan^2\left(\frac{c}{2}\right)+b} + \frac{2\log\left(\tan^2\left(\frac{bx}{2}\right)+1\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{2\log\left(\tan\left(\frac{bx}{2}\right)-\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)-1}-\frac{1}{\tan\left(\frac{c}{2}\right)-1}\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} - \frac{2\log\left(\tan\left(\frac{bx}{2}\right)+\frac{\tan\left(\frac{c}{2}\right)}{\tan\left(\frac{c}{2}\right)+1}-\frac{1}{\tan\left(\frac{c}{2}\right)+1}\right)\tan\left(\frac{c}{2}\right)}{b\tan^2\left(\frac{c}{2}\right)+b} & \text{otherwise} \end{cases}\right)\cos(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c),x)
```

```
[Out] -Piecewise((-x, Eq(c, pi/2)), (x, Eq(c, -pi/2)), (0, Eq(b, 0)), (-2*b*x*tan
(c/2)/(b*tan(c/2)**2 + b) - log(tan(b*x/2)**2 + 1)*tan(c/2)**2/(b*tan(c/2)*
*2 + b) + log(tan(b*x/2)**2 + 1)/(b*tan(c/2)**2 + b) + log(tan(b*x/2) - tan
(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**2 + b) -
log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))/(b*tan(c/2)**2
 + b) + log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/
2)**2/(b*tan(c/2)**2 + b) - log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(t
an(c/2) + 1))/(b*tan(c/2)**2 + b), True))*sin(a) + Piecewise((-log(sin(b*x)
)/b, Eq(c, pi/2)), (log(sin(b*x))/b, Eq(c, -pi/2)), (x/cos(c), Eq(b, 0)), (
-b*x*tan(c/2)**2/(b*tan(c/2)**2 + b) + b*x/(b*tan(c/2)**2 + b) + 2*log(tan(
b*x/2)**2 + 1)*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(
tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)/(b*tan(c/2)**2 + b) - 2*log(tan(
b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)/(b*tan(c/2)**
2 + b), True))*cos(a)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(26) = 52$.
time = 0.44, size = 440, normalized size = 16.92

$$\frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)^2+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^2+1\right)(bx+a)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1} - \frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)\log\left(\tan(bx+a)^2+1\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1} + \frac{2\left(\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^2-2\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^4+2\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right)-4\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+2\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)^2-2\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}c\right)^2\right)\log\left(\left|2\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-2\tan(bx+a)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+2\tan(bx+a)\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}a\right)^2-2\tan(bx+a)\tan\left(\frac{1}{2}c\right)+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^2+1\right|\right)}{\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right)^4+\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^4+\tan\left(\frac{1}{2}a\right)^3-\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c),x, algorithm="giac")
```

```
[Out] ((tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(
1/2*c)^2 + 1)*(b*x + a)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2
*c)^2 + 1) - (tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a
) - tan(1/2*c))*log(tan(b*x + a)^2 + 1)/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/
2*a)^2 + tan(1/2*c)^2 + 1) + 2*(tan(1/2*a)^4*tan(1/2*c)^2 - 2*tan(1/2*a)^3*
tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c)^4 + 2*tan(1/2*a)^3*tan(1/2*c) - 4*ta
n(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)^2 - 2*tan(
```

```
1/2*a)*tan(1/2*c) + tan(1/2*c)^2)*log(abs(2*tan(b*x + a)*tan(1/2*a)^2*tan(1
/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2
+ 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*
tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1))/(tan(1/2*a)^4*tan(1/2*c)^3 - tan
(1/2*a)^3*tan(1/2*c)^4 + tan(1/2*a)^4*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^4
+ tan(1/2*a)^3 - tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c)))/b
```

Mupad [B]
time = 1.04, size = 109, normalized size = 4.19

$$x\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}}{2}-\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}}{2}\right)+x\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}}{2}\right)+\frac{\ln\left(\mathrm{e}^{a\,2\mathrm{i}-c\,2\mathrm{i}}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\right)\,\left(\frac{\mathrm{e}^{-a\,1\mathrm{i}+c\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{a\,1\mathrm{i}-c\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/cos(c + b*x),x)
```

```
[Out] x*(exp(c*1i - a*1i)/2 - exp(a*1i - c*1i)/2) + x*(exp(c*1i - a*1i)/2 + exp(a
*1i - c*1i)/2) + (log(exp(a*2i - c*2i) + exp(a*2i + b*x*2i))*((exp(c*1i - a
*1i)*1i)/2 - (exp(a*1i - c*1i)*1i)/2))/b
```

3.242 $\int \cos(a+bx)\sec^2(c+bx)\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sec(c+bx)\sin(a-c)}{b}$$

```
[Out] arctanh(sin(b*x+c))*cos(a-c)/b-sec(b*x+c)*sin(a-c)/b
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {4679, 2686, 8, 3855}

$$\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a-c)\sec(bx+c)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sec[c + b*x]^2,x]
```

```
[Out] (ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - (Sec[c + b*x]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4679

```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Dist[-Sin[v - w], Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Dist[Cos[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
 && FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \cos(a+bx)\sec^2(c+bx)\,dx = \cos(a-c)\int \sec(c+bx)\,dx - \sin(a-c)\int \sec(c+bx)\tan(c+bx)\,dx$$
$$= \frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b} - \frac{\sin(a-c)\text{Subst}(\int 1\,dx, x, \sec(c+bx))}{b}$$
$$= \frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b} - \frac{\sec(c+bx)\sin(a-c)}{b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 89, normalized size = 2.54

$$-\frac{2i\text{ArcTan}\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b} - \frac{\sec(c+bx)\sin(a-c)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sec[c + b*x]^2,x]
```

```
[Out] ((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)
/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Sec[c
 + b*x]*Sin[a - c])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(35) = 70$.
time = 0.69, size = 407, normalized size = 11.63

method	result
risch	$\frac{i\left(e^{i(bx+3a)}-e^{i(bx+a+2c)}\right)}{b\left(e^{2i(bx+a+c)}+e^{2ia}\right)}+\frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\cos(a-c)}{b}-\frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\cos(a-c)}{b}$
default	$-\frac{2\left(-\frac{\left(\left(\cos^2(c)\right)\left(\sin^2(a)\right)-2\cos(a)\cos(c)\sin(a)\sin(c)+\left(\cos^2(a)\right)\left(\sin^2(c)\right)\right)\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{\left(\left(\cos^2(a)\right)\left(\cos^2(c)\right)+\left(\cos^2(c)\right)\left(\sin^2(a)\right)+\left(\cos^2(a)\right)\left(\sin^2(c)\right)+\left(\sin^2(a)\right)\left(\sin^2(c)\right)\right)(\cos(a)\cos(c)+\sin(a)\sin(c))}-\frac{\cdots}{\left(\cos^2(a)\right)\left(\cos^2(c)\right)+\left(c\cdots\right.}\right.}{\cos(a)\cos(c)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin(a)\sin(c)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\cos(a)\sin(c)-2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\sin(a)\cos\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sec(b*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-2*(-(cos(c)^2*sin(a)^2-2*cos(a)*cos(c)*sin(a)*sin(c)+cos(a)^2*sin(c)^
2)/(cos(a)^2*cos(c)^2+cos(c)^2*sin(a)^2+cos(a)^2*sin(c)^2+sin(a)^2*sin(c)^2
)/(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2*b*x+1/2*a)-1/(cos(a)^2*cos(c)^2+cos
(c)^2*sin(a)^2+cos(a)^2*sin(c)^2+sin(a)^2*sin(c)^2)*(sin(a)*cos(c)-cos(a)*s
in(c)))/(cos(a)*cos(c)*tan(1/2*b*x+1/2*a)^2+sin(a)*sin(c)*tan(1/2*b*x+1/2*a
)^2+2*tan(1/2*b*x+1/2*a)*cos(a)*sin(c)-2*tan(1/2*b*x+1/2*a)*sin(a)*cos(c)-c
```

```
os(a)*cos(c)-sin(a)*sin(c))-2*(cos(a)*cos(c)+sin(a)*sin(c))/(cos(a)^2*cos(c
)^2+cos(c)^2*sin(a)^2+cos(a)^2*sin(c)^2+sin(a)^2*sin(c)^2)/(-cos(c)^2*sin(a
)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin(c)^2-cos(a)^2*sin(c)^2)^(1/2)*arctan(1/2
*(2*(cos(a)*cos(c)+sin(a)*sin(c))*tan(1/2*b*x+1/2*a)-2*sin(a)*cos(c)+2*cos(
a)*sin(c))/(-cos(c)^2*sin(a)^2-cos(a)^2*cos(c)^2-sin(a)^2*sin(c)^2-cos(a)^2
*sin(c)^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(35) = 70$.
time = 0.56, size = 391, normalized size = 11.17

$$-\frac{2\left(\sin\left(bx+2a\right)-\sin\left(bx+2c\right)\right)\cos\left(2bx+a+2c\right)+\left(\cos\left(2bx+a+2c\right)^{2}\cos\left(-a+c\right)+2\cos\left(2bx+a+2c\right)\cos\left(a\right)\cos\left(-a+c\right)+\cos\left(-a+c\right)\sin\left(2bx+a+2c\right)^{2}+2\cos\left(-a+c\right)\sin\left(2bx+a+2c\right)\sin\left(a\right)+\left(\cos\left(a\right)^{2}+\sin\left(a\right)^{2}\right)\cos\left(-a+c\right)\right)\log\left(\frac{\cos(bx+2c)^{2}+\cos(c)^{2}-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^{2}+2\cos(bx+2c)\sin(c)+\sin(c)^{2}}{\cos(bx+2c)^{2}+\cos(c)^{2}+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^{2}-2\cos(bx+2c)\sin(c)+\sin(c)^{2}}\right)-2\left(\cos\left(bx+2a\right)-\cos\left(bx+2c\right)\right)\sin\left(2bx+a+2c\right)+2\cos\left(a\right)\sin\left(bx+2a\right)-2\cos\left(a\right)\sin\left(bx+2c\right)-2\cos\left(bx+2a\right)\sin\left(a\right)+2\cos\left(bx+2c\right)\sin\left(a\right)}{2\left(b\cos\left(2bx+a+2c\right)^{2}+2b\cos\left(2bx+a+2c\right)\cos\left(a\right)+b\sin\left(2bx+a+2c\right)^{2}+2b\sin\left(2bx+a+2c\right)\sin\left(a\right)+\left(\cos\left(a\right)^{2}+\sin\left(a\right)^{2}\right)b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(sin(b*x + 2*a) - sin(b*x + 2*c))*cos(2*b*x + a + 2*c) + (cos(2*b*x
 + a + 2*c)^2*cos(-a + c) + 2*cos(2*b*x + a + 2*c)*cos(a)*cos(-a + c) + cos
(-a + c)*sin(2*b*x + a + 2*c)^2 + 2*cos(-a + c)*sin(2*b*x + a + 2*c)*sin(a)
 + (cos(a)^2 + sin(a)^2)*cos(-a + c))*log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*
cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)
^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)
^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) - 2*(cos(b*x + 2*a) - cos(b*x + 2
*c))*sin(2*b*x + a + 2*c) + 2*cos(a)*sin(b*x + 2*a) - 2*cos(a)*sin(b*x + 2*
c) - 2*cos(b*x + 2*a)*sin(a) + 2*cos(b*x + 2*c)*sin(a))/(b*cos(2*b*x + a +
2*c)^2 + 2*b*cos(2*b*x + a + 2*c)*cos(a) + b*sin(2*b*x + a + 2*c)^2 + 2*b*s
in(2*b*x + a + 2*c)*sin(a) + (cos(a)^2 + sin(a)^2)*b)
```

Fricas [A]
time = 1.31, size = 69, normalized size = 1.97

$$\frac{\cos\left(bx+c\right)\cos\left(-a+c\right)\log\left(\sin\left(bx+c\right)+1\right)-\cos\left(bx+c\right)\cos\left(-a+c\right)\log\left(-\sin\left(bx+c\right)+1\right)+2\sin\left(-a+c\right)}{2\,b\cos\left(bx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(cos(b*x + c)*cos(-a + c)*log(sin(b*x + c) + 1) - cos(b*x + c)*cos(-a +
 c)*log(-sin(b*x + c) + 1) + 2*sin(-a + c))/(b*cos(b*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4095 vs. $2(27) = 54$.
time = 88.16, size = 5545, normalized size = 158.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)**2,x)

[Out] -Piecewise((log(tan(b*x/2))/b, Eq(c, -pi/2) | Eq(c, pi/2)), (0, Eq(b, 0)),
(-2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**
3*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)
**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(tan
(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**3/(b*tan(c/
2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(
c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 8*log(tan(b*x/2) - tan(c/2)/(tan(c
/2) - 1) - 1/(tan(c/2) - 1))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/
2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2
) - b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(
tan(c/2) - 1))*tan(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(
c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/
2)**2 + b) - 2*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan(c/2) - 1))
*tan(c/2)/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*ta
n(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*log(tan(b*x/2
) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3*tan(b*x/2)**2/(
b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) -
4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 2*log(tan(b*x/2) + tan(c/2
)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)**3/(b*tan(c/2)**4*tan(b*x/2)*
*2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) -
 b*tan(b*x/2)**2 + b) - 8*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan
(c/2) + 1))*tan(c/2)**2*tan(b*x/2)/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2
)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)*
*2 + b) - 2*log(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*ta
n(c/2)*tan(b*x/2)**2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan
(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) + 2*lo
g(tan(b*x/2) + tan(c/2)/(tan(c/2) + 1) - 1/(tan(c/2) + 1))*tan(c/2)/(b*tan(
c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*ta
n(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 + b) - 2*tan(c/2)**4/(b*tan(c/2)**4*tan
(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(
b*x/2) - b*tan(b*x/2)**2 + b) - 4*tan(c/2)**3*tan(b*x/2)/(b*tan(c/2)**4*tan
(b*x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(
b*x/2) - b*tan(b*x/2)**2 + b) - 4*tan(c/2)*tan(b*x/2)/(b*tan(c/2)**4*tan(b*
x/2)**2 - b*tan(c/2)**4 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x
/2) - b*tan(b*x/2)**2 + b) + 2/(b*tan(c/2)**4*tan(b*x/2)**2 - b*tan(c/2)**4
 - 4*b*tan(c/2)**3*tan(b*x/2) - 4*b*tan(c/2)*tan(b*x/2) - b*tan(b*x/2)**2 +
 b), True))*sin(a) + Piecewise((-1/(b*sin(b*x)), Eq(c, -pi/2) | Eq(c, pi/2)
), (x/cos(c)**2, Eq(b, 0)), (-log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/
(tan(c/2) - 1))*tan(c/2)**6*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*
tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*
tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*ta
n(b*x/2) + b*tan(b*x/2)**2 - b) + log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1)
- 1/(tan(c/2) - 1))*tan(c/2)**6/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)**
```

```
6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)**
4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2) +
 b*tan(b*x/2)**2 - b) + 4*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(tan
(c/2) - 1))*tan(c/2)**5*tan(b*x/2)/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2
)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2
)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2
) + b*tan(b*x/2)**2 - b) + 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(
tan(c/2) - 1))*tan(c/2)**4*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*t
an(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*t
an(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan
(b*x/2) + b*tan(b*x/2)**2 - b) - 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1)
 - 1/(tan(c/2) - 1))*tan(c/2)**4/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/2)*
*6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/2)*
*4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/2)
+ b*tan(b*x/2)**2 - b) - 8*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/(ta
n(c/2) - 1))*tan(c/2)**3*tan(b*x/2)/(b*tan(c/2)**6*tan(b*x/2)**2 - b*tan(c/
2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*tan(c/
2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*tan(b*x/
2) + b*tan(b*x/2)**2 - b) - 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1) - 1/
(tan(c/2) - 1))*tan(c/2)**2*tan(b*x/2)**2/(b*tan(c/2)**6*tan(b*x/2)**2 - b*
tan(c/2)**6 - 4*b*tan(c/2)**5*tan(b*x/2) - b*tan(c/2)**4*tan(b*x/2)**2 + b*
tan(c/2)**4 - b*tan(c/2)**2*tan(b*x/2)**2 + b*tan(c/2)**2 + 4*b*tan(c/2)*ta
n(b*x/2) + b*tan(b*x/2)**2 - b) + 3*log(tan(b*x/2) - tan(c/2)/(tan(c/2) - 1
) - 1/(tan(c/2) - 1))*tan(c/2)**2/(b*tan(c/2)**...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1341 vs. $2(35) = 70$.
time = 0.50, size = 1341, normalized size = 38.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^2,x, algorithm="giac")
```

```
[Out] -((tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/2*a)^2*tan
(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2
*a)*tan(1/2*c)^3 + tan(1/2*a)^3 - 5*tan(1/2*a)^2*tan(1/2*c) + 5*tan(1/2*a)*
tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) - tan(
1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1
/2*a)*tan(1/2*c) + tan(1/2*b*x + 1/2*a)*tan(1/2*a) - tan(1/2*b*x + 1/2*a)*t
an(1/2*c) + tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan
(1/2*c) + 1))/(tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^2 + tan(
1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c)^2
 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3 + tan(1/2*a)^2*tan(1/2*c) - tan(1
/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 + tan(1/2*a)^2 + tan(1/2*a)*tan(1/2*c) +
```

```
tan(1/2*c)^2 - tan(1/2*a) + tan(1/2*c) + 1) - (tan(1/2*a)^3*tan(1/2*c)^3 +
tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^3*tan(1/
2*c) + 5*tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)^3
 + 5*tan(1/2*a)^2*tan(1/2*c) - 5*tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*c)^3 - t
an(1/2*a)^2 + 5*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + tan(1/2*a) - tan(1/2
*c) + 1)*log(abs(-tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) - tan(1/2*b*x
+ 1/2*a)*tan(1/2*a) + tan(1/2*b*x + 1/2*a)*tan(1/2*c) - tan(1/2*a)*tan(1/2*
c) - tan(1/2*b*x + 1/2*a) + tan(1/2*a) - tan(1/2*c) - 1))/(tan(1/2*a)^3*tan
(1/2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2
*a)^3*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 + ta
n(1/2*a)^3 - tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)
^3 + tan(1/2*a)^2 + tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 + tan(1/2*a) - tan
(1/2*c) + 1) - 4*(2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^4*tan(1/2*c)^2 - 4*tan(
1/2*b*x + 1/2*a)*tan(1/2*a)^3*tan(1/2*c)^3 + tan(1/2*a)^4*tan(1/2*c)^3 + 2*
tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^4 - tan(1/2*a)^3*tan(1/2*c)^4
+ 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^3*tan(1/2*c) - tan(1/2*a)^4*tan(1/2*c)
- 8*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(1/2*a)^3*tan(1/2
*c)^2 + 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c)^3 - 6*tan(1/2*a)^2*tan
(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2 -
 tan(1/2*a)^3 - 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)*tan(1/2*c) + 6*tan(1/2*a)
^2*tan(1/2*c) + 2*tan(1/2*b*x + 1/2*a)*tan(1/2*c)^2 - 6*tan(1/2*a)*tan(1/2*
c)^2 + tan(1/2*c)^3 + tan(1/2*a) - tan(1/2*c))/((tan(1/2*b*x + 1/2*a)^2*tan
(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*b*x + 1/2*a)^2*tan(1/2*a)^2 + 4*tan(1/2*b*
x + 1/2*a)^2*tan(1/2*a)*tan(1/2*c) - 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a)^2*ta
n(1/2*c) - tan(1/2*b*x + 1/2*a)^2*tan(1/2*c)^2 + 4*tan(1/2*b*x + 1/2*a)*tan
(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*b*x + 1/2*a)^2 -
 4*tan(1/2*b*x + 1/2*a)*tan(1/2*a) + tan(1/2*a)^2 + 4*tan(1/2*b*x + 1/2*a)*
tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 - 1)*(tan(1/2*a)^2*tan(
1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1)))/b
```

Mupad [B]
time = 6.52, size = 246, normalized size = 7.03

$$\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}+\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}-1\right)\,1i}{b\,\left(e^{a\,2i-c\,2i}+e^{a\,2i+b\,x\,2i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/cos(c + b*x)^2,x)
```

```
[Out] (log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1) - (exp(a*2i)*exp(-c
*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i
 - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log((exp(a*2i)*exp(-c*2i)*(e
xp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b
*x*1i)*(exp(a*2i)*exp(-c*2i) + 1))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i -
c*2i)^(1/2)) + (exp(a*1i + b*x*1i)*(exp(a*2i - c*2i) - 1)*1i)/(b*(exp(a*2i
- c*2i) + exp(a*2i + b*x*2i)))
```

3.243 $\int \cos(a+bx)\sec^3(c+bx)\,dx$

Optimal. Leaf size=38

$$-\frac{\sec^2(c+bx)\sin(a-c)}{2b}+\frac{\cos(a-c)\tan(c+bx)}{b}$$

```
[Out] -1/2*sec(b*x+c)^2*sin(a-c)/b+cos(a-c)*tan(b*x+c)/b
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4679, 2686, 30, 3852, 8}

$$\frac{\cos(a-c)\tan(bx+c)}{b}-\frac{\sin(a-c)\sec^2(bx+c)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sec[c + b*x]^3,x]
```

```
[Out] -1/2*(Sec[c + b*x]^2*Sin[a - c])/b + (Cos[a - c]*Tan[c + b*x])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4679

```
Int[Cos[v_]*Sec[w_]^(n_.), x_Symbol] :> Dist[-Sin[v - w], Int[Tan[w]*Sec[w]
^(n - 1), x], x] + Dist[Cos[v - w], Int[Sec[w]^(n - 1), x], x] /; GtQ[n, 0]
```

```
&& FreeQ[v - w, x] && NeQ[w, v]
```

Rubi steps

$$\int \cos(a+bx)\sec^3(c+bx)\,dx = \cos(a-c)\int \sec^2(c+bx)\,dx - \sin(a-c)\int \sec^2(c+bx)\tan(c+bx)\,dx$$
$$= -\frac{\cos(a-c)\text{Subst}(\int 1\,dx, x, -\tan(c+bx))}{b} - \frac{\sin(a-c)\text{Subst}(\int x\,dx, x, \text{se}}{b}$$
$$= -\frac{\sec^2(c+bx)\sin(a-c)}{2b} + \frac{\cos(a-c)\tan(c+bx)}{b}$$

Mathematica [**A**]
time = 0.22, size = 35, normalized size = 0.92

$$-\frac{\sec(c)\sec^2(c+bx)(\sin(a)-\cos(a-c)\sin(c+2bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sec[c + b*x]^3,x]
[Out] -1/2*(Sec[c]*Sec[c + b*x]^2*(Sin[a] - Cos[a - c]*Sin[c + 2*b*x]))/b
```

Maple [**A**]
time = 0.90, size = 56, normalized size = 1.47

method	result	size
default	$-\frac{1}{2b(\sin(a)\cos(c)-\cos(a)\sin(c))(-\tan(bx+a)\cos(a)\sin(c)+\tan(bx+a)\sin(a)\cos(c)+\cos(a)\cos(c)+\sin(a)\sin(c))^2}$	56
risch	$\frac{i\left(2\,e^{i(2bx+5a+c)}+e^{i(5a-c)}+e^{i(3a+c)}\right)}{\left(e^{2i(bx+a+c)}+e^{2ia}\right)^2 b}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sec(b*x+c)^3,x,method=_RETURNVERBOSE)
[Out] -1/2/b/(sin(a)*cos(c)-cos(a)*sin(c))/(-tan(b*x+a)*cos(a)*sin(c)+tan(b*x+a)*
sin(a)*cos(c)+cos(a)*cos(c)+sin(a)*sin(c))^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(36) = 72.
time = 0.29, size = 382, normalized size = 10.05

$$-\frac{(2\sin(2bx+2a+2c)+\sin(2a)+\sin(2c))\cos(4bx+a+5c)+2(2\sin(2bx+2a+2c)+\sin(2a)+\sin(2c))\cos(2bx+a+3c)+(\sin(2a)+\sin(2c))\cos(a+c)-(2\cos(2bx+2a+2c)+\cos(2a)+\cos(2c))\sin(4bx+a+5c)+2\cos(a+c)\sin(2bx+2a+2c)-2(2\cos(2bx+2a+2c)+\cos(2a)+\cos(2c))\sin(2bx+a+3c)-(\cos(2a)+\cos(2c))\sin(a+c)-2\cos(2bx+2a+2c)\sin(a+c)}{b\cos(4bx+a+5c)^2+4b\cos(2bx+a+3c)^2+4b\cos(2bx+a+3c)\cos(a+c)+b\cos(a+c)^2+b\sin(4bx+a+5c)^2+4b\sin(2bx+a+3c)^2+4b\sin(2bx+a+3c)\sin(a+c)+b\sin(a+c)^2+2(2b\cos(2bx+a+3c)+b\cos(a+c))\cos(4bx+a+5c)+2(2b\sin(2bx+a+3c)+b\sin(a+c))\sin(4bx+a+5c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] -((2*sin(2*b*x + 2*a + 2*c) + sin(2*a) + sin(2*c))*cos(4*b*x + a + 5*c) + 2
*(2*sin(2*b*x + 2*a + 2*c) + sin(2*a) + sin(2*c))*cos(2*b*x + a + 3*c) + (s
in(2*a) + sin(2*c))*cos(a + c) - (2*cos(2*b*x + 2*a + 2*c) + cos(2*a) + cos
(2*c))*sin(4*b*x + a + 5*c) + 2*cos(a + c)*sin(2*b*x + 2*a + 2*c) - 2*(2*co
s(2*b*x + 2*a + 2*c) + cos(2*a) + cos(2*c))*sin(2*b*x + a + 3*c) - (cos(2*a
) + cos(2*c))*sin(a + c) - 2*cos(2*b*x + 2*a + 2*c)*sin(a + c))/(b*cos(4*b*
x + a + 5*c)^2 + 4*b*cos(2*b*x + a + 3*c)^2 + 4*b*cos(2*b*x + a + 3*c)*cos(
a + c) + b*cos(a + c)^2 + b*sin(4*b*x + a + 5*c)^2 + 4*b*sin(2*b*x + a + 3*
c)^2 + 4*b*sin(2*b*x + a + 3*c)*sin(a + c) + b*sin(a + c)^2 + 2*(2*b*cos(2*
b*x + a + 3*c) + b*cos(a + c))*cos(4*b*x + a + 5*c) + 2*(2*b*sin(2*b*x + a
+ 3*c) + b*sin(a + c))*sin(4*b*x + a + 5*c))
```

Fricas [A]
time = 2.05, size = 40, normalized size = 1.05

$$\frac{2\,\cos\left(bx+c\right)\cos\left(-a+c\right)\sin\left(bx+c\right)+\sin\left(-a+c\right)}{2\,b\cos\left(bx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c) + sin(-a + c))/(b*cos(b*x + c)
^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(36) = 72.
time = 0.46, size = 315, normalized size = 8.29

$$-\frac{\tan\left(\frac{1}{2}\,a\right)^{6}\tan\left(\frac{1}{2}\,c\right)^{6}+3\,\tan\left(\frac{1}{2}\,a\right)^{6}\tan\left(\frac{1}{2}\,c\right)^{4}+3\,\tan\left(\frac{1}{2}\,a\right)^{4}\tan\left(\frac{1}{2}\,c\right)^{6}+3\,\tan\left(\frac{1}{2}\,a\right)^{6}\tan\left(\frac{1}{2}\,c\right)^{2}+9\,\tan\left(\frac{1}{2}\,a\right)^{4}\tan\left(\frac{1}{2}\,c\right)^{4}+3\,\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{6}+\tan\left(\frac{1}{2}\,a\right)^{6}+9\,\tan\left(\frac{1}{2}\,a\right)^{4}\tan\left(\frac{1}{2}\,c\right)^{2}+9\,\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{4}+\tan\left(\frac{1}{2}\,c\right)^{6}+3\,\tan\left(\frac{1}{2}\,a\right)^{4}+9\,\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+3\,\tan\left(\frac{1}{2}\,c\right)^{4}+3\,\tan\left(\frac{1}{2}\,a\right)^{2}+3\,\tan\left(\frac{1}{2}\,c\right)^{2}+1}{4\,{\left(2\,\tan\left(bx+a\right)\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)-2\,\tan\left(bx+a\right)\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+2\,\tan\left(bx+a\right)\tan\left(\frac{1}{2}\,a\right)-\tan\left(\frac{1}{2}\,a\right)^{2}-2\,\tan\left(bx+a\right)\tan\left(\frac{1}{2}\,c\right)+4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,c\right)^{2}+1\right)}^{2}{\left(\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2}+\tan\left(\frac{1}{2}\,a\right)-\tan\left(\frac{1}{2}\,c\right)\right)}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sec(b*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*(tan(1/2*a)^6*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^4 + 3*tan(1/2*a
)^4*tan(1/2*c)^6 + 3*tan(1/2*a)^6*tan(1/2*c)^2 + 9*tan(1/2*a)^4*tan(1/2*c)^
```

```
4 + 3*tan(1/2*a)^2*tan(1/2*c)^6 + tan(1/2*a)^6 + 9*tan(1/2*a)^4*tan(1/2*c)^
2 + 9*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*c)^6 + 3*tan(1/2*a)^4 + 9*tan(1/2
*a)^2*tan(1/2*c)^2 + 3*tan(1/2*c)^4 + 3*tan(1/2*a)^2 + 3*tan(1/2*c)^2 + 1)/
((2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/
2*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)
^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 + 1
)^2*(tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1
/2*c))*b)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/cos(c + b*x)^3,x)
```

```
[Out] \text{Hanged}
```

3.244 $\int \cos^2(a+bx)\cos^3(c+dx)\,dx$

Optimal. Leaf size=144

$$\frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)}+\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sin(c+dx)}{8d}+\frac{\sin(3c+3dx)}{24d}+\frac{3\sin(2a+c+(2b+d)x)}{16(2b+d)}$$

```
[Out] 1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)+3
/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d+3/16*sin(2*a+c+(2*b+d)*x)/(2*b+d)+1/1
6*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Rubi [A]
time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4666, 2717}

$$\frac{\sin(2a+x(2b-3d)-3c)}{16(2b-3d)}+\frac{3\sin(2a+x(2b-d)-c)}{16(2b-d)}+\frac{3\sin(2a+x(2b+d)+c)}{16(2b+d)}+\frac{\sin(2a+x(2b+3d)+3c)}{16(2b+3d)}+\frac{3\sin(c+dx)}{8d}+\frac{\sin(3c+3dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cos[c + d*x]^3,x]
```

```
[Out] Sin[2*a - 3*c + (2*b - 3*d)*x]/(16*(2*b - 3*d)) + (3*Sin[2*a - c + (2*b - d
)*x])/(16*(2*b - d)) + (3*Sin[c + d*x])/(8*d) + Sin[3*c + 3*d*x]/(24*d) + (
3*Sin[2*a + c + (2*b + d)*x])/(16*(2*b + d)) + Sin[2*a + 3*c + (2*b + 3*d)*
x]/(16*(2*b + 3*d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4666

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\cos^3(c+dx)\,dx &= \int \left(\frac{1}{16}\cos(2a-3c+(2b-3d)x)+\frac{3}{16}\cos(2a-c+(2b-d)x)+\frac{3}{8}\cos(c\right.\\ &= \frac{1}{16}\int \cos(2a-3c+(2b-3d)x)\,dx+\frac{1}{16}\int \cos(2a+3c+(2b+3d)x)\,dx+\\ &= \frac{\sin(2a-3c+(2b-3d)x)}{16(2b-3d)}+\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sin(c+dx)}{8d}+\end{aligned}$$

Mathematica [A]
time = 1.66, size = 158, normalized size = 1.10

$$\frac{1}{48}\left(\frac{18\cos(dx)\sin(c)}{d}+\frac{2\cos(3dx)\sin(3c)}{d}+\frac{18\cos(c)\sin(dx)}{d}+\frac{2\cos(3c)\sin(3dx)}{d}+\frac{3\sin(2a-3c+2bx-3dx)}{2b-3d}+\frac{9\sin(2a-c+2bx-dx)}{2b-d}+\frac{9\sin(2a+c+2bx+dx)}{2b+d}+\frac{3\sin(2a+3c+2bx+3dx)}{2b+3d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cos[c + d*x]^3,x]
```

```
[Out] ((18*Cos[d*x]*Sin[c])/d + (2*Cos[3*d*x]*Sin[3*c])/d + (18*Cos[c]*Sin[d*x])/
d + (2*Cos[3*c]*Sin[3*d*x])/d + (3*Sin[2*a - 3*c + 2*b*x - 3*d*x])/(2*b - 3
*d) + (9*Sin[2*a - c + 2*b*x - d*x])/(2*b - d) + (9*Sin[2*a + c + 2*b*x + d
*x])/(2*b + d) + (3*Sin[2*a + 3*c + 2*b*x + 3*d*x])/(2*b + 3*d))/48
```

Maple [A]
time = 0.28, size = 133, normalized size = 0.92

method	result
default	$\frac{\sin(2a-3c+(2b-3d)x)}{32b-48d}+\frac{3\sin(2a-c+(2b-d)x)}{16(2b-d)}+\frac{3\sin(dx+c)}{8d}+\frac{\sin(3dx+3c)}{24d}+\frac{3\sin(2a+c+(2b+d)x)}{16(2b+d)}+\frac{\sin(2a+3c+(2b+3}{32b+48d}$
risch	$\frac{3\sin(dx+c)b^2}{2d(2b-d)(2b+d)}-\frac{3d\sin(dx+c)}{8(2b-d)(2b+d)}+\frac{\sin(2bx-3dx+2a-3c)b}{8(2b-3d)(2b+3d)}+\frac{3d\sin(2bx-3dx+2a-3c)}{16(2b-3d)(2b+3d)}+\frac{3\sin(2bx-dx+2a-c)b}{8(2b-d)(2b+d)}+\frac{3d\sin(}{16(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*cos(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*sin(2*a-3*c+(2*b-3*d)*x)/(2*b-3*d)+3/16*sin(2*a-c+(2*b-d)*x)/(2*b-d)+3
/8*sin(d*x+c)/d+1/24*sin(3*d*x+3*c)/d+3/16*sin(2*a+c+(2*b+d)*x)/(2*b+d)+1/1
6*sin(2*a+3*c+(2*b+3*d)*x)/(2*b+3*d)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. 2(132) = 264.
time = 0.35, size = 1362, normalized size = 9.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/96*(3*(8*b^3*d*sin(3*c) - 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4
*sin(3*c))*cos((2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*sin(3*c) - 12*b^2*d^
2*sin(3*c) - 2*b*d^3*sin(3*c) + 3*d^4*sin(3*c))*cos((2*b + 3*d)*x + 2*a) +
9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*
c))*cos((2*b + d)*x + 2*a + 4*c) - 9*(8*b^3*d*sin(3*c) - 4*b^2*d^2*sin(3*c)
 - 18*b*d^3*sin(3*c) + 9*d^4*sin(3*c))*cos((2*b + d)*x + 2*a - 2*c) - 9*(8*
b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 18*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*c
os(-(2*b - d)*x - 2*a + 4*c) + 9*(8*b^3*d*sin(3*c) + 4*b^2*d^2*sin(3*c) - 1
```

```
8*b*d^3*sin(3*c) - 9*d^4*sin(3*c))*cos(-(2*b - d)*x - 2*a - 2*c) - 3*(8*b^3
*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(
-(2*b - 3*d)*x - 2*a + 6*c) + 3*(8*b^3*d*sin(3*c) + 12*b^2*d^2*sin(3*c) - 2
*b*d^3*sin(3*c) - 3*d^4*sin(3*c))*cos(-(2*b - 3*d)*x - 2*a) - 2*(16*b^4*sin
(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x) + 2*(16*b^4*sin(3*
c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(3*d*x + 6*c) + 18*(16*b^4*si
n(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x + 4*c) - 18*(16*b^4*
sin(3*c) - 40*b^2*d^2*sin(3*c) + 9*d^4*sin(3*c))*cos(d*x - 2*c) - 3*(8*b^3*
d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((
2*b + 3*d)*x + 2*a + 6*c) - 3*(8*b^3*d*cos(3*c) - 12*b^2*d^2*cos(3*c) - 2*b
*d^3*cos(3*c) + 3*d^4*cos(3*c))*sin((2*b + 3*d)*x + 2*a) - 9*(8*b^3*d*cos(3
*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) + 9*d^4*cos(3*c))*sin((2*b + d
)*x + 2*a + 4*c) - 9*(8*b^3*d*cos(3*c) - 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(
3*c) + 9*d^4*cos(3*c))*sin((2*b + d)*x + 2*a - 2*c) + 9*(8*b^3*d*cos(3*c) +
 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c) - 9*d^4*cos(3*c))*sin(-(2*b - d)*x
- 2*a + 4*c) + 9*(8*b^3*d*cos(3*c) + 4*b^2*d^2*cos(3*c) - 18*b*d^3*cos(3*c)
 - 9*d^4*cos(3*c))*sin(-(2*b - d)*x - 2*a - 2*c) + 3*(8*b^3*d*cos(3*c) + 12
*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c) - 3*d^4*cos(3*c))*sin(-(2*b - 3*d)*x -
 2*a + 6*c) + 3*(8*b^3*d*cos(3*c) + 12*b^2*d^2*cos(3*c) - 2*b*d^3*cos(3*c)
- 3*d^4*cos(3*c))*sin(-(2*b - 3*d)*x - 2*a) - 2*(16*b^4*cos(3*c) - 40*b^2*d
^2*cos(3*c) + 9*d^4*cos(3*c))*sin(3*d*x) - 2*(16*b^4*cos(3*c) - 40*b^2*d^2*
cos(3*c) + 9*d^4*cos(3*c))*sin(3*d*x + 6*c) - 18*(16*b^4*cos(3*c) - 40*b^2*
d^2*cos(3*c) + 9*d^4*cos(3*c))*sin(d*x + 4*c) - 18*(16*b^4*cos(3*c) - 40*b^
2*d^2*cos(3*c) + 9*d^4*cos(3*c))*sin(d*x - 2*c))/(9*(cos(3*c)^2 + sin(3*c)^
2)*d^5 - 40*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^3 + 16*(b^4*cos(3*c)^2 + b^
4*sin(3*c)^2)*d)
```

Fricas [**A**]
time = 1.67, size = 163, normalized size = 1.13

$$-\frac{6\left(6\,bd^{3}\cos\left(bx+a\right)\cos\left(dx+c\right)-\left(4\,b^{3}d-bd^{3}\right)\cos\left(bx+a\right)\cos\left(dx+c\right)^{3}\right)\sin\left(bx+a\right)-\left(18\,d^{4}\cos\left(bx+a\right)^{2}+16\,b^{4}-40\,b^{2}d^{2}+\left(8\,b^{4}-2\,b^{2}d^{2}-9\left(4\,b^{2}d^{2}-d^{4}\right)\cos\left(bx+a\right)^{2}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{3\left(16\,b^{4}d-40\,b^{2}d^{3}+9\,d^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(6*(6*b*d^3*cos(b*x + a)*cos(d*x + c) - (4*b^3*d - b*d^3)*cos(b*x + a)
*cos(d*x + c)^3)*sin(b*x + a) - (18*d^4*cos(b*x + a)^2 + 16*b^4 - 40*b^2*d^
2 + (8*b^4 - 2*b^2*d^2 - 9*(4*b^2*d^2 - d^4)*cos(b*x + a)^2)*cos(d*x + c)^2
)*sin(d*x + c))/(16*b^4*d - 40*b^2*d^3 + 9*d^5)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2003 vs. 2(116) = 232.
time = 6.53, size = 2003, normalized size = 13.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*cos(d*x+c)**3,x)

[Out] Piecewise((x*cos(a)**2*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - 3*d*x/
2)**2*sin(c + d*x)**2*cos(c + d*x)/16 - x*sin(a - 3*d*x/2)**2*cos(c + d*x)*
*3/16 + x*sin(a - 3*d*x/2)*sin(c + d*x)**3*cos(a - 3*d*x/2)/8 - 3*x*sin(a -
 3*d*x/2)*sin(c + d*x)*cos(a - 3*d*x/2)*cos(c + d*x)**2/8 - 3*x*sin(c + d*x
)**2*cos(a - 3*d*x/2)**2*cos(c + d*x)/16 + x*cos(a - 3*d*x/2)**2*cos(c + d*
x)**3/16 + 9*sin(a - 3*d*x/2)**2*sin(c + d*x)**3/(16*d) - 5*sin(a - 3*d*x/2
)*sin(c + d*x)**2*cos(a - 3*d*x/2)*cos(c + d*x)/(4*d) + sin(a - 3*d*x/2)*co
s(a - 3*d*x/2)*cos(c + d*x)**3/(24*d) + 5*sin(c + d*x)**3*cos(a - 3*d*x/2)*
*2/(48*d) + sin(c + d*x)*cos(a - 3*d*x/2)**2*cos(c + d*x)**2/d, Eq(b, -3*d/
2)), (-3*x*sin(a - d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 - 3*x*sin(a -
d*x/2)**2*cos(c + d*x)**3/16 - 3*x*sin(a - d*x/2)*sin(c + d*x)**3*cos(a - d
*x/2)/8 - 3*x*sin(a - d*x/2)*sin(c + d*x)*cos(a - d*x/2)*cos(c + d*x)**2/8
+ 3*x*sin(c + d*x)**2*cos(a - d*x/2)**2*cos(c + d*x)/16 + 3*x*cos(a - d*x/2
)**2*cos(c + d*x)**3/16 + sin(a - d*x/2)**2*sin(c + d*x)**3/(48*d) + sin(a
- d*x/2)*sin(c + d*x)**2*cos(a - d*x/2)*cos(c + d*x)/(4*d) + 3*sin(a - d*x/
2)*cos(a - d*x/2)*cos(c + d*x)**3/(8*d) + 31*sin(c + d*x)**3*cos(a - d*x/2)
**2/(48*d) + sin(c + d*x)*cos(a - d*x/2)**2*cos(c + d*x)**2/d, Eq(b, -d/2))
, (-3*x*sin(a + d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 - 3*x*sin(a + d*x
/2)**2*cos(c + d*x)**3/16 + 3*x*sin(a + d*x/2)*sin(c + d*x)**3*cos(a + d*x/
2)/8 + 3*x*sin(a + d*x/2)*sin(c + d*x)*cos(a + d*x/2)*cos(c + d*x)**2/8 + 3
*x*sin(c + d*x)**2*cos(a + d*x/2)**2*cos(c + d*x)/16 + 3*x*cos(a + d*x/2)**
2*cos(c + d*x)**3/16 + 49*sin(a + d*x/2)**2*sin(c + d*x)**3/(48*d) + sin(a
+ d*x/2)**2*sin(c + d*x)*cos(c + d*x)**2/d + 7*sin(a + d*x/2)*sin(c + d*x)*
*2*cos(a + d*x/2)*cos(c + d*x)/(4*d) + 13*sin(a + d*x/2)*cos(a + d*x/2)*cos
(c + d*x)**3/(8*d) - 17*sin(c + d*x)**3*cos(a + d*x/2)**2/(48*d), Eq(b, d/2
)), (3*x*sin(a + 3*d*x/2)**2*sin(c + d*x)**2*cos(c + d*x)/16 - x*sin(a + 3*
d*x/2)**2*cos(c + d*x)**3/16 - x*sin(a + 3*d*x/2)*sin(c + d*x)**3*cos(a + 3
*d*x/2)/8 + 3*x*sin(a + 3*d*x/2)*sin(c + d*x)*cos(a + 3*d*x/2)*cos(c + d*x)
**2/8 - 3*x*sin(c + d*x)**2*cos(a + 3*d*x/2)**2*cos(c + d*x)/16 + x*cos(a +
 3*d*x/2)**2*cos(c + d*x)**3/16 + 9*sin(a + 3*d*x/2)**2*sin(c + d*x)**3/(16
*d) + 5*sin(a + 3*d*x/2)*sin(c + d*x)**2*cos(a + 3*d*x/2)*cos(c + d*x)/(4*d
) - sin(a + 3*d*x/2)*cos(a + 3*d*x/2)*cos(c + d*x)**3/(24*d) + 5*sin(c + d*
x)**3*cos(a + 3*d*x/2)**2/(48*d) + sin(c + d*x)*cos(a + 3*d*x/2)**2*cos(c +
 d*x)**2/d, Eq(b, 3*d/2)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + si
n(a + b*x)*cos(a + b*x)/(2*b))*cos(c)**3, Eq(d, 0)), (16*b**4*sin(a + b*x)*
*2*sin(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) + 24*b**4*sin(a +
b*x)**2*sin(c + d*x)*cos(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5)
+ 16*b**4*sin(c + d*x)**3*cos(a + b*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d
**5) + 24*b**4*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)**2/(48*b**4*d - 12
0*b**2*d**3 + 27*d**5) + 24*b**3*d*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**
3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 40*b**2*d**2*sin(a + b*x)**2*sin(
c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 42*b**2*d**2*sin(a + b*
x)**2*sin(c + d*x)*cos(c + d*x)**2/(48*b**4*d - 120*b**2*d**3 + 27*d**5) -
```

```
40*b**2*d**2*sin(c + d*x)**3*cos(a + b*x)**2/(48*b**4*d - 120*b**2*d**3 + 2
7*d**5) - 78*b**2*d**2*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)**2/(48*b**
4*d - 120*b**2*d**3 + 27*d**5) - 36*b*d**3*sin(a + b*x)*sin(c + d*x)**2*cos
(a + b*x)*cos(c + d*x)/(48*b**4*d - 120*b**2*d**3 + 27*d**5) - 42*b*d**3*si
n(a + b*x)*cos(a + b*x)*cos(c + d*x)**3/(48*b**4*d - 120*b**2*d**3 + 27*d**
5) + 18*d**4*sin(c + d*x)**3*cos(a + b*x)**2/(48*b**4*d - 120*b**2*d**3 + 2
7*d**5) + 27*d**4*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)**2/(48*b**4*d -
 120*b**2*d**3 + 27*d**5), True))
```

Giac [A]
time = 0.41, size = 129, normalized size = 0.90

$$\frac{\sin(2bx+3dx+2a+3c)}{16(2b+3d)}+\frac{3\sin(2bx+dx+2a+c)}{16(2b+d)}+\frac{3\sin(2bx-dx+2a-c)}{16(2b-d)}+\frac{\sin(2bx-3dx+2a-3c)}{16(2b-3d)}+\frac{\sin(3dx+3c)}{24d}+\frac{3\sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*sin(2*b*x + 3*d*x + 2*a + 3*c)/(2*b + 3*d) + 3/16*sin(2*b*x + d*x + 2*
a + c)/(2*b + d) + 3/16*sin(2*b*x - d*x + 2*a - c)/(2*b - d) + 1/16*sin(2*b
*x - 3*d*x + 2*a - 3*c)/(2*b - 3*d) + 1/24*sin(3*d*x + 3*c)/d + 3/8*sin(d*x
 + c)/d
```

Mupad [B]
time = 2.44, size = 495, normalized size = 3.44

$$-e^{a\,2i-c\,1i+b\,x\,2i-d\,x\,1i}\left(\frac{e^{-a\,2i-b\,x\,2i}\,(24\,b^2-6\,d^2)}{b^2\,d\,128i-d^3\,32i}-\frac{3\,d\,(2\,b+d)}{b^2\,d\,128i-d^3\,32i}+\frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b-d)}{b^2\,d\,128i-d^3\,32i}\right)+e^{a\,2i+c\,1i+b\,x\,2i+d\,x\,1i}\left(\frac{3\,d\,(2\,b-d)}{b^2\,d\,128i-d^3\,32i}+\frac{e^{-a\,2i-b\,x\,2i}\,(24\,b^2-6\,d^2)}{b^2\,d\,128i-d^3\,32i}-\frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b+d)}{b^2\,d\,128i-d^3\,32i}\right)-e^{a\,2i-c\,3i+b\,x\,2i-d\,x\,3i}\left(-\frac{3\,d\,(2\,b+3\,d)}{b^2\,d\,384i-d^3\,864i}+\frac{e^{-a\,2i-b\,x\,2i}\,(8\,b^2-18\,d^2)}{b^2\,d\,384i-d^3\,864i}+\frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b-3\,d)}{b^2\,d\,384i-d^3\,864i}\right)+e^{a\,2i+c\,3i+b\,x\,2i+d\,x\,3i}\left(\frac{3\,d\,(2\,b-3\,d)}{b^2\,d\,384i-d^3\,864i}+\frac{e^{-a\,2i-b\,x\,2i}\,(8\,b^2-18\,d^2)}{b^2\,d\,384i-d^3\,864i}-\frac{3\,d\,e^{-a\,4i-b\,x\,4i}\,(2\,b+3\,d)}{b^2\,d\,384i-d^3\,864i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cos(c + d*x)^3,x)
```

```
[Out] exp(a*2i + c*1i + b*x*2i + d*x*1i)*((3*d*(2*b - d))/(b^2*d*128i - d^3*32i)
+ (exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(b^2*d*128i - d^3*32i) - (3*d*exp
(- a*4i - b*x*4i)*(2*b + d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*1i + b*
x*2i - d*x*1i)*((exp(- a*2i - b*x*2i)*(24*b^2 - 6*d^2))/(b^2*d*128i - d^3*3
2i) - (3*d*(2*b + d))/(b^2*d*128i - d^3*32i) + (3*d*exp(- a*4i - b*x*4i)*(2
*b - d))/(b^2*d*128i - d^3*32i)) - exp(a*2i - c*3i + b*x*2i - d*x*3i)*((exp
(- a*2i - b*x*2i)*(8*b^2 - 18*d^2))/(b^2*d*384i - d^3*864i) - (3*d*(2*b + 3
*d))/(b^2*d*384i - d^3*864i) + (3*d*exp(- a*4i - b*x*4i)*(2*b - 3*d))/(b^2*
d*384i - d^3*864i)) + exp(a*2i + c*3i + b*x*2i + d*x*3i)*((3*d*(2*b - 3*d))
/(b^2*d*384i - d^3*864i) + (exp(- a*2i - b*x*2i)*(8*b^2 - 18*d^2))/(b^2*d*3
84i - d^3*864i) - (3*d*exp(- a*4i - b*x*4i)*(2*b + 3*d))/(b^2*d*384i - d^3*
864i))
```

3.245 $\int \cos^2(a+bx)\cos^2(c+dx)\,dx$

Optimal. Leaf size=88

$$\frac{x}{4}+\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)}+\frac{\sin(2c+2dx)}{8d}+\frac{\sin(2(a+c)+2(b+d)x)}{16(b+d)}$$

```
[Out] 1/4*x+1/8*sin(2*b*x+2*a)/b+1/16*sin(2*a-2*c+2*(b-d)*x)/(b-d)+1/8*sin(2*d*x+
2*c)/d+1/16*sin(2*a+2*c+2*(b+d)*x)/(b+d)
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4666, 2717}

$$\frac{\sin(2(a-c)+2x(b-d))}{16(b-d)}+\frac{\sin(2(a+c)+2x(b+d))}{16(b+d)}+\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2c+2dx)}{8d}+\frac{x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Cos[c + d*x]^2,x]
```

```
[Out] x/4 + Sin[2*a + 2*b*x]/(8*b) + Sin[2*(a - c) + 2*(b - d)*x]/(16*(b - d)) +
Sin[2*c + 2*d*x]/(8*d) + Sin[2*(a + c) + 2*(b + d)*x]/(16*(b + d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4666

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\cos^2(c+dx)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cos(2a+2bx)+\frac{1}{8}\cos(2(a-c)+2(b-d)x)+\frac{1}{4}\cos(2c+2dx\right.\\ &= \frac{x}{4}+\frac{1}{8}\int\cos(2(a-c)+2(b-d)x)\,dx+\frac{1}{8}\int\cos(2(a+c)+2(b+d)x)\,d\\ &= \frac{x}{4}+\frac{\sin(2a+2bx)}{8b}+\frac{\sin(2(a-c)+2(b-d)x)}{16(b-d)}+\frac{\sin(2c+2dx)}{8d}+\frac{\sin(2\end{aligned}$$

Mathematica [A]
time = 0.78, size = 105, normalized size = 1.19

$$\frac{2d(b^2-d^2)\sin(2(a+bx))+bd(b+d)\sin(2(a-c+(b-d)x))+b(b-d)(2(b+d)\sin(2(c+dx))+d(4(b+d)x+\sin(2(a+c+(b+d)x))))}{16b(b-d)d(b+d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Cos[c + d*x]^2,x]
```

```
[Out] (2*d*(b^2 - d^2)*Sin[2*(a + b*x)] + b*d*(b + d)*Sin[2*(a - c + (b - d)*x)]
+ b*(b - d)*(2*(b + d)*Sin[2*(c + d*x)] + d*(4*(b + d)*x + Sin[2*(a + c + (
b + d)*x)])))/(16*b*(b - d)*d*(b + d))
```

Maple [A]
time = 0.18, size = 83, normalized size = 0.94

method	result
default	$\frac{x}{4}+\frac{\sin(2bx+2a)}{8b}+\frac{\sin(2dx+2c)}{8d}+\frac{\sin((2b-2d)x+2a-2c)}{16b-16d}+\frac{\sin((2b+2d)x+2a+2c)}{16b+16d}$
risch	$\frac{x}{4}+\frac{\sin(2bx+2a)}{8b}+\frac{\sin(2dx+2c)b^2}{8(b-d)d(b+d)}-\frac{d\sin(2dx+2c)}{8(b-d)(b+d)}+\frac{\sin(2bx-2dx+2a-2c)b}{16(b-d)(b+d)}+\frac{d\sin(2bx-2dx+2a-2c)}{16(b-d)(b+d)}+\frac{\sin(2bx+2dx+2a}{16(b-d)(b+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*cos(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x+1/8*sin(2*b*x+2*a)/b+1/8*sin(2*d*x+2*c)/d+1/16/(b-d)*sin((2*b-2*d)*x+
2*a-2*c)+1/16/(b+d)*sin((2*b+2*d)*x+2*a+2*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 620 vs. 2(78) = 156.
time = 0.30, size = 620, normalized size = 7.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(8*((b*cos(2*c)^2 + b*sin(2*c)^2)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)
^2)*d)*x + (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a + 4*c) -
 (b^2*d*sin(2*c) - b*d^2*sin(2*c))*cos(2*(b + d)*x + 2*a) - (b^2*d*sin(2*c)
 + b*d^2*sin(2*c))*cos(-2*(b - d)*x - 2*a + 4*c) + (b^2*d*sin(2*c) + b*d^2*
sin(2*c))*cos(-2*(b - d)*x - 2*a) + 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2
*b*x + 2*a + 2*c) - 2*(b^2*d*sin(2*c) - d^3*sin(2*c))*cos(2*b*x + 2*a - 2*c
) - 2*(b^3*sin(2*c) - b*d^2*sin(2*c))*cos(2*d*x) + 2*(b^3*sin(2*c) - b*d^2*
sin(2*c))*cos(2*d*x + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d
)*x + 2*a + 4*c) - (b^2*d*cos(2*c) - b*d^2*cos(2*c))*sin(2*(b + d)*x + 2*a)
 + (b^2*d*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a + 4*c) + (b^2*d
```

```
*cos(2*c) + b*d^2*cos(2*c))*sin(-2*(b - d)*x - 2*a) - 2*(b^2*d*cos(2*c) - d
^3*cos(2*c))*sin(2*b*x + 2*a + 2*c) - 2*(b^2*d*cos(2*c) - d^3*cos(2*c))*sin
(2*b*x + 2*a - 2*c) - 2*(b^3*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x) - 2*(b^3
*cos(2*c) - b*d^2*cos(2*c))*sin(2*d*x + 4*c))/((b*cos(2*c)^2 + b*sin(2*c)^2
)*d^3 - (b^3*cos(2*c)^2 + b^3*sin(2*c)^2)*d)
```

Fricas [A]
time = 2.59, size = 105, normalized size = 1.19

$$-\frac{\left(2\,bd^2\cos\left(bx+a\right)^2-b^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(b^3d-bd^3\right)x-\left(2\,b^2d\cos\left(bx+a\right)\cos\left(dx+c\right)^2-d^3\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{4\left(b^3d-bd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*((2*b*d^2*cos(b*x + a)^2 - b^3)*cos(d*x + c)*sin(d*x + c) - (b^3*d - b
*d^3)*x - (2*b^2*d*cos(b*x + a)*cos(d*x + c)^2 - d^3*cos(b*x + a))*sin(b*x
+ a))/(b^3*d - b*d^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. 2(76) = 152.
time = 1.90, size = 1027, normalized size = 11.67

$$\begin{cases} x\cos^2(a)\cos^2(c) & \text{for}\: b=0 \wedge d=0 \\ \left(\frac{x\sin^2(c+dx)}{2}+\frac{x\cos^2(c+dx)}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\cos^2(a) & \text{for}\: b=0 \\ \frac{3x\sin^2(a-dx)\sin^2(c+dx)}{8}+\frac{x\sin^2(a-dx)\cos^2(c+dx)}{8}-\frac{x\sin(a-dx)\sin(c+dx)\cos(a-dx)\cos(c+dx)}{2}+\frac{x\sin^2(c+dx)\cos^2(a-dx)}{8}+\frac{3x\cos^2(a-dx)\cos^2(c+dx)}{8}+\frac{\sin^2(a-dx)\sin(c+dx)\cos(c+dx)}{2d}+\frac{\sin(a-dx)\sin^2(c+dx)\cos(a-dx)}{8d}-\frac{5\sin(a-dx)\cos(a-dx)\cos^2(c+dx)}{8d} & \text{for}\: b=-d \\ \frac{3x\sin^2(a+dx)\sin^2(c+dx)}{8}+\frac{x\sin^2(a+dx)\cos^2(c+dx)}{8}+\frac{x\sin(a+dx)\sin(c+dx)\cos(a+dx)\cos(c+dx)}{2}+\frac{x\sin^2(c+dx)\cos^2(a+dx)}{8}+\frac{3x\cos^2(a+dx)\cos^2(c+dx)}{8}+\frac{3\sin(a+dx)\sin^2(c+dx)\cos(a+dx)}{8d}+\frac{\sin(a+dx)\cos(a+dx)\cos^2(c+dx)}{8d}+\frac{\sin(c+dx)\cos^2(a+dx)\cos(c+dx)}{2d} & \text{for}\: b=d \\ \left(\frac{x\sin^2(a+bx)}{2}+\frac{x\cos^2(a+bx)}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)\cos^2(c) & \text{for}\: d=0 \\ \frac{b^3dx\sin^2(a+bx)\sin^2(c+dx)}{4b^3d-4bd^3}+\frac{b^3dx\sin^2(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3}+\frac{b^3dx\sin^2(c+dx)\cos^2(a+bx)}{4b^3d-4bd^3}+\frac{b^3dx\cos^2(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3}+\frac{b^3\sin^2(a+bx)\sin(c+dx)\cos(c+dx)}{4b^3d-4bd^3}+\frac{b^3\sin(c+dx)\cos^2(a+bx)\cos(c+dx)}{4b^3d-4bd^3}+\frac{2b^2d\sin(a+bx)\cos(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3}-\frac{bd^3x\sin^2(a+bx)\sin^2(c+dx)}{4b^3d-4bd^3}-\frac{bd^3x\sin^2(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3}-\frac{bd^3x\sin^2(c+dx)\cos^2(a+bx)}{4b^3d-4bd^3}-\frac{bd^3x\cos^2(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3}-\frac{2bd^2\sin(c+dx)\cos^2(a+bx)\cos(c+dx)}{4b^3d-4bd^3}-\frac{d^3\sin(a+bx)\sin^2(c+dx)\cos(a+bx)}{4b^3d-4bd^3}-\frac{d^3\sin(a+bx)\cos(a+bx)\cos^2(c+dx)}{4b^3d-4bd^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*cos(d*x+c)**2,x)
```

```
[Out] Piecewise((x*cos(a)**2*cos(c)**2, Eq(b, 0) & Eq(d, 0)), ((x*sin(c + d*x)**2
/2 + x*cos(c + d*x)**2/2 + sin(c + d*x)*cos(c + d*x)/(2*d))*cos(a)**2, Eq(b
, 0)), (3*x*sin(a - d*x)**2*sin(c + d*x)**2/8 + x*sin(a - d*x)**2*cos(c + d
*x)**2/8 - x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)*cos(c + d*x)/2 + x*sin(
c + d*x)**2*cos(a - d*x)**2/8 + 3*x*cos(a - d*x)**2*cos(c + d*x)**2/8 + sin
(a - d*x)**2*sin(c + d*x)*cos(c + d*x)/(2*d) + sin(a - d*x)*sin(c + d*x)**2
*cos(a - d*x)/(8*d) - 5*sin(a - d*x)*cos(a - d*x)*cos(c + d*x)**2/(8*d), Eq
(b, -d)), (3*x*sin(a + d*x)**2*sin(c + d*x)**2/8 + x*sin(a + d*x)**2*cos(c
+ d*x)**2/8 + x*sin(a + d*x)*sin(c + d*x)*cos(a + d*x)*cos(c + d*x)/2 + x*s
in(c + d*x)**2*cos(a + d*x)**2/8 + 3*x*cos(a + d*x)**2*cos(c + d*x)**2/8 +
3*sin(a + d*x)*sin(c + d*x)**2*cos(a + d*x)/(8*d) + sin(a + d*x)*cos(a + d*
x)*cos(c + d*x)**2/(8*d) + sin(c + d*x)*cos(a + d*x)**2*cos(c + d*x)/(2*d),
 Eq(b, d)), ((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(a + b*x)*cos(
a + b*x)/(2*b))*cos(c)**2, Eq(d, 0)), (b**3*d*x*sin(a + b*x)**2*sin(c + d*x
)**2/(4*b**3*d - 4*b*d**3) + b**3*d*x*sin(a + b*x)**2*cos(c + d*x)**2/(4*b*
*3*d - 4*b*d**3) + b**3*d*x*sin(c + d*x)**2*cos(a + b*x)**2/(4*b**3*d - 4*b
*d**3) + b**3*d*x*cos(a + b*x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) + b
```

```
**3*sin(a + b*x)**2*sin(c + d*x)*cos(c + d*x)/(4*b**3*d - 4*b*d**3) + b**3*
sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x)/(4*b**3*d - 4*b*d**3) + 2*b**2*d*
sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*
sin(a + b*x)**2*sin(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(a + b*
x)**2*cos(c + d*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*sin(c + d*x)**2*cos(
a + b*x)**2/(4*b**3*d - 4*b*d**3) - b*d**3*x*cos(a + b*x)**2*cos(c + d*x)**
2/(4*b**3*d - 4*b*d**3) - 2*b*d**2*sin(c + d*x)*cos(a + b*x)**2*cos(c + d*x
)/(4*b**3*d - 4*b*d**3) - d**3*sin(a + b*x)*sin(c + d*x)**2*cos(a + b*x)/(4
*b**3*d - 4*b*d**3) - d**3*sin(a + b*x)*cos(a + b*x)*cos(c + d*x)**2/(4*b**
3*d - 4*b*d**3), True))
```

Giac [A]
time = 0.41, size = 80, normalized size = 0.91

$$\frac{1}{4}\,x + \frac{\sin\left(2\,bx + 2\,dx + 2\,a + 2\,c\right)}{16\,(b + d)} + \frac{\sin\left(2\,bx - 2\,dx + 2\,a - 2\,c\right)}{16\,(b - d)} + \frac{\sin\left(2\,bx + 2\,a\right)}{8\,b} + \frac{\sin\left(2\,dx + 2\,c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cos(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*x + 1/16*sin(2*b*x + 2*d*x + 2*a + 2*c)/(b + d) + 1/16*sin(2*b*x - 2*d*
x + 2*a - 2*c)/(b - d) + 1/8*sin(2*b*x + 2*a)/b + 1/8*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 1.06, size = 177, normalized size = 2.01

$$\frac{2\,b^3\,\sin\left(2\,c+2\,d\,x\right)-2\,d^3\,\sin\left(2\,a+2\,b\,x\right)+b\,d^2\,\sin\left(2\,a-2\,c+2\,b\,x-2\,d\,x\right)-b\,d^2\,\sin\left(2\,a+2\,c+2\,b\,x+2\,d\,x\right)+b^2\,d\,\sin\left(2\,a-2\,c+2\,b\,x-2\,d\,x\right)+b^2\,d\,\sin\left(2\,a+2\,c+2\,b\,x+2\,d\,x\right)+2\,b^2\,d\,\sin\left(2\,a+2\,b\,x\right)-2\,b\,d^2\,\sin\left(2\,c+2\,d\,x\right)-4\,b\,d^3\,x+4\,b^3\,d\,x}{16\,b\,d\,\left(b^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cos(c + d*x)^2,x)
```

```
[Out] (2*b^3*sin(2*c + 2*d*x) - 2*d^3*sin(2*a + 2*b*x) + b*d^2*sin(2*a - 2*c + 2*
b*x - 2*d*x) - b*d^2*sin(2*a + 2*c + 2*b*x + 2*d*x) + b^2*d*sin(2*a - 2*c +
 2*b*x - 2*d*x) + b^2*d*sin(2*a + 2*c + 2*b*x + 2*d*x) + 2*b^2*d*sin(2*a +
2*b*x) - 2*b*d^2*sin(2*c + 2*d*x) - 4*b*d^3*x + 4*b^3*d*x)/(16*b*d*(b^2 - d
^2))
```

3.246 $\int \cos^3(a+bx)\cos^3(c+dx)\,dx$

Optimal. Leaf size=195

$$\frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}+\frac{\sin(3(a-c)+3(b-d)x)}{96(b-d)}+\frac{3\sin(3a-c+(3b-d)x)}{32(3b-d)}+$$

```
[Out] 3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sin(a-c+(b-d)*x)/(b-d)+1/96*sin(3*a-
3*c+3*(b-d)*x)/(b-d)+3/32*sin(3*a-c+(3*b-d)*x)/(3*b-d)+9/32*sin(a+c+(b+d)*x
)/(b+d)+1/96*sin(3*a+3*c+3*(b+d)*x)/(b+d)+3/32*sin(3*a+c+(3*b+d)*x)/(3*b+d)
+3/32*sin(a+3*c+(b+3*d)*x)/(b+3*d)
```

Rubi [A]

time = 0.09, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4666, 2717}

$$\frac{3\sin(a+x(b-3d)-3c)}{32(b-3d)}+\frac{9\sin(a+x(b-d)-c)}{32(b-d)}+\frac{\sin(3(a-c)+3x(b-d))}{96(b-d)}+\frac{3\sin(3a+x(3b-d)-c)}{32(3b-d)}+\frac{9\sin(a+x(b+d)+c)}{32(b+d)}+\frac{\sin(3(a+c)+3x(b+d))}{96(b+d)}+\frac{3\sin(3a+x(3b+d)+c)}{32(3b+d)}+\frac{3\sin(a+x(b+3d)+3c)}{32(b+3d)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Cos[c + d*x]^3,x]
```

```
[Out] (3*Sin[a - 3*c + (b - 3*d)*x])/(32*(b - 3*d)) + (9*Sin[a - c + (b - d)*x])/
(32*(b - d)) + Sin[3*(a - c) + 3*(b - d)*x]/(96*(b - d)) + (3*Sin[3*a - c +
 (3*b - d)*x])/(32*(3*b - d)) + (9*Sin[a + c + (b + d)*x])/(32*(b + d)) + S
in[3*(a + c) + 3*(b + d)*x]/(96*(b + d)) + (3*Sin[3*a + c + (3*b + d)*x])/(
32*(3*b + d)) + (3*Sin[a + 3*c + (b + 3*d)*x])/(32*(b + 3*d))
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4666

```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\cos^3(c+dx)\,dx &= \int \left(\frac{3}{32}\cos(a-3c+(b-3d)x)+\frac{9}{32}\cos(a-c+(b-d)x)+\frac{1}{32}\cos(3(a\right.\\ &= \frac{1}{32}\int \cos(3(a-c)+3(b-d)x)\,dx+\frac{1}{32}\int \cos(3(a+c)+3(b+d)x)\,dx\\ &= \frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}+\frac{\sin(3(a-c)+3(b\right.}{96(b-d)}\end{aligned}$$

Mathematica [A]
time = 1.71, size = 176, normalized size = 0.90

$$\frac{1}{96}\left(\frac{9\sin(a-3c+bx-3dx)}{b-3d}+\frac{27\sin(a-c+bx-dx)}{b-d}+\frac{\sin(3(a-c+bx-dx))}{b-d}+\frac{9\sin(3a-c+3bx-dx)}{3b-d}+\frac{9\sin(3a+c+3bx+dx)}{3b+d}+\frac{9\sin(a+3c+bx+3dx)}{b+3d}+\frac{27\sin(a+c+(b+d)x)}{b+d}+\frac{\sin(3(a+c+(b+d)x))}{b+d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Cos[c + d*x]^3,x]
```

```
[Out] ((9*Sin[a - 3*c + b*x - 3*d*x])/(b - 3*d) + (27*Sin[a - c + b*x - d*x])/(b
- d) + Sin[3*(a - c + b*x - d*x)]/(b - d) + (9*Sin[3*a - c + 3*b*x - d*x])/
(3*b - d) + (9*Sin[3*a + c + 3*b*x + d*x])/(3*b + d) + (9*Sin[a + 3*c + b*x
 + 3*d*x])/(b + 3*d) + (27*Sin[a + c + (b + d)*x])/(b + d) + Sin[3*(a + c +
 (b + d)*x)]/(b + d))/96
```

Maple [A]
time = 0.38, size = 184, normalized size = 0.94

method	result
default	$\frac{3\sin(a-3c+(b-3d)x)}{32(b-3d)}+\frac{9\sin(a-c+(b-d)x)}{32(b-d)}+\frac{9\sin(a+c+(b+d)x)}{32(b+d)}+\frac{3\sin(a+3c+(b+3d)x)}{32(b+3d)}+\frac{\sin((3b-3d)x+3a-3c)}{96b-96d}+\frac{3\sin(}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*cos(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/32*sin(a-3*c+(b-3*d)*x)/(b-3*d)+9/32*sin(a-c+(b-d)*x)/(b-d)+9/32*sin(a+c+
(b+d)*x)/(b+d)+3/32*sin(a+3*c+(b+3*d)*x)/(b+3*d)+1/96/(b-d)*sin((3*b-3*d)*x
+3*a-3*c)+3/32*sin(3*a-c+(3*b-d)*x)/(3*b-d)+3/32*sin(3*a+c+(3*b+d)*x)/(3*b+
d)+1/96/(b+d)*sin((3*b+3*d)*x+3*a+3*c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2614 vs. $2(179) = 358$.
time = 0.42, size = 2614, normalized size = 13.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*d
^3*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a + 4
*c) - 9*(3*b^5*sin(3*c) - b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) + 10*b^2*d^3
*sin(3*c) + 27*b*d^4*sin(3*c) - 9*d^5*sin(3*c))*cos((3*b + d)*x + 3*a - 2*c
) - 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*d^3*s
in(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a + 4*c)
```

+ 9*(3*b^5*sin(3*c) + b^4*d*sin(3*c) - 30*b^3*d^2*sin(3*c) - 10*b^2*d^3*si
n(3*c) + 27*b*d^4*sin(3*c) + 9*d^5*sin(3*c))*cos(-(3*b - d)*x - 3*a - 2*c)
+ 9*(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^3*
sin(3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a + 6*c) - 9*
(9*b^5*sin(3*c) - 27*b^4*d*sin(3*c) - 10*b^3*d^2*sin(3*c) + 30*b^2*d^3*sin(
3*c) + b*d^4*sin(3*c) - 3*d^5*sin(3*c))*cos((b + 3*d)*x + a) + (9*b^5*sin(3
*c) - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^
4*sin(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a + 6*c) - (9*b^5*sin(3*c)
 - 9*b^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*s
in(3*c) - 9*d^5*sin(3*c))*cos(3*(b + d)*x + 3*a) + 27*(9*b^5*sin(3*c) - 9*b
^4*d*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c
) - 9*d^5*sin(3*c))*cos((b + d)*x + a + 4*c) - 27*(9*b^5*sin(3*c) - 9*b^4*d
*sin(3*c) - 82*b^3*d^2*sin(3*c) + 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) -
9*d^5*sin(3*c))*cos((b + d)*x + a - 2*c) - 27*(9*b^5*sin(3*c) + 9*b^4*d*sin
(3*c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^
5*sin(3*c))*cos(-(b - d)*x - a + 4*c) + 27*(9*b^5*sin(3*c) + 9*b^4*d*sin(3*
c) - 82*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*s
in(3*c))*cos(-(b - d)*x - a - 2*c) - (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c) - 8
2*b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin(3*c
))*cos(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*sin(3*c) + 9*b^4*d*sin(3*c) - 82*
b^3*d^2*sin(3*c) - 82*b^2*d^3*sin(3*c) + 9*b*d^4*sin(3*c) + 9*d^5*sin(3*c))
cos(-3(b - d)*x - 3*a) - 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^3*d
^2*sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*cos(-(
b - 3*d)*x - a + 6*c) + 9*(9*b^5*sin(3*c) + 27*b^4*d*sin(3*c) - 10*b^3*d^2*
sin(3*c) - 30*b^2*d^3*sin(3*c) + b*d^4*sin(3*c) + 3*d^5*sin(3*c))*cos(-(b -
 3*d)*x - a) - 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 1
0*b^2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin((3*b + d)*x +
3*a + 4*c) - 9*(3*b^5*cos(3*c) - b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) + 10*
b^2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin((3*b + d)*x + 3*
a - 2*c) + 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^
2*d^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*sin(-(3*b - d)*x - 3*a
 + 4*c) + 9*(3*b^5*cos(3*c) + b^4*d*cos(3*c) - 30*b^3*d^2*cos(3*c) - 10*b^2
*d^3*cos(3*c) + 27*b*d^4*cos(3*c) + 9*d^5*cos(3*c))*sin(-(3*b - d)*x - 3*a
- 2*c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b
^2*d^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*sin((b + 3*d)*x + a + 6*
c) - 9*(9*b^5*cos(3*c) - 27*b^4*d*cos(3*c) - 10*b^3*d^2*cos(3*c) + 30*b^2*d
^3*cos(3*c) + b*d^4*cos(3*c) - 3*d^5*cos(3*c))*sin((b + 3*d)*x + a) - (9*b^
5*cos(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) +
 9*b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin(3*(b + d)*x + 3*a + 6*c) - (9*b^5*c
os(3*c) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*
b*d^4*cos(3*c) - 9*d^5*cos(3*c))*sin(3*(b + d)*x + 3*a) - 27*(9*b^5*cos(3*c
) - 9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*
cos(3*c) - 9*d^5*cos(3*c))*sin((b + d)*x + a + 4*c) - 27*(9*b^5*cos(3*c) -
9*b^4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) + 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(
3*c) - 9*d^5*cos(3*c))*sin((b + d)*x + a - 2*c) + 27*(9*b^5*cos(3*c) + 9*b^

```
4*d*cos(3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c)
 + 9*d^5*cos(3*c))*sin(-(b - d)*x - a + 4*c) + 27*(9*b^5*cos(3*c) + 9*b^4*d
*cos(3*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) +
9*d^5*cos(3*c))*sin(-(b - d)*x - a - 2*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3
*c) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*
cos(3*c))*sin(-3*(b - d)*x - 3*a + 6*c) + (9*b^5*cos(3*c) + 9*b^4*d*cos(3*c
) - 82*b^3*d^2*cos(3*c) - 82*b^2*d^3*cos(3*c) + 9*b*d^4*cos(3*c) + 9*d^5*co
s(3*c))*sin(-3*(b - d)*x - 3*a) + 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 1
0*b^3*d^2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))
*sin(-(b - 3*d)*x - a + 6*c) + 9*(9*b^5*cos(3*c) + 27*b^4*d*cos(3*c) - 10*b
^3*d^2*cos(3*c) - 30*b^2*d^3*cos(3*c) + b*d^4*cos(3*c) + 3*d^5*cos(3*c))*si
n(-(b - 3*d)*x - a))/(9*b^6*cos(3*c)^2 + 9*b^6*sin(3*c)^2 - 9*(cos(3*c)^2 +
 sin(3*c)^2)*d^6 + 91*(b^2*cos(3*c)^2 + b^2*sin(3*c)^2)*d^4 - 91*(b^4*cos(3
*c)^2 + b^4*sin(3*c)^2)*d^2)
```

Fricas [**A**]
time = 3.15, size = 240, normalized size = 1.23

$$\frac{\left(\left(18\,b^5-2\,b^3d^2+\left(9\,b^5-82\,b^3d^2+9\,bd^4\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)^3-6\,\left(20\,b^3d^2+\left(b^3d^2-9\,bd^4\right)\cos\left(bx+a\right)^2\right)\cos\left(dx+c\right)\right)\sin\left(bx+a\right)+\left(120\,b^2d^3\cos\left(bx+a\right)+2\,\left(b^2d^3-9\,d^5\right)\cos\left(bx+a\right)^3-\left(\left(9\,b^4d-82\,b^2d^3+9\,d^5\right)\cos\left(bx+a\right)^3+6\,\left(9\,b^4d-b^2d^3\right)\cos\left(bx+a\right)\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{3\,\left(9\,b^6-91\,b^4d^2+91\,b^2d^4-9\,d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(((18*b^5 - 2*b^3*d^2 + (9*b^5 - 82*b^3*d^2 + 9*b*d^4)*cos(b*x + a)^2)*
cos(d*x + c)^3 - 6*(20*b^3*d^2 + (b^3*d^2 - 9*b*d^4)*cos(b*x + a)^2)*cos(d*
x + c))*sin(b*x + a) + (120*b^2*d^3*cos(b*x + a) + 2*(b^2*d^3 - 9*d^5)*cos(
b*x + a)^3 - ((9*b^4*d - 82*b^2*d^3 + 9*d^5)*cos(b*x + a)^3 + 6*(9*b^4*d -
b^2*d^3)*cos(b*x + a))*cos(d*x + c)^2)*sin(d*x + c))/(9*b^6 - 91*b^4*d^2 +
91*b^2*d^4 - 9*d^6)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3582 vs. $2(172) = 344$.
time = 21.03, size = 3582, normalized size = 18.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*cos(d*x+c)**3,x)
```

```
[Out] Piecewise((x*cos(a)**3*cos(c)**3, Eq(b, 0) & Eq(d, 0)), (3*x*sin(a - 3*d*x)
**3*sin(c + d*x)**3/32 - 9*x*sin(a - 3*d*x)**3*sin(c + d*x)*cos(c + d*x)**2
/32 - 9*x*sin(a - 3*d*x)**2*sin(c + d*x)**2*cos(a - 3*d*x)*cos(c + d*x)/32
+ 3*x*sin(a - 3*d*x)**2*cos(a - 3*d*x)*cos(c + d*x)**3/32 + 3*x*sin(a - 3*d
*x)*sin(c + d*x)**3*cos(a - 3*d*x)**2/32 - 9*x*sin(a - 3*d*x)*sin(c + d*x)*
cos(a - 3*d*x)**2*cos(c + d*x)**2/32 - 9*x*sin(c + d*x)**2*cos(a - 3*d*x)**
3*cos(c + d*x)/32 + 3*x*cos(a - 3*d*x)**3*cos(c + d*x)**3/32 - 3*sin(a - 3*
```

```
d*x)**3*sin(c + d*x)**2*cos(c + d*x)/(320*d) - sin(a - 3*d*x)**3*cos(c + d*
x)**3/(4*d) - 11*sin(a - 3*d*x)**2*sin(c + d*x)**3*cos(a - 3*d*x)/(320*d) -
 3*sin(a - 3*d*x)**2*sin(c + d*x)*cos(a - 3*d*x)*cos(c + d*x)**2/(20*d) - 1
17*sin(a - 3*d*x)*cos(a - 3*d*x)**2*cos(c + d*x)**3/(320*d) - sin(c + d*x)*
*3*cos(a - 3*d*x)**3/(30*d) - 61*sin(c + d*x)*cos(a - 3*d*x)**3*cos(c + d*x
)**2/(320*d), Eq(b, -3*d)), (-5*x*sin(a - d*x)**3*sin(c + d*x)**3/16 - 3*x*
sin(a - d*x)**3*sin(c + d*x)*cos(c + d*x)**2/16 + 9*x*sin(a - d*x)**2*sin(c
 + d*x)**2*cos(a - d*x)*cos(c + d*x)/16 + 3*x*sin(a - d*x)**2*cos(a - d*x)*
cos(c + d*x)**3/16 - 3*x*sin(a - d*x)*sin(c + d*x)**3*cos(a - d*x)**2/16 -
9*x*sin(a - d*x)*sin(c + d*x)*cos(a - d*x)**2*cos(c + d*x)**2/16 + 3*x*sin(
c + d*x)**2*cos(a - d*x)**3*cos(c + d*x)/16 + 5*x*cos(a - d*x)**3*cos(c + d
*x)**3/16 + 3*sin(a - d*x)**3*sin(c + d*x)**2*cos(c + d*x)/(16*d) + sin(a -
 d*x)**3*cos(c + d*x)**3/(16*d) + sin(a - d*x)**2*sin(c + d*x)**3*cos(a - d
*x)/(2*d) - 3*sin(a - d*x)*sin(c + d*x)**2*cos(a - d*x)**2*cos(c + d*x)/(4*
d) + 7*sin(c + d*x)**3*cos(a - d*x)**3/(48*d) + 11*sin(c + d*x)*cos(a - d*x
)**3*cos(c + d*x)**2/(16*d), Eq(b, -d)), (3*x*sin(a - d*x/3)**3*sin(c + d*x
)**3/32 + 3*x*sin(a - d*x/3)**3*sin(c + d*x)*cos(c + d*x)**2/32 - 9*x*sin(a
 - d*x/3)**2*sin(c + d*x)**2*cos(a - d*x/3)*cos(c + d*x)/32 - 9*x*sin(a - d
*x/3)**2*cos(a - d*x/3)*cos(c + d*x)**3/32 - 9*x*sin(a - d*x/3)*sin(c + d*x
)**3*cos(a - d*x/3)**2/32 - 9*x*sin(a - d*x/3)*sin(c + d*x)*cos(a - d*x/3)*
*2*cos(c + d*x)**2/32 + 3*x*sin(c + d*x)**2*cos(a - d*x/3)**3*cos(c + d*x)/
32 + 3*x*cos(a - d*x/3)**3*cos(c + d*x)**3/32 + 33*sin(a - d*x/3)**3*sin(c
+ d*x)**2*cos(c + d*x)/(320*d) + sin(a - d*x/3)**3*cos(c + d*x)**3/(10*d) +
 9*sin(a - d*x/3)**2*sin(c + d*x)**3*cos(a - d*x/3)/(320*d) + 9*sin(a - d*x
/3)*sin(c + d*x)**2*cos(a - d*x/3)**2*cos(c + d*x)/(20*d) + 183*sin(a - d*x
/3)*cos(a - d*x/3)**2*cos(c + d*x)**3/(320*d) + 3*sin(c + d*x)**3*cos(a - d
*x/3)**3/(4*d) + 351*sin(c + d*x)*cos(a - d*x/3)**3*cos(c + d*x)**2/(320*d)
, Eq(b, -d/3)), (-3*x*sin(a + d*x/3)**3*sin(c + d*x)**3/32 - 3*x*sin(a + d*
x/3)**3*sin(c + d*x)*cos(c + d*x)**2/32 - 9*x*sin(a + d*x/3)**2*sin(c + d*x
)**2*cos(a + d*x/3)*cos(c + d*x)/32 - 9*x*sin(a + d*x/3)**2*cos(a + d*x/3)*
cos(c + d*x)**3/32 + 9*x*sin(a + d*x/3)*sin(c + d*x)**3*cos(a + d*x/3)**2/3
2 + 9*x*sin(a + d*x/3)*sin(c + d*x)*cos(a + d*x/3)**2*cos(c + d*x)**2/32 +
3*x*sin(c + d*x)**2*cos(a + d*x/3)**3*cos(c + d*x)/32 + 3*x*cos(a + d*x/3)*
*3*cos(c + d*x)**3/32 - 33*sin(a + d*x/3)**3*sin(c + d*x)**2*cos(c + d*x)/(
320*d) - sin(a + d*x/3)**3*cos(c + d*x)**3/(10*d) + 9*sin(a + d*x/3)**2*sin
(c + d*x)**3*cos(a + d*x/3)/(320*d) - 9*sin(a + d*x/3)*sin(c + d*x)**2*cos(
a + d*x/3)**2*cos(c + d*x)/(20*d) - 183*sin(a + d*x/3)*cos(a + d*x/3)**2*co
s(c + d*x)**3/(320*d) + 3*sin(c + d*x)**3*cos(a + d*x/3)**3/(4*d) + 351*sin
(c + d*x)*cos(a + d*x/3)**3*cos(c + d*x)**2/(320*d), Eq(b, d/3)), (5*x*sin(
a + d*x)**3*sin(c + d*x)**3/16 + 3*x*sin(a + d*x)**3*sin(c + d*x)*cos(c + d
*x)**2/16 + 9*x*sin(a + d*x)**2*sin(c + d*x)**2*cos(a + d*x)*cos(c + d*x)/1
6 + 3*x*sin(a + d*x)**2*cos(a + d*x)*cos(c + d*x)**3/16 + 3*x*sin(a + d*x)*
sin(c + d*x)**3*cos(a + d*x)**2/16 + 9*x*sin(a + d*x)*sin(c + d*x)*cos(a +
d*x)**2*cos(c + d*x)**2/16 + 3*x*sin(c + d*x)**2*cos(a + d*x)**3*cos(c + d*
x)/16 + 5*x*cos(a + d*x)**3*cos(c + d*x)**3/16 + 5*sin(a + d*x)**3*sin(c +
```

```
d*x)**2*cos(c + d*x)/(16*d) + 13*sin(a + d*x)**3*cos(c + d*x)**3/(48*d) + 3
*sin(a + d*x)*sin(c + d*x)**2*cos(a + d*x)**2*cos(c + d*x)/(4*d) + sin(a +
d*x)*cos(a + d*x)**2*cos(c + d*x)**3/(2*d) - 3*sin(c + d*x)**3*cos(a + d*x)
**3/(16*d) + 3*sin(c + d*x)*cos(a + d*x)**3*cos(c + d*x)**2/(16*d), Eq(b, d
)), (-3*x*sin(a + 3*d*x)**3*sin(c + d*x)**3/32 + 9*x*sin(a + 3*d*x)**3*sin(
c + d*x)*cos(c + d*x)**2/32 - 9*x*sin(a + 3*d*x)**2*sin(c + d*x)**2*cos(a +
 3*d*x)*cos(c + d*x)/32 + 3*x*sin(a + 3*d*x)**2*cos(a + 3*d*x)*cos(c + d*x)
**3/32 - 3*x*sin(a + 3*d*x)*sin(c + d*x)**3*cos(a + 3*d*x)**2/32 + 9*x*sin(
a + 3*d*x)*sin(c + d*x)*cos(a + 3*d*x)**2*cos(c + d*x)**2/32 - 9*x*sin(c +
d*x)**2*cos(a + 3*d*x)**3*cos(c + d*x)/32 + 3*x*cos(a + 3*d*x)**3*cos(c + d
*x)**3/32 + 51*sin(a + 3*d*x)**3*sin(c + d*x)**2*cos(c + d*x)/(320*d) + sin
(a + 3*d*x)**3*cos(c + d*x)**3/(5*d) - 27*sin(a + 3*d*x)**2*sin(c + d*x)**3
*cos(a + 3*d*x)/(320*d) + 3*sin(a + 3*d*x)*sin(c + d*x)**2*cos(a + 3*d*x)**
2*cos(c + d*x)/(20*d) + 101*sin(a + 3*d*x)*cos(a + 3*d*x)**2*cos(c + d*x)**
3/(320*d) - sin(c + d*x)**3*cos(a + 3*d*x)**3/(...
```

Giac [A]
time = 0.42, size = 181, normalized size = 0.93

$$\frac{\sin\left(3\,bx+3\,dx+3\,a+3\,c\right)}{96\,(b+d)}+\frac{3\,\sin\left(3\,bx+dx+3\,a+c\right)}{32\,(3\,b+d)}+\frac{3\,\sin\left(3\,bx-dx+3\,a-c\right)}{32\,(3\,b-d)}+\frac{\sin\left(3\,bx-3\,dx+3\,a-3\,c\right)}{96\,(b-d)}+\frac{3\,\sin\left(bx+3\,dx+a+3\,c\right)}{32\,(b+3\,d)}+\frac{9\,\sin\left(bx+dx+a+c\right)}{32\,(b+d)}+\frac{9\,\sin\left(bx-dx+a-c\right)}{32\,(b-d)}+\frac{3\,\sin\left(bx-3\,dx+a-3\,c\right)}{32\,(b-3\,d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*cos(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/96*sin(3*b*x + 3*d*x + 3*a + 3*c)/(b + d) + 3/32*sin(3*b*x + d*x + 3*a +
c)/(3*b + d) + 3/32*sin(3*b*x - d*x + 3*a - c)/(3*b - d) + 1/96*sin(3*b*x -
 3*d*x + 3*a - 3*c)/(b - d) + 3/32*sin(b*x + 3*d*x + a + 3*c)/(b + 3*d) + 9
/32*sin(b*x + d*x + a + c)/(b + d) + 9/32*sin(b*x - d*x + a - c)/(b - d) +
3/32*sin(b*x - 3*d*x + a - 3*c)/(b - 3*d)
```

Mupad [B]
time = 4.54, size = 999, normalized size = 5.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*cos(c + d*x)^3,x)
```

```
[Out] - exp(a*3i - c*1i + b*x*3i - d*x*1i)*((9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3)/(
b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*6i - b*x*6i)*(9*b*d^2 + 3*b^2
*d - 9*b^3 - 3*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i) + (exp(- a*2i - b*
x*2i)*(9*b*d^2 - 81*b^2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*
640i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))/(b^4*5
76i + d^4*64i - b^2*d^2*640i)) - exp(a*3i + c*1i + b*x*3i + d*x*1i)*((9*b*d
^2 + 3*b^2*d - 9*b^3 - 3*d^3)/(b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(-
a*6i - b*x*6i)*(9*b*d^2 - 3*b^2*d - 9*b^3 + 3*d^3))/(b^4*576i + d^4*64i - b
```

```
^2*d^2*640i) + (exp(- a*2i - b*x*2i)*(9*b*d^2 + 81*b^2*d - 81*b^3 - 9*d^3))
/(b^4*576i + d^4*64i - b^2*d^2*640i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 - 81*
b^2*d - 81*b^3 + 9*d^3))/(b^4*576i + d^4*64i - b^2*d^2*640i)) - exp(a*3i -
c*3i + b*x*3i - d*x*3i)*((9*b*d^2 - b^2*d - b^3 + 9*d^3)/(b^4*192i + d^4*17
28i - b^2*d^2*1920i) - (exp(- a*6i - b*x*6i)*(9*b*d^2 + b^2*d - b^3 - 9*d^3
))/(b^4*192i + d^4*1728i - b^2*d^2*1920i) + (exp(- a*2i - b*x*2i)*(9*b*d^2
- 27*b^2*d - 9*b^3 + 27*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i) - (exp
(- a*4i - b*x*4i)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(b^4*192i + d^4*17
28i - b^2*d^2*1920i)) - exp(a*3i + c*3i + b*x*3i + d*x*3i)*((9*b*d^2 + b^2*
d - b^3 - 9*d^3)/(b^4*192i + d^4*1728i - b^2*d^2*1920i) - (exp(- a*6i - b*x
*6i)*(9*b*d^2 - b^2*d - b^3 + 9*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i
) + (exp(- a*2i - b*x*2i)*(9*b*d^2 + 27*b^2*d - 9*b^3 - 27*d^3))/(b^4*192i
+ d^4*1728i - b^2*d^2*1920i) - (exp(- a*4i - b*x*4i)*(9*b*d^2 - 27*b^2*d -
9*b^3 + 27*d^3))/(b^4*192i + d^4*1728i - b^2*d^2*1920i))
```

3.247 $\int \cos(a+bx)\tan^3(c+bx)\,dx$

Optimal. Leaf size=72

$$\frac{\cos(a+bx)}{b}+\frac{\cos(a-c)\sec(c+bx)}{b}+\frac{3\tanh^{-1}(\sin(c+bx))\sin(a-c)}{2b}-\frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b}$$

```
[Out] cos(b*x+a)/b+cos(a-c)*sec(b*x+c)/b+3/2*arctanh(sin(b*x+c))*sin(a-c)/b-1/2*s
ec(b*x+c)*sin(a-c)*tan(b*x+c)/b
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$,
Rules used = {4675, 4672, 2718, 3855, 2686, 8, 2691}

$$\frac{3\sin(a-c)\tanh^{-1}(\sin(bx+c))}{2b}+\frac{\cos(a-c)\sec(bx+c)}{b}-\frac{\sin(a-c)\tan(bx+c)\sec(bx+c)}{2b}+\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Tan[c + b*x]^3,x]
```

```
[Out] Cos[a + b*x]/b + (Cos[a - c]*Sec[c + b*x])/b + (3*ArcTanh[Sin[c + b*x]]*Sin
[a - c])/(2*b) - (Sec[c + b*x]*Sin[a - c]*Tan[c + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4672

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4675

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\int \cos(a+bx)\tan^3(c+bx)\,dx = -\left(\sin(a-c)\int \sec(c+bx)\tan^2(c+bx)\,dx\right) + \int \sin(a+bx)\tan^2(c+bx$$

$$= -\frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b} + \cos(a-c)\int \sec(c+bx)\tan(c+bx$$

$$= \frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{2b} - \frac{\sec(c+bx)\sin(a-c)\tan(c+bx)}{2b} + \cos$$

$$= \frac{\cos(a+bx)}{b} + \frac{\cos(a-c)\sec(c+bx)}{b} + \frac{3\tanh^{-1}(\sin(c+bx))\sin(a-c)}{2b} -$$

Mathematica [A]
time = 0.41, size = 70, normalized size = 0.97

$$\frac{(2\cos(a-2c-bx)+5\cos(a+bx)+\cos(a+2c+3bx))\sec^2(c+bx)+12\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{bx}{2}\right)\right)\sin(a-c)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Tan[c + b*x]^3,x]
```

```
[Out] ((2*Cos[a - 2*c - b*x] + 5*Cos[a + b*x] + Cos[a + 2*c + 3*b*x])*Sec[c + b*x
]^2 + 12*ArcTanh[Sin[c] + Cos[c]*Tan[(b*x)/2]]*Sin[a - c])/(4*b)
```

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 181, normalized size = 2.51

method	result
risch	$\frac{e^{i(bx+a)}}{2b}+\frac{e^{-i(bx+a)}}{2b}+\frac{3\,e^{i(3bx+5a+2c)}+e^{i(3bx+3a+4c)}+e^{i(bx+5a)}+3\,e^{i(bx+3a+2c)}}{2b\left(e^{2i(bx+a+c)}+e^{2ia}\right)^2}+\frac{3\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\sin(a-c)}{2b}-\frac{3\ln(e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*tan(b*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+1/2/b/(exp(2*I*(b*x+a+c))+exp(2*
I*a))^2*(3*exp(I*(3*b*x+5*a+2*c))+exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+5*a))+3
*exp(I*(b*x+3*a+2*c)))+3/2*ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*sin(a-c)-3/2
*ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(68) = 136$.
time = 0.59, size = 1027, normalized size = 14.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(cos(5*b*x + a + 4*c) + 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*cos(6
*b*x + 2*a + 4*c) + 2*(5*cos(4*b*x + 2*a + 2*c) + 2*cos(4*b*x + 4*c) + 2*co
s(2*b*x + 2*a) + 5*cos(2*b*x + 2*c) + 1)*cos(5*b*x + a + 4*c) + 10*(2*cos(3
*b*x + a + 2*c) + cos(b*x + a))*cos(4*b*x + 2*a + 2*c) + 4*(2*cos(3*b*x + a
 + 2*c) + cos(b*x + a))*cos(4*b*x + 4*c) + 4*(2*cos(2*b*x + 2*a) + 5*cos(2*
b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 4*cos(2*b*x + 2*a)*cos(b*x + a) + 10
*cos(2*b*x + 2*c)*cos(b*x + a) + 3*(cos(5*b*x + a + 4*c)^2*sin(-a + c) + 4*
cos(3*b*x + a + 2*c)^2*sin(-a + c) + 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*si
n(-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(5*b*x + a + 4*c)^2*sin(-a + c)
 + 4*sin(3*b*x + a + 2*c)^2*sin(-a + c) + 4*sin(3*b*x + a + 2*c)*sin(b*x +
a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c) + 2*(2*cos(3*b*x + a + 2*c)*sin
(-a + c) + cos(b*x + a)*sin(-a + c))*cos(5*b*x + a + 4*c) + 2*(2*sin(3*b*x
+ a + 2*c)*sin(-a + c) + sin(b*x + a)*sin(-a + c))*sin(5*b*x + a + 4*c))*lo
g((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2
 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos
(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)
) + 2*(sin(5*b*x + a + 4*c) + 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*sin(6*
b*x + 2*a + 4*c) + 2*(5*sin(4*b*x + 2*a + 2*c) + 2*sin(4*b*x + 4*c) + 2*sin
(2*b*x + 2*a) + 5*sin(2*b*x + 2*c))*sin(5*b*x + a + 4*c) + 10*(2*sin(3*b*x
+ a + 2*c) + sin(b*x + a))*sin(4*b*x + 2*a + 2*c) + 4*(2*sin(3*b*x + a + 2*
c) + sin(b*x + a))*sin(4*b*x + 4*c) + 4*(2*sin(2*b*x + 2*a) + 5*sin(2*b*x +
 2*c))*sin(3*b*x + a + 2*c) + 4*sin(2*b*x + 2*a)*sin(b*x + a) + 10*sin(2*b*
x + 2*c)*sin(b*x + a) + 2*cos(b*x + a))/(b*cos(5*b*x + a + 4*c)^2 + 4*b*cos
```

```
(3*b*x + a + 2*c)^2 + 4*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a
)^2 + b*sin(5*b*x + a + 4*c)^2 + 4*b*sin(3*b*x + a + 2*c)^2 + 4*b*sin(3*b*x
 + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2 + 2*(2*b*cos(3*b*x + a + 2*c) +
 b*cos(b*x + a))*cos(5*b*x + a + 4*c) + 2*(2*b*sin(3*b*x + a + 2*c) + b*sin
(b*x + a))*sin(5*b*x + a + 4*c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(68) = 136$.
time = 3.09, size = 366, normalized size = 5.08

$$\frac{16 \cos\left(bx+a\right)^{3} \cos\left(-2\,a+2\,c\right) - 4 \left(4 \cos\left(bx+a\right)^{2}+1\right) \sin\left(bx+a\right) \sin\left(-2\,a+2\,c\right) - 4 \left(\cos\left(-2\,a+2\,c\right)-5\right) \cos\left(bx+a\right) + \frac{3\sqrt{2}\left(2\left(\cos(-2\,a+2\,c)^{2}-1\right)\cos(bx+a)\sin(bx+a)+\left(2\cos(bx+a)^{2}\cos(-2\,a+2\,c)-\cos(-2\,a+2\,c)+1\right)\sin(-2\,a+2\,c)\right)\log\left(-\frac{2\cos(bx+a)^{2}\cos(-2\,a+2\,c)-2\cos(bx+a)\sin(bx+a)\sin(-2\,a+2\,c)+\frac{2\sqrt{2}\left((\cos(-2\,a+2\,c)+1)\sin(bx+a)+\cos(bx+a)\sin(-2\,a+2\,c)\right)}{\sqrt{\cos\left(-2\,a+2\,c\right)+1}}-\cos(-2\,a+2\,c)-3}{2\cos(bx+a)^{2}\cos(-2\,a+2\,c)-2\cos(bx+a)\sin(bx+a)\sin(-2\,a+2\,c)-\cos(-2\,a+2\,c)+1}\right)}{\sqrt{\cos\left(-2\,a+2\,c\right)+1}}}{8 \left(2\,b \cos\left(bx+a\right)^{2} \cos\left(-2\,a+2\,c\right) - 2\,b \cos\left(bx+a\right) \sin\left(bx+a\right) \sin\left(-2\,a+2\,c\right) - b \cos\left(-2\,a+2\,c\right) + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(16*cos(b*x + a)^3*cos(-2*a + 2*c) - 4*(4*cos(b*x + a)^2 + 1)*sin(b*x +
 a)*sin(-2*a + 2*c) - 4*(cos(-2*a + 2*c) - 5)*cos(b*x + a) + 3*sqrt(2)*(2*(
cos(-2*a + 2*c)^2 - 1)*cos(b*x + a)*sin(b*x + a) + (2*cos(b*x + a)^2*cos(-2
*a + 2*c) - cos(-2*a + 2*c) + 1)*sin(-2*a + 2*c))*log(-(2*cos(b*x + a)^2*co
s(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((c
os(-2*a + 2*c) + 1)*sin(b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-
2*a + 2*c) + 1) - cos(-2*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) -
2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(co
s(-2*a + 2*c) + 1))/(2*b*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*b*cos(b*x + a)*
sin(b*x + a)*sin(-2*a + 2*c) - b*cos(-2*a + 2*c) + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos\left(a+bx\right) \tan^{3}\left(bx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)**3,x)
```

```
[Out] Integral(cos(a + b*x)*tan(b*x + c)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*tan(b*x + c)^3, x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*tan(c + b*x)^3,x)
[Out] \text{Hanged}
```

3.248 $\int \cos(a+bx)\tan^2(c+bx)\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sec(c+bx)\sin(a-c)}{b}-\frac{\sin(a+bx)}{b}$$

```
[Out] arctanh(sin(b*x+c))*cos(a-c)/b-sec(b*x+c)*sin(a-c)/b-sin(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.400$, Rules used = {4675, 4672, 2717, 3855, 2686, 8}

$$-\frac{\sin(a-c)\sec(bx+c)}{b}+\frac{\cos(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Tan[c + b*x]^2,x]
```

```
[Out] (ArcTanh[Sin[c + b*x]]*Cos[a - c])/b - (Sec[c + b*x]*Sin[a - c])/b - Sin[a
+ b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4672

```
Int[Sin[v_]*Tan[w_]^(n_.), x_Symbol] :> -Int[Cos[v]*Tan[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
```

```
w, x] && NeQ[w, v]
```

Rule 4675

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\tan^2(c+bx)\,dx &= -\left(\sin(a-c)\int \sec(c+bx)\tan(c+bx)\,dx\right)+\int \sin(a+bx)\tan(c+bx)\,dx\\ &= \cos(a-c)\int \sec(c+bx)\,dx-\frac{\sin(a-c)\mathrm{Subst}\left(\int 1\,dx,x,\sec(c+bx)\right)}{b}-\int c\\ &= \frac{\tanh^{-1}(\sin(c+bx))\cos(a-c)}{b}-\frac{\sec(c+bx)\sin(a-c)}{b}-\frac{\sin(a+bx)}{b}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 111, normalized size = 2.41

$$-\frac{2i\mathrm{ArcTan}\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b}-\frac{\cos(bx)\sin(a)}{b}-\frac{\sec(c+bx)\sin(a-c)}{b}-\frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Tan[c + b*x]^2,x]
```

```
[Out] ((-2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*Sin[c] + Cos[c]*Sin[(b*x)
/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b - (Cos[b
*x]*Sin[a])/b - (Sec[c + b*x]*Sin[a - c])/b - (Cos[a]*Sin[b*x])/b
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 149, normalized size = 3.24

method	result
risch	$\frac{i e^{i(bx+a)}}{2b}-\frac{i e^{-i(bx+a)}}{2b}-\frac{i\left(-e^{i(bx+3a)}+e^{i(bx+a+2c)}\right)}{b\left(e^{2i(bx+a+c)}+e^{2ia}\right)}-\frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\cos(a-c)}{b}+\frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\cos(a-c)}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*tan(b*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))-I/b/(exp(2*I*(b*x+a+c))+exp(
2*I*a))*(-exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))-ln(exp(I*(b*x+a))-I*exp(I*(a
-c)))/b*cos(a-c)+ln(exp(I*(b*x+a))+I*exp(I*(a-c)))/b*cos(a-c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(46) = 92$.
time = 0.55, size = 526, normalized size = 11.43

$$\frac{\left(\sin\left(3\,bx+a+2\,c\right)+\sin\left(bx+a\right)\right)\cos\left(4\,bx+2\,a+2\,c\right)-3\,\left(\sin\left(2\,bx+2\,a\right)-\sin\left(2\,bx+2\,c\right)\right)\cos\left(3\,bx+a+2\,c\right)-\left(\cos\left(3\,bx+a+2\,c\right)^{2}\cos\left(-a+c\right)+2\,\cos\left(3\,bx+a+2\,c\right)\cos\left(bx+a\right)\cos\left(-a+c\right)+\cos\left(bx+a\right)^{2}\cos\left(-a+c\right)+\cos\left(-a+c\right)\sin\left(3\,bx+a+2\,c\right)^{2}+2\,\cos\left(-a+c\right)\sin\left(3\,bx+a+2\,c\right)\sin\left(bx+a\right)+\cos\left(-a+c\right)\sin\left(bx+a\right)^{2}\right)\log\left(\frac{\cos\left(bx+2\,c\right)^{2}+\cos\left(c\right)^{2}-2\,\cos\left(c\right)\sin\left(bx+2\,c\right)+\sin\left(bx+2\,c\right)^{2}+2\,\cos\left(bx+2\,c\right)\sin\left(c\right)+\sin\left(c\right)^{2}}{\cos\left(bx+2\,c\right)^{2}+\cos\left(c\right)^{2}+2\,\cos\left(c\right)\sin\left(bx+2\,c\right)+\sin\left(bx+2\,c\right)^{2}-2\,\cos\left(bx+2\,c\right)\sin\left(c\right)+\sin\left(c\right)^{2}}\right)-\left(\cos\left(3\,bx+a+2\,c\right)+\cos\left(bx+a\right)\right)\sin\left(4\,bx+2\,a+2\,c\right)+\left(3\,\cos\left(2\,bx+2\,a\right)-3\,\cos\left(2\,bx+2\,c\right)-1\right)\sin\left(3\,bx+a+2\,c\right)-3\,\cos\left(bx+a\right)\sin\left(2\,bx+2\,a\right)+3\,\cos\left(bx+a\right)\sin\left(2\,bx+2\,c\right)+3\,\cos\left(2\,bx+2\,a\right)\sin\left(bx+a\right)-3\,\cos\left(2\,bx+2\,c\right)\sin\left(bx+a\right)-\sin\left(bx+a\right)}{2\,\left(b\cos\left(3\,bx+a+2\,c\right)^{2}+2\,b\cos\left(3\,bx+a+2\,c\right)\cos\left(bx+a\right)+b\cos\left(bx+a\right)^{2}+b\sin\left(3\,bx+a+2\,c\right)^{2}+2\,b\sin\left(3\,bx+a+2\,c\right)\sin\left(bx+a\right)+b\sin\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((sin(3*b*x + a + 2*c) + sin(b*x + a))*cos(4*b*x + 2*a + 2*c) - 3*(sin(
2*b*x + 2*a) - sin(2*b*x + 2*c))*cos(3*b*x + a + 2*c) - (cos(3*b*x + a + 2*
c)^2*cos(-a + c) + 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos(-a + c) + cos(b*
x + a)^2*cos(-a + c) + cos(-a + c)*sin(3*b*x + a + 2*c)^2 + 2*cos(-a + c)*s
in(3*b*x + a + 2*c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2)*log((cos(b*x
 + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 + 2*cos(b
*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2 + 2*cos(c)*sin(b*
x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)) - (cos(3
*b*x + a + 2*c) + cos(b*x + a))*sin(4*b*x + 2*a + 2*c) + (3*cos(2*b*x + 2*a
) - 3*cos(2*b*x + 2*c) - 1)*sin(3*b*x + a + 2*c) - 3*cos(b*x + a)*sin(2*b*x
 + 2*a) + 3*cos(b*x + a)*sin(2*b*x + 2*c) + 3*cos(2*b*x + 2*a)*sin(b*x + a)
 - 3*cos(2*b*x + 2*c)*sin(b*x + a) - sin(b*x + a))/(b*cos(3*b*x + a + 2*c)^
2 + 2*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x
+ a + 2*c)^2 + 2*b*sin(3*b*x + a + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(46) = 92$.
time = 2.66, size = 316, normalized size = 6.87

$$\frac{4\,\left(\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+\frac{\sqrt{2}\left(\left(\cos\left(-2\,a+2\,c\right)+1\right)\sin\left(bx+a\right)\sin\left(-2\,a+2\,c\right)-\left(\cos\left(-2\,a+2\,c\right)^{2}+2\,\cos\left(-2\,a+2\,c\right)+1\right)\cos\left(bx+a\right)\right)\log\left(\frac{2\,\cos\left(bx+a\right)^{2}\cos\left(-2\,a+2\,c\right)-2\,\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(-2\,a+2\,c\right)-\frac{2\,\sqrt{2}\left(\left(\cos\left(-2\,a+2\,c\right)+1\right)\sin\left(bx+a\right)+\cos\left(bx+a\right)\sin\left(-2\,a+2\,c\right)\right)}{\sqrt{\cos\left(-2\,a+2\,c\right)+1}}-\cos\left(-2\,a+2\,c\right)-3}{2\,\cos\left(bx+a\right)^{2}\cos\left(-2\,a+2\,c\right)-2\,\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(-2\,a+2\,c\right)-\cos\left(-2\,a+2\,c\right)+1}\right)}{\sqrt{\cos\left(-2\,a+2\,c\right)+1}}+4\,\left(\cos\left(bx+a\right)^{2}-2\right)\sin\left(-2\,a+2\,c\right)}{4\,\left(b\sin\left(bx+a\right)\sin\left(-2\,a+2\,c\right)-\left(b\cos\left(-2\,a+2\,c\right)+b\right)\cos\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a) + sqrt(2)*((cos(-2*a
 + 2*c) + 1)*sin(b*x + a)*sin(-2*a + 2*c) - (cos(-2*a + 2*c)^2 + 2*cos(-2*a
 + 2*c) + 1)*cos(b*x + a))*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*
x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(
b*x + a) + cos(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2
*a + 2*c) - 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x +
 a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) + 1))/sqrt(cos(-2*a + 2*c) + 1) + 4*(
cos(b*x + a)^2 - 2)*sin(-2*a + 2*c))/(b*sin(b*x + a)*sin(-2*a + 2*c) - (b*c
os(-2*a + 2*c) + b)*cos(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(a+bx)\tan^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*tan(b*x + c)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*tan(b*x + c)^2, x)
```

Mupad [**B**]
time = 5.24, size = 285, normalized size = 6.20

$$-\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b}-\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}-1\right)}{b\,\left(e^{a\,2i-c\,2i}\,1i+e^{a\,2i+b\,x\,2i}\,1i\right)}+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*tan(c + b*x)^2,x)
```

```
[Out] (exp(a*1i + b*x*1i)*1i)/(2*b) - (exp(- a*1i - b*x*1i)*1i)/(2*b) - (exp(a*1i
 + b*x*1i)*(exp(a*2i - c*2i) - 1))/(b*(exp(a*2i - c*2i)*1i + exp(a*2i + b*x
*2i)*1i)) + (log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1) - (exp(
a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2
))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)) - (log((exp(a*2i)*e
xp(-c*2i)*(exp(a*2i)*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp
(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) + 1))*(exp(a*2i - c*2i) + 1))/(2*b
*exp(a*2i - c*2i)^(1/2))
```

3.249 $\int \cos(a+bx)\tan(c+bx)\,dx$

Optimal. Leaf size=30

$$-\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b}$$

```
[Out] -cos(b*x+a)/b-arctanh(sin(b*x+c))*sin(a-c)/b
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4675, 2718, 3855}

$$-\frac{\sin(a-c)\tanh^{-1}(\sin(bx+c))}{b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Tan[c + b*x],x]
```

```
[Out] -(Cos[a + b*x]/b) - (ArcTanh[Sin[c + b*x]]*Sin[a - c])/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4675

```
Int[Cos[v_]*Tan[w_]^(n_.), x_Symbol] :> Int[Sin[v]*Tan[w]^(n - 1), x] - Dis
t[Sin[v - w], Int[Sec[w]*Tan[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\tan(c+bx)\,dx &= -\left(\sin(a-c)\int \sec(c+bx)\,dx\right)+\int \sin(a+bx)\,dx\\ &= -\frac{\cos(a+bx)}{b}-\frac{\tanh^{-1}(\sin(c+bx))\sin(a-c)}{b}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 93, normalized size = 3.10

$$-\frac{\cos(a)\cos(bx)}{b}+\frac{2i\text{ArcTan}\left(\frac{(i\cos(c)+\sin(c))\left(\cos\left(\frac{bx}{2}\right)\sin(c)+\cos(c)\sin\left(\frac{bx}{2}\right)\right)}{\cos(c)\cos\left(\frac{bx}{2}\right)-i\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b}+\frac{\sin(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Tan[c + b*x],x]
[Out] -((Cos[a]*Cos[b*x])/b) + ((2*I)*ArcTan[((I*Cos[c] + Sin[c])*(Cos[(b*x)/2]*S
in[c] + Cos[c]*Sin[(b*x)/2]))/(Cos[c]*Cos[(b*x)/2] - I*Cos[(b*x)/2]*Sin[c])
]*Sin[a - c])/b + (Sin[a]*Sin[b*x])/b
```

Maple [C] Result contains complex when optimal does not.
time = 0.10, size = 97, normalized size = 3.23

method	result	size
risch	$-\frac{e^{i(bx+a)}}{2b}-\frac{e^{-i(bx+a)}}{2b}-\frac{\ln\left(e^{i(bx+a)}+ie^{i(a-c)}\right)\sin(a-c)}{b}+\frac{\ln\left(e^{i(bx+a)}-ie^{i(a-c)}\right)\sin(a-c)}{b}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*tan(b*x+c),x,method=_RETURNVERBOSE)
[Out] -1/2*exp(I*(b*x+a))/b-1/2/b*exp(-I*(b*x+a))-ln(exp(I*(b*x+a))+I*exp(I*(a-c)
))/b*sin(a-c)+ln(exp(I*(b*x+a))-I*exp(I*(a-c)))/b*sin(a-c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(30) = 60$.
time = 0.55, size = 131, normalized size = 4.37

$$-\frac{\log\left(\frac{\cos(bx+2c)^2+\cos(c)^2-2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2+2\cos(bx+2c)\sin(c)+\sin(c)^2}{\cos(bx+2c)^2+\cos(c)^2+2\cos(c)\sin(bx+2c)+\sin(bx+2c)^2-2\cos(bx+2c)\sin(c)+\sin(c)^2}\right)\sin(-a+c)+2\cos(bx+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="maxima")
[Out] -1/2*(log((cos(b*x + 2*c)^2 + cos(c)^2 - 2*cos(c)*sin(b*x + 2*c) + sin(b*x
+ 2*c)^2 + 2*cos(b*x + 2*c)*sin(c) + sin(c)^2)/(cos(b*x + 2*c)^2 + cos(c)^2
 + 2*cos(c)*sin(b*x + 2*c) + sin(b*x + 2*c)^2 - 2*cos(b*x + 2*c)*sin(c) + s
in(c)^2))*sin(-a + c) + 2*cos(b*x + a))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(30) = 60$.
time = 3.16, size = 196, normalized size = 6.53

$$\frac{\frac{\sqrt{2}\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\frac{2\sqrt{2}((\cos(-2a+2c)+1)\sin(bx+a)+\cos(bx+a)\sin(-2a+2c))}{\sqrt{\cos(-2a+2c)+1}}-\cos(-2a+2c)-3}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)+1}\right)\sin(-2a+2c)}{\sqrt{\cos(-2a+2c)+1}}-4\cos(bx+a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x
 + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a) + cos
(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) - 3)
/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a +
 2*c) - cos(-2*a + 2*c) + 1))*sin(-2*a + 2*c)/sqrt(cos(-2*a + 2*c) + 1) - 4
*cos(b*x + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(a+bx)\tan(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*tan(b*x + c), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*tan(b*x + c), x)
```

Mupad [B]
time = 4.73, size = 237, normalized size = 7.90

$$-\frac{e^{-a\,1i-b\,x\,1i}}{2\,b}-\frac{e^{a\,1i+b\,x\,1i}}{2\,b}+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i-i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*tan(c + b*x),x)
```

```
[Out] (log(- exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i - 1i) - (exp(a*2i)*ex
p(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp
(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) - exp(a*1i + b*x*1i)/(2
*b) - exp(- a*1i - b*x*1i)/(2*b) - (log((exp(a*2i)*exp(-c*2i)*(exp(a*2i)*ex
p(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b*x*1i)*(ex
p(a*2i)*exp(-c*2i)*1i - 1i))*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i
))^(1/2))
```

3.250 $\int \cos(a+bx)\cot(c+bx)\,dx$

Optimal. Leaf size=29

$$-\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b}+\frac{\cos(a+bx)}{b}$$

```
[Out] -arctanh(cos(b*x+c))*cos(a-c)/b+cos(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4673, 2718, 3855}

$$\frac{\cos(a+bx)}{b}-\frac{\cos(a-c)\tanh^{-1}(\cos(bx+c))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[c + b*x],x]
```

```
[Out] -((ArcTanh[Cos[c + b*x]]*Cos[a - c])/b) + Cos[a + b*x]/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4673

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cot(c+bx)\,dx &= \cos(a-c)\int \csc(c+bx)\,dx-\int \sin(a+bx)\,dx\\ &= -\frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{b}+\frac{\cos(a+bx)}{b}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 94, normalized size = 3.24

$$-\frac{2i\text{ArcTan}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\cos(a-c)}{b}+\frac{\cos(a)\cos(bx)}{b}-\frac{\sin(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[c + b*x],x]
```

```
[Out] ((-2*I)*ArcTan[((Cos[c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)
/2]))/(I*Cos[c]*Cos[(b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Cos[a - c])/b + (Cos[a
]*Cos[b*x])/b - (Sin[a]*Sin[b*x])/b
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 93, normalized size = 3.21

method	result	size
risch	$\frac{e^{i(bx+a)}}{2b}+\frac{e^{-i(bx+a)}}{2b}+\frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\cos(a-c)}{b}-\frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos(a-c)}{b}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(I*(b*x+a))/b+1/2/b*exp(-I*(b*x+a))+ln(exp(I*(b*x+a))-exp(I*(a-c)))/
b*cos(a-c)-ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(29) = 58.
time = 0.28, size = 105, normalized size = 3.62

$$-\frac{\cos(-a+c)\log\left(\cos(bx)^2+2\cos(bx)\cos(c)+\cos(c)^2+\sin(bx)^2-2\sin(bx)\sin(c)+\sin(c)^2\right)-\cos(-a+c)\log\left(\cos(bx)^2-2\cos(bx)\cos(c)+\cos(c)^2+\sin(bx)^2+2\sin(bx)\sin(c)+\sin(c)^2\right)-2\cos(bx+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(-a + c)*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^
2 - 2*sin(b*x)*sin(c) + sin(c)^2) - cos(-a + c)*log(cos(b*x)^2 - 2*cos(b*x)
*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) - 2*cos(b*x
 + a))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(29) = 58.
time = 2.74, size = 190, normalized size = 6.55

$$\frac{\sqrt{2}\sqrt{\cos(-2a+2c)+1}\log\left(-\frac{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\frac{2\sqrt{2}((\cos(-2a+2c)+1)\cos(bx+a)-\sin(bx+a)\sin(-2a+2c))}{\sqrt{\cos(-2a+2c)+1}}-\cos(-2a+2c)+3}{2\cos(bx+a)^2\cos(-2a+2c)-2\cos(bx+a)\sin(bx+a)\sin(-2a+2c)-\cos(-2a+2c)-1}\right)+4\cos(bx+a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*sqrt(cos(-2*a + 2*c) + 1)*log(-(2*cos(b*x + a)^2*cos(-2*a + 2*
c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a + 2
*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c)
+ 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x +
 a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1)) + 4*cos(b*x + a))/
b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos (a + bx) \cot (bx + c) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*cot(b*x + c), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(29) = 58.
time = 0.43, size = 234, normalized size = 8.07

$$-\frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right)-1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}c\right)}-\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)^2+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^2+1\right)\log\left(\left|\tan\left(\frac{1}{2}bx\right)+\tan\left(\frac{1}{2}c\right)\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}+\frac{2\left(2\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)+\tan\left(\frac{1}{2}a\right)^2-1\right)}{\left(\tan\left(\frac{1}{2}bx\right)^2+1\right)\left(\tan\left(\frac{1}{2}a\right)^2+1\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c),x, algorithm="giac")
```

```
[Out] -((tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^2*tan(1/2*c) + 4*tan(1/2*a)*tan(1
/2*c)^2 - tan(1/2*c)^3 + tan(1/2*c))*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/
(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1
/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*tan(1/2*c
) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan
(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 2*(2*tan(1/2*b*x)*tan(1/2*a)
 + tan(1/2*a)^2 - 1)/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 + 1)))/b
```

Mupad [B]
time = 5.39, size = 231, normalized size = 7.97

$$\frac{e^{-a\,1i-b\,x\,1i}}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}}{2\,b}-\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}+\frac{\ln\left(-e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}\,1i+1i\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}+1\right)\,1i}{\sqrt{e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}+1\right)}{2\,b\,\sqrt{e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(c + b*x),x)
```

```
[Out] exp(- a*1i - b*x*1i)/(2*b) + exp(a*1i + b*x*1i)/(2*b) - (log(- exp(a*1i)*ex
p(b*x*1i)*(exp(a*2i)*exp(-c*2i)*1i + 1i) - (exp(a*2i)*exp(-c*2i)*(exp(a*2i)
*exp(-c*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) + 1))/
(2*b*exp(a*2i - c*2i)^(1/2)) + (log((exp(a*2i)*exp(-c*2i)*(exp(a*2i)*exp(-c
*2i) + 1)*1i)/(exp(a*2i)*exp(-c*2i))^(1/2) - exp(a*1i)*exp(b*x*1i)*(exp(a*2
i)*exp(-c*2i)*1i + 1i))*(exp(a*2i - c*2i) + 1))/(2*b*exp(a*2i - c*2i)^(1/2)
)
```

3.251 $\int \cos(a+bx)\cot^2(c+bx)\,dx$

Optimal. Leaf size=46

$$-\frac{\cos(a-c)\csc(c+bx)}{b}+\frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b}-\frac{\sin(a+bx)}{b}$$

```
[Out] -cos(a-c)*csc(b*x+c)/b+arctanh(cos(b*x+c))*sin(a-c)/b-sin(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4673, 4674, 2717, 3855, 2686, 8}

$$-\frac{\cos(a-c)\csc(bx+c)}{b}+\frac{\sin(a-c)\tanh^{-1}(\cos(bx+c))}{b}-\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[c + b*x]^2,x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) + (ArcTanh[Cos[c + b*x]]*Sin[a - c])/b - Sin
[a + b*x]/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4673

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
```

```
w, x] && NeQ[w, v]
```

Rule 4674

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\cot^2(c+bx)\,dx &= \cos(a-c)\int \cot(c+bx)\csc(c+bx)\,dx - \int \cot(c+bx)\sin(a+bx)\,dx\\ &= -\frac{\cos(a-c)\text{Subst}(\int 1\,dx, x, \csc(c+bx))}{b} - \sin(a-c)\int \csc(c+bx)\,dx - \\ &= -\frac{\cos(a-c)\csc(c+bx)}{b} + \frac{\tanh^{-1}(\cos(c+bx))\sin(a-c)}{b} - \frac{\sin(a+bx)}{b}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 112, normalized size = 2.43

$$-\frac{\cos(a-c)\csc(c+bx)}{b} - \frac{\cos(bx)\sin(a)}{b} + \frac{2i\text{ArcTan}\left(\frac{(\cos(c)-i\sin(c))\left(\cos(c)\cos\left(\frac{bx}{2}\right)-\sin(c)\sin\left(\frac{bx}{2}\right)\right)}{i\cos(c)\cos\left(\frac{bx}{2}\right)+\cos\left(\frac{bx}{2}\right)\sin(c)}\right)\sin(a-c)}{b} - \frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[c + b*x]^2,x]
```

```
[Out] -((Cos[a - c]*Csc[c + b*x])/b) - (Cos[b*x]*Sin[a])/b + ((2*I)*ArcTan[((Cos[
c] - I*Sin[c])*(Cos[c]*Cos[(b*x)/2] - Sin[c]*Sin[(b*x)/2]))/(I*Cos[c]*Cos[(
b*x)/2] + Cos[(b*x)/2]*Sin[c])]*Sin[a - c])/b - (Cos[a]*Sin[b*x])/b
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 145, normalized size = 3.15

method	result	si
risch	$\frac{i e^{i(bx+a)}}{2b} - \frac{i e^{-i(bx+a)}}{2b} + \frac{i\left(e^{i(bx+3a)}+e^{i(bx+a+2c)}\right)}{b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)} - \frac{\ln\left(e^{i(bx+a)}-e^{i(a-c)}\right)\sin(a-c)}{b} + \frac{\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\sin(a-c)}{b}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*exp(I*(b*x+a))/b-1/2*I/b*exp(-I*(b*x+a))+I/b/(-exp(2*I*(b*x+a+c))+exp
(2*I*a))*(exp(I*(b*x+3*a))+exp(I*(b*x+a+2*c)))-ln(exp(I*(b*x+a))-exp(I*(a-c
)))/b*sin(a-c)+ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*sin(a-c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(46) = 92$.
time = 0.31, size = 613, normalized size = 13.33

$$\frac{\left(\sin\left(3bx+a+2c\right)-\sin\left(bx+a\right)\right)\cos\left(4bx+2a+2c\right)+3\left(\sin\left(2bx+2a\right)+\sin\left(2bx+2c\right)\right)\cos\left(3bx+a+2c\right)-\left(\cos\left(3bx+a+2c\right)^{2}\sin\left(-a+c\right)-2\cos\left(3bx+a+2c\right)\cos\left(bx+a\right)\sin\left(-a+c\right)+\cos\left(bx+a\right)^{2}\sin\left(-a+c\right)+\sin\left(3bx+a+2c\right)^{2}\sin\left(-a+c\right)-2\sin\left(3bx+a+2c\right)\sin\left(bx+a\right)\sin\left(-a+c\right)+\sin\left(bx+a\right)^{2}\sin\left(-a+c\right)\right)\log\left(\cos\left(bx\right)^{2}+2\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^{2}+\sin\left(bx\right)^{2}-2\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^{2}\right)+\left(\cos\left(3bx+a+2c\right)^{2}\sin\left(-a+c\right)-2\cos\left(3bx+a+2c\right)\cos\left(bx+a\right)\sin\left(-a+c\right)+\cos\left(bx+a\right)^{2}\sin\left(-a+c\right)+\sin\left(3bx+a+2c\right)^{2}\sin\left(-a+c\right)-2\sin\left(3bx+a+2c\right)\sin\left(bx+a\right)\sin\left(-a+c\right)+\sin\left(bx+a\right)^{2}\sin\left(-a+c\right)\right)\log\left(\cos\left(bx\right)^{2}-2\cos\left(bx\right)\cos\left(c\right)+\cos\left(c\right)^{2}+\sin\left(bx\right)^{2}+2\sin\left(bx\right)\sin\left(c\right)+\sin\left(c\right)^{2}\right)-\left(\cos\left(3bx+a+2c\right)-\cos\left(bx+a\right)\right)\sin\left(4bx+2a+2c\right)-\left(3\cos\left(2bx+2a\right)+3\cos\left(2bx+2c\right)-1\right)\sin\left(3bx+a+2c\right)-3\cos\left(bx+a\right)\sin\left(2bx+2a\right)-3\cos\left(bx+a\right)\sin\left(2bx+2c\right)+3\cos\left(2bx+2a\right)\sin\left(bx+a\right)+3\cos\left(2bx+2c\right)\sin\left(bx+a\right)-\sin\left(bx+a\right)}{2\left(b\cos\left(3bx+a+2c\right)^{2}-2b\cos\left(3bx+a+2c\right)\cos\left(bx+a\right)+b\cos\left(bx+a\right)^{2}+b\sin\left(3bx+a+2c\right)^{2}-2b\sin\left(3bx+a+2c\right)\sin\left(bx+a\right)+b\sin\left(bx+a\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((sin(3*b*x + a + 2*c) - sin(b*x + a))*cos(4*b*x + 2*a + 2*c) + 3*(sin(
2*b*x + 2*a) + sin(2*b*x + 2*c))*cos(3*b*x + a + 2*c) - (cos(3*b*x + a + 2*
c)^2*sin(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x + a)*sin(-a + c) + cos(b*
x + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c)^2*sin(-a + c) - 2*sin(3*b*x + a
 + 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*sin(-a + c))*log(cos(b*x)
^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin(c)
^2) + (cos(3*b*x + a + 2*c)^2*sin(-a + c) - 2*cos(3*b*x + a + 2*c)*cos(b*x
+ a)*sin(-a + c) + cos(b*x + a)^2*sin(-a + c) + sin(3*b*x + a + 2*c)^2*sin(
-a + c) - 2*sin(3*b*x + a + 2*c)*sin(b*x + a)*sin(-a + c) + sin(b*x + a)^2*
sin(-a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2
*sin(b*x)*sin(c) + sin(c)^2) - (cos(3*b*x + a + 2*c) - cos(b*x + a))*sin(4*
b*x + 2*a + 2*c) - (3*cos(2*b*x + 2*a) + 3*cos(2*b*x + 2*c) - 1)*sin(3*b*x
+ a + 2*c) - 3*cos(b*x + a)*sin(2*b*x + 2*a) - 3*cos(b*x + a)*sin(2*b*x + 2
*c) + 3*cos(2*b*x + 2*a)*sin(b*x + a) + 3*cos(2*b*x + 2*c)*sin(b*x + a) - s
in(b*x + a))/(b*cos(3*b*x + a + 2*c)^2 - 2*b*cos(3*b*x + a + 2*c)*cos(b*x +
 a) + b*cos(b*x + a)^2 + b*sin(3*b*x + a + 2*c)^2 - 2*b*sin(3*b*x + a + 2*c
)*sin(b*x + a) + b*sin(b*x + a)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(46) = 92$.
time = 3.23, size = 316, normalized size = 6.87

$$\frac{4\left(\cos\left(-2a+2c\right)+1\right)\cos\left(bx+a\right)^{2}-4\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(-2a+2c\right)+\frac{\sqrt{2}\left(\left(\cos\left(-2a+2c\right)+1\right)\sin\left(bx+a\right)\sin\left(-2a+2c\right)-\left(\cos\left(-2a+2c\right)^{2}-1\right)\cos\left(bx+a\right)\right)\log\left(-\frac{2\cos\left(bx+a\right)^{2}\cos\left(-2a+2c\right)-2\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(-2a+2c\right)-\frac{2\sqrt{2}\left(\left(\cos\left(-2a+2c\right)+1\right)\cos\left(bx+a\right)-\sin\left(bx+a\right)\sin\left(-2a+2c\right)\right)}{\sqrt{\cos\left(-2a+2c\right)+1}}-\cos\left(-2a+2c\right)+3}{2\cos\left(bx+a\right)^{2}\cos\left(-2a+2c\right)-2\cos\left(bx+a\right)\sin\left(bx+a\right)\sin\left(-2a+2c\right)-\cos\left(-2a+2c\right)-1}\right)}{\sqrt{\cos\left(-2a+2c\right)+1}}-8\cos\left(-2a+2c\right)-8}{4\left(b\cos\left(bx+a\right)\sin\left(-2a+2c\right)+\left(b\cos\left(-2a+2c\right)+b\right)\sin\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(cos(-2*a + 2*c) + 1)*cos(b*x + a)^2 - 4*cos(b*x + a)*sin(b*x + a)*s
in(-2*a + 2*c) + sqrt(2)*((cos(-2*a + 2*c) + 1)*sin(b*x + a)*sin(-2*a + 2*c
) - (cos(-2*a + 2*c)^2 - 1)*cos(b*x + a))*log(-(2*cos(b*x + a)^2*cos(-2*a +
 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*sqrt(2)*((cos(-2*a
+ 2*c) + 1)*cos(b*x + a) - sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*
c) + 1) - cos(-2*a + 2*c) + 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*
x + a)*sin(b*x + a)*sin(-2*a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-2*a +
 2*c) + 1) - 8*cos(-2*a + 2*c) - 8)/(b*cos(b*x + a)*sin(-2*a + 2*c) + (b*co
s(-2*a + 2*c) + b)*sin(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(a+bx)\cot^2(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*cot(b*x + c)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 627 vs. 2(46) = 92.
time = 0.45, size = 627, normalized size = 13.63

$$\frac{\frac{4\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}c\right)^2\right)\log\left(\left|\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right)-1\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}c\right)}-\frac{4\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)-\tan\left(\frac{1}{2}c\right)\right)\log\left(\left|\tan\left(\frac{1}{2}bx\right)+\tan\left(\frac{1}{2}c\right)\right|\right)}{\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1}-\frac{\tan\left(\frac{1}{2}bx\right)^3\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^4-6\tan\left(\frac{1}{2}bx\right)^3\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+4\tan\left(\frac{1}{2}bx\right)^3\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^3-6\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}bx\right)^3\tan\left(\frac{1}{2}c\right)^4+\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^4+\tan\left(\frac{1}{2}bx\right)^3\tan\left(\frac{1}{2}a\right)^2-4\tan\left(\frac{1}{2}bx\right)^3\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+6\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+6\tan\left(\frac{1}{2}bx\right)^3\tan\left(\frac{1}{2}c\right)^2+2\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+6\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}c\right)^3+12\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^3-2\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right)^4-\tan\left(\frac{1}{2}bx\right)^3+\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)^2-6\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}c\right)-12\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+2\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-2\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right)^2-16\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^2+2\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}bx\right)-2\tan\left(\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}bx\right)^3\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}bx\right)^3+\tan\left(\frac{1}{2}bx\right)\tan\left(\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}bx\right)-\tan\left(\frac{1}{2}c\right)\right)\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+\tan\left(\frac{1}{2}c\right)\right)}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*ta
n(1/2*c) - tan(1/2*c)^2)*log(abs(tan(1/2*b*x)*tan(1/2*c) - 1))/(tan(1/2*a)^
2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 + tan(1/2*c)) - 4*(
tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c)
)*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*
a)^2 + tan(1/2*c)^2 + 1) - (tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^4 - 6*ta
n(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^2 + 4*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/
2*c)^3 - 6*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*b*x)^3*tan(1/
2*c)^4 + tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^4 + tan(1/2*b*x)^3*tan(1/2*a)
^2 - 4*tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c) + 6*tan(1/2*b*x)^2*tan(1/2*a)^2
*tan(1/2*c) + 6*tan(1/2*b*x)^3*tan(1/2*c)^2 + 2*tan(1/2*b*x)*tan(1/2*a)^2*t
an(1/2*c)^2 + 6*tan(1/2*b*x)^2*tan(1/2*c)^3 + 12*tan(1/2*b*x)*tan(1/2*a)*ta
n(1/2*c)^3 - 2*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*b*x)*tan(1/2*c)^4 - tan(
1/2*b*x)^3 + tan(1/2*b*x)*tan(1/2*a)^2 - 6*tan(1/2*b*x)^2*tan(1/2*c) - 12*t
an(1/2*b*x)*tan(1/2*a)*tan(1/2*c) + 2*tan(1/2*a)^2*tan(1/2*c) - 2*tan(1/2*b
*x)*tan(1/2*c)^2 - 16*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*c)^3 - tan(1/2*b*
x) - 2*tan(1/2*c))/((tan(1/2*b*x)^4*tan(1/2*c) + tan(1/2*b*x)^3*tan(1/2*c)^
2 - tan(1/2*b*x)^3 + tan(1/2*b*x)*tan(1/2*c)^2 - tan(1/2*b*x) - tan(1/2*c))
*(tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c))))/b
```

Mupad [B]
time = 5.44, size = 289, normalized size = 6.28

$$-\frac{e^{-a\,1i-b\,x\,1i}\,1i}{2\,b}+\frac{e^{a\,1i+b\,x\,1i}\,1i}{2\,b}-\frac{e^{a\,1i+b\,x\,1i}\,\left(e^{a\,2i-c\,2i}+1\right)}{b\,\left(e^{a\,2i-c\,2i}\,1i-e^{a\,2i+b\,x\,2i}\,1i\right)}-\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)-\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}+\frac{\ln\left(e^{a\,1i}\,e^{b\,x\,1i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)+\frac{e^{a\,2i}\,e^{-c\,2i}\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)\,1i}{\sqrt{-e^{a\,2i}\,e^{-c\,2i}}}\right)\,\left(e^{a\,2i-c\,2i}-1\right)}{2\,b\,\sqrt{-e^{a\,2i-c\,2i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(c + b*x)^2,x)
```

```
[Out] (exp(a*1i + b*x*1i)*1i)/(2*b) - (exp(- a*1i - b*x*1i)*1i)/(2*b) - (exp(a*1i
 + b*x*1i)*(exp(a*2i - c*2i) + 1))/(b*(exp(a*2i - c*2i)*1i - exp(a*2i + b*x
*2i)*1i)) - (log(exp(a*1i)*exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) - (exp(a*
2i)*exp(-c*2i)*(exp(a*2i)*exp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2)
)*(exp(a*2i - c*2i) - 1))/(2*b*(-exp(a*2i - c*2i))^(1/2)) + (log(exp(a*1i)*
exp(b*x*1i)*(exp(a*2i)*exp(-c*2i) - 1) + (exp(a*2i)*exp(-c*2i)*(exp(a*2i)*e
xp(-c*2i) - 1)*1i)/(-exp(a*2i)*exp(-c*2i))^(1/2))*(exp(a*2i - c*2i) - 1))/(
2*b*(-exp(a*2i - c*2i))^(1/2))
```

3.252 $\int \cos(a+bx)\cot^3(c+bx)\,dx$

Optimal. Leaf size=73

$$\frac{3\tanh^{-1}(\cos(c+bx))\cos(a-c)}{2b}-\frac{\cos(a+bx)}{b}-\frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b}+\frac{\csc(c+bx)\sin(a-c)}{b}$$

```
[Out] 3/2*arctanh(cos(b*x+c))*cos(a-c)/b-cos(b*x+a)/b-1/2*cos(a-c)*cot(b*x+c)*csc
(b*x+c)/b+csc(b*x+c)*sin(a-c)/b
```

Rubi [A]
time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {4673, 4674, 2718, 3855, 2686, 8, 2691}

$$\frac{3\cos(a-c)\tanh^{-1}(\cos(bx+c))}{2b}+\frac{\sin(a-c)\csc(bx+c)}{b}-\frac{\cos(a-c)\cot(bx+c)\csc(bx+c)}{2b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[c + b*x]^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + b*x]]*Cos[a - c])/(2*b) - Cos[a + b*x]/b - (Cos[a - c]*C
ot[c + b*x]*Csc[c + b*x])/(2*b) + (Csc[c + b*x]*Sin[a - c])/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4673

```
Int[Cos[v_]*Cot[w_]^(n_.), x_Symbol] :> -Int[Sin[v]*Cot[w]^(n - 1), x] + Di
st[Cos[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v -
w, x] && NeQ[w, v]
```

Rule 4674

```
Int[Cot[w_]^(n_.)*Sin[v_], x_Symbol] :> Int[Cos[v]*Cot[w]^(n - 1), x] + Dis
t[Sin[v - w], Int[Csc[w]*Cot[w]^(n - 1), x], x] /; GtQ[n, 0] && FreeQ[v - w
, x] && NeQ[w, v]
```

Rubi steps

$$\int \cos(a+bx)\cot^3(c+bx)\,dx = \cos(a-c)\int \cot^2(c+bx)\csc(c+bx)\,dx - \int \cot^2(c+bx)\sin(a+bx)\,dx$$
$$= -\frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} - \frac{1}{2}\cos(a-c)\int \csc(c+bx)\,dx - \sin(a$$
$$= \frac{\tanh^{-1}(\cos(c+bx))\cos(a-c)}{2b} - \frac{\cos(a-c)\cot(c+bx)\csc(c+bx)}{2b} - \cos(a$$
$$= \frac{3\tanh^{-1}(\cos(c+bx))\cos(a-c)}{2b} - \frac{\cos(a+bx)}{b} - \frac{\cos(a-c)\cot(c+bx)\csc}{2b}$$

Mathematica [A]
time = 0.37, size = 71, normalized size = 0.97

$$\frac{12\tanh^{-1}\left(\cos(c)-\sin(c)\tan\left(\frac{bx}{2}\right)\right)\cos(a-c)+(2\cos(a-2c-bx)-5\cos(a+bx)+\cos(a+2c+3bx))\csc^2(c+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[c + b*x]^3,x]
```

```
[Out] (12*ArcTanh[Cos[c] - Sin[c]*Tan[(b*x)/2]]*Cos[a - c] + (2*Cos[a - 2*c - b*x
] - 5*Cos[a + b*x] + Cos[a + 2*c + 3*b*x])*Csc[c + b*x]^2)/(4*b)
```

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 179, normalized size = 2.45

method	result
risch	$-\frac{e^{i(bx+a)}}{2b}-\frac{e^{-i(bx+a)}}{2b}-\frac{-3\,e^{i(3bx+5a+2c)}+e^{i(3bx+3a+4c)}+e^{i(bx+5a)}-3\,e^{i(bx+3a+2c)}}{2b\left(-e^{2i(bx+a+c)}+e^{2ia}\right)^{2}}+\frac{3\ln\left(e^{i(bx+a)}+e^{i(a-c)}\right)\cos(a-c)}{2b}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*exp(I*(b*x+a))/b-1/2/b*exp(-I*(b*x+a))-1/2/b/(-exp(2*I*(b*x+a+c))+exp(
2*I*a))^2*(-3*exp(I*(3*b*x+5*a+2*c))+exp(I*(3*b*x+3*a+4*c))+exp(I*(b*x+5*a)
)-3*exp(I*(b*x+3*a+2*c)))+3/2*ln(exp(I*(b*x+a))+exp(I*(a-c)))/b*cos(a-c)-3/
2*ln(exp(I*(b*x+a))-exp(I*(a-c)))/b*cos(a-c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1254 vs. $2(69) = 138$.
time = 0.32, size = 1254, normalized size = 17.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(cos(5*b*x + a + 4*c) - 2*cos(3*b*x + a + 2*c) + cos(b*x + a))*cos(
6*b*x + 2*a + 4*c) - 2*(5*cos(4*b*x + 2*a + 2*c) - 2*cos(4*b*x + 4*c) - 2*c
os(2*b*x + 2*a) + 5*cos(2*b*x + 2*c) - 1)*cos(5*b*x + a + 4*c) + 10*(2*cos(
3*b*x + a + 2*c) - cos(b*x + a))*cos(4*b*x + 2*a + 2*c) - 4*(2*cos(3*b*x +
a + 2*c) - cos(b*x + a))*cos(4*b*x + 4*c) - 4*(2*cos(2*b*x + 2*a) - 5*cos(2
*b*x + 2*c) + 1)*cos(3*b*x + a + 2*c) + 4*cos(2*b*x + 2*a)*cos(b*x + a) - 1
0*cos(2*b*x + 2*c)*cos(b*x + a) - 3*(cos(5*b*x + a + 4*c)^2*cos(-a + c) + 4
*cos(3*b*x + a + 2*c)^2*cos(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*c
os(-a + c) + cos(b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x + a + 4*c)^
2 + 4*cos(-a + c)*sin(3*b*x + a + 2*c)^2 - 4*cos(-a + c)*sin(3*b*x + a + 2*
c)*sin(b*x + a) + cos(-a + c)*sin(b*x + a)^2 - 2*(2*cos(3*b*x + a + 2*c)*co
s(-a + c) - cos(b*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*cos(-a +
c)*sin(3*b*x + a + 2*c) - cos(-a + c)*sin(b*x + a))*sin(5*b*x + a + 4*c))*l
og(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(
c) + sin(c)^2) + 3*(cos(5*b*x + a + 4*c)^2*cos(-a + c) + 4*cos(3*b*x + a +
2*c)^2*cos(-a + c) - 4*cos(3*b*x + a + 2*c)*cos(b*x + a)*cos(-a + c) + cos(
b*x + a)^2*cos(-a + c) + cos(-a + c)*sin(5*b*x + a + 4*c)^2 + 4*cos(-a + c)
*sin(3*b*x + a + 2*c)^2 - 4*cos(-a + c)*sin(3*b*x + a + 2*c)*sin(b*x + a) +
 cos(-a + c)*sin(b*x + a)^2 - 2*(2*cos(3*b*x + a + 2*c)*cos(-a + c) - cos(b
*x + a)*cos(-a + c))*cos(5*b*x + a + 4*c) - 2*(2*cos(-a + c)*sin(3*b*x + a
+ 2*c) - cos(-a + c)*sin(b*x + a))*sin(5*b*x + a + 4*c))*log(cos(b*x)^2 - 2
*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2) +
2*(sin(5*b*x + a + 4*c) - 2*sin(3*b*x + a + 2*c) + sin(b*x + a))*sin(6*b*x
```

```
+ 2*a + 4*c) - 2*(5*sin(4*b*x + 2*a + 2*c) - 2*sin(4*b*x + 4*c) - 2*sin(2*b
*x + 2*a) + 5*sin(2*b*x + 2*c))*sin(5*b*x + a + 4*c) + 10*(2*sin(3*b*x + a
+ 2*c) - sin(b*x + a))*sin(4*b*x + 2*a + 2*c) - 4*(2*sin(3*b*x + a + 2*c) -
 sin(b*x + a))*sin(4*b*x + 4*c) - 4*(2*sin(2*b*x + 2*a) - 5*sin(2*b*x + 2*c
))*sin(3*b*x + a + 2*c) + 4*sin(2*b*x + 2*a)*sin(b*x + a) - 10*sin(2*b*x +
2*c)*sin(b*x + a) + 2*cos(b*x + a))/(b*cos(5*b*x + a + 4*c)^2 + 4*b*cos(3*b
*x + a + 2*c)^2 - 4*b*cos(3*b*x + a + 2*c)*cos(b*x + a) + b*cos(b*x + a)^2
+ b*sin(5*b*x + a + 4*c)^2 + 4*b*sin(3*b*x + a + 2*c)^2 - 4*b*sin(3*b*x + a
 + 2*c)*sin(b*x + a) + b*sin(b*x + a)^2 - 2*(2*b*cos(3*b*x + a + 2*c) - b*c
os(b*x + a))*cos(5*b*x + a + 4*c) - 2*(2*b*sin(3*b*x + a + 2*c) - b*sin(b*x
 + a))*sin(5*b*x + a + 4*c))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(69) = 138$.
time = 3.54, size = 385, normalized size = 5.27

$$-\frac{16\cos(bx+a)^3\cos(-2a+2c) - 4\left(4\cos(bx+a)^2+1\right)\sin(bx+a)\sin(-2a+2c) - 4\left(\cos(-2a+2c)+5\right)\cos(bx+a) + \frac{3\sqrt{2}\left(2\left(\cos(-2a+2c)+1\right)\cos(bx+a)\sin(bx+a)\sin(-2a+2c) - 2\left(\cos(-2a+2c)^2+\cos(-2a+2c)\right)\cos(bx+a)^2 + \cos(-2a+2c)^2 + 2\cos(-2a+2c)+1\right)\log\left(\frac{2\cos(bx+a)^2\cos(-2a+2c) - 2\cos(bx+a)\sin(bx+a)\sin(-2a+2c) + \frac{2\sqrt{2}\left((\cos(-2a+2c)+1)\cos(bx+a) - \sin(bx+a)\sin(-2a+2c)\right)}{\sqrt{\cos(-2a+2c)+1}} - \cos(-2a+2c)+3}{2\cos(bx+a)^2\cos(-2a+2c) - 2\cos(bx+a)\sin(bx+a)\sin(-2a+2c) - \cos(-2a+2c) - 1}\right)}{\sqrt{\cos(-2a+2c)+1}}}{8\left(2b\cos(bx+a)^2\cos(-2a+2c) - 2b\cos(bx+a)\sin(bx+a)\sin(-2a+2c) - b\cos(-2a+2c) - b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(16*cos(b*x + a)^3*cos(-2*a + 2*c) - 4*(4*cos(b*x + a)^2 + 1)*sin(b*x
+ a)*sin(-2*a + 2*c) - 4*(cos(-2*a + 2*c) + 5)*cos(b*x + a) + 3*sqrt(2)*(2*
(cos(-2*a + 2*c) + 1)*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - 2*(cos(-2
*a + 2*c)^2 + cos(-2*a + 2*c))*cos(b*x + a)^2 + cos(-2*a + 2*c)^2 + 2*cos(-
2*a + 2*c) + 1)*log((2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(
b*x + a)*sin(-2*a + 2*c) + 2*sqrt(2)*((cos(-2*a + 2*c) + 1)*cos(b*x + a) -
sin(b*x + a)*sin(-2*a + 2*c))/sqrt(cos(-2*a + 2*c) + 1) - cos(-2*a + 2*c) +
 3)/(2*cos(b*x + a)^2*cos(-2*a + 2*c) - 2*cos(b*x + a)*sin(b*x + a)*sin(-2*
a + 2*c) - cos(-2*a + 2*c) - 1))/sqrt(cos(-2*a + 2*c) + 1))/(2*b*cos(b*x +
a)^2*cos(-2*a + 2*c) - 2*b*cos(b*x + a)*sin(b*x + a)*sin(-2*a + 2*c) - b*co
s(-2*a + 2*c) - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(a+bx)\cot^3(bx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)**3,x)
```

```
[Out] Integral(cos(a + b*x)*cot(b*x + c)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(69) = 138$.

time = 0.46, size = 963, normalized size = 13.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(12*(tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)^2*tan(1/2*c) + 4*tan(1/2*a)
*tan(1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*c))*log(abs(tan(1/2*b*x)*tan(1/2*c)
- 1))/(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)^2*tan(1/2*c) + tan(1/2*c)^3 +
 tan(1/2*c)) - 12*(tan(1/2*a)^2*tan(1/2*c)^2 - tan(1/2*a)^2 + 4*tan(1/2*a)*
tan(1/2*c) - tan(1/2*c)^2 + 1)*log(abs(tan(1/2*b*x) + tan(1/2*c)))/(tan(1/2
*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1) + 16*(2*tan(1/2*b*x)*
tan(1/2*a) + tan(1/2*a)^2 - 1)/((tan(1/2*b*x)^2 + 1)*(tan(1/2*a)^2 + 1)) +
(2*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^7 + tan(1/2*b*x)^2*tan(1/2*a)^2*t
an(1/2*c)^8 + 6*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^5 + 2*tan(1/2*b*x)^2
*tan(1/2*a)^2*tan(1/2*c)^6 - 2*tan(1/2*b*x)^3*tan(1/2*c)^7 - 4*tan(1/2*b*x)
^2*tan(1/2*a)*tan(1/2*c)^7 - 2*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^7 - tan
(1/2*b*x)^2*tan(1/2*c)^8 - 6*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(1/2*c)^3 + 16*
tan(1/2*b*x)^3*tan(1/2*a)*tan(1/2*c)^4 - 22*tan(1/2*b*x)^2*tan(1/2*a)^2*tan
(1/2*c)^4 - 6*tan(1/2*b*x)^3*tan(1/2*c)^5 + 20*tan(1/2*b*x)^2*tan(1/2*a)*ta
n(1/2*c)^5 - 14*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c)^5 - 2*tan(1/2*b*x)^2*t
an(1/2*c)^6 + 16*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^6 + 2*tan(1/2*a)^2*tan(
1/2*c)^6 + 2*tan(1/2*b*x)*tan(1/2*c)^7 - 2*tan(1/2*b*x)^3*tan(1/2*a)^2*tan(
1/2*c) + 2*tan(1/2*b*x)^2*tan(1/2*a)^2*tan(1/2*c)^2 + 6*tan(1/2*b*x)^3*tan(
1/2*c)^3 - 20*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/2*c)^3 + 14*tan(1/2*b*x)*tan(
1/2*a)^2*tan(1/2*c)^3 + 22*tan(1/2*b*x)^2*tan(1/2*c)^4 - 16*tan(1/2*b*x)*ta
n(1/2*a)*tan(1/2*c)^4 + 12*tan(1/2*a)^2*tan(1/2*c)^4 + 14*tan(1/2*b*x)*tan(
1/2*c)^5 - 8*tan(1/2*a)*tan(1/2*c)^5 - 2*tan(1/2*c)^6 + tan(1/2*b*x)^2*tan(
1/2*a)^2 + 2*tan(1/2*b*x)^3*tan(1/2*c) + 4*tan(1/2*b*x)^2*tan(1/2*a)*tan(1/
2*c) + 2*tan(1/2*b*x)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(1/2*b*x)^2*tan(1/2*c)
^2 + 16*tan(1/2*b*x)*tan(1/2*a)*tan(1/2*c)^2 + 2*tan(1/2*a)^2*tan(1/2*c)^2
- 14*tan(1/2*b*x)*tan(1/2*c)^3 + 8*tan(1/2*a)*tan(1/2*c)^3 - 12*tan(1/2*c)^
4 - tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*c) - 2*tan(1/2*c)^2)/((tan(1/2*
a)^2*tan(1/2*c)^2 + tan(1/2*c)^2)*(tan(1/2*b*x)^2*tan(1/2*c) + tan(1/2*b*x)
*tan(1/2*c)^2 - tan(1/2*b*x) - tan(1/2*c))^2))/b
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(c + b*x)^3,x)
```

```
[Out] \text{Hanged}
```

3.253 $\int \cos(a+bx)\tan(c+dx)\,dx$

Optimal. Leaf size=134

$$\frac{e^{-i(a+bx)}}{2b}-\frac{e^{i(a+bx)}}{2b}-\frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}+\frac{e^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}$$

```
[Out] 1/2/b/exp(I*(b*x+a))-1/2*exp(I*(b*x+a))/b-hypergeom([1, -1/2*b/d],[1-1/2*b/
d],-exp(2*I*(d*x+c)))/b/exp(I*(b*x+a))+exp(I*(b*x+a))*hypergeom([1, 1/2*b/d
],[1+1/2*b/d],-exp(2*I*(d*x+c)))/b
```

Rubi [A]

time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4656, 2225, 2283}

$$-\frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b}+\frac{e^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;-e^{2i(c+dx)}\right)}{b}+\frac{e^{-i(a+bx)}}{2b}-\frac{e^{i(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Tan[c + d*x],x]
```

```
[Out] 1/(2*b*E^(I*(a + b*x))) - E^(I*(a + b*x))/(2*b) - Hypergeometric2F1[1, -1/2
*b/d, 1 - b/(2*d), -E^((2*I)*(c + d*x))]/(b*E^(I*(a + b*x))) + (E^(I*(a + b
*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), -E^((2*I)*(c + d*x))])/b
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4656

```
Int[Cos[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[(-I)*
(1/(E^(I*(a + b*x))*2)) - I*(E^(I*(a + b*x))/2) + I*(1/(E^(I*(a + b*x))*(1
+ E^(2*I*(c + d*x))))) + I*(E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x)))), x] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(a+bx)\tan(c+dx)\,dx = \int \left(-\frac{1}{2}ie^{-i(a+bx)} - \frac{1}{2}ie^{i(a+bx)} + \frac{ie^{-i(a+bx)}}{1+e^{2i(c+dx)}} + \frac{ie^{i(a+bx)}}{1+e^{2i(c+dx)}}\right) dx$$
$$= -\left(\frac{1}{2}i\int e^{-i(a+bx)}\,dx\right) - \frac{1}{2}i\int e^{i(a+bx)}\,dx + i\int \frac{e^{-i(a+bx)}}{1+e^{2i(c+dx)}}\,dx + i\int \frac{e^{i}}{1+}$$
$$= \frac{e^{-i(a+bx)}}{2b} - \frac{e^{i(a+bx)}}{2b} - \frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};-e^{2i(c+dx)}\right)}{b} + \frac{e^{i(a+bx)}\,{}_2F}{}$$

Mathematica [A]
time = 1.71, size = 197, normalized size = 1.47

$$\frac{e^{-i(a-2c+bx)}\left(b(b+2d)e^{2idx}\left(1+e^{2ic}\right){}_2F_1\left(1,1-\frac{b}{2d};2-\frac{b}{2d};-e^{2i(c+dx)}\right)+(b-2d)\left((b+2d)\left(-1+e^{2i(a+bx)}\right)-be^{2i(a+(b+d)x)}\left(1+e^{2ic}\right){}_2F_1\left(1,1+\frac{b}{2d};2+\frac{b}{2d};-e^{2i(c+dx)}\right)\right)\right)}{\left(b^3-4bd^2\right)\left(1+e^{2ic}\right)}+\frac{\sin(a+bx)\tan(c)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Tan[c + d*x],x]
```

```
[Out] (b*(b + 2*d)*E^((2*I)*d*x)*(1 + E^((2*I)*c))*Hypergeometric2F1[1, 1 - b/(2*
d), 2 - b/(2*d), -E^((2*I)*(c + d*x))] + (b - 2*d)*((b + 2*d)*(-1 + E^((2*I
)*(a + b*x))) - b*E^((2*I)*(a + (b + d)*x))*(1 + E^((2*I)*c))*Hypergeometri
c2F1[1, 1 + b/(2*d), 2 + b/(2*d), -E^((2*I)*(c + d*x))]))/((b^3 - 4*b*d^2)*
E^(I*(a - 2*c + b*x))*(1 + E^((2*I)*c))) + (Sin[a + b*x]*Tan[c])/b
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \cos(bx+a)\tan(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*tan(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*tan(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)*tan(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="fricas")
[Out] integral(cos(b*x + a)*tan(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(a+bx)\tan(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(d*x+c),x)
[Out] Integral(cos(a + b*x)*tan(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*tan(d*x+c),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*tan(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,\tan(c+d\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*tan(c + d*x),x)
[Out] int(cos(a + b*x)*tan(c + d*x), x)
```

3.254 $\int \cos(a+bx)\cot(c+dx)\,dx$

Optimal. Leaf size=130

$$-\frac{e^{-i(a+bx)}}{2b}+\frac{e^{i(a+bx)}}{2b}+\frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b}-\frac{e^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d};e^{2i(c+dx)}\right)}{b}$$

```
[Out] -1/2/b/exp(I*(b*x+a))+1/2*exp(I*(b*x+a))/b+hypergeom([1, -1/2*b/d],[1-1/2*b
/d],exp(2*I*(d*x+c)))/b/exp(I*(b*x+a))-exp(I*(b*x+a))*hypergeom([1, 1/2*b/d
],[1+1/2*b/d],exp(2*I*(d*x+c)))/b
```

Rubi [A]

time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4654, 2225, 2283}

$$\frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b}-\frac{e^{i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};\frac{b}{2d}+1;e^{2i(c+dx)}\right)}{b}-\frac{e^{-i(a+bx)}}{2b}+\frac{e^{i(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Cot[c + d*x],x]
```

```
[Out] -1/2*1/(b*E^(I*(a + b*x))) + E^(I*(a + b*x))/(2*b) + Hypergeometric2F1[1, -
1/2*b/d, 1 - b/(2*d), E^((2*I)*(c + d*x))]/(b*E^(I*(a + b*x))) - (E^(I*(a +
 b*x))*Hypergeometric2F1[1, b/(2*d), 1 + b/(2*d), E^((2*I)*(c + d*x))])/b
```

Rule 2225

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2283

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 4654

```
Int[Cos[(a_.) + (b_.)*(x_)]*Cot[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[I*(1/
(E^(I*(a + b*x))*2)) + I*(E^(I*(a + b*x))/2) - I*(1/(E^(I*(a + b*x))*(1 - E
^(2*I*(c + d*x))))) - I*(E^(I*(a + b*x))/(1 - E^(2*I*(c + d*x)))), x] /; Fr
eeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(a+bx)\cot(c+dx)\,dx = \int \left(\frac{1}{2}ie^{-i(a+bx)} + \frac{1}{2}ie^{i(a+bx)} - \frac{ie^{-i(a+bx)}}{1-e^{2i(c+dx)}} - \frac{ie^{i(a+bx)}}{1-e^{2i(c+dx)}}\right)dx$$
$$= \frac{1}{2}i\int e^{-i(a+bx)}\,dx + \frac{1}{2}i\int e^{i(a+bx)}\,dx - i\int \frac{e^{-i(a+bx)}}{1-e^{2i(c+dx)}}\,dx - i\int \frac{e^{i(a+bx)}}{1-e^{2i(c+dx}}$$
$$= -\frac{e^{-i(a+bx)}}{2b} + \frac{e^{i(a+bx)}}{2b} + \frac{e^{-i(a+bx)}\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)}{b} - \frac{e^{i(a+bx)}\,{}_2F_1(}{}$$

Mathematica [A]
time = 1.88, size = 108, normalized size = 0.83

$$\frac{e^{-i(a+bx)}\left(-1+e^{2i(a+bx)}+2\,{}_2F_1\left(1,-\frac{b}{2d};1-\frac{b}{2d};e^{2i(c+dx)}\right)-2e^{2i(a+bx)}\,{}_2F_1\left(1,\frac{b}{2d};1+\frac{b}{2d};e^{2i(c+dx)}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Cot[c + d*x],x]
[Out] (-1 + E^((2*I)*(a + b*x)) + 2*Hypergeometric2F1[1, -1/2*b/d, 1 - b/(2*d), E
^((2*I)*(c + d*x))] - 2*E^((2*I)*(a + b*x))*Hypergeometric2F1[1, b/(2*d), 1
 + b/(2*d), E^((2*I)*(c + d*x))])/(2*b*E^(I*(a + b*x)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \cos(bx+a)\cot(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(d*x+c),x)
[Out] int(cos(b*x+a)*cot(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)*cot(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*cot(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos(a+bx)\cot(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*cot(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\cot(c+d\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(c + d*x),x)
```

```
[Out] int(cos(a + b*x)*cot(c + d*x), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```